Chapter 18

Lecture Presentation

Chapter 18

Ray Optics
Suggested Videos for Chapter 18

• Prelecture Videos
  • Mirrors and Reflection
  • Refraction
  • Lenses and Images

• Class Videos
  • Scattering
  • Index of Refraction
  • Real and Virtual Images
  • Diverging Mirror

• Video Tutor Solutions
  • Ray Optics

• Video Tutor Demos
  • Partially Covering a Lens
Suggested Simulations for Chapter 18

- ActivPhysics
  - 15.1–15.11

- PhETs
  - Geometric Optics
Chapter 18 Ray Optics

**Chapter Goal:** To understand and apply the ray model of light.
Chapter 18 Preview
Looking Ahead: Reflection

• Light rays can bounce, or **reflect**, off a surface. Rays from the bird’s head reflect from the water, forming an upside-down image.

You’ll learn how the **law of reflection** can be used to understand image formation by mirrors.
The two images of the turtle are due to **refraction**, the bending of light rays as they travel from one material into another.

- You’ll learn **Snell’s law** for refraction and how images can be formed by refraction.
Chapter 18 Preview
Looking Ahead: Lenses and Mirrors

• Rays refracting at the surfaces of this lens form a magnified image of the girl behind it.

• You’ll learn how to locate and characterize the images formed by lenses and mirrors.
Chapter 18 Preview
Looking Ahead

**Reflection**
Light rays can bounce, or reflect, off a surface. Rays from the bird’s head reflect from the water, forming an upside-down image.

**Refraction**
The two images of the turtle are due to refraction, the bending of light rays as they travel from one material into another.

**Lenses and Mirrors**
Rays refracting at the surfaces of this lens form a magnified image of the girl behind it.

You’ll learn how the law of reflection can be used to understand image formation by mirrors.

You’ll learn Snell’s law for refraction and how images can be formed by refraction.

You’ll learn how to locate and characterize the images formed by lenses and mirrors.

Text p. 565
Chapter 18 Preview
Looking Back: The Ray Model of Light

• In Chapter 17, you learned that light spreads out as it passes through a narrow slit, but travels straight forward through wide openings.

• In this chapter, we’ll study the behavior of light in the ray model, applicable when light interacts with objects of everyday size such as mirrors or lenses.
The dark screen has a 2-mm-diameter hole. The bulb is the only source of light. What do you see on the viewing screen?
Reading Question 18.1

When an object like a tree is illuminated by the sun, and you are looking toward the tree, light rays leave the object

A. Only from points at the top and base of the tree, but in every direction.
B. From every point on the surface of the tree, but only toward your eyes.
C. Only from points at the top and base of the tree, but only toward your eyes.
D. From every point on the surface of the tree, and in every direction.
Reading Question 18.1

When an object like a tree is illuminated by the sun, and you are looking toward the tree, light rays leave the object

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C. Only from points at the top and base of the tree, but only toward your eyes.
D. From every point on the surface of the tree, and in every direction.

✔ D. From every point on the surface of the tree, and in every direction.
Reading Question 18.2

The image seen in a plane mirror is located

A. In front of the mirror.
B. Behind the mirror.
C. At the surface of the mirror.
D. At the position of the object.
Reading Question 18.2

The image seen in a plane mirror is located

A. In front of the mirror.

B. Behind the mirror. **✔**

C. At the surface of the mirror.

D. At the position of the object.
Reading Question 18.3

A light ray can change direction when going from one material into another. That phenomenon is known as

A. Reflection.
B. Absorption.
C. Refraction.
D. Scattering.
Reading Question 18.3

A light ray can change direction when going from one material into another. That phenomenon is known as

A. Reflection.
B. Absorption.
C. Refraction. ✅
D. Scattering.
Reading Question 18.4

A virtual image is

A. The cause of optical illusions.
B. A point from which rays appear to diverge.
C. An image that only seems to exist.
D. The image that is left in space after you remove the viewing screen.
Reading Question 18.4

A virtual image is

A. The cause of optical illusions.

✅ B. A point from which rays appear to diverge.

C. An image that only seems to exist.

D. The image that is left in space after you remove the viewing screen.
Reading Question 18.5

The focal length of a converging lens is

A. The distance at which an image is formed.
B. The distance at which an object must be placed to form an image.
C. The distance at which parallel light rays are focused.
D. The distance from the front surface of the lens to the back surface.
Reading Question 18.5

The focal length of a converging lens is

A. The distance at which an image is formed.
B. The distance at which an object must be placed to form an image.

C. The distance at which parallel light rays are focused.

D. The distance from the front surface of the lens to the back surface.

C. The distance at which parallel light rays are focused.
Section 18.1 The Ray Model of Light
The Ray Model of Light

- The ray model of light, which ignores diffraction, is valid as long as any apertures through which the light passes are larger than about 1 mm.

- A **light ray** is a line in the direction along which energy of light is flowing.

- A laser beam is a good approximation for light rays.
The Ray Model of Light

Light rays travel in straight lines.

Light travels through a vacuum or a transparent material in straight lines called light rays. The speed of light in a material is $v = c/n$, where $n$ is the index of refraction of the material.

Light rays can cross.

Light rays do not interact with each other. Two rays can cross without either being affected in any way.
A light ray travels forever unless it interacts with matter.

A light ray continues forever unless it has an interaction with matter that causes the ray to change direction or to be absorbed. Light interacts with matter in four different ways:

- At an interface between two materials, light can be reflected, refracted, or both.
- Within a material, light can be either scattered or absorbed.

These interactions are discussed later in the chapter.
An object is a source of light rays.

An object is a source of light rays. Rays originate from every point on the object, and each point sends rays in all directions. Objects may be self-luminous—they create light rays—or they may be reflective objects that reflect only rays that originate elsewhere.
The eye sees by focusing a bundle of rays.

The eye sees an object when *diverging* bundles of rays from each point on the object enter the pupil and are focused to an image on the retina. Imaging is discussed later in the chapter, and the eye will be treated in much greater detail in Chapter 19.
Sources of Light Rays

- **Self-luminous objects** (or sources) directly create light rays. Self-luminous objects include lightbulbs and the sun.
- **Reflective objects** are objects that reflect rays originating from self-luminous objects. These objects include a piece of paper or a tree.
Since a light ray is an idealization, there are no true ray sources. Still, the thin beam of a laser is often a good approximation of a single ray.
A point source is also an idealized source of light. It is infinitely small and emits light rays in every direction. The tiny filaments of these bulbs approximate point sources.
Sources of Light Rays: Self-Luminous Objects

An extended source

This is the most common light source. The *entire surface* of an extended source is luminous, so that every point of an extended source acts as a point source. Lightbulbs, flames, and the sun are extended sources.
Sources of Light Rays: Self-Luminous Objects

A parallel-ray source

Certain sources, such as flashlights and movie projectors, produce a bundle of parallel rays. Rays from a very distant object, such as a star, are very nearly parallel.
Ray Diagrams

• A ray diagram is a diagram that shows a few light rays in order to simplify the situation.
• In reality, rays originate from every point on an object and travel in all directions.

These are just a few of the infinitely many rays leaving the object.
QuickCheck 18.1

The dark screen has a small hole, \( \approx 2 \text{ mm} \) in diameter. The lightbulb is the only source of light. What do you see on the viewing screen?

A.  
B.  
C.
QuickCheck 18.1

The dark screen has a small hole, \( \approx 2 \) mm in diameter. The lightbulb is the only source of light. What do you see on the viewing screen?

A. 

B. [Correct Answer]

C.
QuickCheck 18.2

Two point sources of light illuminate a narrow vertical aperture in a dark screen. What do you see on the viewing screen?

A.  
B.  
C.  
D.  
E.  

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QuickCheck 18.2

Two point sources of light illuminate a narrow vertical aperture in a dark screen. What do you see on the viewing screen?
Seeing Objects

• In order for our eye to see an object, rays from that object must enter the eye.

• You cannot see a laser beam traveling across the room because no light from the laser enters the eye. The beam is invisible to you.

• This is the case for a ray or a parallel ray source.
Seeing Objects

• A point source and an extended source emit rays in every direction, and some of the rays will enter the eye no matter where it is located.

• Thus a point source or an extended source is visible to all observers.
Seeing Objects

- **Diffuse reflection** is the process of reflecting incident light in all directions.
- **Scattering** is a process in which single rays are broken into many weaker rays that leave in all directions.

An incident ray breaks into many weaker rays that scatter in every direction. Some scattered rays enter the eye, so the point is visible. Other points on the page are visible in the same way.
Seeing Objects

• When reading a book, every point on the surface of the page is struck by a ray from the lamp.
• Then, because of diffuse reflection, these rays scatter in every direction, some of which enter your eye.

An incident ray breaks into many weaker rays that scatter in every direction.

Incident ray

Some scattered rays enter the eye, so the point is visible.

Other points on the page are visible in the same way.
Seeing Objects

• Lasers are visible when small particles, such as dust, smoke, or water droplets, scatter the rays from the laser in every direction.

• Some of the rays are scattered in the direction of your eye, making the particles in the path of the laser visible.
Shadows

• An opaque object can intercept rays from a point source, leaving a dark area, or *shadow*, behind it.

• With a point source, the shadow is completely dark and the edges of the shadow are sharp.
Shadows

- An extended source is a large number of point sources, each of which casts a shadow.
- The shadows overlap, so the overall shadow is no longer sharp.

(b) Extended source

The shadow is not sharp because many points on the screen are struck by only some of the rays from the source.
Shadows

- Depending on the size of the source, there is often a true shadow that no light reaches, surrounded by a fuzzy region of increasing brightness.

(c) View of bulb as seen from three points on the screen

A

The whole bulb is visible from point A. Point A is fully illuminated.

B

At B, the disk partially obscures the bulb. Point B is in partial shadow.

C

At C, the disk completely blocks the bulb. Point C is dark.
Example Problem

If the aperture is very small, how far apart on the screen built into the left side of the box are the images of the point-like red and green light sources?

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Section 18.2 Reflection
Reflection

- **Specular reflection** is the reflection from a smooth, shiny surface such as a mirror or a piece of polished metal.
- A three-dimensional perspective shows that the incident and reflected rays are both in a plane that is normal to the surface.
Reflection

- It is customary to represent reflection with a simpler view.
- The incident and reflected rays are in the plane of the page. The reflective surface extends into and out of the page.
- A *single* light ray represents the entire bundle of parallel rays. This is oversimplified, but it keeps the figure and the analysis clear.
Reflection

- The angle of incidence, $\theta_i$, is the angle between the incident ray and the line perpendicular to the surface.
- The angle of reflection, $\theta_r$, is the angle between the reflected ray and the normal to the surface.
Reflection

The law of reflection states:

1. The incident ray and the reflected ray are both in the same plane, which is perpendicular to the surface, and

2. The angle of reflection equals the angle of incidence:

\[ \theta_r = \theta_i \]
Example 18.1 Light reflecting from a mirror

A full-length mirror on a closet door is 2.0 m tall. The bottom touches the floor. A bare lightbulb hangs 1.0 m from the closet door, 0.5 m above the top of the mirror. How long is the streak of reflected light across the floor?
Example 18.1 Light reflecting from a mirror (cont.)

**PREPARE** Treat the lightbulb as a point source and use the ray model of light. FIGURE 18.9 is a visual overview of the light rays. We need to consider only the two rays that strike the edges of the mirror. All other reflected rays will fall between these two.
Example 18.1 Light reflecting from a mirror (cont.)

**SOLVE** The ray that strikes the bottom of the mirror reflects from it and hits the floor just where the mirror meets the floor. For the top ray, Figure 18.9 has used the law of reflection to set the angle of reflection equal to the angle of incidence; we call both $\theta$. By simple geometry, the other angles shown are also equal to $\theta$. From the small triangle at the upper right,

$$\theta = \tan^{-1}\left(\frac{0.5 \text{ m}}{1.0 \text{ m}}\right) = 26.6^\circ$$
Example 18.1 Light reflecting from a mirror (cont.)

But we also have \( \tan \theta = (2.0 \text{ m})/l \), or

\[
l = \frac{2.0 \text{ m}}{\tan \theta} = \frac{2.0 \text{ m}}{\tan 26.6^\circ} = 4.0 \text{ m}
\]

Since the lower ray struck right at the mirror’s base, the total length of the reflected streak is 4.0 m.
QuickCheck 18.3

You are looking at the image of a pencil in a mirror. What do you see in the mirror if the top half of the mirror is covered with a piece of dark paper?

A. The full image of the pencil
B. The top half only of the pencil
C. The bottom half only of the pencil
D. No pencil, only the paper
QuickCheck 18.3

You are looking at the image of a pencil in a mirror. What do you see in the mirror if the top half of the mirror is covered with a piece of dark paper?

A. The full image of the pencil
B. The top half only of the pencil
C. The bottom half only of the pencil
D. No pencil, only the paper

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Diffuse Reflection

- On the microscopic scale, the surface of a diffuse reflector (paper or cloth) is rough.
- The law of reflection holds, but the irregularities of the surface cause the reflected rays to leave in all directions.

Each ray obeys the law of reflection at that point, but the irregular surface causes the reflected rays to leave in many random directions.

Magnified view of surface
The Plane Mirror

• A **plane mirror** is a flat mirror.

• Rays from point P will reflect according to the law of reflection.

![Diagram of rays reflecting off a plane mirror](image)

Rays from P reflect from the mirror. Each ray obeys the law of reflection.
The dashed lines indicate that the rays appear to have come from point $P'$. All reflected rays appear to come from point $P'$. 

These reflected rays appear to have come from point $P'$. 

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The Plane Mirror

Point $P'$, from which the reflected rays diverge, is called the **virtual image** of $P$.

The image is virtual because no rays actually leave point $P'$, however the light waves act exactly as if they were.

The reflected rays *all* diverge from $P'$, which appears to be the source of the reflected rays. Your eye collects the bundle of diverging rays and “sees” the light coming from $P'$. 
The Plane Mirror

- The image distance $s'$ is equal to the object distance $s$:
  
  $$s' = s$$  
  (plane mirror)
The Plane Mirror

• The eye captures and focuses diverging bundles of rays from each point of the image of an extended object.

The rays from P and Q that reach your eye reflect from different areas of the mirror.
The Plane Mirror

1. Rays from each point on the object spread out in all directions and strike every point on the mirror. Only a very few of these rays enter your eye, but the other rays are very real and might be seen by other observers.

2. Rays from points P and Q enter your eye after reflecting from different areas of the mirror. This is why you can’t always see the full image in a very small mirror.
QuickCheck 18.4

An object is placed in front of a mirror. The observer is positioned as shown. Which of the points shown best indicates where the observer would perceive the image to be located?
An object is placed in front of a mirror. The observer is positioned as shown. Which of the points shown best indicates where the observer would perceive the image to be located?

![Diagram showing mirror, observer, and objects]

- Option A
- Option B
- Option C

Correct answer: C
Example 18.2 How high is the mirror?

If your height is \( h \), what is the shortest mirror on the wall in which you can see your full image? Where must the top of the mirror be hung?

**PREPARE** Use the ray model of light. FIGURE 18.13 is a visual overview of the light rays. We need to consider only the two rays that leave the top of your head and your feet and reflect into your eye.
**Example 18.2 How high is the mirror? (cont.)**

**SOLVE** Let the distance from your eyes to the top of your head be $l_1$ and the distance to your feet be $l_2$. Your height is $h = l_1 + l_2$. A light ray from the top of your head that reflects from the mirror at $\theta_r = \theta_i$ and enters your eye must, by congruent triangles, strike the mirror a distance $\frac{1}{2}l_1$ above your eyes.
Example 18.2 How high is the mirror? (cont.)

Similarly, a ray from your foot to your eye strikes the mirror a distance $\frac{1}{2}l_2$ below your eyes. The distance between these two points on the mirror is

$$\frac{1}{2}l_1 + \frac{1}{2}l_2 = \frac{1}{2}h.$$ 

A ray from anywhere else on your body will reach your eye if it strikes the mirror between these two points. Pieces of the mirror outside these two points are irrelevant, not because rays don’t strike them but because the reflected rays don’t reach your eye.
Example 18.2 How high is the mirror? (cont.)

Thus the shortest mirror in which you can see your full reflection is $\frac{1}{2}h$. But this will work only if the top of the mirror is hung midway between your eyes and the top of your head.

**ASSESS** It is interesting that the answer does not depend on how far you are from the mirror.
Section 18.3 Refraction
Refraction

Two things happen when a light ray crosses the boundary between the air and the glass:

1. Part of the light *reflects* from the boundary, obeying the law of reflection. This is how you see the reflections from pools of water or storefront windows, even though water and glass are transparent.
2. Part of the light continues into the second medium. It is transmitted rather than reflected, but the transmitted ray changes direction as it crosses the boundary. The transmission of light from one medium to another, but with a change in direction, is called **refraction**.
Refraction

- This figure shows the refraction of light rays from a parallel beam of light, such as a laser beam, and rays from a source.

- An infinite number of rays are incident on the boundary, although for simplicity we focus on a single light ray.

- Reflection occurs at the boundary, but is usually very weak and is ignored here.
Refraction

• The angle between the incident ray and the normal is the *angle of incidence*.

• The angle on the transmitted side, *measured from the normal*, is called the *angle of refraction*.
Refraction

- The angles are the same for the ray entering Medium 2 in the first figure and the ray exiting Medium 2 in the second figure.
Refraction

• In 1621, Dutch scientist Willebrord Snell proposed a mathematical statement of the “law of refraction” now called Snell’s Law:

\[ n_1 \sin \theta_1 = n_2 \sin \theta_2 \]

Snell’s law for refraction between two media

• The index of refraction determines how much a light ray is bent when crossing the boundary between two different media (a consequence of the change in the speed of light as it crosses a boundary.)
QuickCheck 18.5

A laser beam passing from medium 1 to medium 2 is refracted as shown. Which is true?

A. $n_1 < n_2$
B. $n_1 > n_2$
C. There’s not enough information to compare $n_1$ and $n_2$
QuickCheck 18.5

A laser beam passing from medium 1 to medium 2 is refracted as shown. Which is true?

A. \( n_1 < n_2 \)
B. \( n_1 > n_2 \)
C. There’s not enough information to compare \( n_1 \) and \( n_2 \)

![Diagram showing the direction of the refracted beam](image)

Bends away from the normal.
### TABLE 18.1 Indices of refraction

<table>
<thead>
<tr>
<th>Medium</th>
<th>( n )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vacuum</td>
<td>1 exactly</td>
</tr>
<tr>
<td>Air (actual)</td>
<td>1.0003</td>
</tr>
<tr>
<td>Air (accepted)*</td>
<td>1.00</td>
</tr>
<tr>
<td>Water</td>
<td>1.33</td>
</tr>
<tr>
<td>Ethyl alcohol</td>
<td>1.36</td>
</tr>
<tr>
<td>Oil</td>
<td>1.46</td>
</tr>
<tr>
<td>Glass (typical)</td>
<td>1.50</td>
</tr>
<tr>
<td>Polystyrene plastic</td>
<td>1.59</td>
</tr>
<tr>
<td>Cubic zirconia</td>
<td>2.18</td>
</tr>
<tr>
<td>Diamond</td>
<td>2.42</td>
</tr>
<tr>
<td>Silicon (infrared)</td>
<td>3.50</td>
</tr>
</tbody>
</table>

*Use this value in problems.
Examples of Refraction

Snell’s law shows:

• When a ray is transmitted into a material with a higher index of refraction, it bends to make a smaller angle with the normal.

• When a ray is transmitted into a material with a lower index of refraction, it bends to make a larger angle with the normal.
Examples of Refraction

TACTICS BOX 18.1 Analyzing refraction

1. **Draw a ray diagram.** Represent the light beam with one ray.
2. **Draw a line normal (perpendicular) to the boundary.** Do this at each point where the ray intersects a boundary.
3. **Show the ray bending in the correct direction.** The angle is larger on the side with the smaller index of refraction. This is the qualitative application of Snell’s law.
4. **Label angles of incidence and refraction.** Measure all angles from the normal.
5. **Use Snell’s law.** Calculate the unknown angle or unknown index of refraction.

Exercises 10–13

Text: p. 573
A light ray enters a glass prism as shown. Which is a possible path for the ray through the prism?
A light ray enters a glass prism as shown. Which is a possible path for the ray through the prism?

A light ray enters a glass prism as shown. Which is a possible path for the ray through the prism?
Example Problem

What is the index of refraction of the plastic if a ray is refracted as in the figure?
Total Internal Reflection

- **Total internal reflection (TIR)** occurs when a light ray is unable to refract through a boundary. Instead, 100% of the light *reflects* from the boundary.
Total Internal Reflection

- Crossing a boundary into a material with a lower index of refraction causes the ray to bend away from the normal.

- As angle $\theta_1$ increases, the refraction angle $\theta_2$ approaches $90^\circ$.

- The fraction of light energy that is transmitted decreases while the fraction reflected increases.
Total Internal Reflection

- A critical angle $\theta_c$ is reached when $\theta_2 = 90^\circ$.
- The refracted light vanishes at the critical angle; there is only reflected light.

$$\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right)$$

Critical angle of incidence for total internal reflection

- There is no critical angle and no total internal reflection if $n_2 > n_1$. 

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Total Internal Reflection

- In a pair of binoculars, the lenses are much farther apart than your eyes, so the light rays need to be brought together.

- Binoculars use a pair of prisms, forcing the light to undergo two TIRs before emerging from the eyepiece.

Angles of incidence exceed the critical angle.
QuickCheck 18.6

A laser beam undergoes two refractions plus total internal reflection at the interface between medium 2 and medium 3. Which is true?

A. $n_1 < n_3$
B. $n_1 > n_3$
C. There’s not enough information to compare $n_1$ and $n_3$
QuickCheck 18.6

A laser beam undergoes two refractions plus total internal reflection at the interface between medium 2 and medium 3. Which is true?

A. \( n_1 < n_3 \)

✓ B. \( n_1 > n_3 \)

C. There’s not enough information to compare \( n_1 \) and \( n_3 \)

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Example 18.5 Seeing a submerged light

A lightbulb is set in the bottom of a 3.0-m-deep swimming pool. What is the diameter of the circle inside which a duck swimming on the surface could see the bulb?
Example 18.5 Seeing a submerged light (cont.)

**PREPARE** Represent the lightbulb as a point source and use the ray model of light. FIGURE 18.22 is a visual overview of the light rays. The lightbulb emits rays at all angles, but only some of the rays refract into the air where they can be seen from above. Rays striking the surface at greater than the critical angle undergo TIR back down into the water. The diameter of the circle of light is the distance $D$ between the two points at which rays strike the surface at the critical angle.
Example 18.5 Seeing a submerged light (cont.)

**SOLVE** From trigonometry, the circle diameter is
\[ D = 2h \tan \theta_c, \]
where \( h \) is the depth of the water. The critical angle for a water-air boundary is
\[ \theta_c = \sin^{-1}(1.00/1.33) = 48.7^\circ. \]
Thus
\[ D = 2(3.0 \text{ m}) \tan 48.7^\circ = 6.8 \text{ m} \]

![Diagram showing the critical angle and circle diameter](image)
Example 18.5 Seeing a submerged light (cont.)

**ASSESS** Light rays emerging at the edge of the circle actually skim the surface of the water. By reversing the direction of the rays, we can understand what a diver sees when she’s underwater. This idea is explored further in the discussion below.

![Diagram of light rays](image)

- **Air,** $n_2 = 1.00$
- **Water,** $n_1 = 1.33$
- $h = 3.0 \text{ m}$

Rays at the critical angle $\theta_c$ form the edge of the circle of light seen from above.
A fish in an aquarium with flat sides looks out at a hungry cat. To the fish, the distance to the cat appears to be

A. Less than the actual distance.
B. Equal to the actual distance.
C. More than the actual distance.
QuickCheck 18.8

A fish in an aquarium with flat sides looks out at a hungry cat. To the fish, the distance to the cat appears to be

A. Less than the actual distance.
B. Equal to the actual distance.
C. More than the actual distance.

✔️ C. More than the actual distance.
Fiber Optics

- Fiber optics use total internal reflection for the transmission of light through optical fibers.
- Light rays pass into the narrow-diameter glass fiber, but then strike the inside wall of the fiber at an angle of incidence approaching 90°. This is larger than the critical angle, so the light undergoes TIR and remains inside the glass.
Fiber Optics

- When the light rays reach the flat end of the fiber, the angle of incidence is lower and the light can cross the boundary.
- To protect it from external damage, a glass *cladding* surrounds the glass *core*. Light undergoes TIR at the cladding boundary, and remains within the core.
- Endoscopes made from optical fibers are used for *anthroscopic surgery*.

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Section 18.4 Image Formation by Refraction
Image Formation by Refraction

• The part of the ruler submerged in water in the photograph appears closer than the part above water.

• The rays from the submerged portion refract at the water-air boundary.
Image Formation by Refraction

- To your eye, the rays appear to diverge not from the object at point P, but instead from point P'.
- The ruler appears closer than it really is because of refraction of light at the boundary.

(b) Finding the image of the ruler

Diverging rays appear to come from this point. This is a virtual image.
Image Formation by Refraction

- The **optical axis** is the line through the object and perpendicular to the boundary.
- The distance $l$ is common to both the incident and refracted rays:

$$l = s \tan \theta_1 = s' \tan \theta_2$$

$$s' = \frac{\tan \theta_1}{\tan \theta_2} s$$

Snell’s Law relates the angles:

$$\frac{\sin \theta_1}{\sin \theta_2} = \frac{n_2}{n_1}$$

Rays diverge from the virtual image at $P'$. © 2015 Pearson Education, Inc.
Image Formation by Refraction

• The small-angle approximation shows \( \sin \theta \approx \tan \theta \).

Therefore

\[
\frac{\tan \theta_1}{\tan \theta_2} \approx \frac{\sin \theta_1}{\sin \theta_2} = \frac{n_2}{n_1}
\]

• The image distance is

The index of refraction of the medium that the object is in

\( s' = \frac{n_2}{n_1} s \)

The index of refraction of the medium that the observer is in
Example 18.6 An air bubble window

A fish and a sailor look at each other through a 5.0-cm-thick glass porthole in a submarine. There happens to be a small air bubble right in the center of the glass. How far behind the surface of the glass does the air bubble appear to the fish? To the sailor?
Example 18.6 An air bubble window (cont.)

**PREPARE** Represent the air bubble as a point source and use the ray model of light. Light rays from the bubble refract into the air on one side and into the water on the other. The ray diagram looks like Figure 18.25.

![Ray Diagram](image)

- **Object**
- **Virtual image**
- **Optical axis**
- **θ₁**
- **θ₂**
- **s**
- **s'**
- **l**

Rays diverge from the virtual image at P'.
Example 18.6 An air bubble window (cont.)

**SOLVE** The index of refraction of the glass is \( n_1 = 1.50 \). The bubble is in the center of the window, so the object distance from either side of the window is \( s = 2.5 \) cm. From the water side, the fish sees the bubble at an image distance

\[
 s' = \frac{n_2}{n_1} s = \frac{1.33}{1.50} (2.5 \text{ cm}) = 2.2 \text{ cm}
\]

The observer (the fish) is in water. The object (the bubble) is in glass.
Example 18.6 An air bubble window (cont.)

This is the apparent depth of the bubble. The sailor, in air, sees the bubble at an image distance

\[ s' = \frac{n_2}{n_1} s = \frac{1.00}{1.50} (2.5 \text{ cm}) = 1.7 \text{ cm} \]

**ASSESS** The image distance is *shorter* for the sailor because of the *larger* difference between the two indices of refraction.
Section 18.5 Thin Lenses: Ray Tracing
Thin Lenses: Ray Tracing

- A **lens** is a transparent material that uses refraction of light rays at *curved* surfaces to form an image.
- **Ray tracing** is a pictorial method used to understand image formation.
Thin Lenses: Ray Tracing

• A **converging lens** causes the rays to refract *toward* the optical axis.

• A **diverging lens** causes the rays to refract *away* from the axis.

(a) Converging lenses, which are thicker in the center than at the edges, refract parallel rays toward the optical axis.

(b) Diverging lenses, which are thinner in the center than at the edges, refract parallel rays away from the optical axis.
Thin Lenses: Ray Tracing

• In a converging lens, an incoming ray refracts toward the optical axis at both the first (air-to-glass) boundary and the second (glass-to-air) boundary.
Thin Lenses: Ray Tracing

- The incoming rays initially parallel to the optical axis converge at the same point, the focal point of the lens.
- The distance of the focal point from the lens is called the focal length $f$ of the lens.

(a) Converging lens

Parallel rays

Optical axis

The near focal point is also a distance $f$ from the lens.

Focal length $f$

This is the far focal point. Rays actually converge at this point.

Rays continue after passing through the focal point.
Thin Lenses: Ray Tracing

- There are focal points on both sides of the lens.
- The focal point on the side from which the light is incident is the *near focal point*; the focal point on the other side is the *far focal point*.

![Diagram of ray tracing through a converging lens](image)

(a) **Converging lens**

- Parallel rays
- Optical axis
- Focal length $f$

- This is the far focal point. Rays actually converge at this point.
- The near focal point is also a distance $f$ from the lens.
- Rays continue after passing through the focal point.
Thin Lenses: Ray Tracing

- For a diverging lens, the **focal length** is the distance from the lens to the point at which rays parallel to the optical axis converge or from which they appear to diverge.

(b) Diverging lens

- Focal length \( f \)
- Parallel rays
- Far focal point
- Optical axis

This is the near focal point. Rays appear to diverge from this point.
Converging Lenses

- A **thin lens** is an idealized lens whose thickness is zero and that lies entirely in a plane called the **lens plane**.
- Within the **thin-lens approximation**, **all refraction occurs as the rays cross the lens plane**, and **all distances are measured from the lens plane**.
Converging Lenses

(a) Lens plane
Far focal point

Parallel rays $f$

Any ray initially parallel to the optical axis will refract through the focal point on the far side of the lens.

(b) Lens plane
Near focal point

$f$
Parallel rays

Any ray passing through the near focal point emerges from the lens parallel to the optical axis.

(c)

Center of lens
Rays are not bent.

Any ray directed at the center of the lens passes through in a straight line.
Real Images

- If rays diverge from an object at point P and interact with a lens such that they *converge* at point P′, then we call P′ a **real image** of point P.

- A *virtual image* is at a point from which rays appear to *diverge*, but through which no rays actually pass.
Real Images

Rays leave P (and Q and R) in every direction. For clarity, only a few are shown.

All the rays leaving one point in the object plane (P) that reach the lens are refracted by it and converge to one point in the image plane (P').

This is a real image.
Real Images

- All points on the object that are in the same plane, the object plane, converge to image points in the image plane.

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Real Images

- The image is called an **inverted image** because it is upside down with respect to the object. It is a standard characteristic of real-image formation.

- Rays from point P *fill* the entire lens surface. A larger lens “collects” more rays, and therefore makes a brighter image.
Real Images

• The rays don’t stop at $P'$ unless we place a screen in the image plane. When we do, the image is sharp and well-focused.

• If the screen is placed other than in the image plane, an image is produced but it is blurry and out of focus.
Real Images

- Although you can draw many lines in a ray-tracing diagram, only three are necessary for locating the image.

**TACTICS BOX 18.2** Ray tracing for a converging lens

1. **Draw an optical axis.** Use graph paper or a ruler! Establish an appropriate scale.
2. **Center the lens on the axis.** Draw the lens plane perpendicular to the axis through the center of the lens. Mark and label the focal points at distance $f$ on either side.
3. **Represent the object with an upright arrow at distance $s$.** It’s usually best to place the base of the arrow on the axis and to draw the arrow about half the radius of the lens.
4. **Draw the three “special rays” from the tip of the arrow.** Use a straight-edge or a ruler. The rays refract at the lens plane, not at the surfaces of the lens.
   a. A ray initially parallel to the axis refracts through the far focal point.
   b. A ray that enters the lens along a line through the near focal point emerges parallel to the axis.
   c. A ray through the center of the lens does not bend.
5. **Extend the rays until they converge.** The rays converge at the image point. Draw the rest of the image in the image plane. If the base of the object is on the axis, then the base of the image will also be on the axis.
6. **Measure the image distance $s’$.** Also, if needed, measure the image height relative to the object height. The magnification can be found from Equation 18.8.
Example 18.7 Finding the image of a flower

A 4.0-cm-diameter flower is 200 cm from the 50-cm-focal-length lens of a camera. How far should the plane of the camera’s light detector be placed behind the lens to record a well-focused image? What is the diameter of the image on the detector?
Example 18.7 Finding the image of a flower (cont.)

**PREPARE** The flower is in the object plane. Use ray tracing to locate the image.

1. Lay out the optical axis, with a scale.
2. Draw the lens and lens plane and mark the focal points.
3. Draw the object as an arrow with its base on the axis.
4. Draw the 3 special rays from the tip of the arrow.
   - a. Parallel to the axis
   - b. Through the near focal point
   - c. Through the center of the lens
5. The convergence point is the tip of the image. Draw the rest of the image.
6. Measure the image distance.

The rays refract at the lens plane.
Example 18.7 Finding the image of a flower (cont.)

**SOLVE** FIGURE 18.33 shows the ray-tracing diagram and the steps of Tactics Box 18.2. The image has been drawn in the plane where the three special rays converge. You can see *from the drawing* that the image distance is \( s' \approx 65 \text{ cm} \).

1. Lay out the optical axis, with a scale.
2. Draw the lens and lens plane and mark the focal points.
3. Draw the object as an arrow with its base on the axis.
4. Draw the 3 special rays from the tip of the arrow.
   - a. Parallel to the axis
   - b. Through the near focal point
   - c. Through the center of the lens
   *The rays refract at the lens plane.*
5. The convergence point is the tip of the image. Draw the rest of the image.
6. Measure the image distance.

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Example 18.7 Finding the image of a flower (cont.)

This is where the detector needs to be placed to record a focused image. The heights of the object and image are labeled $h$ and $h'$. The ray through the center of the lens is a straight line; thus the object and image both subtend the same angle $\theta$.

1. Lay out the optical axis, with a scale.
2. Draw the lens and lens plane and mark the focal points.
3. Draw the object as an arrow with its base on the axis.
4. Draw the 3 special rays from the tip of the arrow.
   a. Parallel to the axis
   b. Through the near focal point
   c. Through the center of the lens

   The rays refract at the lens plane.
5. The convergence point is the tip of the image. Draw the rest of the image.
6. Measure the image distance.
Example 18.7 Finding the image of a flower (cont.)

From similar triangles,

\[ \frac{h'}{s'} = \frac{h}{s} \]

1. Lay out the optical axis, with a scale.
2. Draw the lens and lens plane and mark the focal points.
3. Draw the object as an arrow with its base on the axis.
4. Draw the 3 special rays from the tip of the arrow.
   a. Parallel to the axis
   b. Through the near focal point
   c. Through the center of the lens
   
   The rays refract at the lens plane.
5. The convergence point is the tip of the image. Draw the rest of the image.
6. Measure the image distance.
Example 18.7 Finding the image of a flower (cont.)

Solving for $h'$ gives

$$h' = h \frac{s'}{s} = (4.0 \text{ cm}) \frac{65 \text{ cm}}{200 \text{ cm}} = 1.3 \text{ cm}$$

The flower’s image has a diameter of 1.3 cm.
Example 18.7 Finding the image of a flower (cont.)

ASSESS We’ve been able to learn a great deal about the image from a simple geometric procedure.

1. Lay out the optical axis, with a scale.
2. Draw the lens and lens plane and mark the focal points.
3. Draw the object as an arrow with its base on the axis.
4. Draw the 3 special rays from the tip of the arrow.
   a. Parallel to the axis
   b. Through the near focal point
   c. Through the center of the lens
   The rays refract at the lens plane.
5. The convergence point is the tip of the image. Draw the rest of the image.
6. Measure the image distance.
Magnification

• The image can be larger or smaller than the object, depending on the location and focal length of the lens.

• The **magnification** $m$ describes the *orientation* of the image relative to the object and its size.

  
  1. The absolute value of $m$ gives the ratio of image height to object height: $h'/h = |m|$.

  2. A positive value of $m$ indicates that the image is upright relative to the object. A negative value of $m$ indicates that the image is inverted.
Virtual Images

- When the object is inside the focal point, a ray passing through the focal point (to the left) would never reach the lens.
- The rays emerging parallel to the axis entered the lens along a line passing through the near focal point.
Virtual Images

• The three refracted rays do not converge.
• The rays appear to **diverge** from point \( P' \).
• Point \( P' \) is a **virtual image** of the object point \( P \).
• It is an **upright image**.
Virtual Images

• Because no rays actually pass through $P'$, placing a screen at the image plane would not produce an image.

• Your eye can still see a virtual image. This is what happens when you look through a magnifying glass or the eyepiece of a microscope or binoculars.
Virtual Images

• The magnification $m = -s'/s$ is positive since the virtual image is upright. That means the ratio $-s/s$ is negative.
• We define the image distance $s'$ to be negative for a virtual image.
• This is a sign convention.
Example 18.8 Magnifying a flower

To see a flower better, you hold a 6.0-cm-focal-length magnifying glass 4.0 cm from the flower. What is the magnification?
Example 18.8 Magnifying a flower (cont.)

**PREPARE** The flower is in the object plane. Use ray tracing to locate the image. Once the image distance is known, Equation 18.8 can be used to find the magnification.

[Diagram showing ray tracing and magnification calculation]
Example 18.8 Magnifying a flower (cont.)

SOLVE FIGURE 18.36 shows the ray-tracing diagram. The three special rays diverge from the lens, but we can use a straightedge to extend the rays backward to the point from which they diverge.
Example 18.8 Magnifying a flower (cont.)

This point, the image point, is seen to be 12 cm to the left of the lens. Because this is a virtual image, the image distance is \( s' = -12 \text{ cm} \). From Equation 18.8 the magnification is

\[
m = -\frac{s'}{s} = -\frac{-12 \text{ cm}}{4.0 \text{ cm}} = 3.0
\]
Example 18.8 Magnifying a flower (cont.)

**ASSESS** The image is three times as large as the object and, as we see from the ray-tracing diagram and the fact that $m > 0$, upright.
Diverging Lenses

• A diverging lens is one that is thinner at its center than at its edge.
• Diverging lenses *always* make virtual images.
Diverging Lenses

Any ray initially parallel to the optical axis diverges along a line through the near focal point.

Any ray directed along a line toward the far focal point emerges from the lens parallel to the optical axis.

Any ray directed at the center of the lens passes through in a straight line.
Ray tracing for a diverging lens

1–3 Follow steps 1 through 3 of Tactics Box 18.2.
4 Draw the three “special rays” from the tip of the arrow. Use a straight-edge or a ruler. The rays refract at the lens plane.
   a. A ray parallel to the axis diverges along a line through the near focal point.
   b. A ray along a line toward the far focal point emerges parallel to the axis.
   c. A ray through the center of the lens does not bend.
5 Trace the diverging rays backward. The point from which they are diverging is the image point, which is always a virtual image.
6 Measure the image distance $s'$, which, because the image is virtual, we will take as a negative number. Also, if needed, measure the image height relative to the object height. The magnification can be found from Equation 18.8.

Text: p. 584
QuickCheck 18.9

You can use the sun’s rays and a lens to start a fire. To do so, you should use

A. A converging lens.
B. A diverging lens.
C. Either a converging or a diverging lens will work if you use it correctly.
QuickCheck 18.9

You can use the sun’s rays and a lens to start a fire. To do so, you should use

✓  A. A converging lens.

   B. A diverging lens.

   C. Either a converging or a diverging lens will work if you use it correctly.
QuickCheck 18.10

A lens produces a sharply focused, inverted image on a screen. What will you see on the screen if the lens is removed?

A. An inverted but blurry image
B. An image that is dimmer but otherwise unchanged
C. A sharp, upright image
D. A blurry, upright image
E. No image at all
A lens produces a sharply focused, inverted image on a screen. What will you see on the screen if the lens is removed?

A. An inverted but blurry image
B. An image that is dimmer but otherwise unchanged
C. A sharp, upright image
D. A blurry, upright image
E. No image at all

✔
A lens produces a sharply focused, inverted image on a screen. What will you see on the screen if a piece of dark paper is lowered to cover the top half of the lens?

A. An inverted but blurry image  
B. An image that is dimmer but otherwise unchanged  
C. Only the top half of the image  
D. Only the bottom half of the image  
E. No image at all
QuickCheck 18.11

A lens produces a sharply focused, inverted image on a screen. What will you see on the screen if a piece of dark paper is lowered to cover the top half of the lens?

A. An inverted but blurry image

B. An image that is dimmer but otherwise unchanged

C. Only the top half of the image

D. Only the bottom half of the image

E. No image at all
A lens produces a sharply focused, inverted image on a screen. What will you see on the screen if the lens is covered by a dark mask having only a small hole in the center?

A. An inverted but blurry image  
B. An image that is dimmer but otherwise unchanged  
C. Only the middle piece of the image  
D. A circular diffraction pattern  
E. No image at all
QuickCheck 18.12

A lens produces a sharply focused, inverted image on a screen. What will you see on the screen if the lens is covered by a dark mask having only a small hole in the center?

A. An inverted but blurry image  
B. An image that is dimmer but otherwise unchanged  
C. Only the middle piece of the image  
D. A circular diffraction pattern  
E. No image at all

✔
QuickCheck 18.13

Which of these ray diagrams is possibly correct?
QuickCheck 18.13

Which of these ray diagrams is possibly correct?

A.  
B.  
C.  
D.  

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Example 18.9 Demagnifying a flower

A diverging lens with a focal length of 50 cm is placed 100 cm from a flower. Where is the image? What is its magnification?
**Example 18.9 Demagnifying a flower (cont.)**

**PREPARE** The flower is in the object plane. Use ray tracing to locate the image. Then Equation 18.8 can be used to find the magnification.
Example 18.9 Demagnifying a flower (cont.)

**SOLVE** FIGURE 18.38 shows the ray-tracing diagram. The three special rays (labeled a, b, and c to match the Tactics Box) do not converge. However, they can be traced backward to an intersection $\approx 33$ cm to the left of the lens.
Example 18.9 Demagnifying a flower (cont.)

Because the rays appear to diverge from the image, this is a virtual image and $s'$ is $< 0$. The magnification is

$$m = -\frac{s'}{s} = -\frac{-33 \text{ cm}}{100 \text{ cm}} = 0.33$$

The image, which can be seen by looking through the lens, is one-third the size of the object and upright.
**Example 18.9 Demagnifying a flower (cont.)**

**ASSESS** Ray tracing with a diverging lens is somewhat trickier than with a converging lens, so this example is worth careful study.
Section 18.6 Image Formation with Spherical Mirrors
Image Formation with Spherical Mirrors

- **Spherical mirrors** are curved mirrors and can be used to form images.
- The mirror in the figure is a **concave mirror**. Parallel waves reflect off the mirror and pass through the focal point.
Image Formation with Spherical Mirrors

- This mirror is a **convex mirror**. The parallel rays that reflect off its surface appear to have come from a point behind the mirror.
Concave Mirrors

- Parallel rays will reflect off a concave mirror through the focal point.

Any ray initially parallel to the optical axis will reflect through the focal point.
Concave Mirrors

• Rays passing through the focal point will reflect from a concave mirror, emerging parallel to the optical axis.

(b)
Focal point

Any ray passing through the focal point will, after reflection, emerge parallel to the optical axis.
Concave Mirrors

- The law of reflection says any incoming ray will reflect at the same angle but on the other side of the optical axis.

\[ \theta_i = \theta_r \]

Any ray directed at the center of the mirror will reflect at an equal angle on the opposite side of the optical axis.
Concave Mirrors

- When an object’s distance $s$ from the mirror is greater than the focal length, the three special rays are enough to locate the position and size of the image.
Concave Mirrors

- Incoming rays reflect off the **mirror plane**, not off the curved surface of the mirror for the ray trace.
Concave Mirrors

- The image is *real* because rays converge at the image point $P'$.
- If the object is inside the focal point, the image is a virtual image.
Concave Mirrors

**TACTICS BOX 18.4** Ray tracing for a concave mirror

1. **Draw an optical axis.** Use graph paper or a ruler! Establish an appropriate scale.
2. **Center the mirror on the axis.** Mark and label the focal point at distance $f$ from the mirror’s surface. Draw the mirror plane through the mirror’s center, perpendicular to the axis.
3. **Represent the object with an upright arrow at distance $s$.** It’s usually best to place the base of the arrow on the axis and to draw the arrow about half the radius of the mirror.
4. **Draw the three “special rays” from the tip of the arrow.** Use a straightedge or a ruler. The rays should reflect off the mirror plane.
   a. A ray parallel to the axis reflects through the focal point.
   b. An incoming ray that passes through the focal point reflects back parallel to the axis.
   c. A ray that strikes the center of the mirror reflects at an equal angle on the opposite side of the optical axis.
5. **Extend the rays until they converge.** The rays converge at the image point. Draw the rest of the image in the image plane. If the base of the object is on the axis, then the base of the image will also be on the axis.
6. **Measure the image distance $s'$**. Also, if needed, measure the image height relative to the object height. The magnification can be found from Equation 18.8.

Exercises 21a, 22

Text: p. 585
Example 18.10 Analyzing a concave mirror

A 3.0-cm-high object is located 60 cm from a concave mirror. The mirror’s focal length is 40 cm. Use ray tracing to find the position, height, and magnification of the image.

**PREPARE** FIGURE 18.42 shows the ray-tracing diagram and the steps of Tactics Box 18.4.

1. Lay out the optical axis, with a scale.
2. Draw the mirror, the mirror plane, and mark the focal point.
3. Draw the object as an arrow with its base on the axis.
4. Draw the 3 special rays from the tip of the arrow.
   a. Parallel to the axis
   b. Through the focal point
   c. Hitting the center of the mirror
5. The convergence point is the tip of the image. Draw the rest of the image.
6. Measure the image distance.
Example 18.10 Analyzing a concave mirror (cont.)

**SOLVE** After preparing a careful drawing, we can use a ruler to find that the image position is \( s' \approx 120 \text{ cm} \). The magnification is thus

\[
m = -\frac{s'}{s} \approx -\frac{120 \text{ cm}}{60 \text{ cm}} = -2.0
\]
Example 18.10 Analyzing a concave mirror (cont.)

The negative sign indicates that the image is inverted. The image height is thus twice the object height, or \( h' \approx 6 \text{ cm} \).

**ASSESS** The image is a *real* image because light rays converge at the image point.
Convex Mirrors

Any ray initially parallel to the optical axis will reflect as though it came from the focal point.

Any ray initially directed toward the focal point will reflect parallel to the optical axis.

Any ray directed at the center of the mirror will reflect at an equal angle on the opposite side of the optical axis.
Convex Mirrors

- We use the three special rays to show that no rays converge at the image point $P'$. 

This ray entered parallel to the optical axis, and thus appears to have come from the focal point.

This ray was heading for the focal point, and thus emerges parallel to the optical axis.

This ray that strikes the center of the mirror reflects at an equal angle on the opposite side of the optical axis.
Convex Mirrors

- Diverging rays *appear* to have come from point \( P' \).
- The image is upright and much smaller than the object.
Convex Mirrors

• Convex mirrors are used as passenger-side rearview mirrors and in stores to keep an eye on customers. Because the image is smaller, you can *see much more of it*.

**Ray tracing for a convex mirror**

1–3 Follow steps 1 through 3 of Tactics Box 18.4.

4 **Draw the three “special rays” from the tip of the arrow.** Use a straight-edge or a ruler. The rays should reflect off the mirror plane.
   a. A ray parallel to the axis reflects as though it came from the focal point.
   b. A ray initially directed toward the focal point reflects parallel to the axis.
   c. A ray that strikes the center of the mirror reflects at an equal angle on the opposite side of the optical axis.

5 **Extend the emerging rays behind the mirror until they converge.** The point of convergence is the image point. Draw the rest of the image in the image plane. If the base of the object is on the axis, then the base of the image will also be on the axis.

6 **Measure the image distance s’**. Also, if needed, measure the image height relative to the object height. The magnification can be found from Equation 18.8.

Exercises 21b, 23

Text: p. 587
QuickCheck 18.19

You see an upright, magnified image of your face when you look into magnifying “cosmetic mirror.” The image is located

A. In front of the mirror’s surface.
B. On the mirror’s surface.
C. Behind the mirror’s surface.
D. Only in your mind because it’s a virtual image.
QuickCheck 18.19

You see an upright, magnified image of your face when you look into magnifying “cosmetic mirror.” The image is located

A. In front of the mirror’s surface.
B. On the mirror’s surface.
C. Behind the mirror’s surface. ✓
D. Only in your mind because it’s a virtual image.
Conceptual Example 18.11 Driver and passenger mirrors

The rearview mirror on the driver’s side of a car is a plane (flat) mirror, while the mirror on the passenger’s side is convex. Why is this?
Conceptual Example 18.11 Driver and passenger mirrors (cont.)

**REASON** It is important for the driver to have a wide field of view from either mirror. He sits close to the driver-side mirror, so it appears large and can reflect a fairly wide view of what’s behind. The passenger-side mirror is quite far from the driver, so it appears relatively small. If it were flat, it would offer only a narrow view of what’s behind. Making it convex, like the security mirror discussed above, provides a wider field of view, but the trade-off is a smaller image. That’s why the passenger-side mirror usually contains a warning: Objects in mirror are closer than they appear!
Section 18.7 The Thin-Lens Equation
The Thin-Lens Equation
The Thin-Lens Equation

• Because the two shaded triangles are right angles, the angle $\theta$ must be the same for both. The triangles are similar.

• For similar triangles, the ratios of any two similar sides are the same:

\[
\frac{h'}{h} = \frac{s' - f}{f}
\]
The Thin-Lens Equation

• We have also shown that

\[ \frac{h'}{h} = \frac{s'}{s} \]

• Combining the equations gives

\[ \frac{s'}{s} = \frac{s' - f}{f} \]

\[ \frac{1}{s} = \frac{s' - f}{sf} = \frac{1}{f} - \frac{1}{s'} \]
The Thin-Lens Equation

• Finally this is written as the **thin-lens equation**:

\[
\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}
\]

**Thin-lens equation (also works for mirrors)**
relating object and image distances to focal length
Sign Conventions for Lenses and Mirrors

- In the thin-lens equation, the sign of the focal length can be either positive or negative, depending on the type of lens or mirror.
### Sign Conventions for Lenses and Mirrors

**SYNTHESIS 18.1 Lenses, mirrors, and their sign conventions**

There are six distinct image situations that can occur for lenses and mirrors. We outline these here, and give the sign of each quantity. Note that the object distance $s$ is always positive.

<table>
<thead>
<tr>
<th>Converging lens or concave mirror</th>
<th>Diverging lens or convex mirror</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Object outside focal point</strong></td>
<td><strong>Object inside focal point</strong></td>
</tr>
<tr>
<td>The focal length $f$ of a converging lens or concave mirror is <em>positive</em>.</td>
<td>The focal length $f$ of a diverging lens or convex mirror is <em>negative</em>.</td>
</tr>
<tr>
<td><img src="image" alt="Converging lens diagram" /></td>
<td><img src="image" alt="Diverging lens diagram" /></td>
</tr>
<tr>
<td>The object is farther from the lens or mirror than the focal point.</td>
<td>For diverging lenses and convex mirrors, there is no distinction between an object inside or outside the focal point.</td>
</tr>
<tr>
<td><img src="image" alt="Converging mirror diagram" /></td>
<td><img src="image" alt="Diverging mirror diagram" /></td>
</tr>
<tr>
<td>Real, inverted image</td>
<td>Virtual, upright image</td>
</tr>
<tr>
<td><img src="image" alt="Converging mirror diagram" /></td>
<td><img src="image" alt="Diverging mirror diagram" /></td>
</tr>
<tr>
<td><img src="image" alt="Converging mirror diagram" /></td>
<td><img src="image" alt="Diverging mirror diagram" /></td>
</tr>
</tbody>
</table>

These two cases give *real* images that are *inverted* (the magnification $m$ is *negative*). The image distance $s'$ is *positive*.

These four cases give *virtual* images that are *upright* (the magnification $m$ is *positive*). The image distance $s'$ is *negative*.

Text: p. 590
QuickCheck 18.14

A lens creates an image as shown. In this situation, the object distance $s$ is

A. Larger than the focal length $f$
B. Equal to the focal length $f$
C. Smaller than focal length $f$
QuickCheck 18.14

A lens creates an image as shown. In this situation, the object distance $s$ is

A. Larger than the focal length $f$
B. Equal to the focal length $f$
C. Smaller than focal length $f$
QuickCheck 18.15

A lens creates an image as shown. In this situation, the image distance $s'$ is

A. Larger than the focal length $f$
B. Equal to the focal length $f$
C. Smaller than focal length $f$
QuickCheck 18.15

A lens creates an image as shown. In this situation, the image distance $s'$ is

A. Larger than the focal length $f$
B. Equal to the focal length $f$
C. Smaller than focal length $f$
QuickCheck 18.16

Light rays are converging to point 1. The lens is inserted into the rays with its focal point at point 1. Which picture shows the rays leaving the lens?
QuickCheck 18.16

Light rays are converging to point 1. The lens is inserted into the rays with its focal point at point 1. Which picture shows the rays leaving the lens?

A.  
B.  
C.  
D.  
E.  

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QuickCheck 18.17

A lens creates an image as shown. In this situation,

A. $s < f$
B. $f < s < 2f$
C. $s > 2f$
D. There’s not enough information to compare $s$ to $f$
QuickCheck 18.17

A lens creates an image as shown. In this situation,

A. $s < f$

B. $f < s < 2f$

C. $s > 2f$

D. There’s not enough information to compare $s$ to $f$

The image is real, which requires $s > f$. The image is taller than the object, and
$s' > s$ requires $s < 2f$. 
QuickCheck 18.18

In this figure the image is produced by a lens. At which position is the lens?
QuickCheck 18.18

In this figure the image is produced by a lens. At which position is the lens?

A. B. C. D. E.
Example 18.12 Analyzing a magnifying lens

A stamp collector uses a magnifying lens that sits 2.0 cm above the stamp. The magnification is 4. What is the focal length of the lens?
Example 18.12 Analyzing a magnifying lens (cont.)

**PREPARE** A magnifying lens is a converging lens with the object distance less than the focal length \((s < f)\). Assume it is a thin lens. The user looks through the lens and sees a virtual image. FIGURE 18.47 shows the lens and a ray-tracing diagram.
Example 18.12 Analyzing a magnifying lens (cont.)

**SOLVE** A virtual image is upright, so $m = +4$. The magnification is $m = -s'/s$; thus

$$s' = -4s = -4(2.0 \text{ cm}) = -8.0 \text{ cm}$$
Example 18.12 Analyzing a magnifying lens (cont.)

We can use $s$ and $s'$ in the thin-lens equation to find the focal length:

$$\frac{1}{f} = \frac{1}{s} + \frac{1}{s'} = \frac{1}{2.0 \text{ cm}} + \frac{1}{-8.0 \text{ cm}} = 0.375 \text{ cm}^{-1}$$
Example 18.12 Analyzing a magnifying lens (cont.)

Thus

$$f = \frac{1}{0.375 \text{ cm}^{-1}} = 2.7 \text{ cm}$$

ASSESS $f > 2 \text{ cm}$, as expected because the object has to be inside the focal point.
Reflection

Law of reflection: $\theta_r = \theta_i$

Reflection can be **specular** (mirror-like) or **diffuse** (from rough surfaces).

Plane mirrors: A virtual image is formed at $P'$ with $s' = s$, where $s$ is the **object distance** and $s'$ is the **image distance**.

Text: p. 593
Summary: General Principles

Refraction

**Snell’s law** of refraction:

\[ n_1 \sin \theta_1 = n_2 \sin \theta_2 \]

Index of refraction is \( n = c/v \). The ray is closer to the normal on the side with the larger index of refraction.

If \( n_2 < n_1 \), **total internal reflection** (TIR) occurs when the angle of incidence \( \theta_1 \) is greater than \( \theta_c = \sin^{-1}(n_2/n_1) \).

Text: p. 593
Summary: Important Concepts

The ray model of light

Light travels along straight lines, called light rays, at speed \( v = c/n \).

A light ray continues forever unless an interaction with matter causes it to reflect, refract, scatter, or be absorbed.

Light rays come from self-luminous or reflective objects. Each point on the object sends rays in all directions.

Ray diagrams use only a few select rays to represent all the rays emitted by an object.

In order for the eye to see an object (or image), rays from the object or image must enter the eye.

Text: p. 593
Summary: Important Concepts

Image formation

If rays diverge from P and, after interacting with a lens or mirror, appear to diverge from P', without actually passing through P', then P' is a virtual image of P.

If rays diverge from P and interact with a lens or mirror so that the refracted rays converge at P', then P' is a real image of P. Rays actually pass through a real image.

Text: p. 593
Summary: Applications

Ray tracing for lenses
Three special rays in three basic situations:

- Converging lens
  - Real image

- Converging lens
  - Virtual image

- Diverging lens
  - Virtual image

Ray tracing for mirrors
Three special rays in three basic situations:

- Concave mirror
  - Real image

- Concave mirror
  - Virtual image

- Convex mirror
  - Virtual image

Text: p. 593
Summary: Applications

The thin-lens equation
For a lens or curved mirror, the object distance $s$, the image distance $s'$, and the focal length $f$ are related by the thin-lens equation:

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

The magnification of a lens or mirror is $m = -s'/s$.

Sign conventions for the thin-lens equation:

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Positive when</th>
<th>Negative when</th>
</tr>
</thead>
<tbody>
<tr>
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<td>$s'$</td>
<td><em>Real</em> image; on opposite side of a lens from object, or in front of a mirror</td>
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</tr>
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<tr>
<td>$m$</td>
<td>Image is upright.</td>
<td>Image is inverted.</td>
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Summary

**GENERAL PRINCIPLES**

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Text: p. 593
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If rays diverge from \( P \) and interact with a lens or mirror so that the refracted rays **converge** at \( P' \), then \( P' \) is a **real image** of \( P \). Rays actually pass through a real image.
### APPLICATIONS

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Three special rays in three basic situations:

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For a lens or curved mirror, the object distance $s$, the image distance $s'$, and the focal length $f$ are related by the thin-lens equation:

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