Planetary Motion and Gravitation

In today’s lesson you will:

• describe the relationship between a planet’s orbital radius and period.
• define and relate Newton’s law of universal gravitation to Kepler’s laws.
• explain the importance of Cavendish’s investigation.

**Main Idea:** The gravitational force between two objects is proportional to the product of their masses divided by the square of the distance between them.
Early Observations

- Copernicus showed that the motion of planets is much more easily understood by assuming that Earth and other planets revolve around the Sun.

- His model helped explain phenomena such as the inner planets Mercury and Venus always appearing near the Sun.

- Tycho realized that the charts of the time did not accurately predict astronomical events.

- He recognized that measurements were required from one location over a long period of time. He is credited with the most accurate measurements of the time.
Kepler’s Laws

- **Kepler’s first law** states that the paths of the planets are ellipses, with the Sun at one focus.

- **Kepler’s second law** states that an imaginary line from the Sun to a planet sweeps out equal areas in equal time intervals.
Kepler’s First and Second Laws

If \( t_1 = t_2 \), then

\[ \text{Area}_1 = \text{Area}_2 \]
Kepler’s Third Law

- A period is the time it takes for one revolution of an orbiting body.

- **Kepler’s third law** states that the square of the ratio of the periods of any two planets revolving about the Sun is equal to the cube of the ratio of their average distances from the Sun.

- Thus, if the periods of the planets are $T_A$ and $T_B$, and their average distances from the Sun are $r_A$ and $r_B$:

  \[
  \left(\frac{T_A}{T_B}\right)^2 = \left(\frac{r_A}{r_B}\right)^3
  \]

  **Kepler’s Third Law**

- Comets are classified as long-period comets and short-period comets, based on orbital periods, often often have very elliptical orbits, and also obey Kepler’s three laws.
## Solar System Data

<table>
<thead>
<tr>
<th>Planet</th>
<th>Minimum Distance from Sun (km)</th>
<th>Maximum Distance from Sun (km)</th>
<th>Average Distance from Sun (km)</th>
<th>Period (Earth Years)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mercury</td>
<td>$4.608 \times 10^7$</td>
<td>$6.982 \times 10^7$</td>
<td>$5.791 \times 10^7$</td>
<td>0.241</td>
</tr>
<tr>
<td>Venus</td>
<td>$1.075 \times 10^8$</td>
<td>$1.089 \times 10^8$</td>
<td>$1.082 \times 10^8$</td>
<td>0.615</td>
</tr>
<tr>
<td>Earth</td>
<td>$1.471 \times 10^8$</td>
<td>$1.521 \times 10^8$</td>
<td>$1.496 \times 10^8$</td>
<td>1.000</td>
</tr>
<tr>
<td>Mars</td>
<td>$2.066 \times 10^8$</td>
<td>$2.492 \times 10^8$</td>
<td>$2.279 \times 10^8$</td>
<td>1.881</td>
</tr>
<tr>
<td>Jupiter</td>
<td>$7.405 \times 10^8$</td>
<td>$8.166 \times 10^8$</td>
<td>$7.786 \times 10^8$</td>
<td>11.860</td>
</tr>
<tr>
<td>Saturn</td>
<td>$1.353 \times 10^9$</td>
<td>$1.515 \times 10^9$</td>
<td>$1.434 \times 10^9$</td>
<td>29.420</td>
</tr>
<tr>
<td>Uranus</td>
<td>$2.741 \times 10^9$</td>
<td>$3.004 \times 10^9$</td>
<td>$2.872 \times 10^9$</td>
<td>84.010</td>
</tr>
<tr>
<td>Neptune</td>
<td>$4.444 \times 10^9$</td>
<td>$4.546 \times 10^9$</td>
<td>$4.495 \times 10^9$</td>
<td>164.790</td>
</tr>
<tr>
<td>Pluto</td>
<td>$4.435 \times 10^9$</td>
<td>$7.304 \times 10^9$</td>
<td>$5.870 \times 10^9$</td>
<td>247.680</td>
</tr>
</tbody>
</table>

Kepler’s Third Law $\left( \frac{T_A^2}{T_B^2} \right) = \left( \frac{r_A}{r_B} \right)^3$
Callisto’s Distance from Jupiter

Galileo measured the orbital radii of Jupiter’s moons using the diameter of Jupiter as a unit of measure. He found that Io, the closest moon to Jupiter, had a period of 1.8 days and was 4.2 units from the center of Jupiter. Callisto, the fourth moon from Jupiter, had a period of 16.7 days. Using the same units that Galileo used, predict Callisto’s distance from Jupiter.

**Step 1: Analyze and Sketch the Problem**

Identify the known and unknown variables.

**Known:**

**Unknown:**
Callisto’s Distance from Jupiter

Galileo found that Io, the closest moon to Jupiter, had a period of 1.8 days and was 4.2 units from the center of Jupiter. Callisto, the fourth moon from Jupiter, had a period of 16.7 days. Using the same units that Galileo used, predict Callisto’s distance from Jupiter.

**Step 2:** Solve for the Unknown

**Known:**
- $T_C = 16.7$ days
- $T_I = 1.8$ days
- $r_I = 4.2$ units

**Unknown:** $r_C = ?$
Callisto’s Distance from Jupiter

Galileo found that Io, the closest moon to Jupiter, had a period of 1.8 days and was 4.2 units from the center of Jupiter. Callisto, the fourth moon from Jupiter, had a period of 16.7 days. Using the same units that Galileo used, predict Callisto’s distance from Jupiter.

**Step 3: Evaluate the Answer**

Are the units correct?

Is the magnitude realistic?
Newton’s Law of Universal Gravitation

- Newton found that the magnitude of the force, $F$, on a planet due to the Sun varies inversely with the square of the distance, $r$, between the centers of the planet and the Sun.

- The force of attraction between two objects must be proportional to the objects’ masses and is known as the **gravitational force**.

- Newton’s **law of universal gravitation** states that objects attract other objects with a force that is proportional to the product of their masses and inversely proportional to the square of the distance between them.

**Law of Universal Gravitation**  
\[ F = G \frac{m_1 m_2}{r^2} \]
Gravitation
Universal Gravitation & Kepler’s Third Law

• Newton stated his law of universal gravitation in terms that applied to the motion of planets around the Sun, which agreed with Kepler’s third law and confirmed that Newton’s law fit the best observations of the day.

• Using the laws of motion, the period of a planet can be found.

   Period of a Planet Orbiting the Sun

   \[ T = \sqrt{\left(\frac{4\pi^2}{Gm_s}\right)r^3} \]

• The period of a planet orbiting the Sun is equal to \(2\pi\) times the square root of the average distance from the Sun cubed, divided by the product of the universal gravitational constant and the mass of the Sun.
Cavendish’s Apparatus

• In 1798 Englishman Henry Cavendish measured the gravitational force between two objects.

• A Cavendish balance uses a light source and a mirror to measure the movement of the spheres.

• By substituting the values for force, mass, and distance into Newton’s law of universal gravitation, an experimental value for $G$ is found: when $m_1$ and $m_2$ are measured in kilograms, $r$ in meters, and $F$ in newtons, $G = 6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2$. 
Importance of G

• Cavendish’s experiment often is called “weighing Earth” because his experiment helped determine Earth’s mass.

• Once the value of G is known, not only the mass of Earth, but also the mass of the Sun can be determined.

• In addition, the gravitational force between any two objects can be calculated using Newton’s law of universal gravitation.

• Cavendish’s investigation determined the value of G, confirmed Newton’s prediction that a gravitational force exists between two objects, and helped calculate the mass of Earth.
Understand Main Ideas

1. **Gravity**  What is the gravitational force between two 15-kg balls whose centers are 35 m apart? What fraction is this of the weight of one package?

2. **Neptune’s Orbital Period**  Neptune orbits the Sun at an average distance of $4.496 \times 10^{12}$ m, which allows gases, such as methane, to condense and form an atmosphere. If the mass of the Sun is $1.99 \times 10^{30}$ kg, calculate the period of Neptune’s orbit.

3. **Universal Gravitational Constant**  Cavendish did his investigation using lead spheres. Suppose he had replaced the lead spheres with copper spheres of equal mass. Would his value of G be the same or different? Explain.
Critical Thinking

Picking up a rock requires less effort on the Moon than on Earth. How will the Moon’s gravitational force affect the path of the rock if it is thrown horizontally? If the thrower accidentally drops the rock on her toe, will it hurt more or less than it would on Earth? Explain.