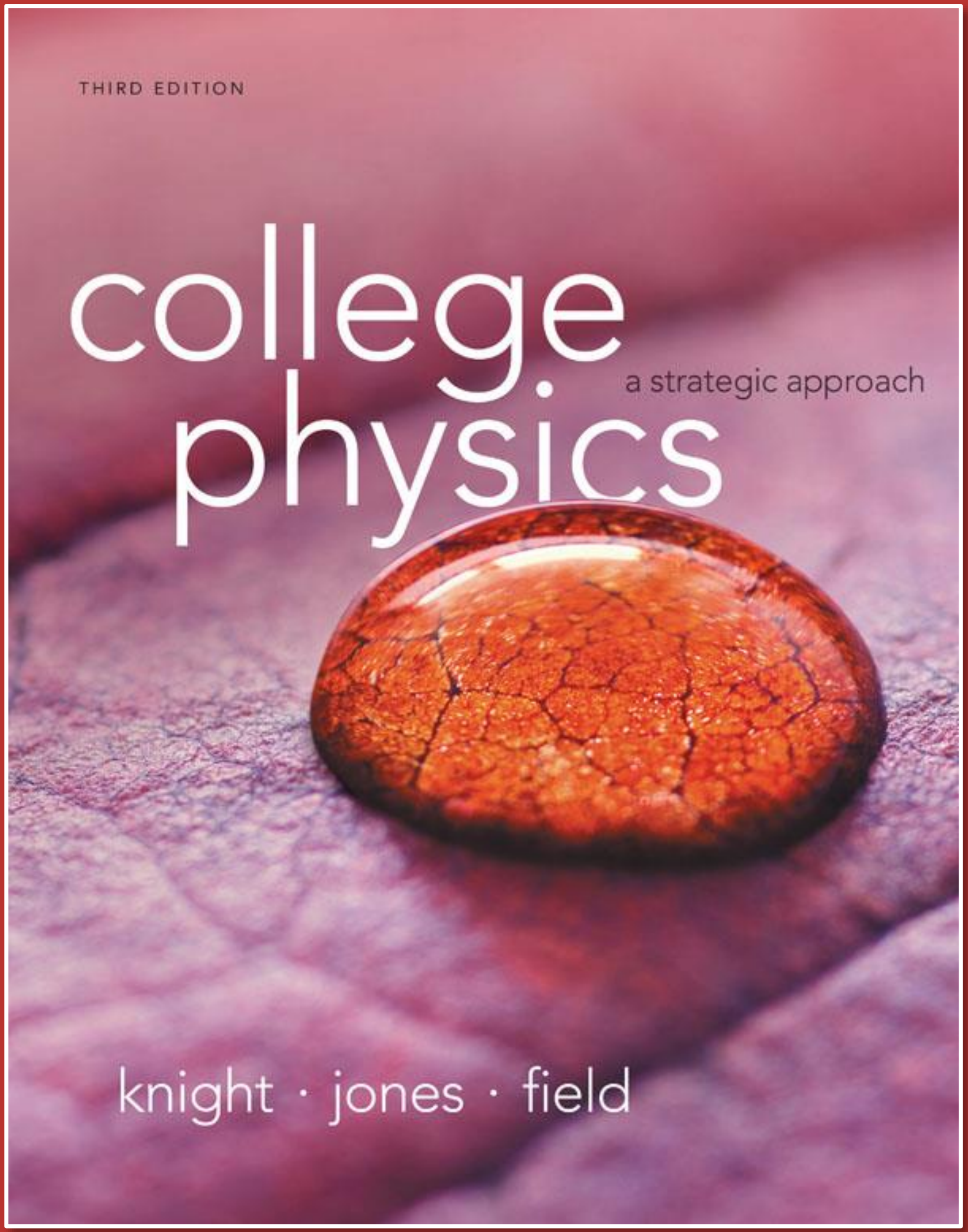


Lecture Presentation

Chapter 23

Circuits

Suggested Videos for Chapter 23

- **Prelecture Videos**
  - *Analyzing Circuits*
  - *Series and Parallel Circuits*
  - *Capacitor Circuits*
- **Video Tutor Solutions**
  - *Circuits*
- **Class Videos**
  - *Circuits: Warm-Up Exercises*
  - *Circuits: Analyzing More Complex Circuits*
- **Video Tutor Demos**
  - *Bulbs Connected in Series and Parallel*
  - *Discharge Speed for Series and Parallel Capacitors*

Suggested Simulations for Chapter 23

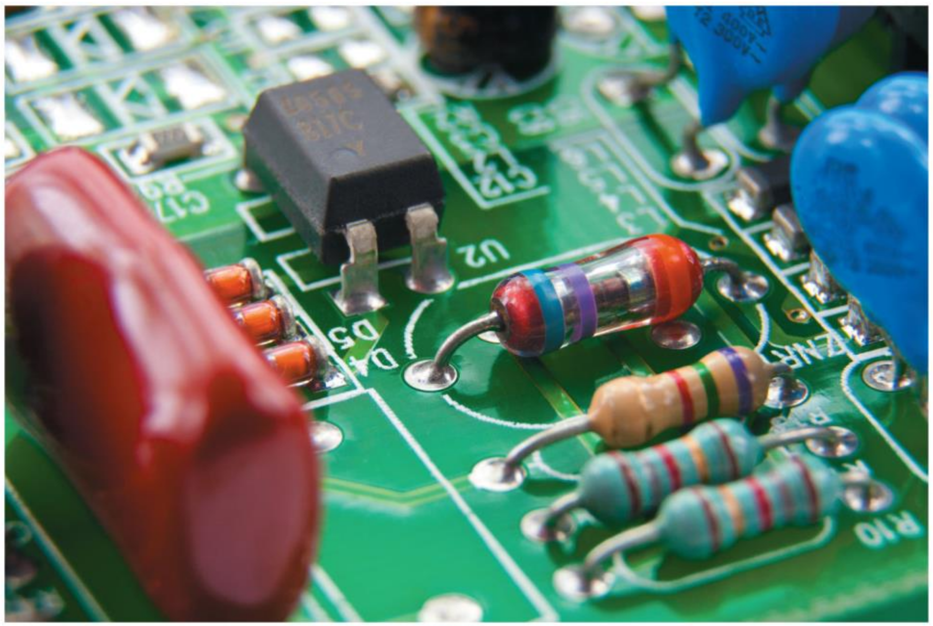
- **ActivPhysics**
  - *12.1–12.5, 12.7, 12.8*
- **PhETs**
  - *Circuit Construction Kit (AC+DC)*
  - *Circuit Construction Kit (DC)*

Chapter 23 Circuits

Chapter Goal: To understand the fundamental physical principles that govern electric circuits.

Chapter 23 Preview
Looking Ahead: Analyzing Circuits

- Practical circuits consist of many elements—resistors, batteries, capacitors—connected together.



- You'll learn how to analyze complex circuits by breaking them into simpler pieces.

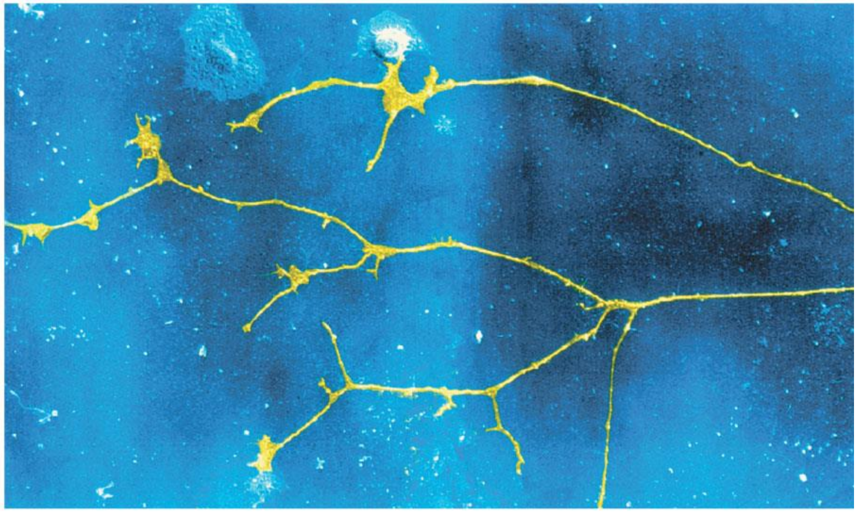
Chapter 23 Preview
Looking Ahead: Series and Parallel Circuits

- There are two basic ways to connect resistors together and capacitors together: **series circuits** and **parallel circuits.**

- You'll learn why holiday lights are wired in series but headlights are in parallel.

Chapter 23 Preview
Looking Ahead: Electricity in the Body

- Your nervous system works by transmitting electrical signals along *axons*, the long nerve fibers shown here.

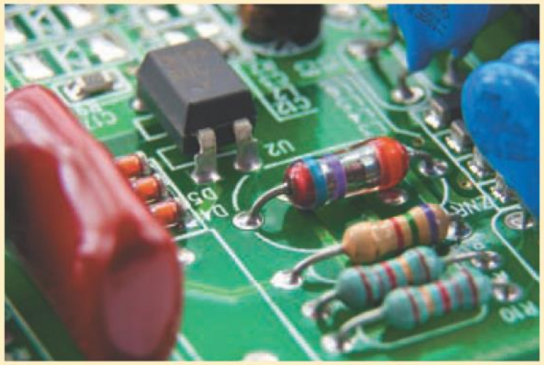


- You'll learn how to understand nerve impulses in terms of the resistance, capacitance, and electric potential of individual nerve cells.

Chapter 23 Preview
Looking Ahead

Analyzing Circuits

Practical circuits consist of many elements—resistors, batteries, capacitors—connected together.



You'll learn how to analyze complex circuits by breaking them into simpler pieces.

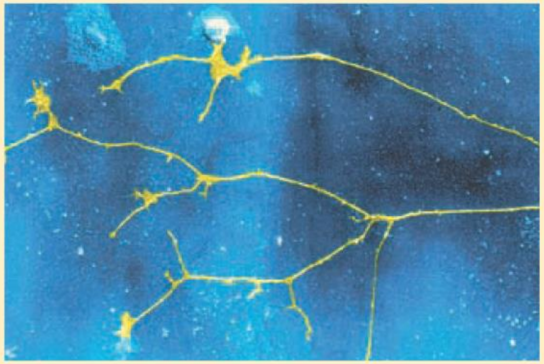
Series and Parallel Circuits

There are two basic ways to connect resistors together and capacitors together: **series circuits** and **parallel circuits.**

You'll learn why holiday lights are wired in series but headlights are in parallel.

Electricity in the Body

Your nervous system works by transmitting electrical signals along *axons*, the long nerve fibers shown here.

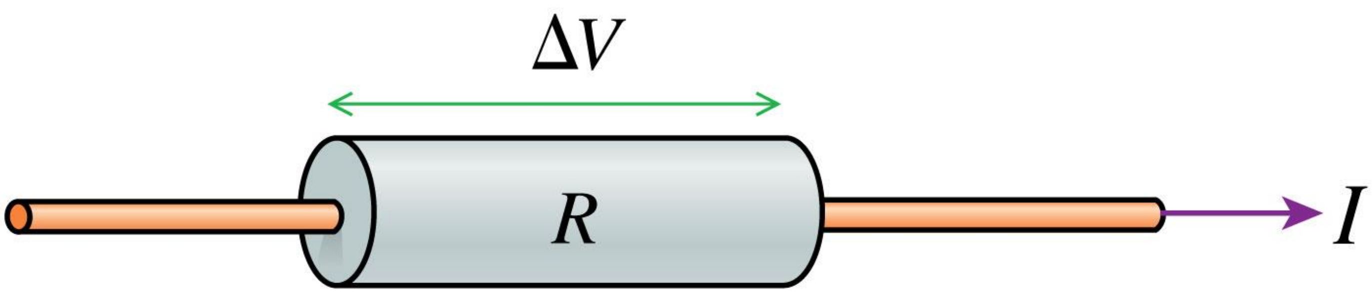


You'll learn how to understand nerve impulses in terms of the resistance, capacitance, and electric potential of individual nerve cells.

Text: p. 727

Chapter 23 Preview
Looking Back: Ohm's Law

- In Section 22.5 you learned Ohm's law, the relationship between the current through a resistor and the potential difference across it.

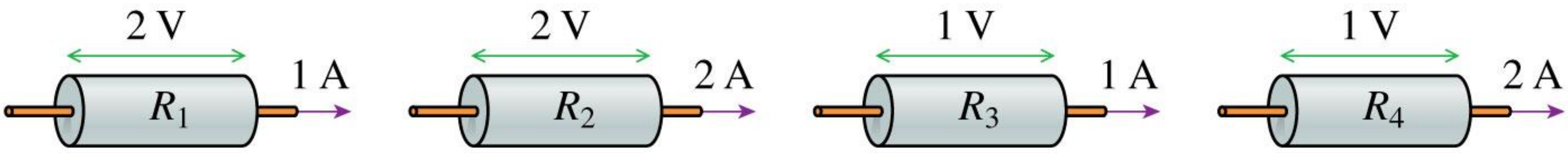


$$I = \frac{\Delta V}{R}$$

- In this chapter, you'll use Ohm's law when analyzing more complex circuits consisting of multiple resistors and batteries.

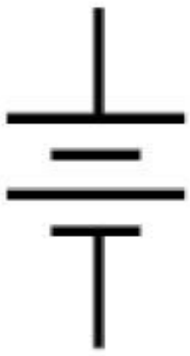
Chapter 23 Preview
Stop to Think

Rank in order, from smallest to largest, the resistances $R1$ to $R4$ of the four resistors.



Reading Question 23.1

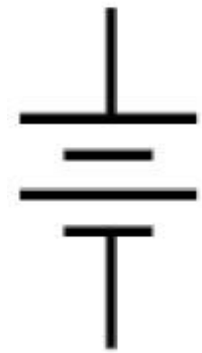
The symbol shown represents a



A. Battery.
B. Resistor.
C. Capacitor.
D. Transistor.

Reading Question 23.1

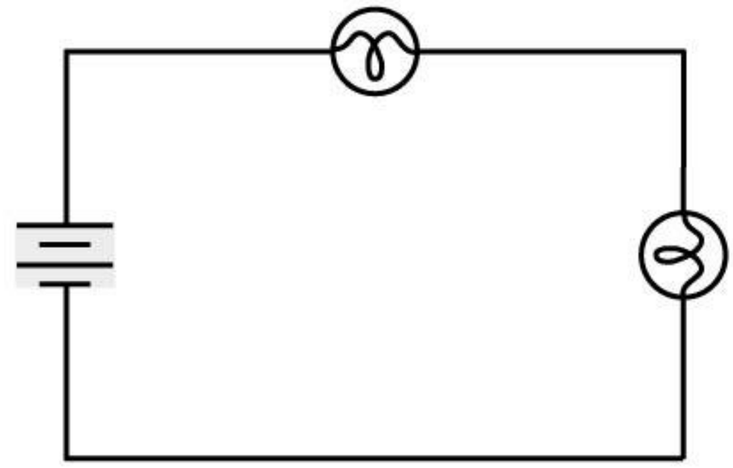
The symbol shown represents a



✔ A. Battery.
B. Resistor.
C. Capacitor.
D. Transistor.

Reading Question 23.2

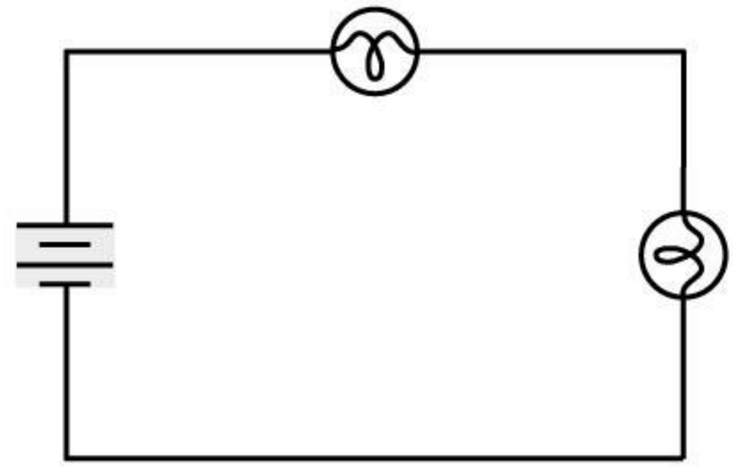
The bulbs in the circuit below are connected ___________.



A. In series
B. In parallel

Reading Question 23.2

The bulbs in the circuit below are connected __________.



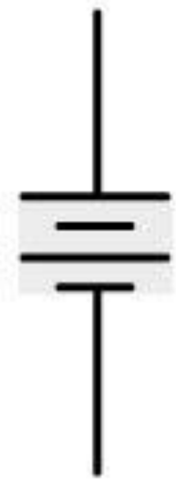
✔ A. In series

B. In parallel

Reading Question 23.3

Which terminal of the battery has a higher potential?

A. The top terminal
B. The bottom terminal

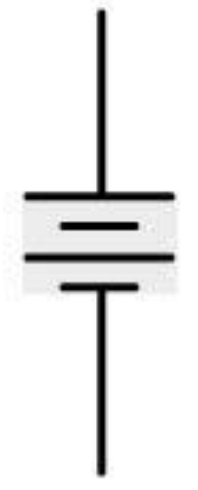


Reading Question 23.3

Which terminal of the battery has a higher potential?

✔ A. The top terminal

B. The bottom terminal



Reading Question 23.4

When three resistors are combined in series the total resistance of the combination is

A. Greater than any of the individual resistance values.
B. Less than any of the individual resistance values.
C. The average of the individual resistance values.

Reading Question 23.4

When three resistors are combined in series the total resistance of the combination is

✔ A. Greater than any of the individual resistance values.
B. Less than any of the individual resistance values.
C. The average of the individual resistance values.

Reading Question 23.5

In an *RC* circuit, what is the name of the quantity represented by the symbol τ?

A. The decay constant
B. The characteristic time
C. The time constant
D. The resistive component
E. The Kirchoff

Reading Question 23.5

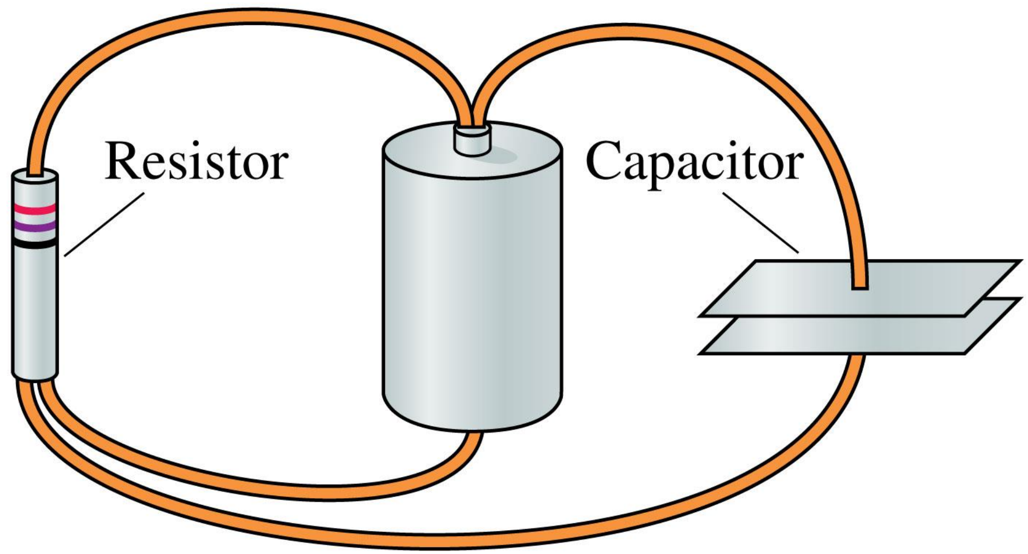
In an *RC* circuit, what is the name of the quantity represented by the symbol τ?

A. The decay constant
B. The characteristic time
✔ C. The time constant
D. The resistive component
E. The Kirchoff

Section 23.1 Circuit Elements and Diagrams

Circuit Elements and Diagrams

- This is an electric circuit in which a resistor and a capacitor are connected by wires to a battery.



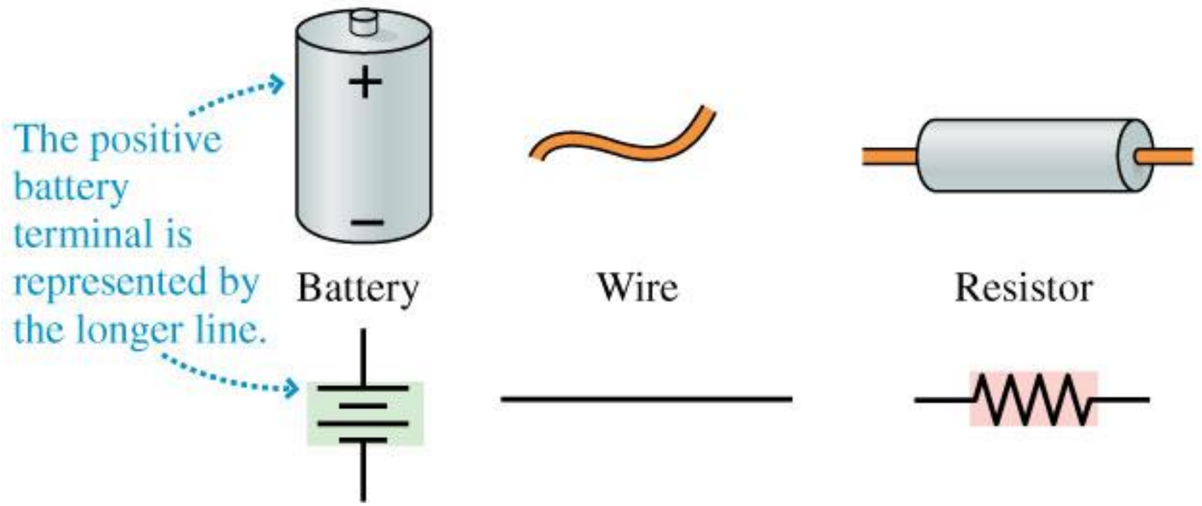
- To understand the operation of the circuit, we do not need to know whether the wires are bent or straight, or whether the battery is to the right or left of the resistor.
- The literal picture provides many irrelevant details.

Circuit Elements and Diagrams

- Rather than drawing a literal picture of circuit to describe or analyze circuits, we use a more abstract picture called a **circuit diagram**.
- A circuit diagram is a *logical* picture of what is connected to what.
- The actual circuit may *look* quite different from the circuit diagram, but it will have the same logic and connections.

Circuit Elements and Diagrams

- Here are the basic symbols used for electric circuit drawings:



Circuit Elements and Diagrams

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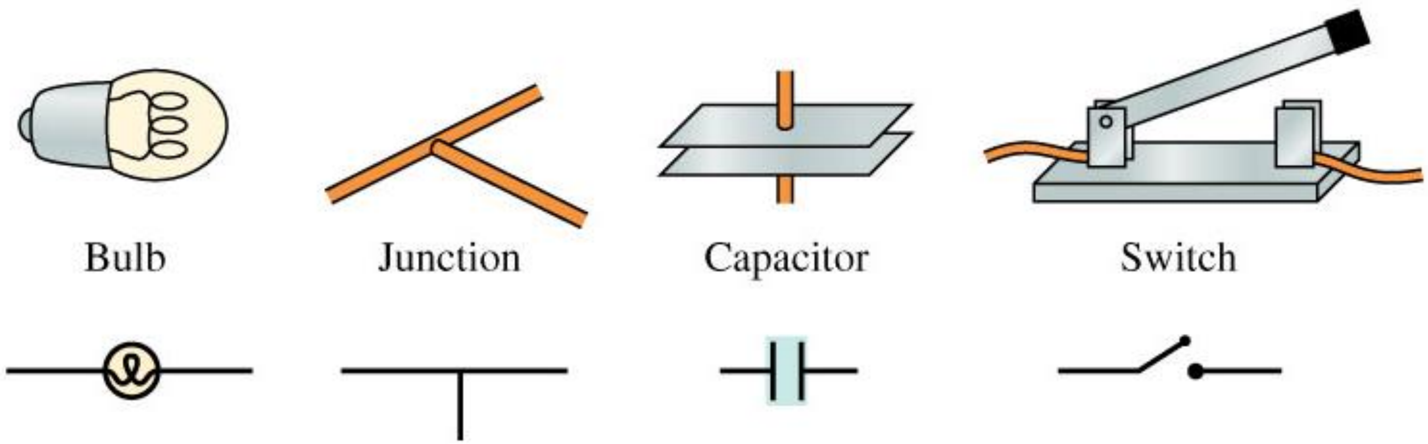
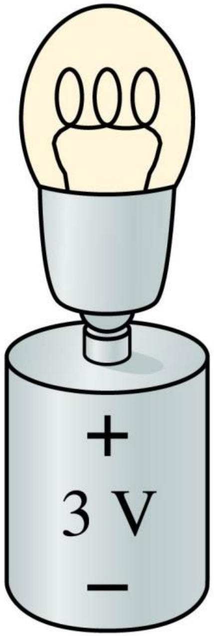


QuickCheck 23.1

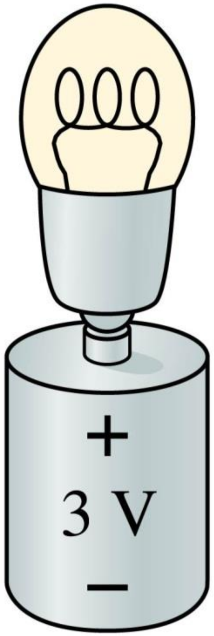
Does the bulb light?

A. Yes
B. No
C. I'm not sure.



QuickCheck 23.1

Does the bulb light?



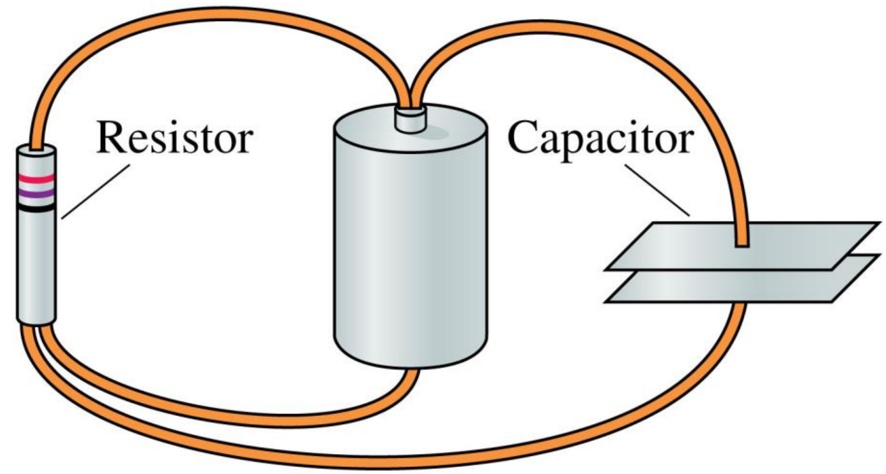
A. Yes

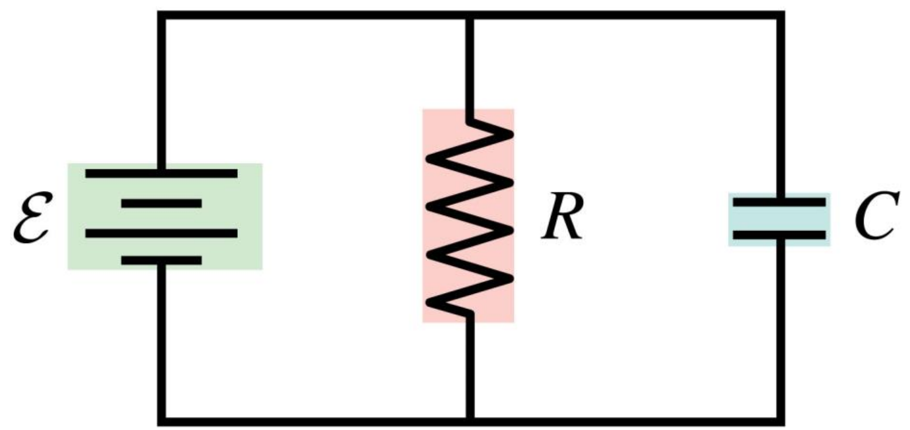
✔ B. No Not a complete circuit

C. I'm not sure.

Circuit Elements and Diagrams

- The circuit diagram for the simple circuit is now shown.
- The battery's emf $\mathcal{E}$, the resistance R, and the capacitance C of the capacitor are written beside the circuit elements.
- The wires, which in practice may bend and curve, are shown as straight-line connections between the circuit elements.





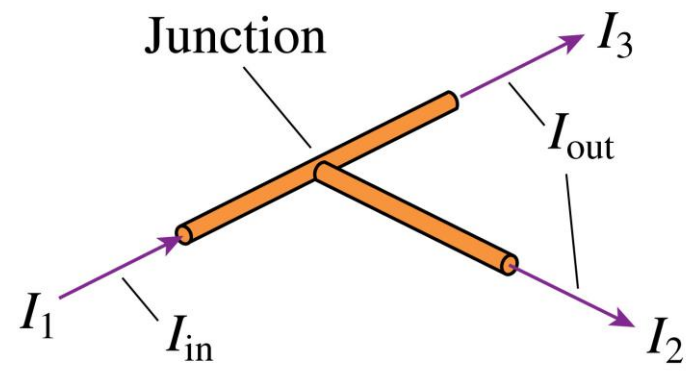
Section 23.2 Kirchhoff’s Law

Kirchhoff's Laws

- *Kirchhoff's junction law*, as we learned in Chapter 22, states that the total current into a junction must equal the total current leaving the junction.
- This is a result of charge and current conservation:

$$\sum I_{\text{in}} = \sum I_{\text{out}}$$

Kirchhoff's junction law

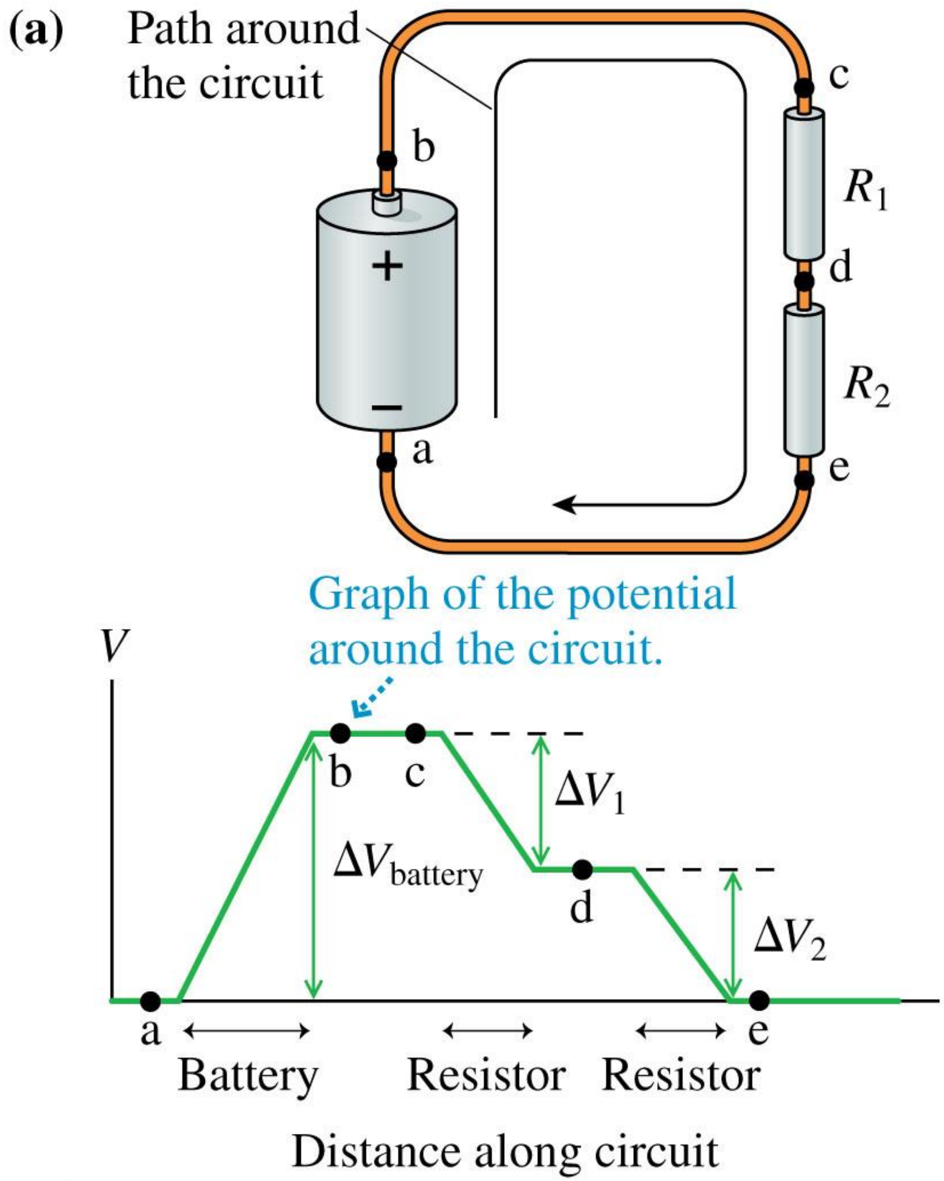


Junction law: $I_1 = I_2 + I_3$

Kirchhoff's Laws

- The gravitational potential energy of an object depends only on its position, not on the path it took to get to that position.
- The same is true of electric potential energy. If a charged particle moves around a closed loop and returns to its starting point, there is no net change in its electric potential energy: $\Delta u_{\text{elec}} = 0$.
- Because $V = U_{\text{elec}}/q$, **the net change in the electric potential around any loop or closed path must be zero** as well.

Kirchhoff's Laws

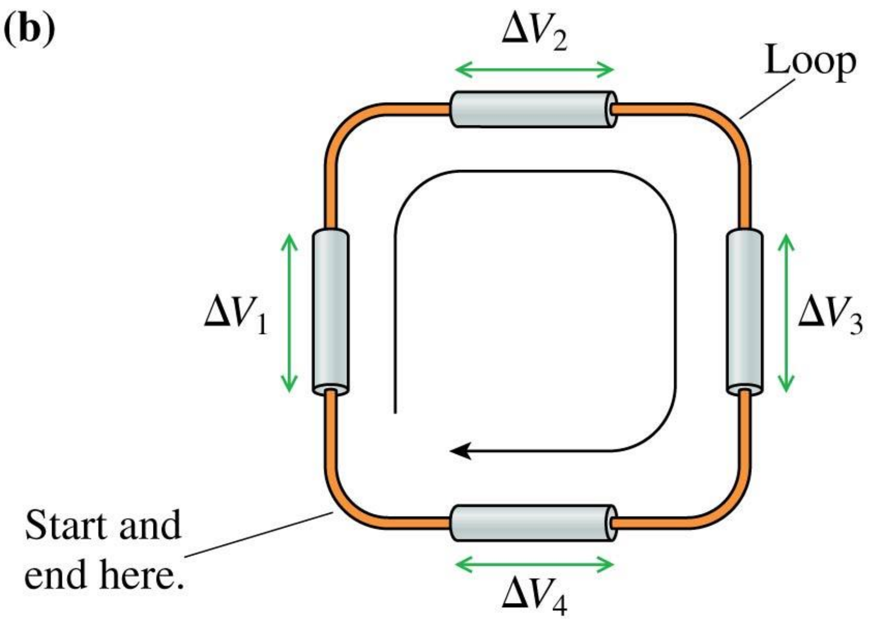


Kirchhoff's Laws

- For any circuit, if we add all of the potential differences around the loop formed by the circuit, the sum must be zero.
- This result is **Kirchhoff's loop law:**

$$\Delta V_{\text{loop}} = \sum_i \Delta V_i = 0$$

Kirchhoff's loop law



Loop law: $\Delta V_1 + \Delta V_2 + \Delta V_3 + \Delta V_4 = 0$

- ΔV_i is the potential difference of the *i*th component of the loop.

Kirchhoff's Laws

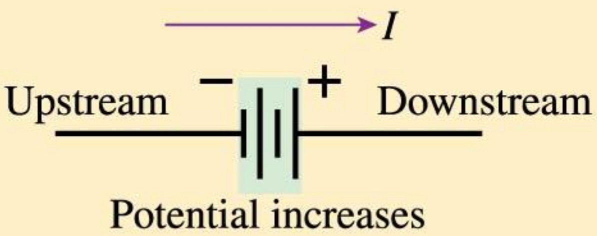
TACTICS BOX 23.1 **Using Kirchhoff's loop law**

1. **Draw a circuit diagram.** Label all known and unknown quantities.
2. **Assign a direction to the current.** Draw and label a current arrow I to show your choice. Choose the direction of the current based on how the batteries or sources of emf "want" the current to go. If you choose the current direction opposite the actual direction, the final value for the current that you calculate will have the correct magnitude but will be negative, letting you know that the direction is opposite the direction you chose.
3. **"Travel" around the loop.** Start at any point in the circuit, then go all the way around the loop in the direction you assigned to the current in step 2.

Text: pp. 729–730

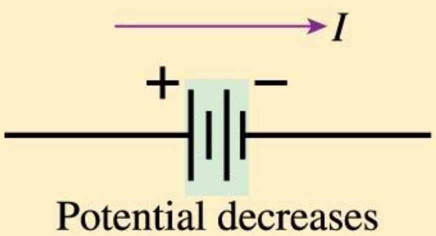
Kirchhoff's Laws

❸ As you go through each circuit element, ΔV is interpreted to mean $\Delta V = V_{\text{downstream}} - V_{\text{upstream}}$.



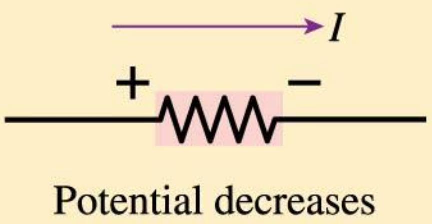
- For a battery with current in the negative-to-positive direction:

$$\Delta V_{\text{bat}} = +\mathcal{E}$$



- For a battery with current in the positive-to-negative direction:

$$\Delta V_{\text{bat}} = -\mathcal{E}$$



- For a resistor: $\Delta V_{\text{R}} = -IR$

❹ **Apply the loop law:** $\sum \Delta V_i = 0$

Exercises 8, 9

Text: p. 730

Kirchhoff's Laws

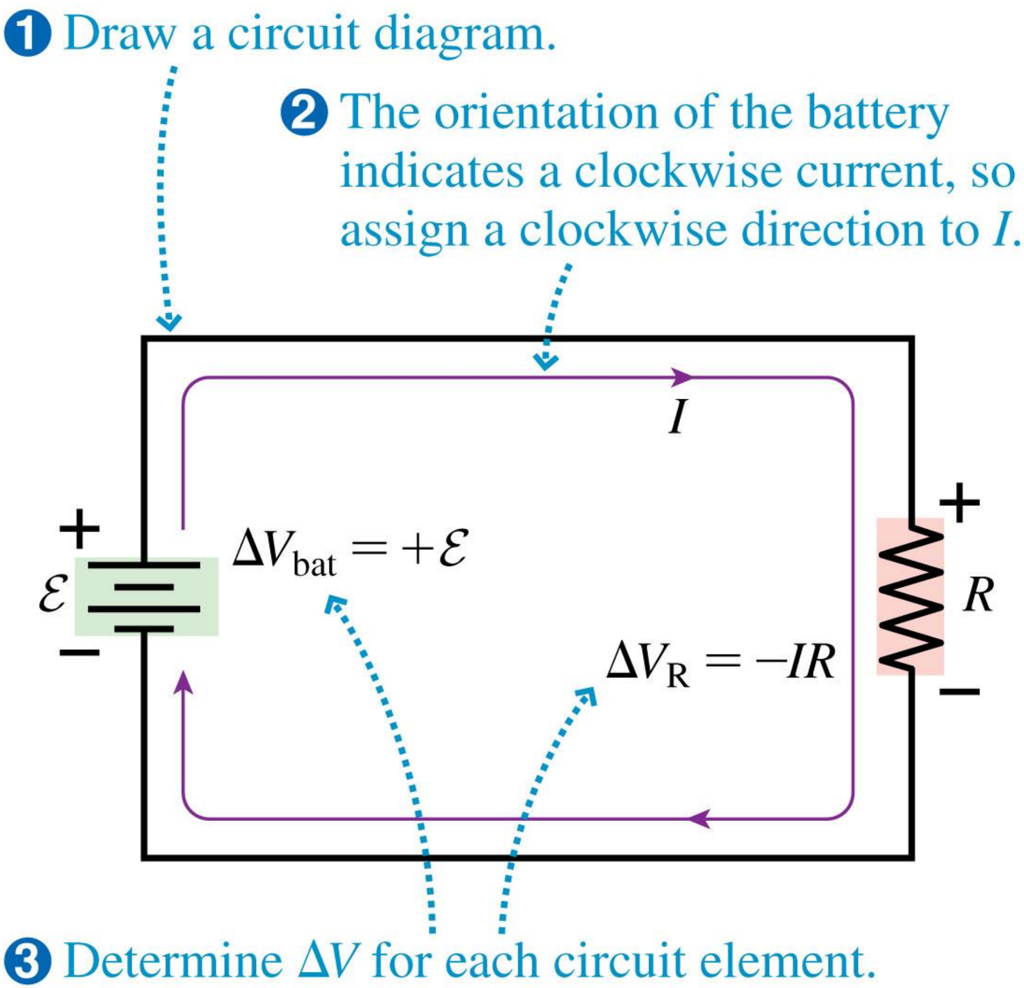
- ΔV_{bat} can be positive or negative for a battery, but ΔV_{R} for a resistor is always negative because the potential in a resistor *decreases* along the direction of the current.
- Because the potential across a resistor always decreases, we often speak of the *voltage drop* across the resistor.

Kirchhoff's Laws

- The most basic electric circuit is a single resistor connected to the two terminals of a battery.
- There are no junctions, so the current is the same in all parts of the circuit.

Kirchhoff's Laws

- The first three steps of the analysis of the basic circuit using Kirchhoff's Laws:



Kirchhoff's Laws

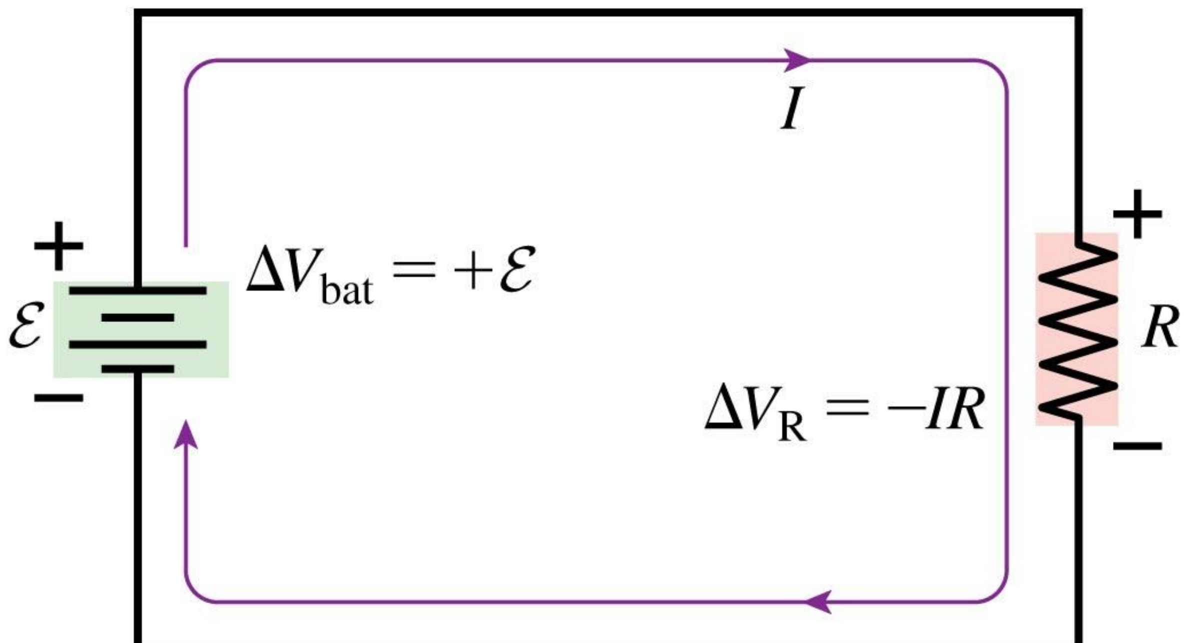
- The fourth step in analyzing a circuit is to apply Kirchhoff's loop law:

$$\Delta V_{\text{loop}} = \sum_i \Delta V_i = \Delta V_{\text{bat}} + \Delta V_{\text{R}} = 0$$

- First we must find the values for ΔV_{bat} and ΔV_{R}.

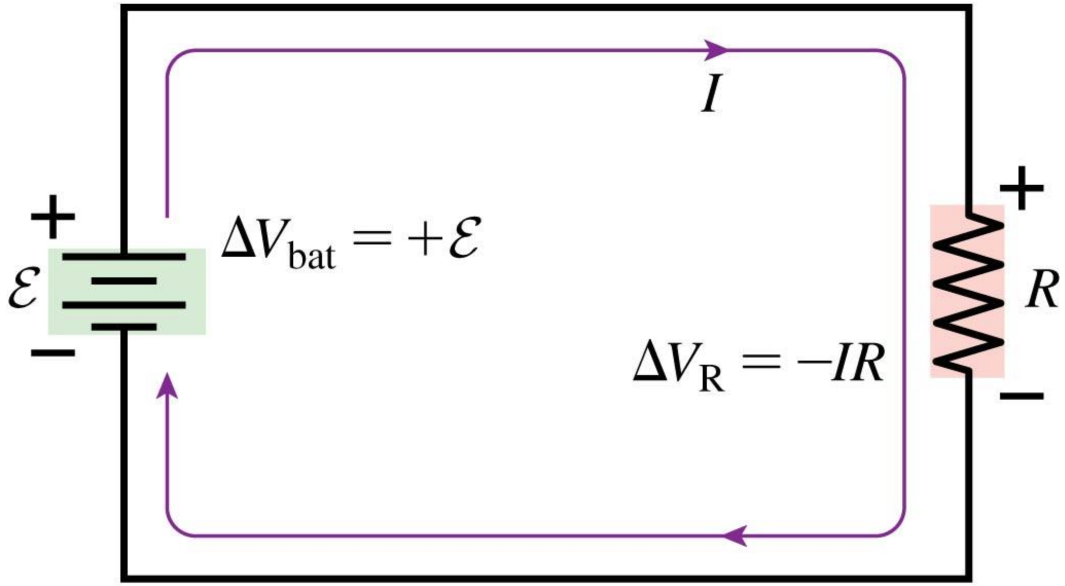
Kirchhoff's Laws

- The potential *increases* as we travel through the battery on our clockwise journey around the loop. We enter the negative terminal and exit the positive terminal after having gained potential $\mathcal{E}$.
- Thus $\Delta V_{\text{bat}} = + \mathcal{E}$.



Kirchhoff's Laws

- The *magnitude* of the potential difference across the resistor is $\Delta V = IR$, but Ohm's law does not tell us whether this should be positive or negative. The potential of a resistor *decreases* in the direction of the current, which is indicated with + and – signs in the figure.
- Thus, $\Delta V_R = -IR$.



Kirchhoff's Laws

- With the values of ΔV_{bat} and ΔV_{R}, we can use Kirchhoff's loop law:

$$\Delta V_{\text{loop}} = \sum_i \Delta V_i = \Delta V_{\text{bat}} + \Delta V_{\text{R}} = 0$$

$$\mathcal{E} - IR = 0$$

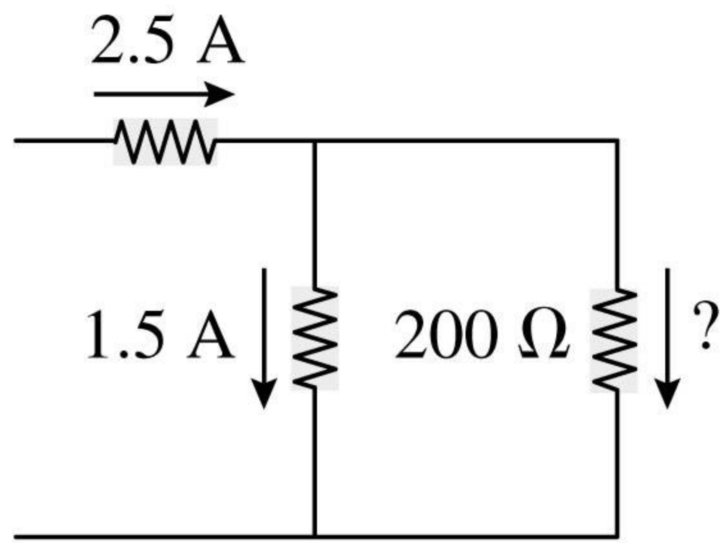
- We can solve for the current in the circuit:

$$I = \frac{\mathcal{E}}{R}$$

QuickCheck 23.6

The diagram below shows a segment of a circuit. What is the current in the 200 Ω resistor?

A. 0.5 A
B. 1.0 A
C. 1.5 A
D. 2.0 A
E. There is not enough information to decide.



QuickCheck 23.6

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A. 0.5 A
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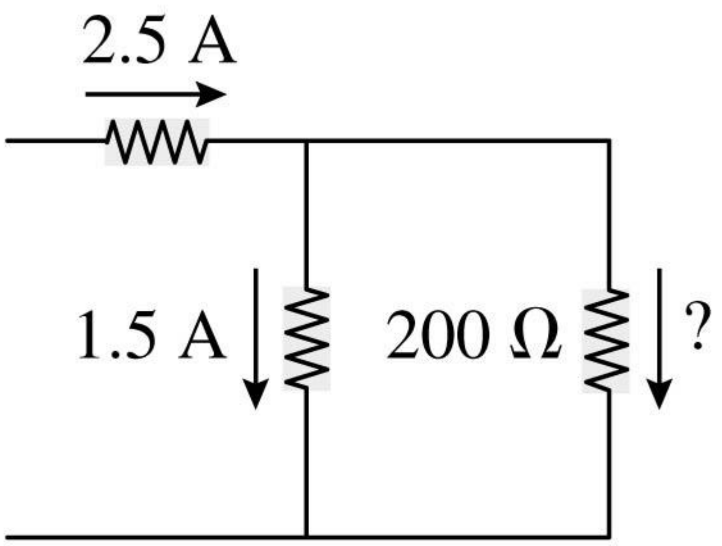
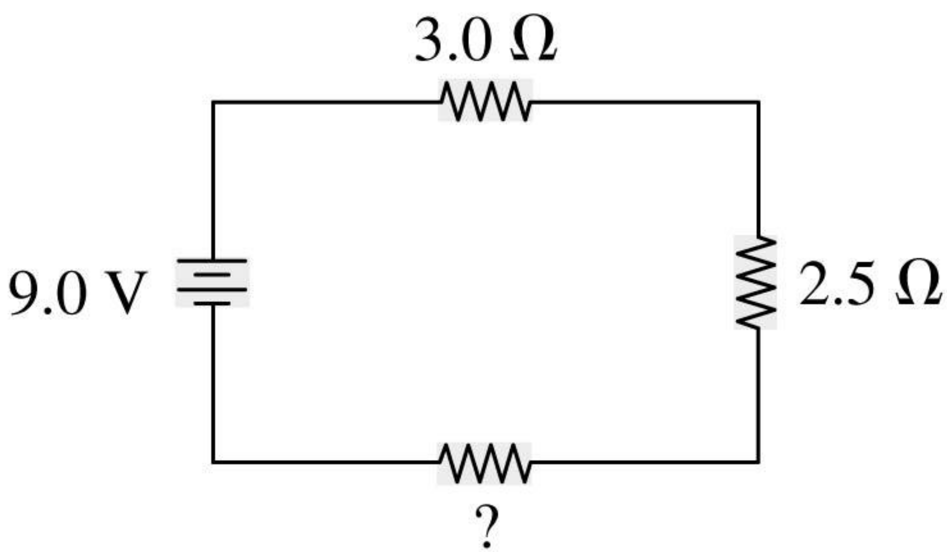


Example Problem

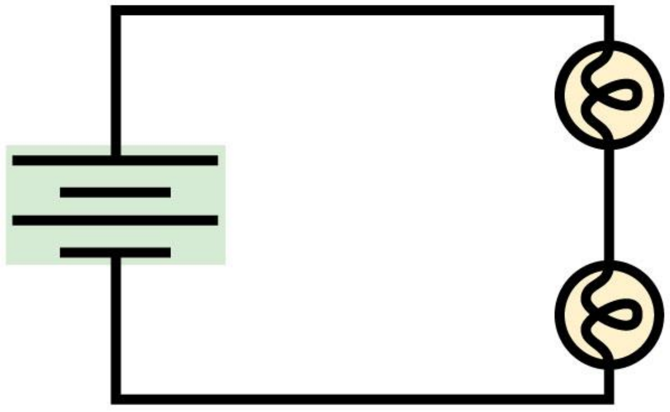
There is a current of 1.0 A in the following circuit. What is the resistance of the unknown circuit element?



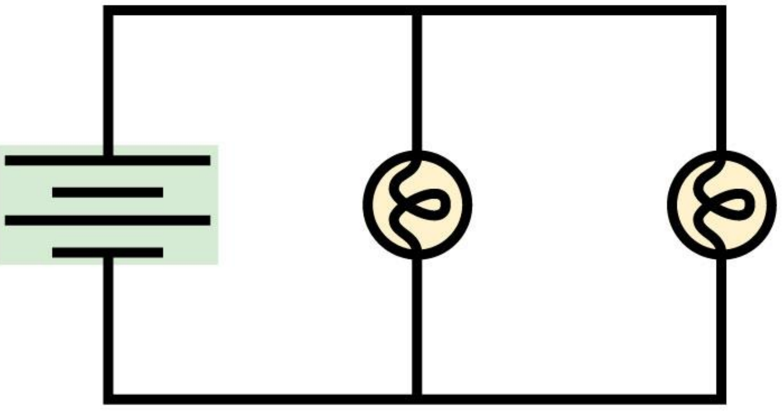
Section 23.3 Series and Parallel Circuits

Series and Parallel Circuits

- There are two possible ways that you can connect the circuit.
- *Series* and *parallel* circuits have very different properties.
- We say two bulbs are connected in **series** if they are connected directly to each other with no junction in between.

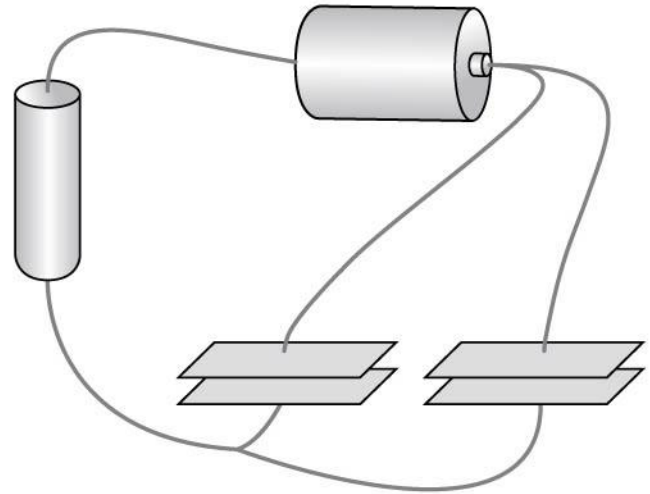


Series



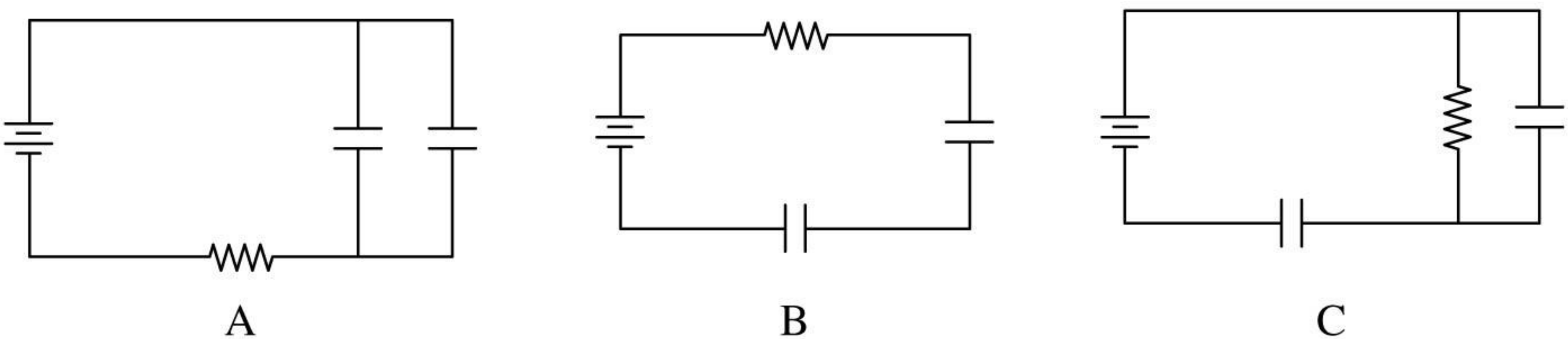
Parallel

QuickCheck 23.4



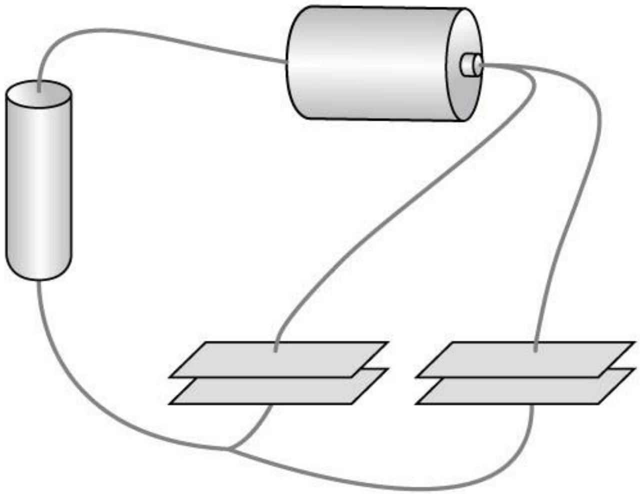
The circuit shown has a battery, two capacitors, and a resistor.

Which of the following circuit diagrams is the best representation of the circuit shown?



QuickCheck 23.4

The circuit shown has a battery, two capacitors, and a resistor.



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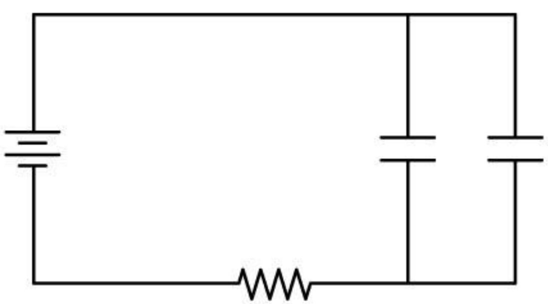
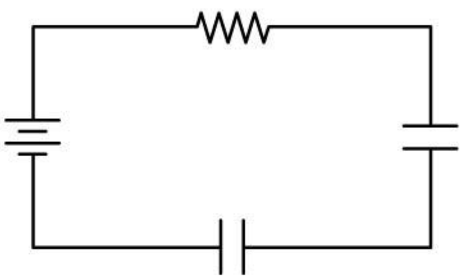
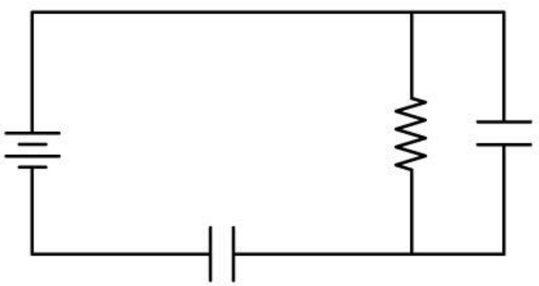


✔ A



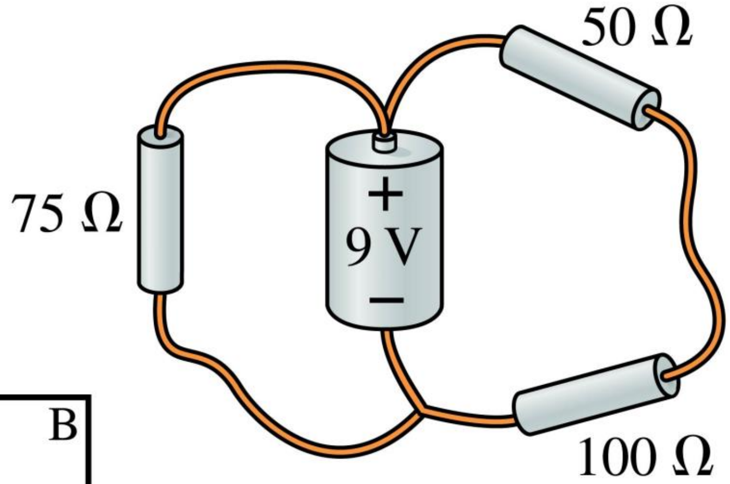
B

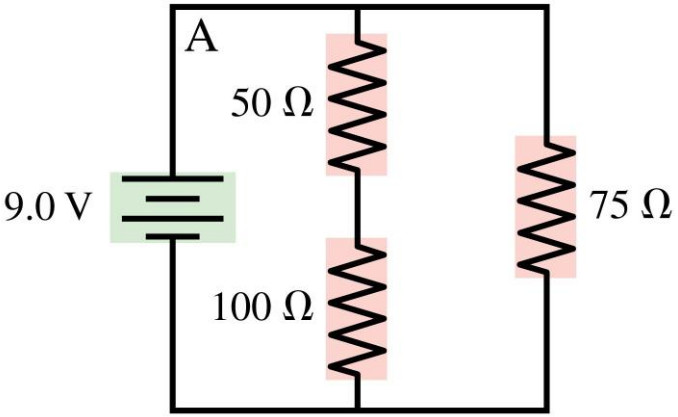


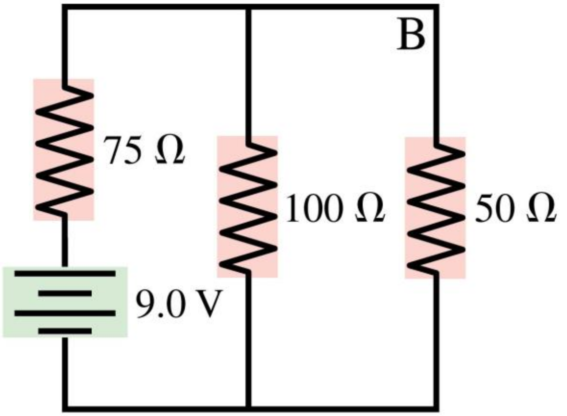
C

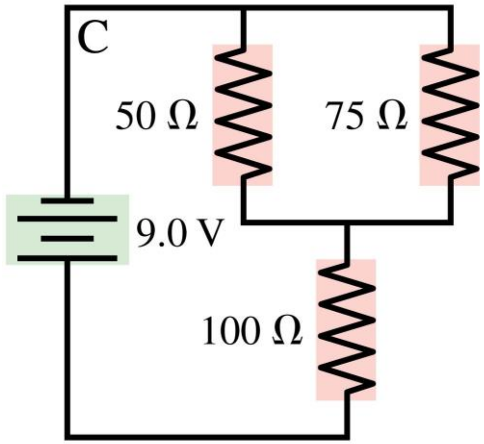
QuickCheck 23.5

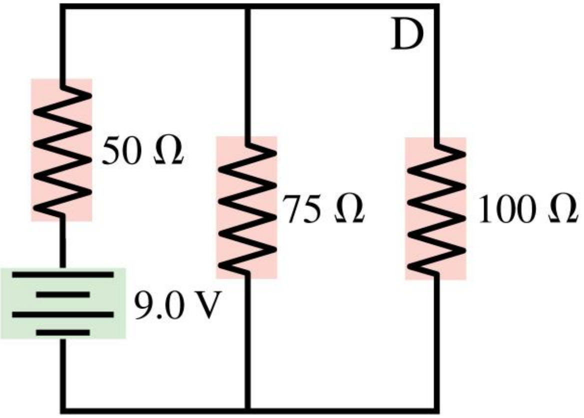
Which is the correct circuit diagram for the circuit shown?











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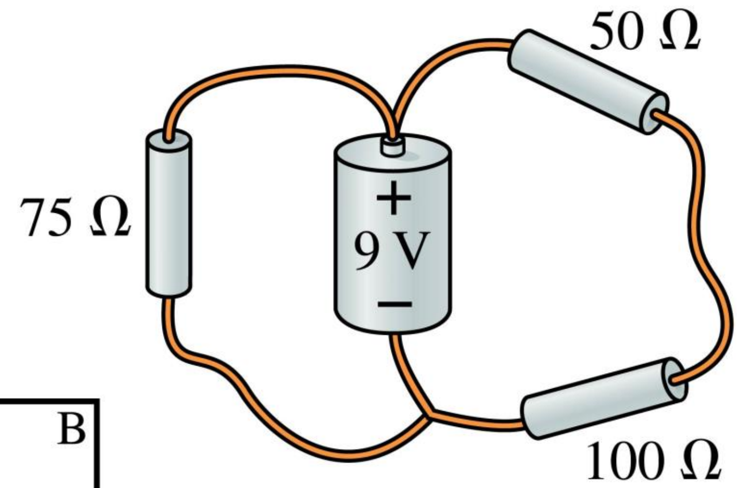
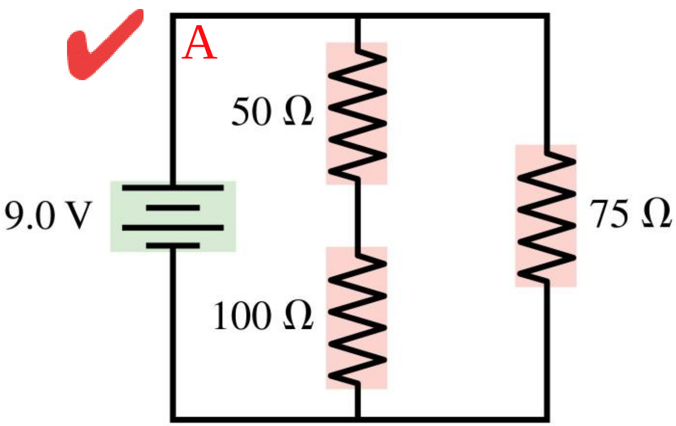
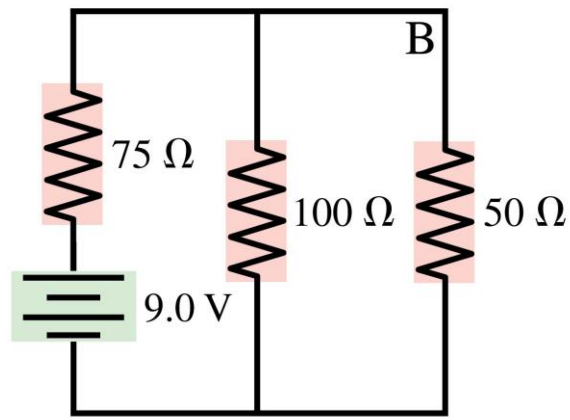
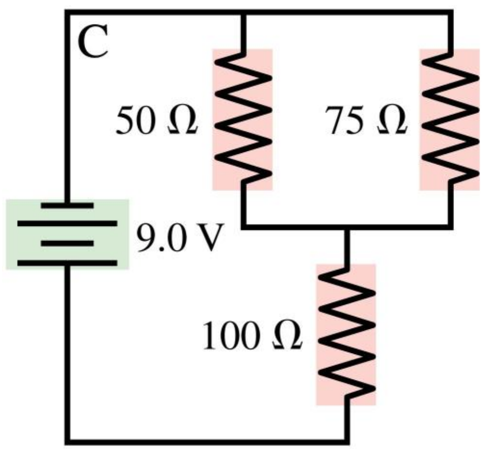
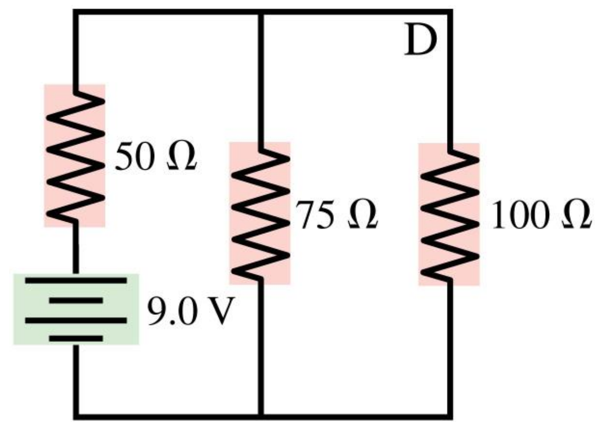




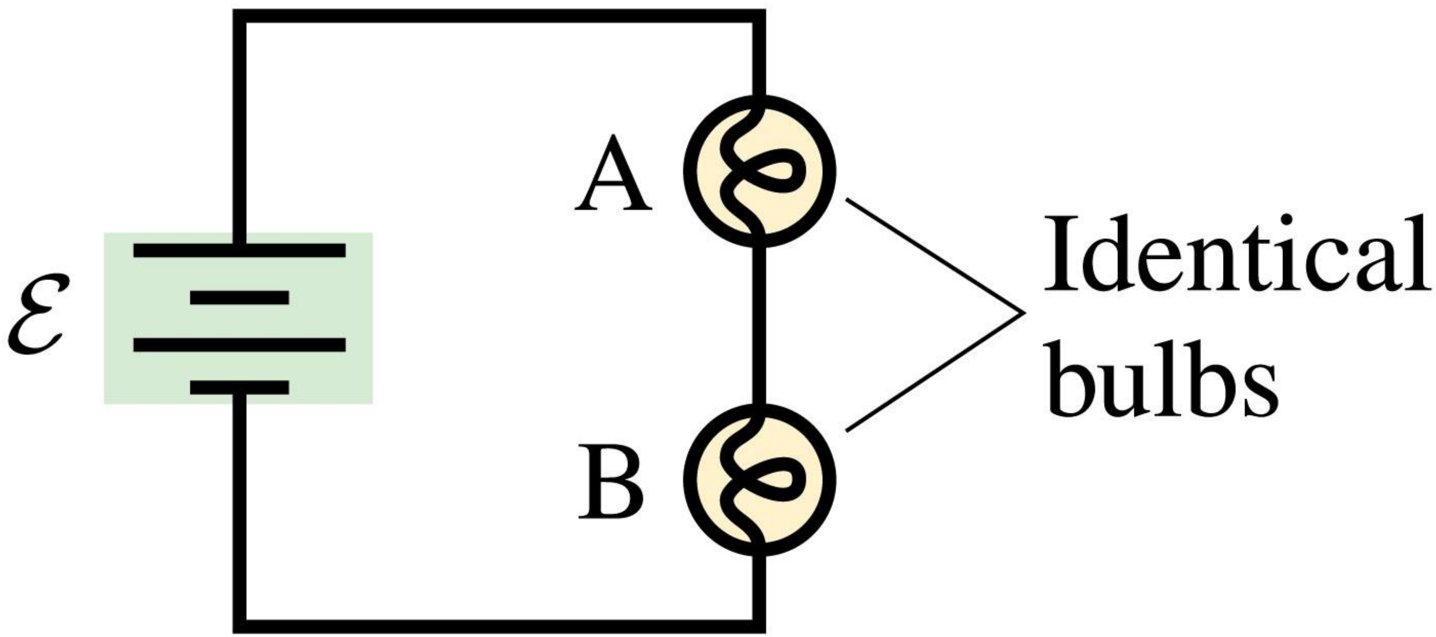






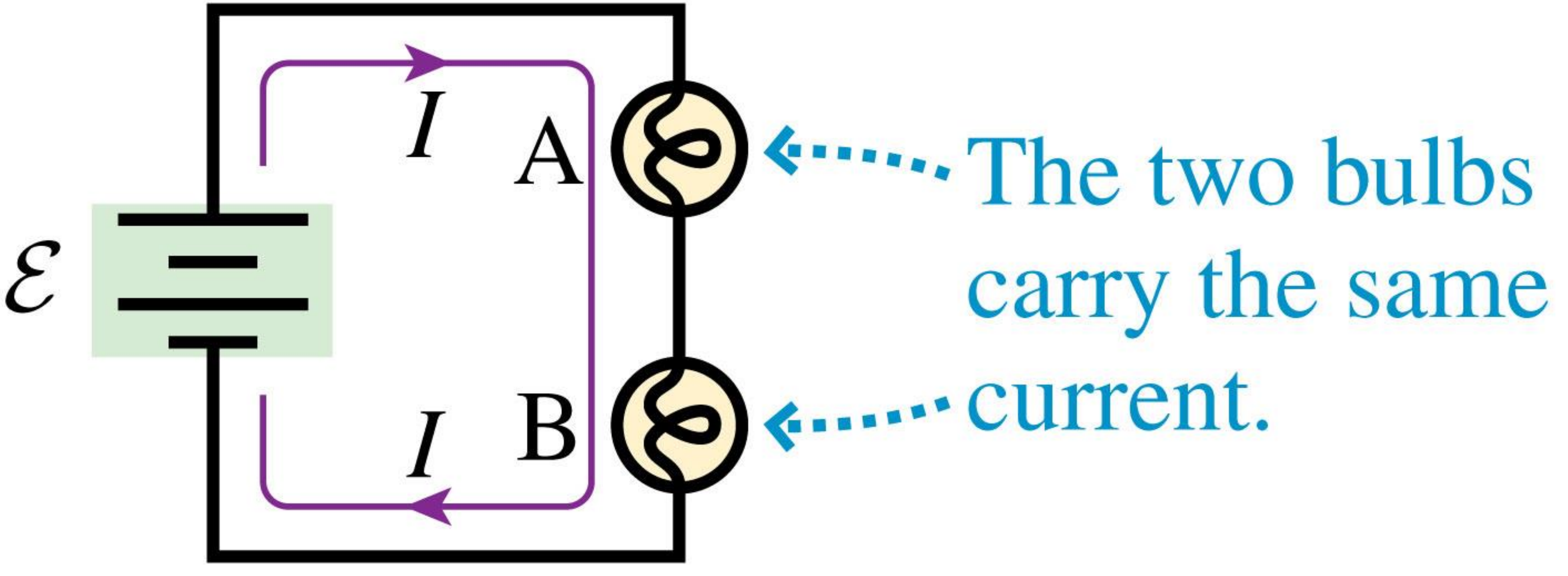
Example 23.2 Brightness of bulbs in series

FIGURE 23.11 shows two identical lightbulbs connected in series. Which bulb is brighter: A or B? Or are they equally bright?



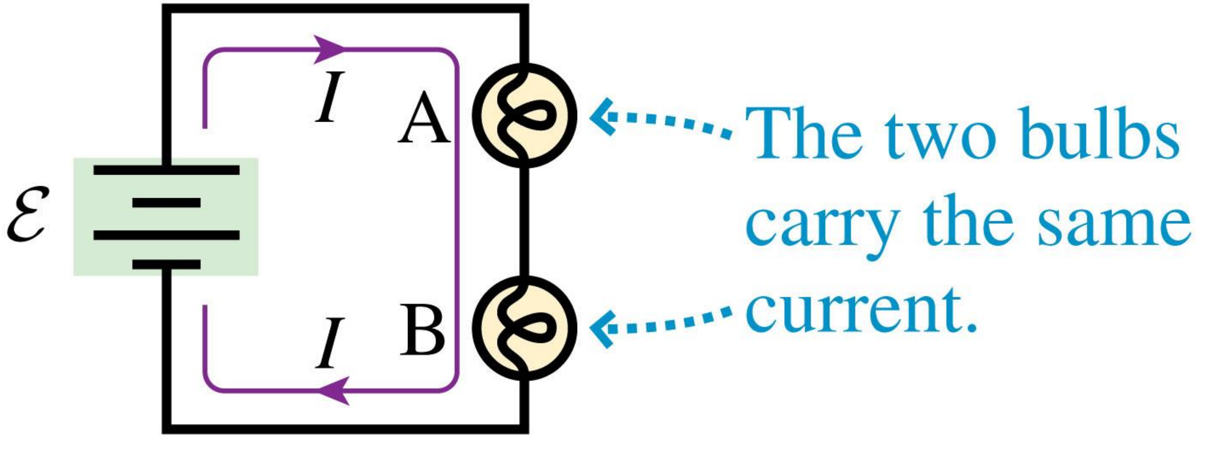
Example 23.2 Brightness of bulbs in series (cont.)

REASON Current is conserved, and there are no junctions in the circuit. Thus, as **FIGURE 23.12** shows, the current is the same at all points.



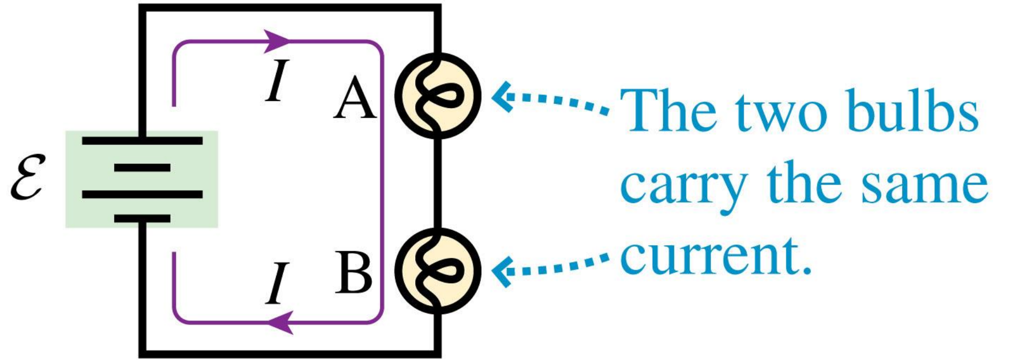
Example 23.2 Brightness of bulbs in series (cont.)

We learned in ◀**SECTION 22.6** that the power dissipated by a resistor is $P = I^2R$. If the two bulbs are identical (i.e., the same resistance) and have the same current through them, the power dissipated by each bulb is the same. This means that the brightness of the bulbs must be the same. The voltage across each of the bulbs will be the same as well because $\Delta V = IR$.



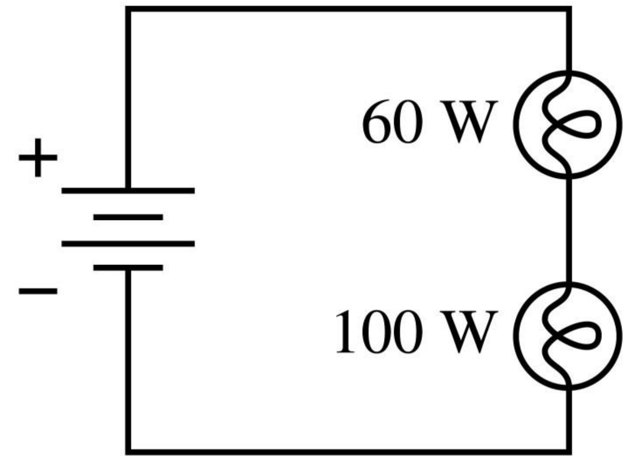
Example 23.2 Brightness of bulbs in series (cont.)

ASSESS It's perhaps tempting to think that bulb A will be brighter than bulb B, thinking that something is "used up" before the current gets to bulb B. It is true that *energy* is being transformed in each bulb, but current must be conserved and so both bulbs dissipate energy at the same rate. We can extend this logic to a special case: If one bulb burns out, and no longer lights, the second bulb will go dark as well. If one bulb can no longer carry a current, neither can the other.



QuickCheck 23.12

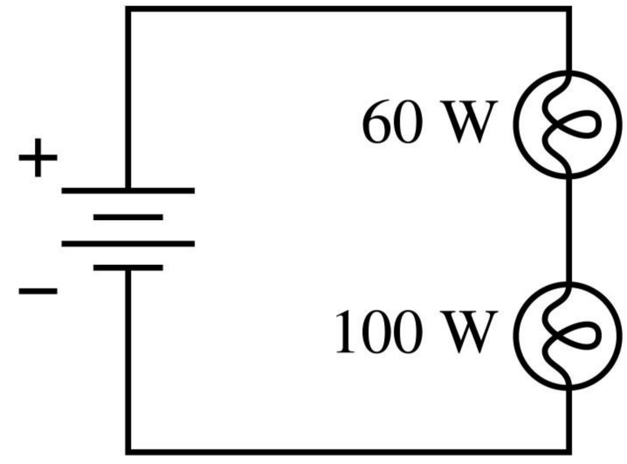
Which bulb is brighter?



A. The 60 W bulb.
B. The 100 W bulb.
C. Their brightnesses are the same.
D. There's not enough information to tell.

QuickCheck 23.12

Which bulb is brighter?

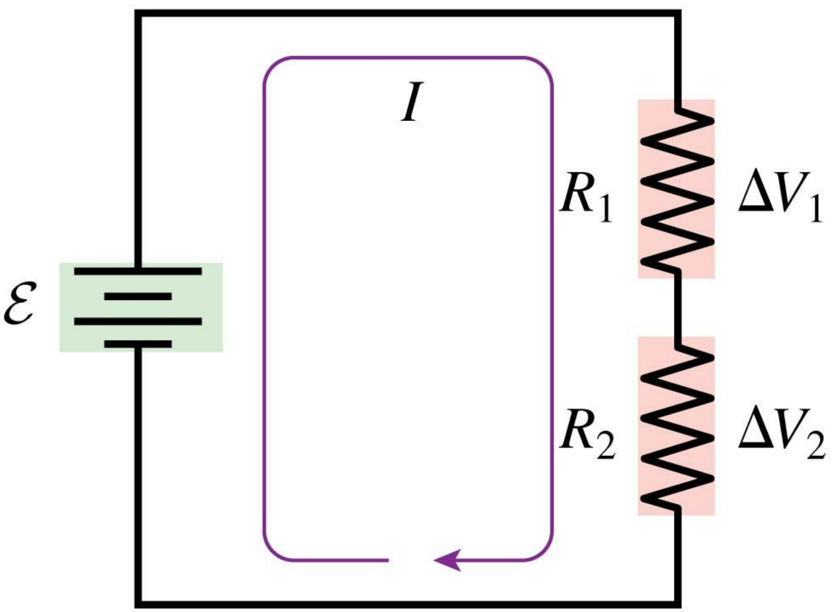


A. The 60 W bulb.
B. The 100 W bulb.
C. Their brightnesses are the same.
D. There's not enough information to tell.

$P = I^2R$ and both have the same current.

Series Resistors

- This figure shows two resistors in series connected to a battery.
- Because there are no junctions, the current *I* must be the same in both resistors.

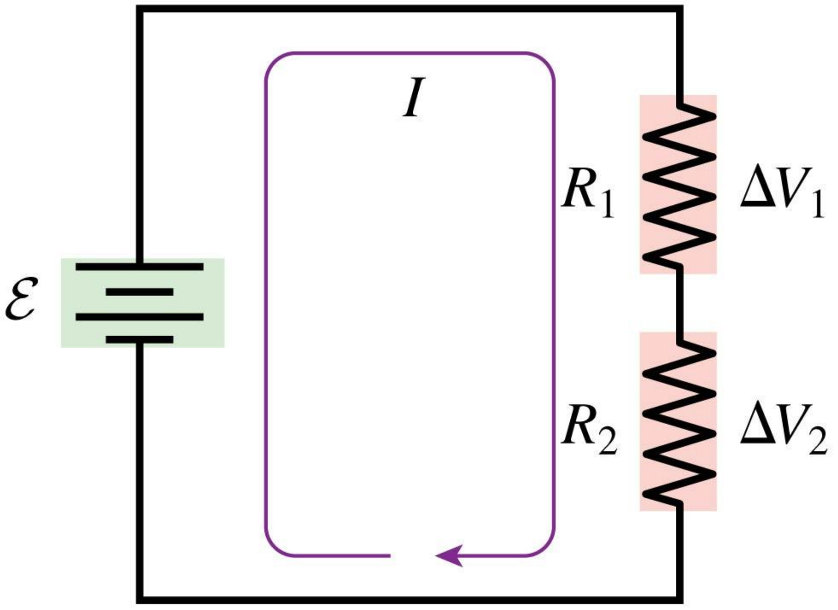


Series Resistors

- We use Kirchhoff's loop law to look at the potential differences.

$$\sum_i \Delta V_i = \mathcal{E} + \Delta V_1 + \Delta V_2 = 0$$

(a) Two resistors in series



Series Resistors

- The voltage drops across the two resistors, in the direction of the current, are $\Delta V_1 = -IR_1$ and $\Delta V_2 = -IR_2$.
- We solve for the current in the circuit:

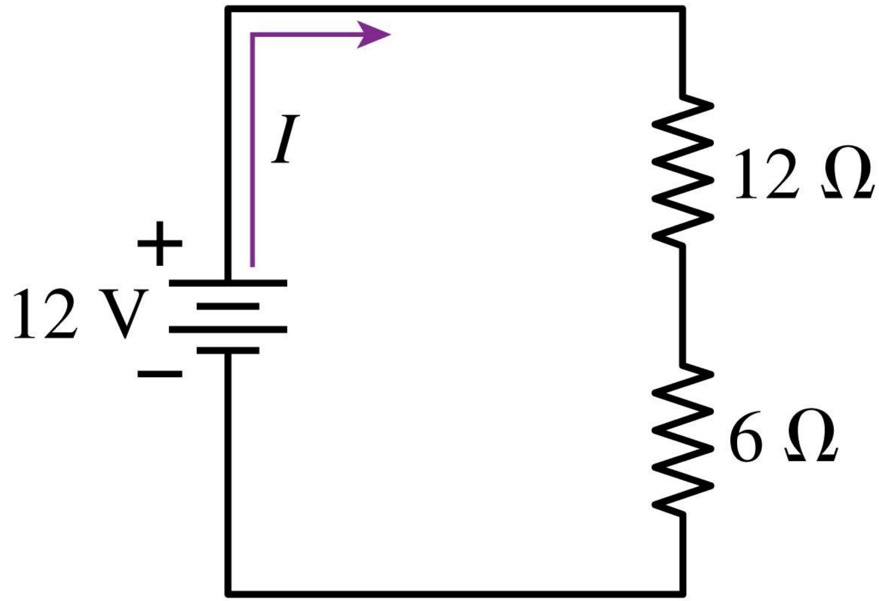
$$\mathcal{E} = -\Delta V_1 - \Delta V_2 = IR_1 + IR_2$$

$$I = \frac{\mathcal{E}}{R_1 + R_2}$$

QuickCheck 23.13

The battery current *I* is

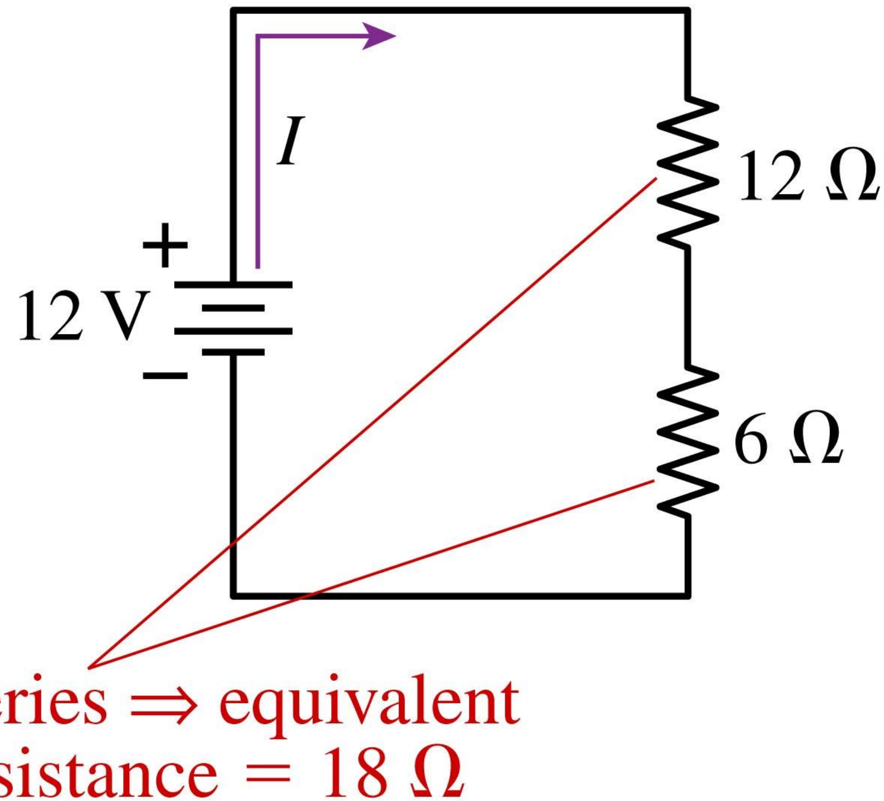
A. 3 A
B. 2 A
C. 1 A
D. 2/3 A
E. 1/2 A



QuickCheck 23.13

The battery current *I* is

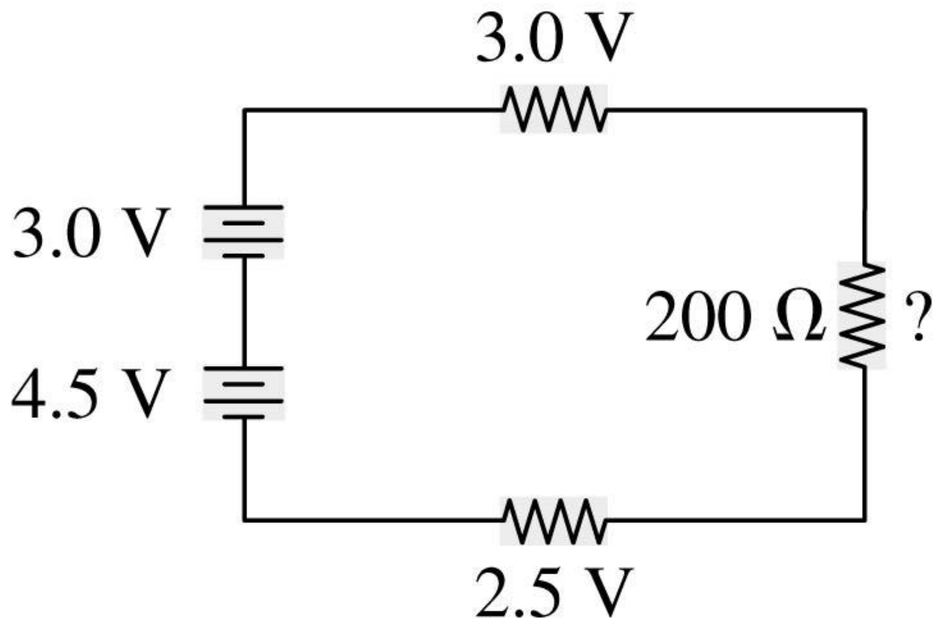
A. 3 A
B. 2 A
C. 1 A
D. 2/3 A ✔
E. 1/2 A



QuickCheck 23.9

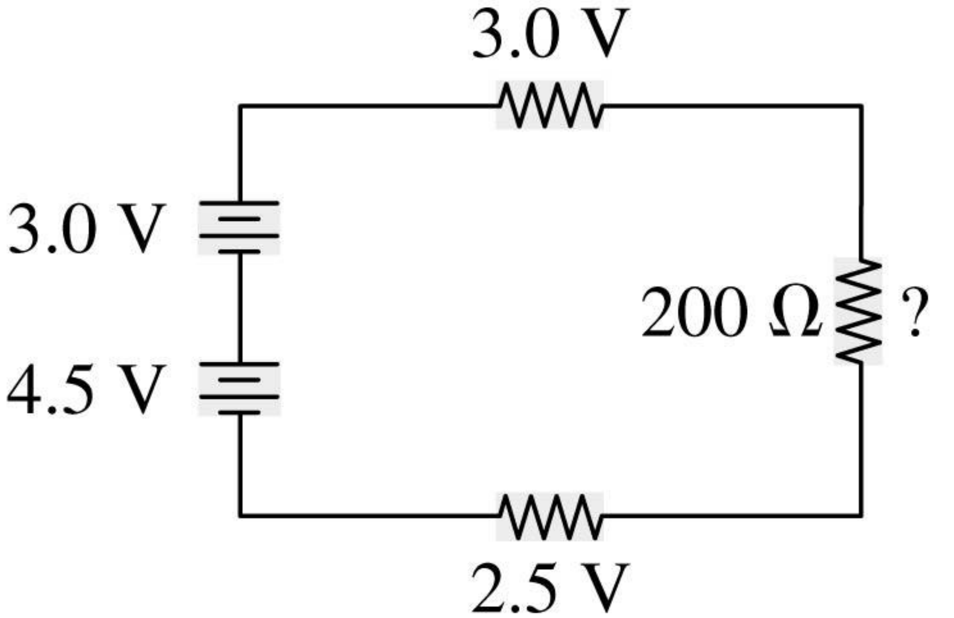
The diagram below shows a circuit with two batteries and three resistors. What is the potential difference across the 200 Ω resistor?

A. 2.0 V
B. 3.0 V
C. 4.5 V
D. 7.5 V
E. There is not enough information to decide.



QuickCheck 23.9

The diagram below shows a circuit with two batteries and three resistors. What is the potential difference across the 200 Ω resistor?



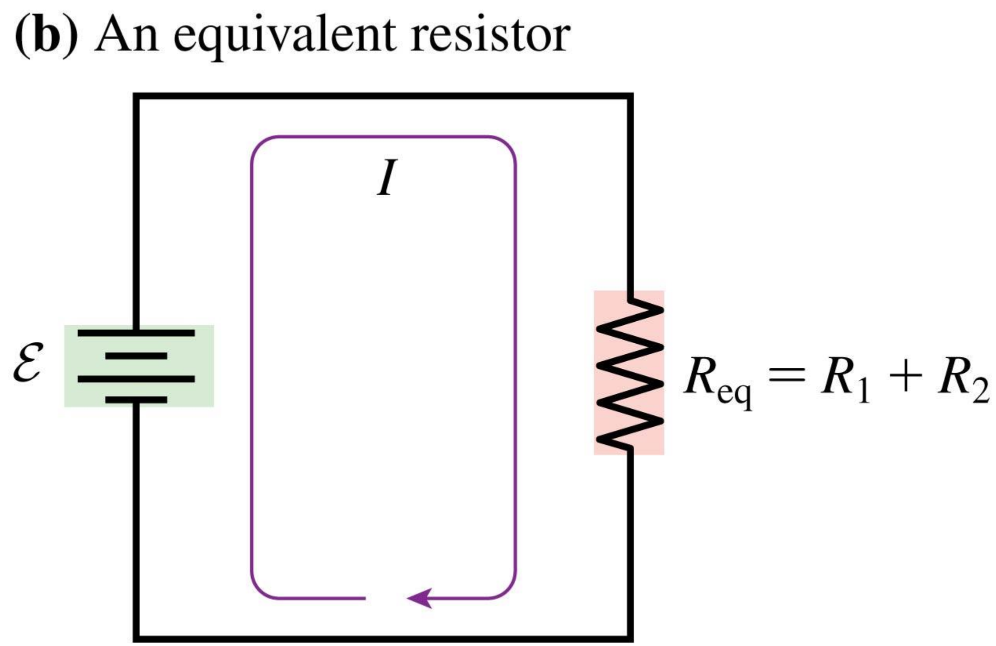
✔ A. 2.0 V
B. 3.0 V
C. 4.5 V
D. 7.5 V
E. There is not enough information to decide.

Series Resistors

- If we replace two resistors with a single resistor having the value

$$R_{eq} = R_1 + R_2$$

the total potential difference across this resistor is still $\mathcal{E}$ because the potential difference is established by the battery.



Series Resistors

- The current in the single resistor circuit is:

$$I = \frac{\mathcal{E}}{R_{eq}} = \frac{\mathcal{E}}{R_1 + R_2}$$

- The single resistor is *equivalent* to the two series resistors in the sense that the circuit's current and potential difference are the same in both cases.
- If we have N resistors in series, their **equivalent resistance** is the sum of the N individual resistances:

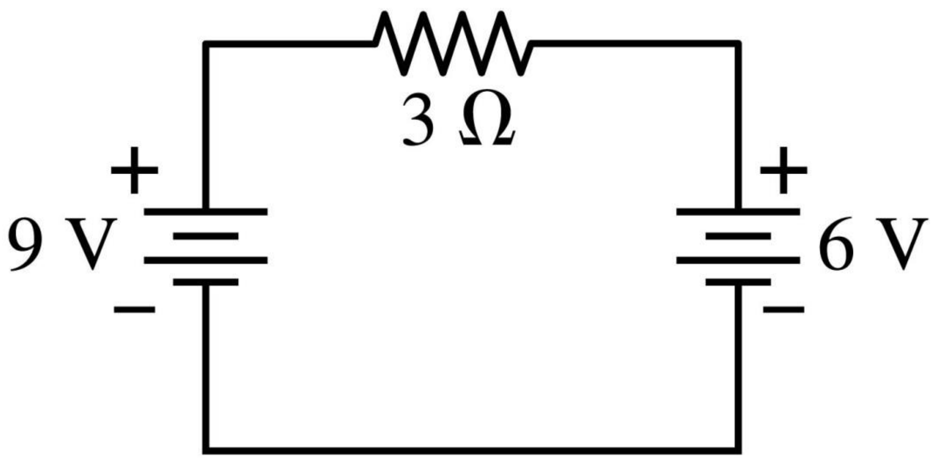
$$R_{eq} = R_1 + R_2 + \cdots + R_N$$

Equivalent resistance of N series resistors

QuickCheck 23.7

The current through the 3 Ω resistor is

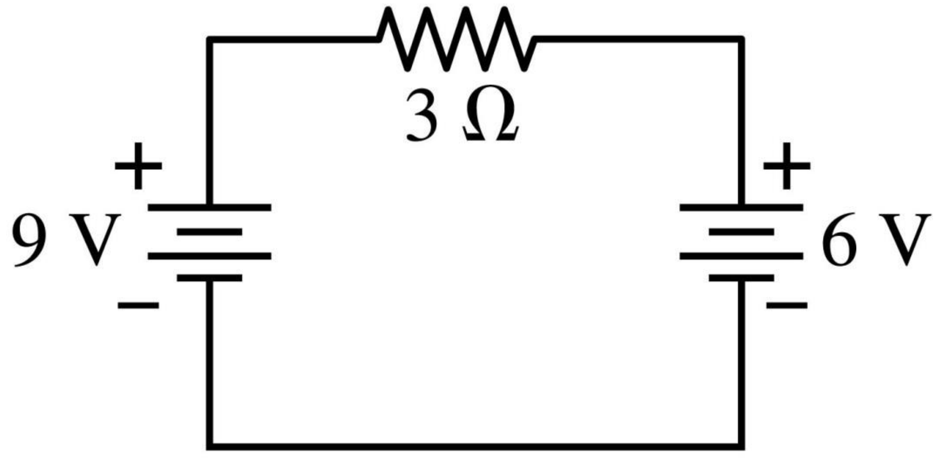
A. 9 A
B. 6 A
C. 5 A
D. 3 A
E. 1 A



QuickCheck 23.7

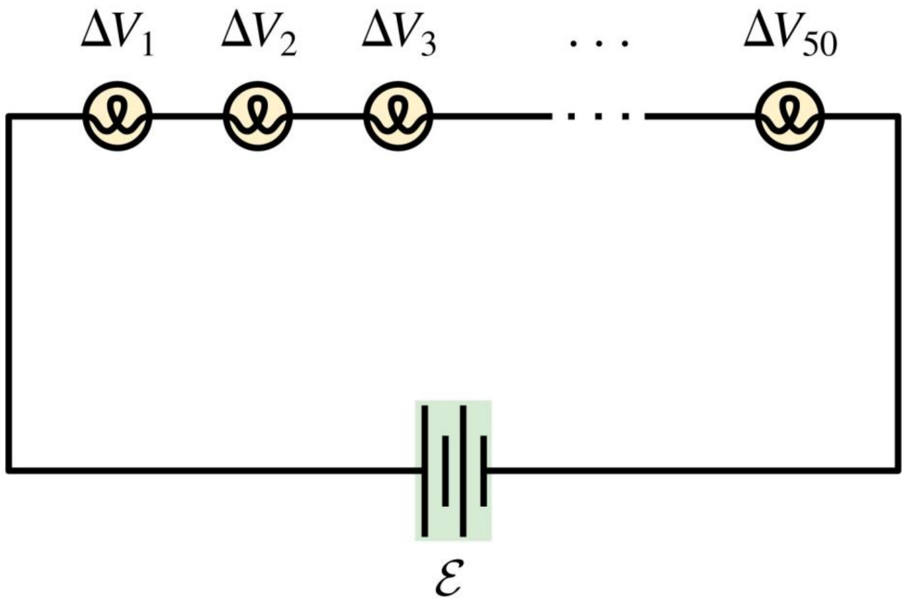
The current through the 3 Ω resistor is

A. 9 A
B. 6 A
C. 5 A
D. 3 A
✔ E. 1 A



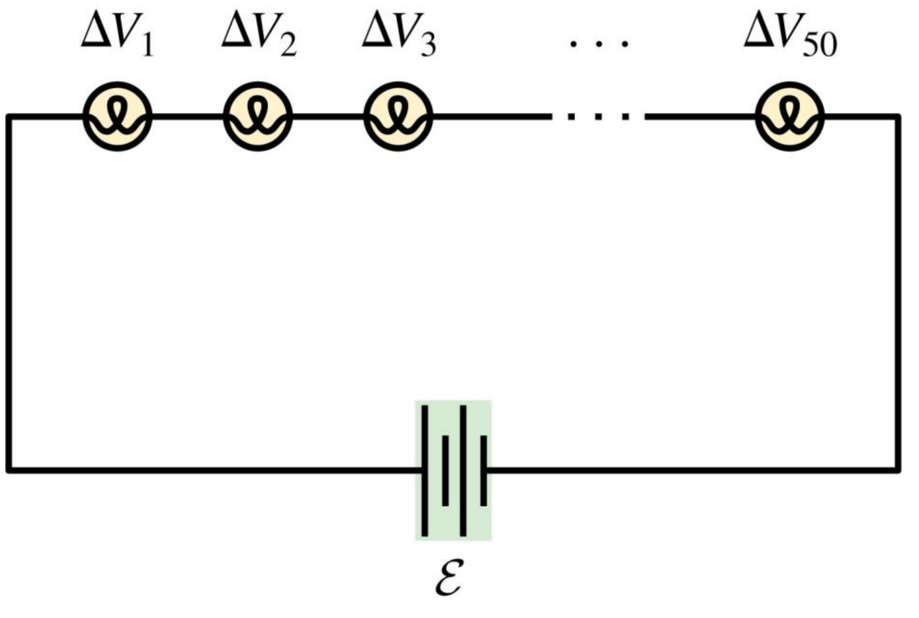
Example 23.3 Potential difference of Christmas-tree minilights

A string of Christmas-tree minilights consists of 50 bulbs wired in series. What is the potential difference across each bulb when the string is plugged into a 120 V outlet?



Example 23.3 Potential difference of Christmas-tree minilights (cont.)

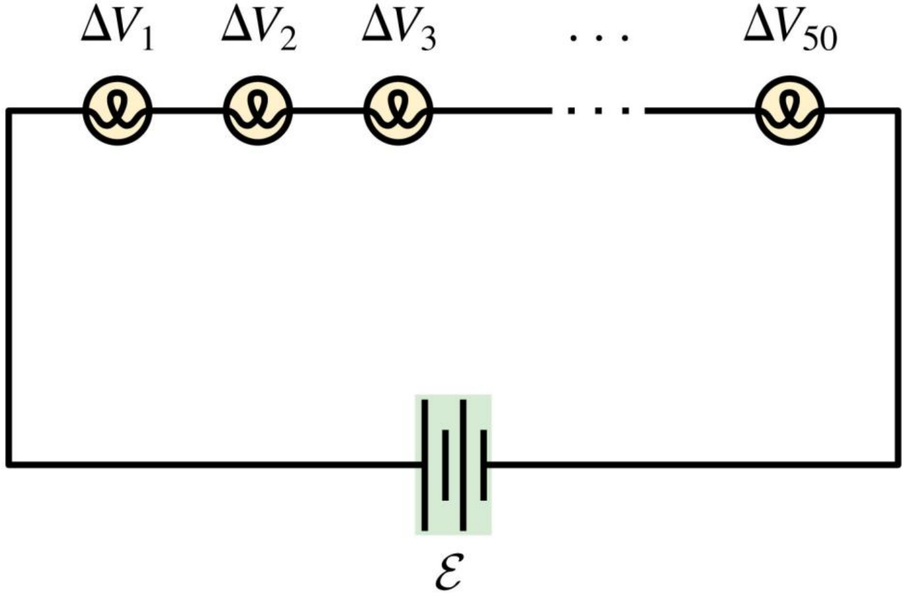
PREPARE **FIGURE 23.14** shows the minilight circuit, which has 50 bulbs in series. The current in each of the bulbs is the same because they are in series.



Example 23.3 Potential difference of Christmas-tree minilights (cont.)

SOLVE Applying Kirchhoff's loop law around the circuit, we find

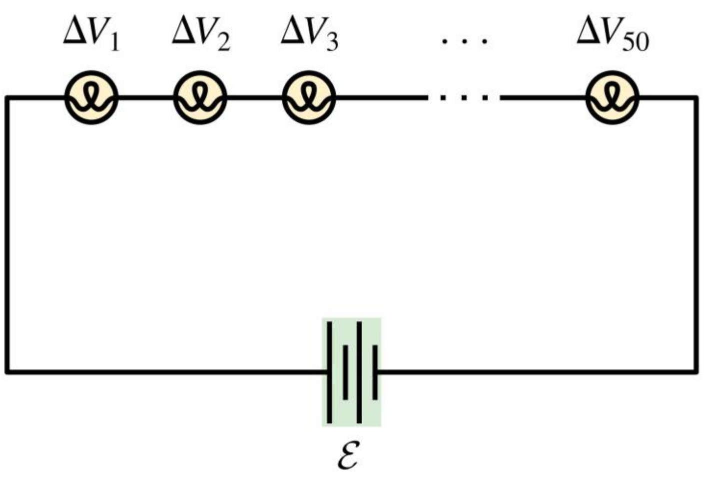
$$\mathcal{E} = \Delta V_1 + \Delta V_2 + \cdots + \Delta V_{50}$$



Example 23.3 Potential difference of Christmas-tree minilights (cont.)

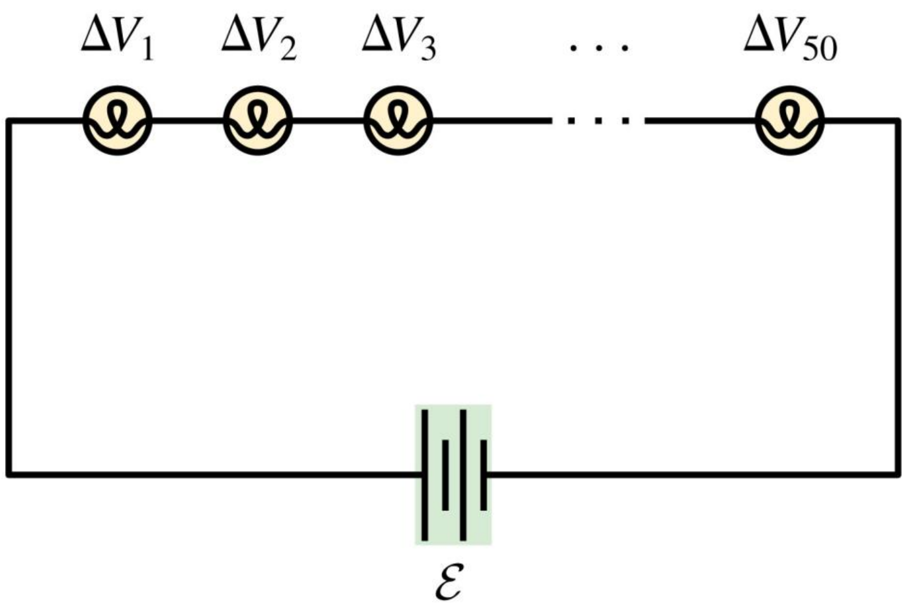
The bulbs are all identical and, because the current in the bulbs is the same, all of the bulbs have the same potential difference. The potential difference across a single bulb is thus

$$\Delta V_1 = \frac{\mathcal{E}}{50} = \frac{120\ \text{V}}{50} = 2.4\ \text{V}$$



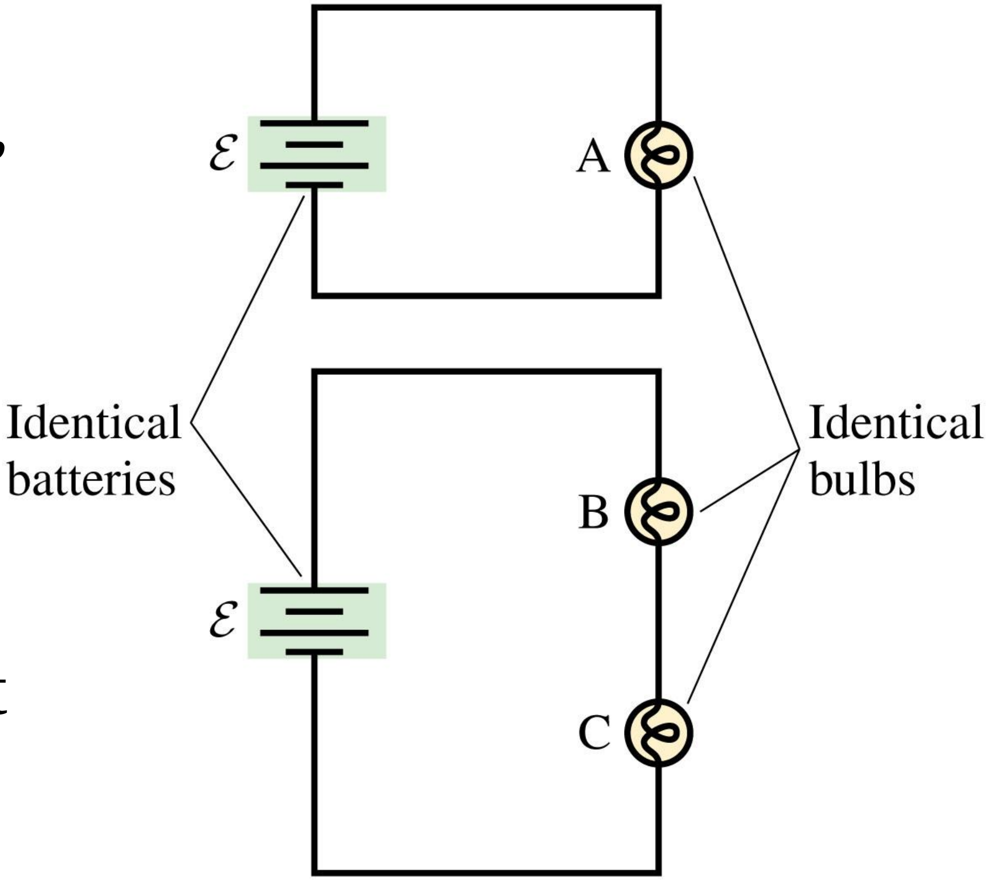
Example 23.3 Potential difference of Christmas-tree minilights (cont.)

ASSESS This result seems reasonable. The potential difference is “shared” by the bulbs in the circuit. Since the potential difference is shared among 50 bulbs, the potential difference across each bulb will be quite small.



Series Resistors

- We compare two circuits: one with a single lightbulb, and the other with two lightbulbs connected in series. All of the batteries and bulbs are identical.
- How does the brightness of the bulbs in the different circuits compare?



Series Resistors

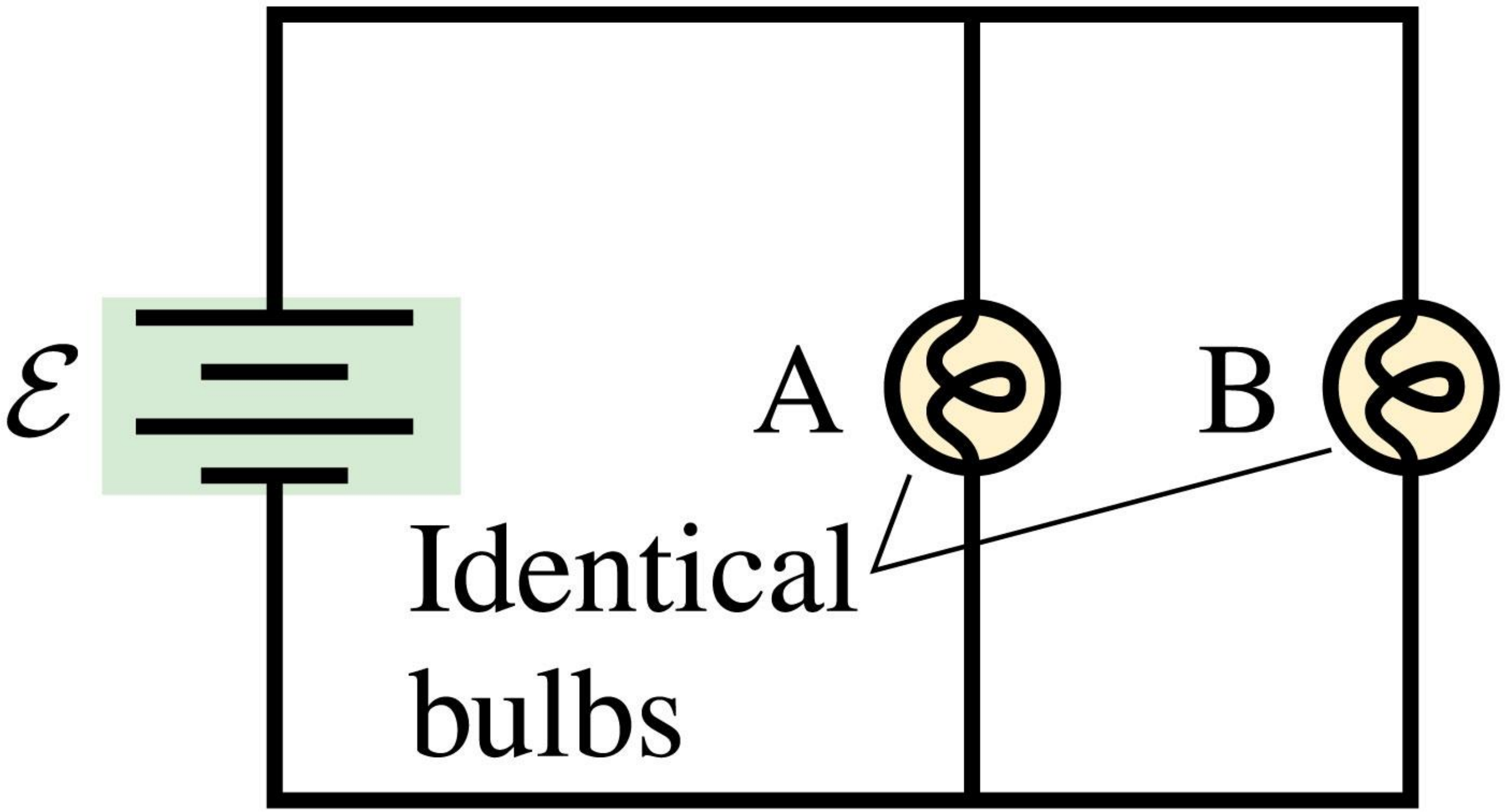
- In a circuit with one bulb, circuit A, a battery drives the current $I_A = \mathcal{E}/R$ through the bulb.
- In a circuit, with two bulbs (in series) with the same resistance R, circuit B, the equivalent resistance is $R_{eq} = 2R$.
- The current running through the bulbs in the circuit B is $I_B = \mathcal{E}/2R$.
- Since the emf from the battery and the resistors are the same in each circuit, $I_B = ½ I_A$.
- The two bulbs in circuit B are equally bright, but they are dimmer than the bulb in circuit A because there is less current.

Series Resistors

- **A battery is a source of potential difference, *not* a source of current.**
- The battery does provide the current in a circuit, but the *amount* of current depends on the resistance.
- **The amount of current depends jointly on the battery's emf *and* the resistance of the circuit attached to the battery.**

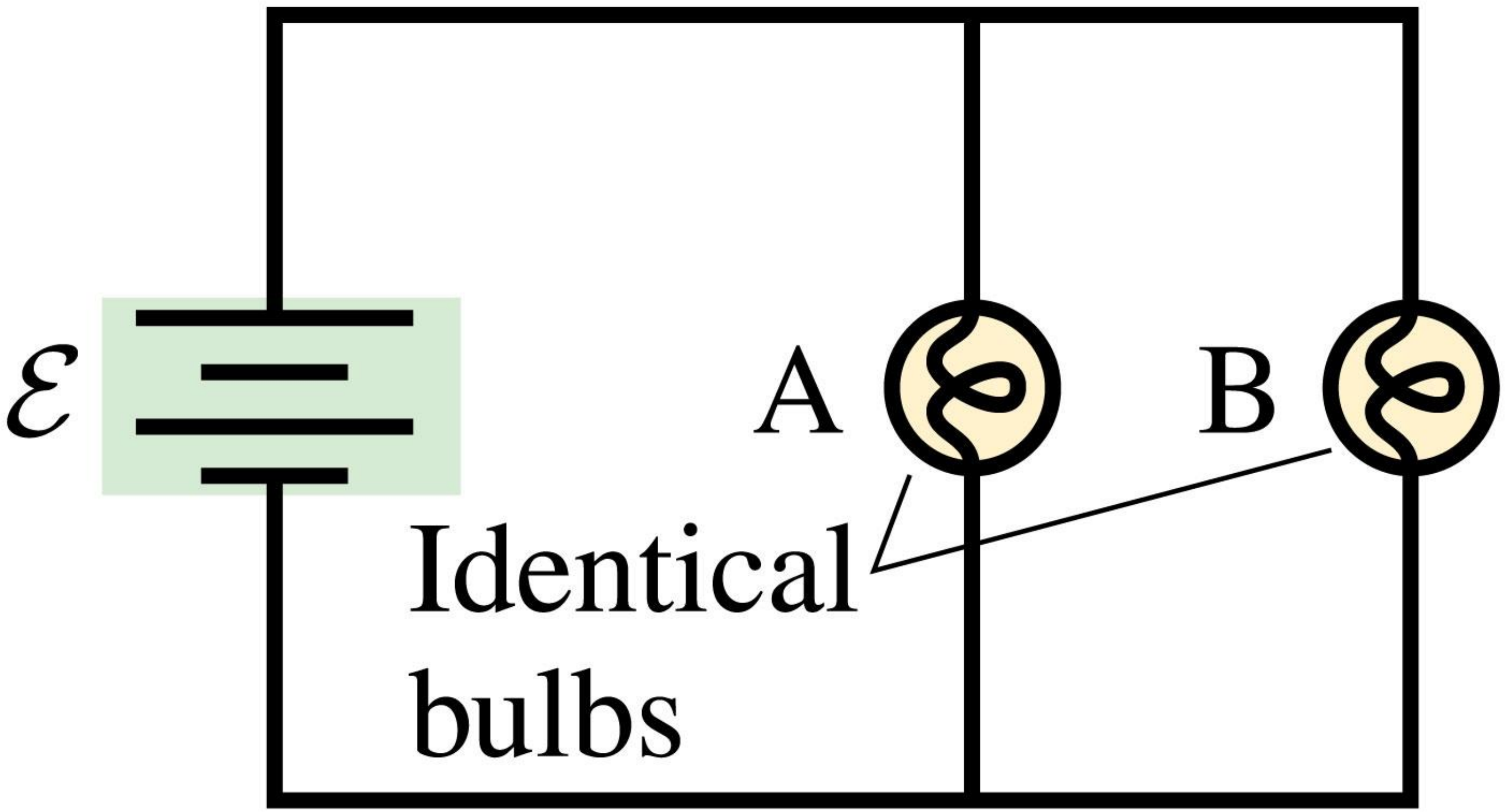
Parallel Resistors

- In a circuit where two bulbs are connected at *both* ends, we say that they are connected in **parallel**.

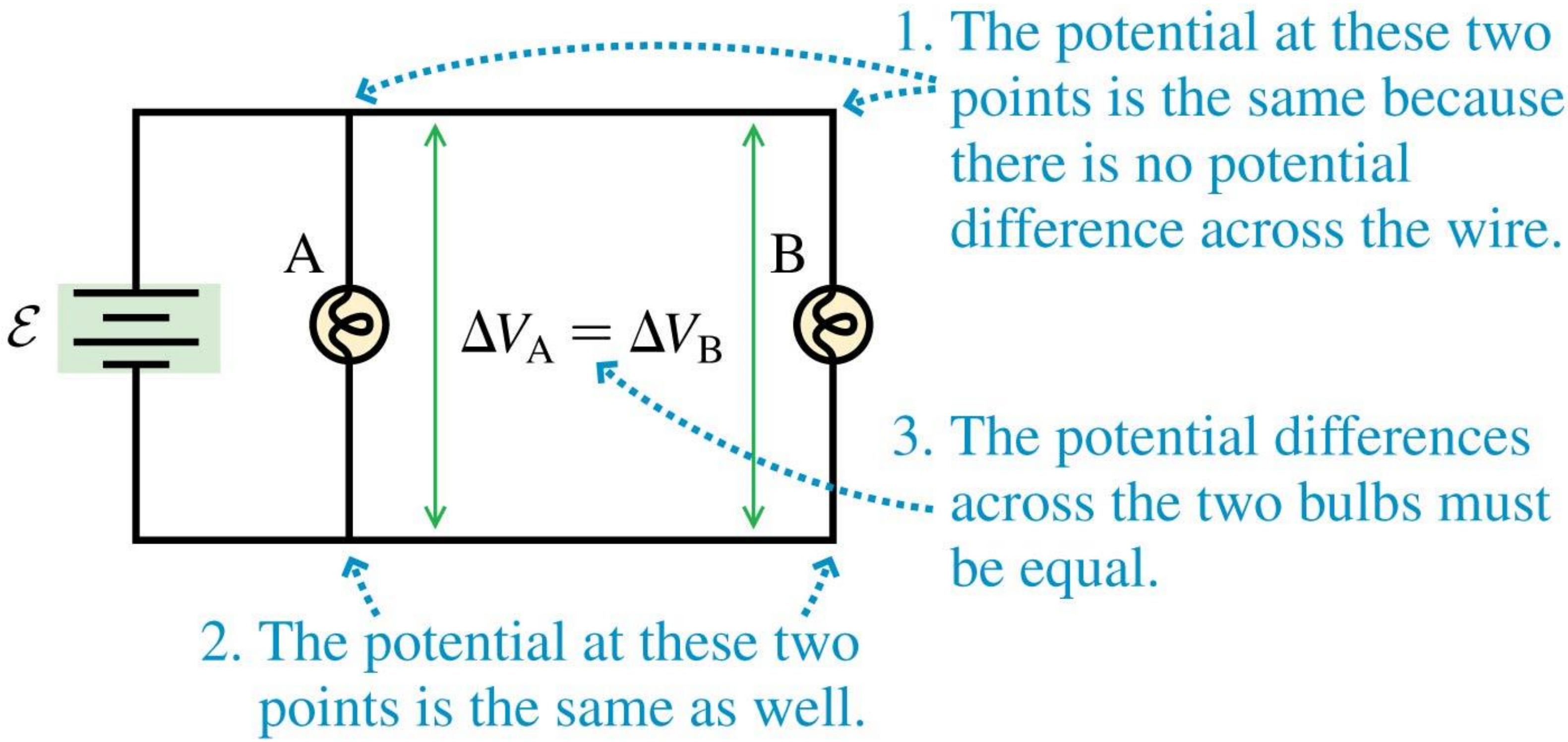


Conceptual Example 23.5 Brightness of bulbs in parallel

Which lightbulb in the circuit of **FIGURE 23.18** is brighter: A or B? Or are they equally bright?



Conceptual Example 23.5 Brightness of bulbs in parallel (cont.)

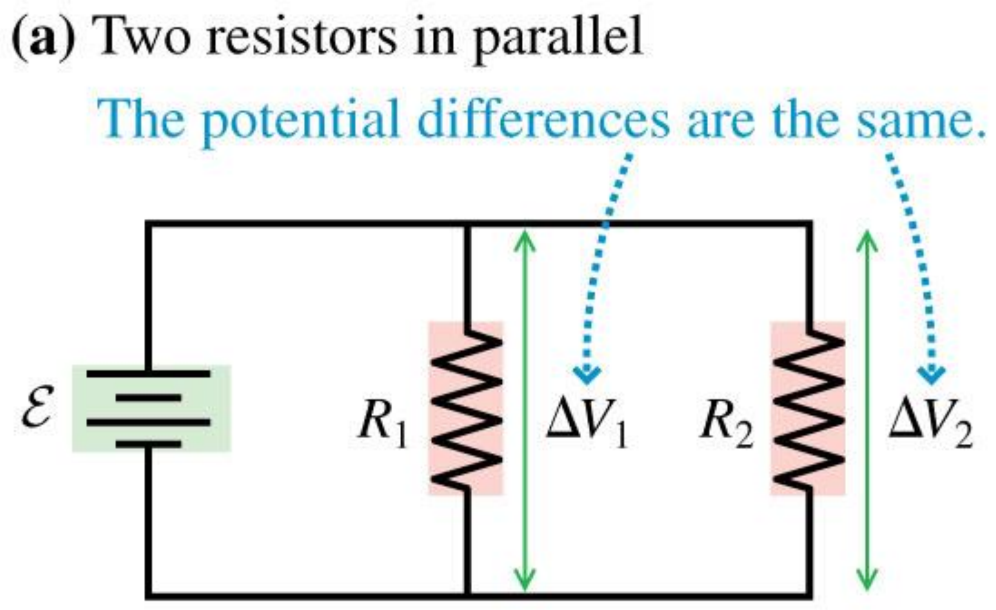


Because the bulbs are identical, the currents through the two bulbs are equal and thus the bulbs are equally bright.

Conceptual Example 23.5 Brightness of bulbs in parallel (cont.)

ASSESS One might think that A would be brighter than B because current takes the “shortest route.” But current is determined by potential difference, and two bulbs connected in parallel have the same potential difference.

Parallel Resistors



- The potential difference across each resistor in parallel is equal to the emf of the battery because both resistors are connected directly to the battery.

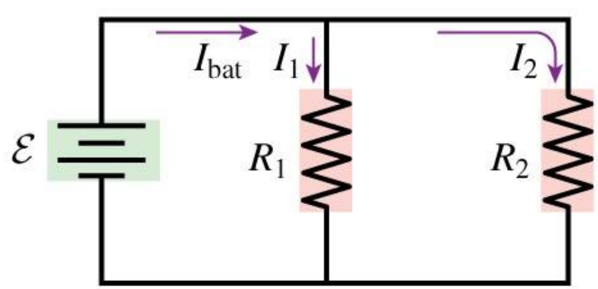
Parallel Resistors

- The current I_{bat} from the battery splits into currents I_1 and I_2 at the top of the junction.
- According to the junction law,

$$I_{bat} = I_1 + I_2$$

- Applying Ohm's law to each resistor, we find that the battery current is

$$I_{bat} = \frac{\Delta V_1}{R_1} + \frac{\Delta V_2}{R_2} = \frac{\mathcal{E}}{R_1} + \frac{\mathcal{E}}{R_2} = \mathcal{E}\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

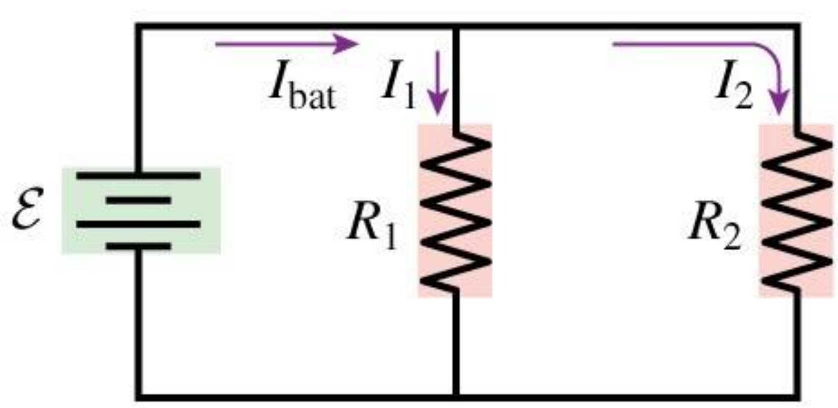


Parallel Resistors

- Can we replace a group of parallel resistors with a single equivalent resistor?
- To be equivalent, ΔV must equal $\mathcal{E}$ and I must equal I_{bat}:

$$R_{eq} = \frac{\Delta V}{I} = \frac{\mathcal{E}}{I_{bat}} = \left(\frac{1}{R_1} + \frac{1}{R_2}\right)^{-1}$$

- This is the *equivalent resistance*, so a single R_{eq} acts exactly the same as multiple resistors.



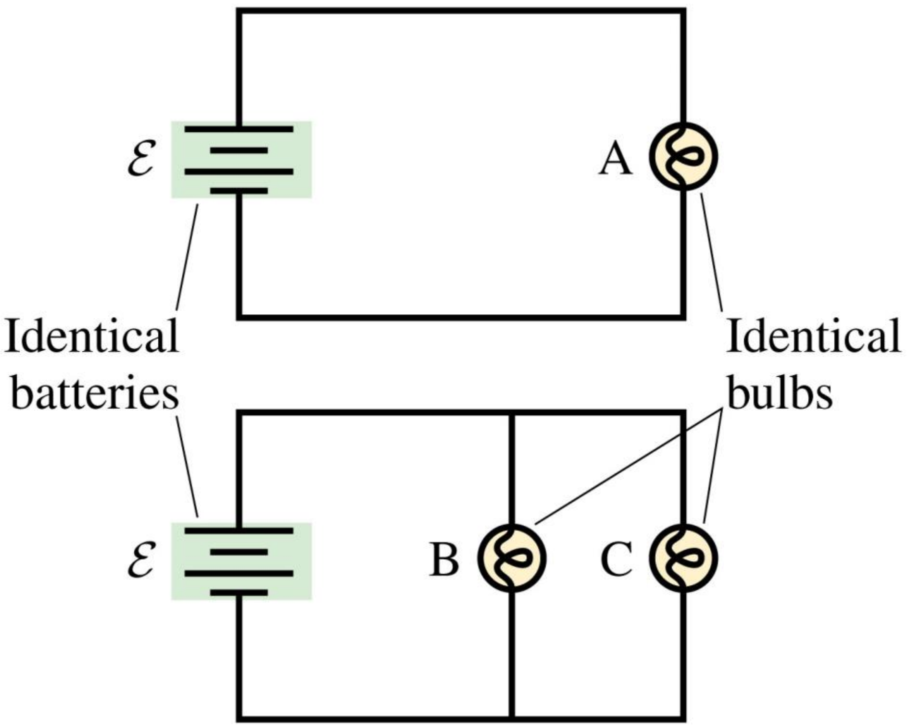
Parallel Resistors

$$R_{eq} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_N}\right)^{-1}$$

Equivalent resistance of *N* parallel resistors

Parallel Resistors

- How does the brightness of bulb B compare to that of bulb A?
- Each bulb is connected to the same potential difference, that of the battery, so they all have the *same* brightness.

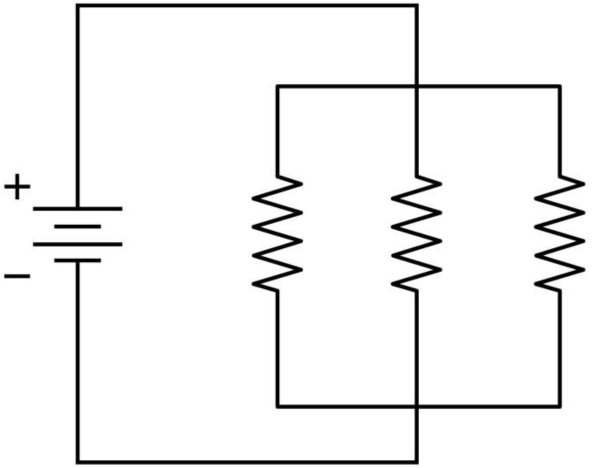


- In the second circuit, the battery must power two lightbulbs, so it provides twice as much current.

QuickCheck 23.10

What things about the resistors in this circuit are the same for all three?

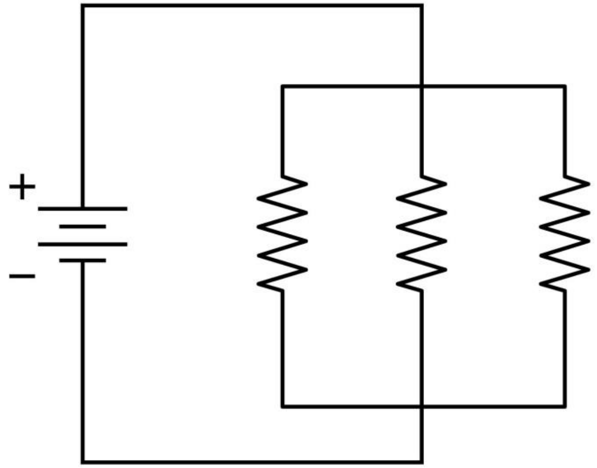
A. Current I
B. Potential difference ΔV
C. Resistance R
D. A and B
E. B and C



QuickCheck 23.10

What things about the resistors in this circuit are the same for all three?

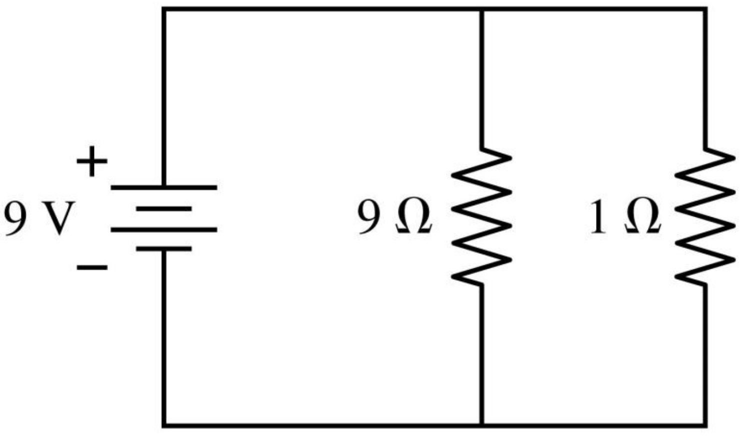
A. Current *I*
✔ B. Potential difference ΔV
C. Resistance *R*
D. A and B
E. B and C



QuickCheck 23.11

Which resistor dissipates more power?

A. The 9 Ω resistor
B. The 1 Ω resistor
C. They dissipate the same power



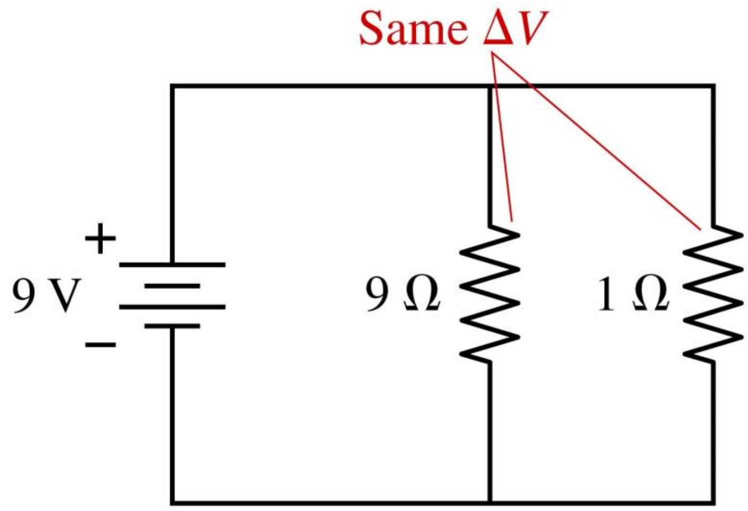
QuickCheck 23.11

Which resistor dissipates more power?

A. The 9 Ω resistor

✔ B. The 1 Ω resistor

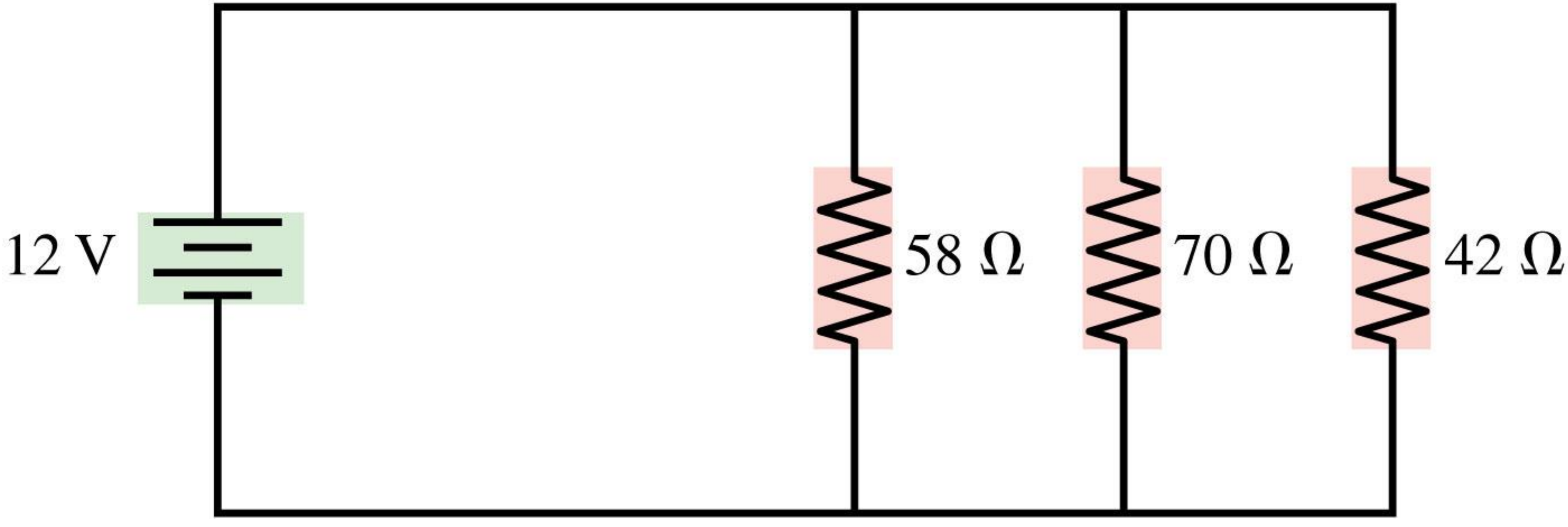
C. They dissipate the same power



$$P = \frac{(\Delta V)^2}{R}$$

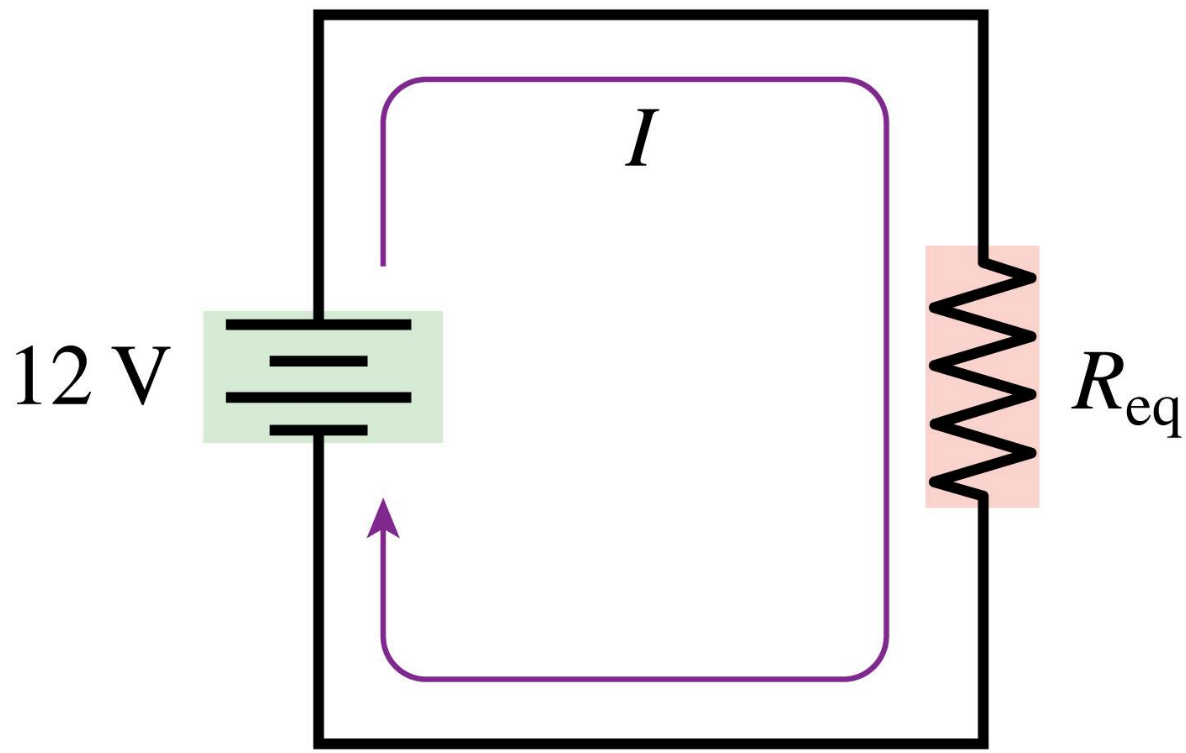
Example 23.6 Current in a parallel resistor circuit

The three resistors of **FIGURE 23.22** are connected to a 12 V battery. What current is provided by the battery?



Example 23.6 Current in a parallel resistor circuit (cont.)

PREPARE The three resistors are in parallel, so we can reduce them to a single equivalent resistor, as in **FIGURE 23.23**.



Example 23.6 Current in a parallel resistor circuit (cont.)

SOLVE We can use Equation 23.12 to calculate the equivalent resistance:

$$R_{eq} = \left(\frac{1}{58\ \Omega} + \frac{1}{70\ \Omega} + \frac{1}{42\ \Omega}\right)^{-1} = 18.1\ \Omega$$

Once we know the equivalent resistance, we can use Ohm's law to calculate the current leaving the battery:

$$I = \frac{\mathcal{E}}{R_{eq}} = \frac{12\ \text{V}}{18.1\ \Omega} = 0.66\ \text{A}$$

Example 23.6 Current in a parallel resistor circuit (cont.)

Because the battery can't tell the difference between the original three resistors and this single equivalent resistor, the battery in Figure 23.22 provides a current of 0.66 A to the circuit.

ASSESS As we'll see, the equivalent resistance of a group of parallel resistors is less than the resistance of any of the resistors in the group. 18 Ω is less than any of the individual values, a good check on our work.

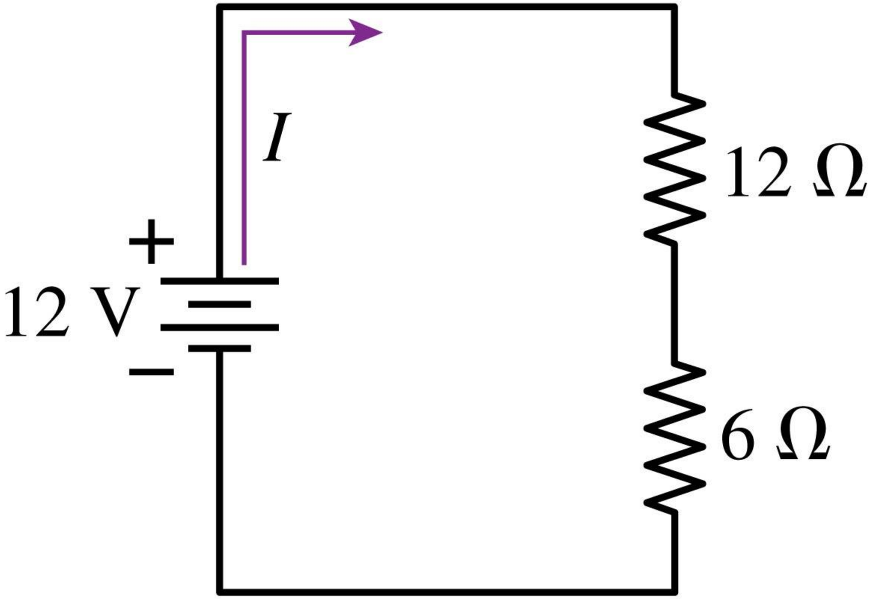
Parallel Resistors

- It would seem that more resistors would imply more resistance. This is true for resistors in series, but not for resistors in parallel.
- Parallel resistors provide more pathways for charge to get through.
- **The equivalent of several resistors in parallel is always *less* than any single resistor in the group.**
- An analogy is driving in heavy traffic. If there is an alternate route for cars to travel, more cars will be able to flow.

QuickCheck 23.13

The battery current *I* is

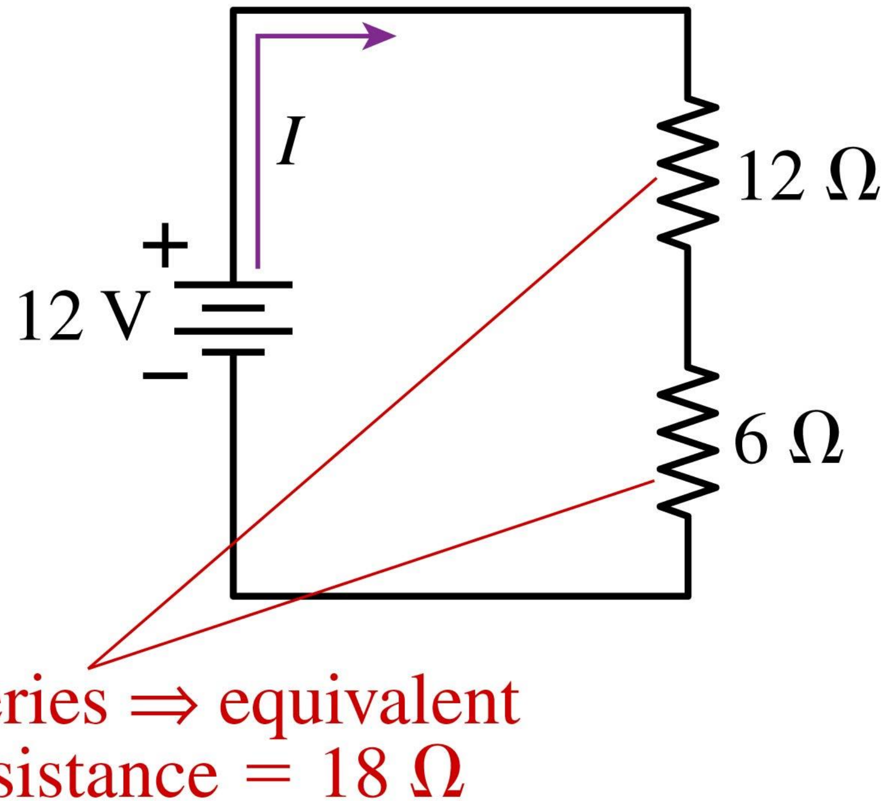
A. 3 A
B. 2 A
C. 1 A
D. 2/3 A
E. 1/2 A



QuickCheck 23.13

The battery current *I* is

A. 3 A
B. 2 A
C. 1 A
✔ D. 2/3 A
E. 1/2 A



QuickCheck 23.14

The battery current I is

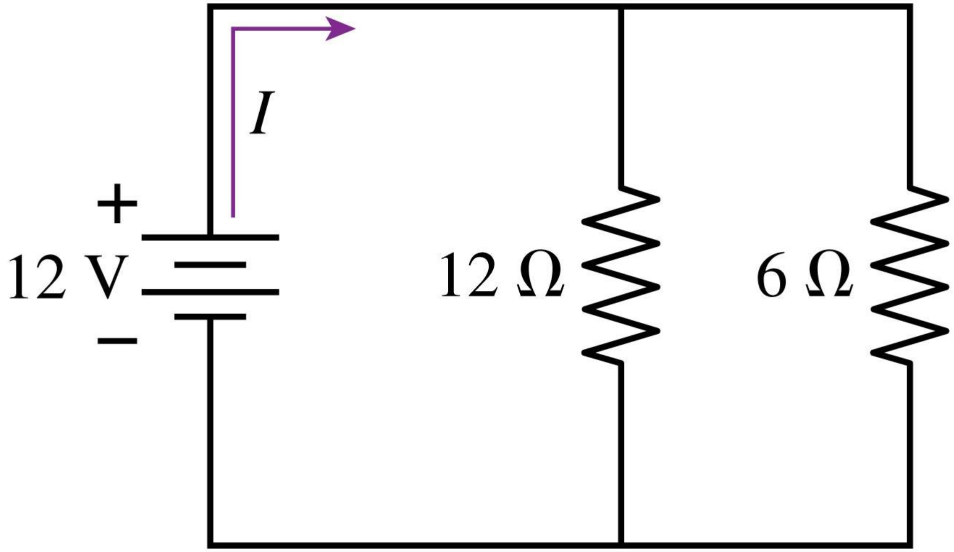
A. 3 A

B. 2 A

C. 1 A

D. 2/3 A

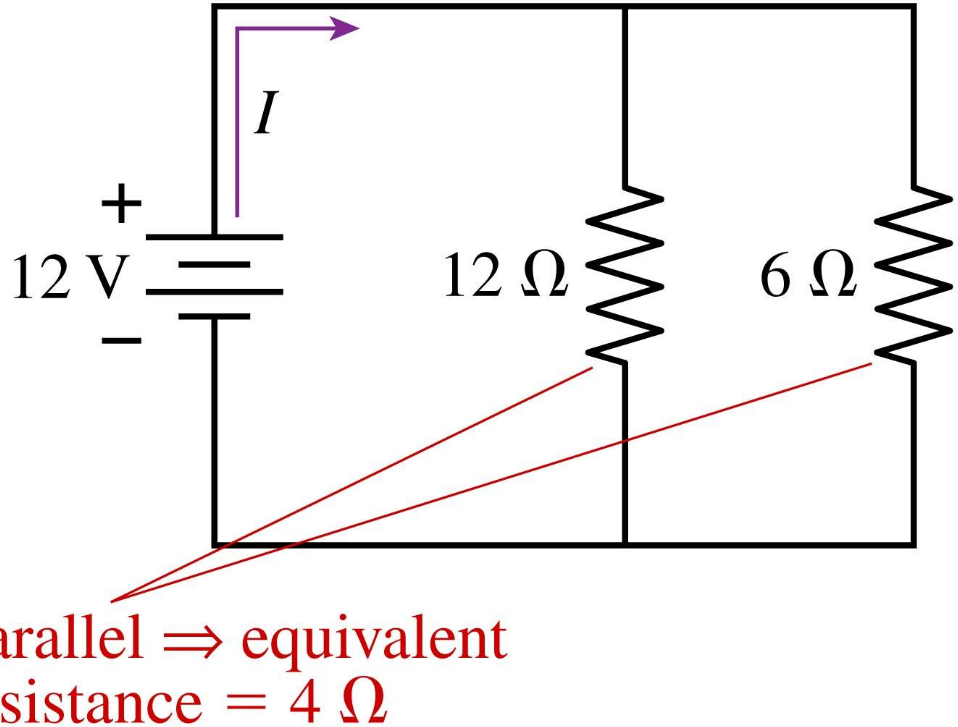
E. 1/2 A



QuickCheck 23.14

The battery current I is

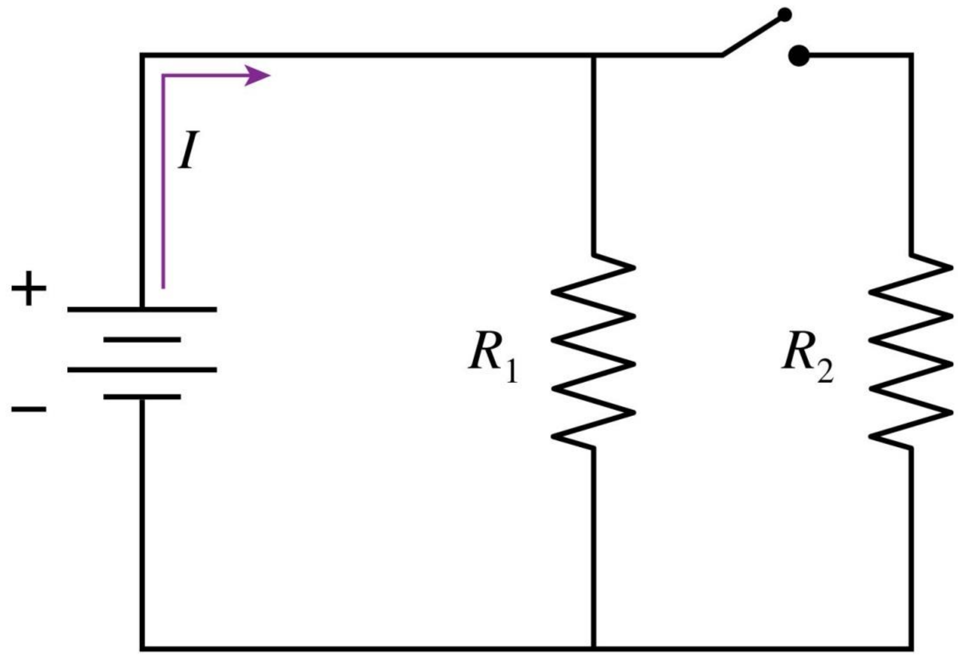
A. 3 A ✔
B. 2 A
C. 1 A
D. 2/3 A
E. 1/2 A



QuickCheck 23.15

When the switch closes, the battery current

A. Increases.
B. Stays the same.
C. Decreases.

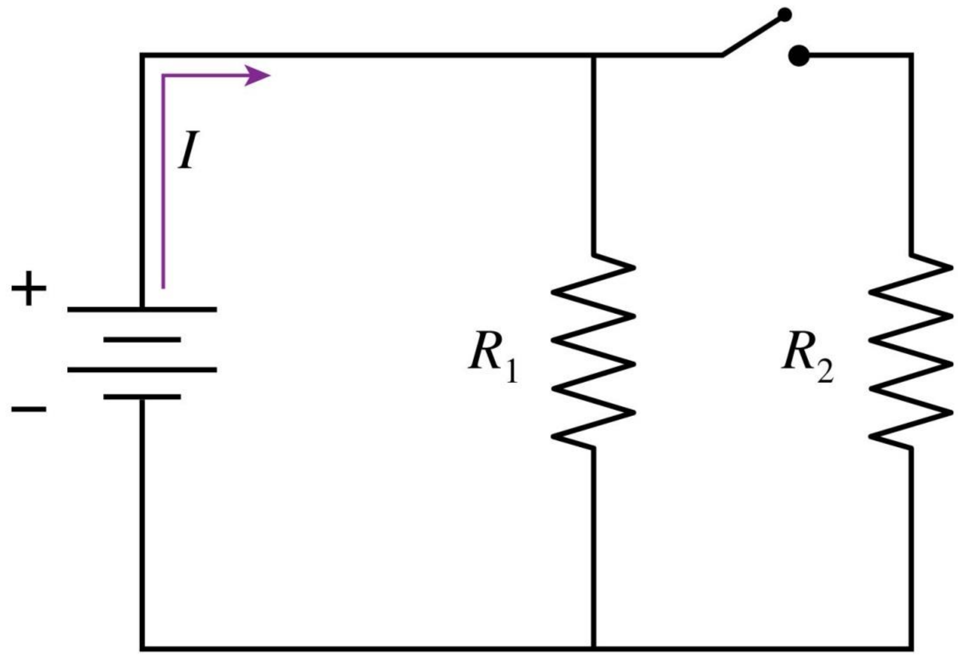


QuickCheck 23.15

When the switch closes, the battery current

A. Increases.
B. Stays the same.
C. Decreases.

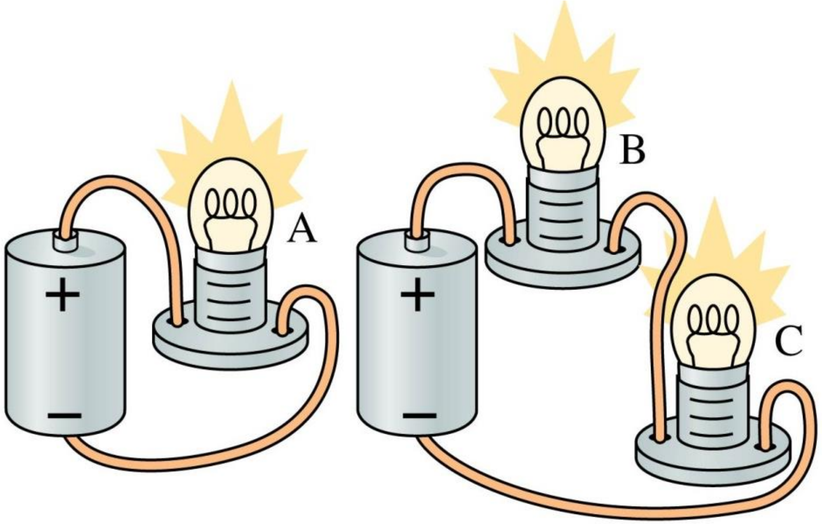
Equivalent resistance decreases.
Potential difference is unchanged.



QuickCheck 23.2

The three bulbs are identical and the two batteries are identical. Compare the brightnesses of the bulbs.

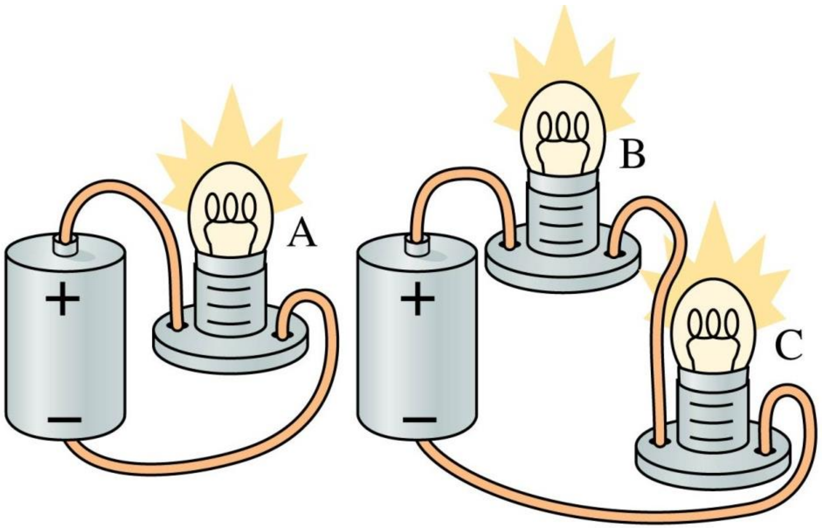
A. A > B > C
B. A > C > B
C. A > B = C
D. A < B = C
E. A = B = C



QuickCheck 23.2

The three bulbs are identical and the two batteries are identical. Compare the brightnesses of the bulbs.

A. A > B > C
B. A > C > B
C. A > B = C
D. A < B = C
E. A = B = C

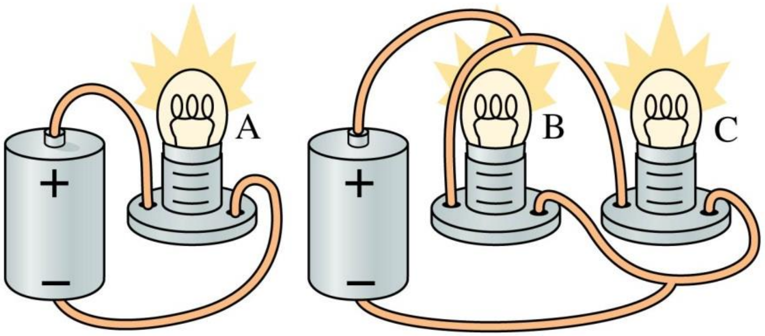


This question is checking your initial intuition.
We'll return to it later.

QuickCheck 23.3

The three bulbs are identical and the two batteries are identical. Compare the brightnesses of the bulbs.

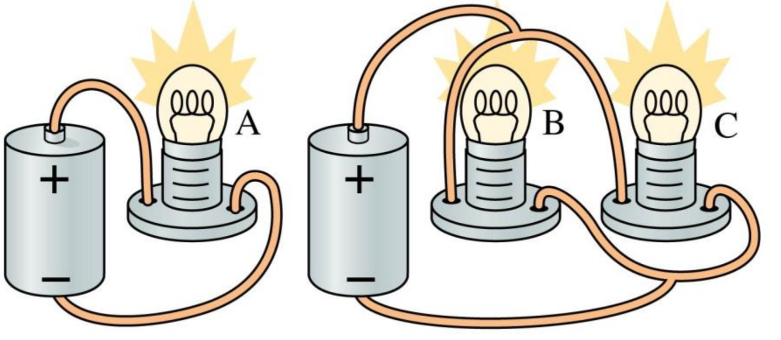
A. A > B > C
B. A > C > B
C. A > B = C
D. A < B = C
E. A = B = C



QuickCheck 23.3

The three bulbs are identical and the two batteries are identical. Compare the brightnesses of the bulbs.

A. A > B > C
B. A > C > B
C. A > B = C
D. A < B = C
E. A = B = C



This question is checking your initial intuition. We'll return to it later.

QuickCheck 23.18

The lightbulbs are identical. Initially both bulbs are glowing. What happens when the switch is closed?

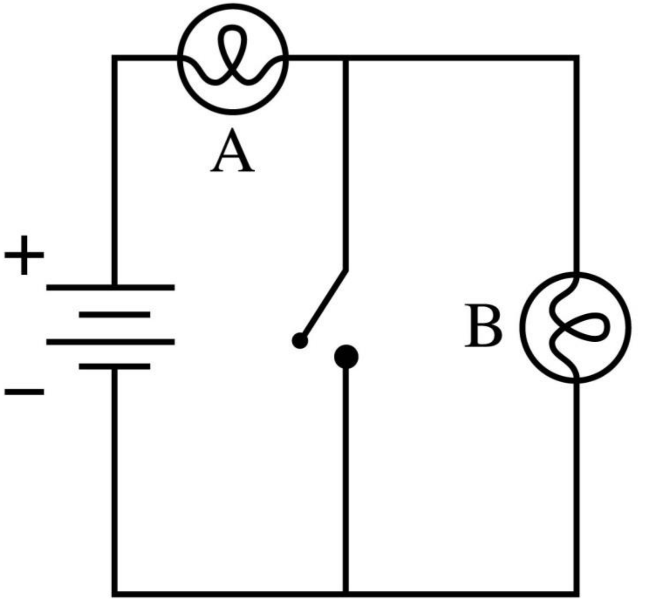
A. Nothing.

B. A stays the same;
B gets dimmer.

C. A gets brighter;
B stays the same.

D. Both get dimmer.

E. A gets brighter;
B goes out.



QuickCheck 23.18

The lightbulbs are identical. Initially both bulbs are glowing. What happens when the switch is closed?

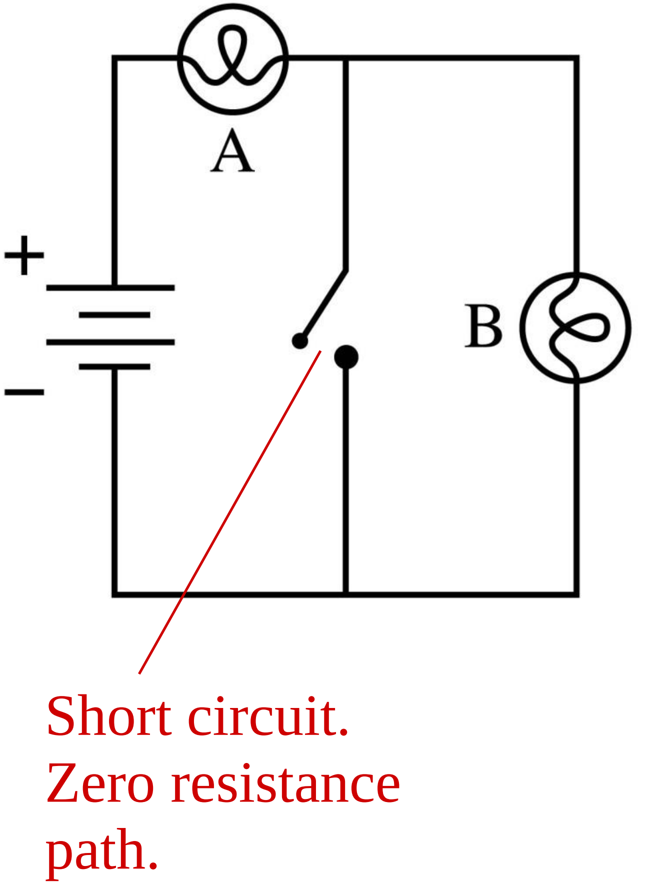
A. Nothing.

B. A stays the same; B gets dimmer.

C. A gets brighter; B stays the same.

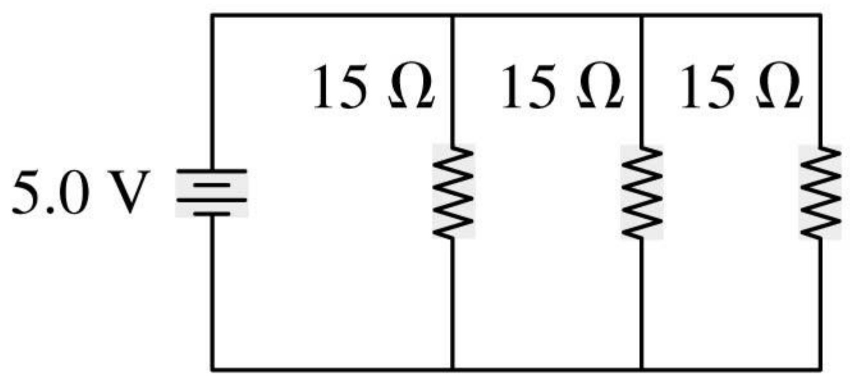
D. Both get dimmer.

✔ E. A gets brighter; B goes out.



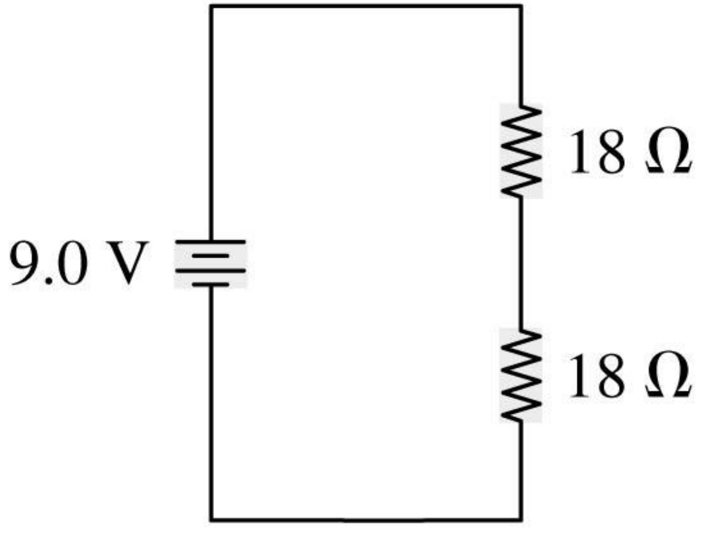
Example Problem

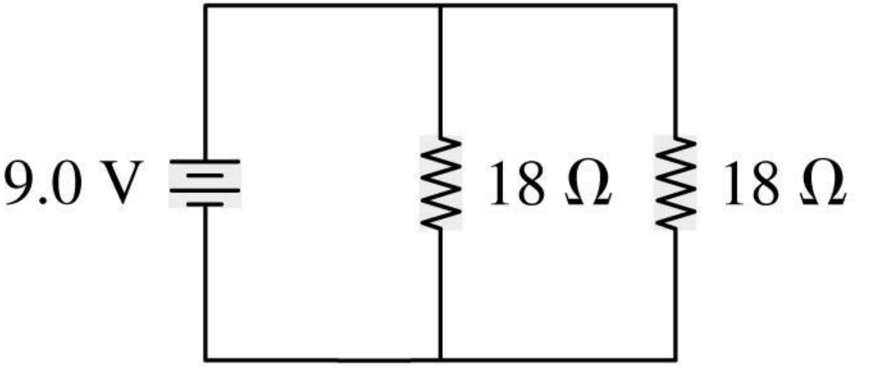
What is the current supplied by the battery in the following circuit?



Example Problem

A resistor connected to a power supply works as a heater. Which of the following two circuits will provide more power?





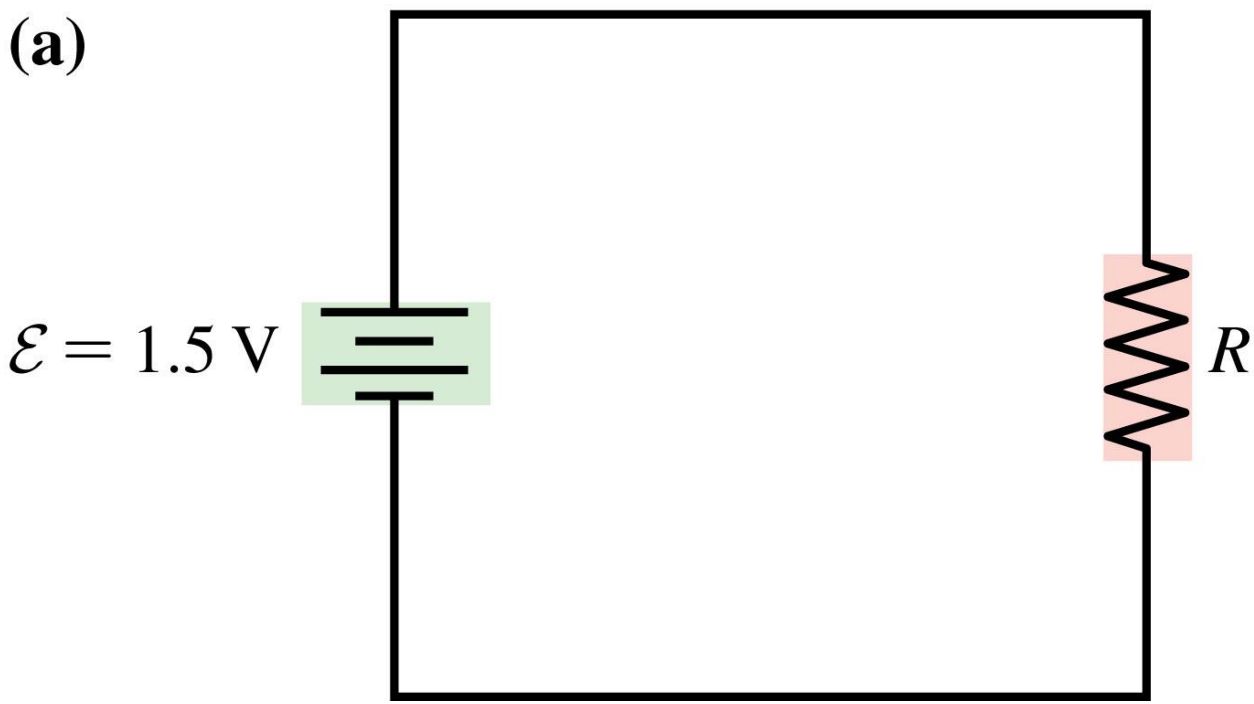
Section 23.4 Measuring Voltage and Current

Measuring Voltage and Current

- An **ammeter** is a device that measures the current in a circuit element.
- Because charge flows *through* circuit elements, an ammeter must be placed *in series* with the circuit element whose current is to be measured.

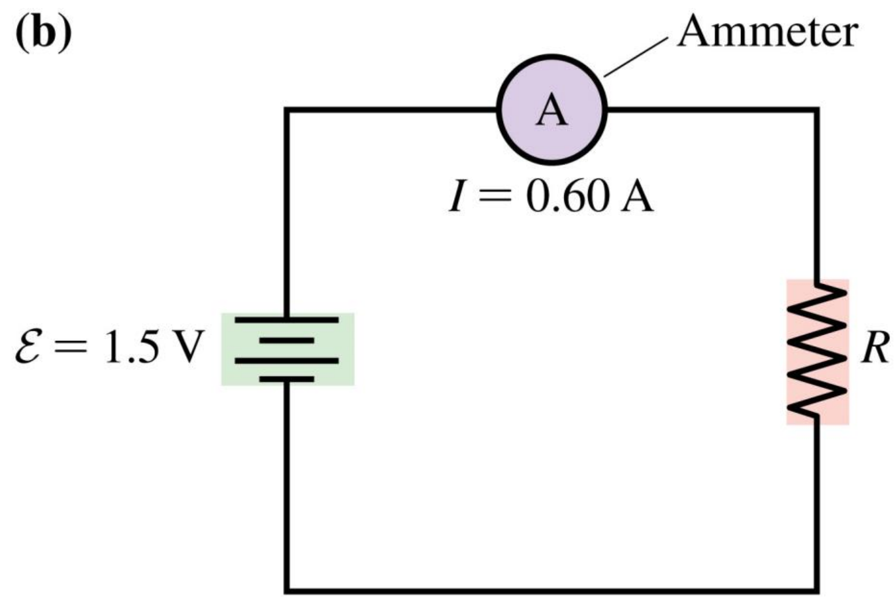
Measuring Voltage and Current

- In order to determine the resistance in this simple, one-resistor circuit with a fixed emf of 1.5 V, we must know the current in the circuit.



Measuring Voltage and Current

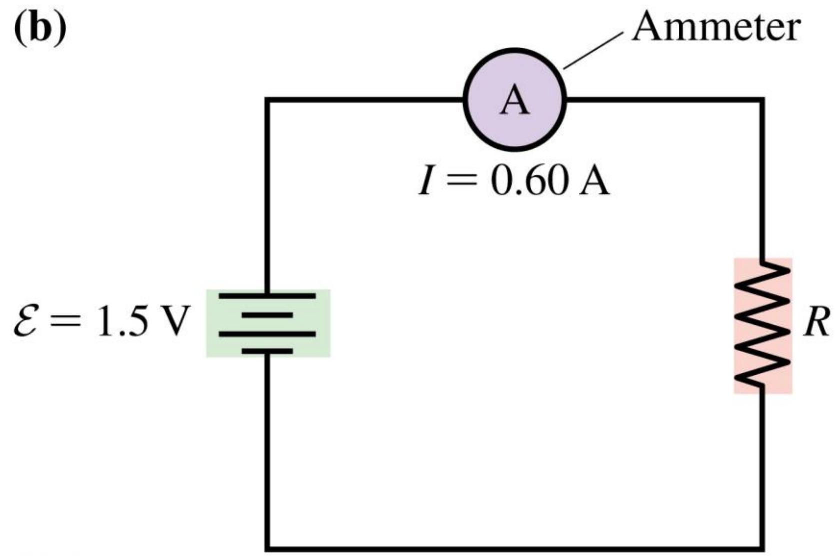
- To determine the current in the circuit, we insert the ammeter. To do so, we must *break the connection* between the battery and the resistor.
- Because they are in series, the ammeter and the resistor have the same current.



- **The resistance of an ideal ammeter is zero** so that it can *measure* the current without *changing* the current.

Measuring Voltage and Current

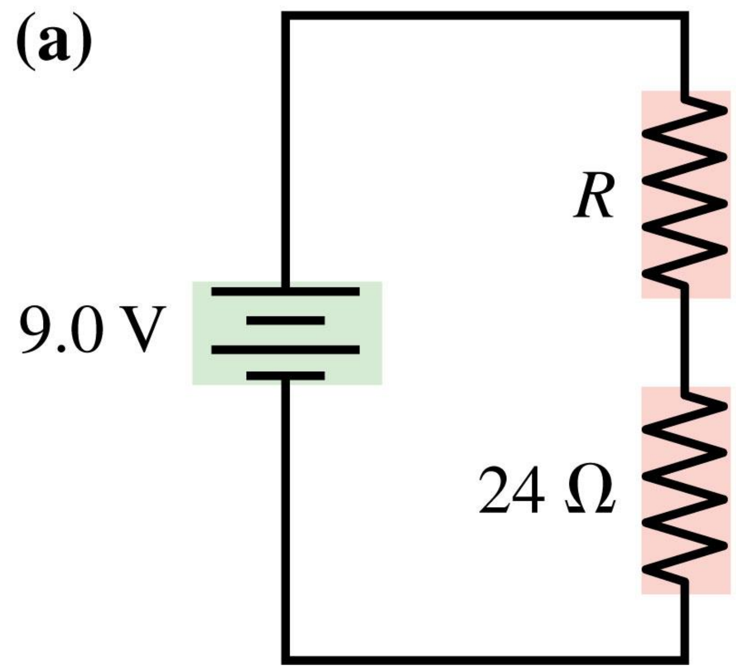
- In this circuit, the ammeter reads a current $I = 0.60$ A.
- If the ammeter is ideal, there is no potential difference across it. The potential difference across the resistor is $\Delta V = \mathcal{E}$.
- The resistance is then calculated:



$$R = \frac{\mathcal{E}}{I} = \frac{1.5\ \text{V}}{0.60\ \text{A}} = 2.5\ \Omega$$

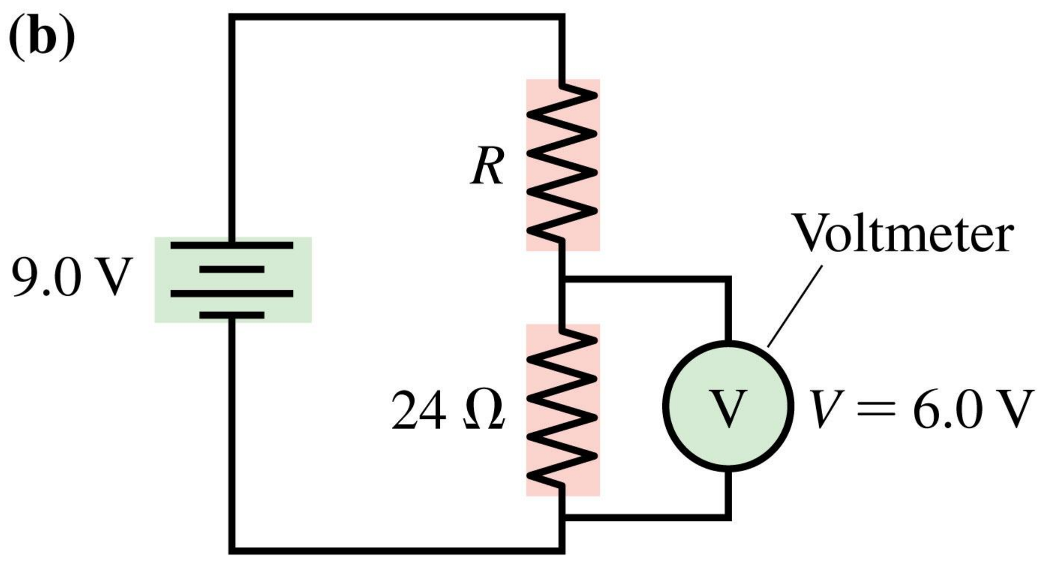
Measuring Voltage and Current

- A **voltmeter** is used to measure the potential differences in a circuit.
- Because the potential difference is measured *across* a circuit element, a voltmeter is placed in *parallel* with the circuit element whose potential difference is to be measured.



Measuring Voltage and Current

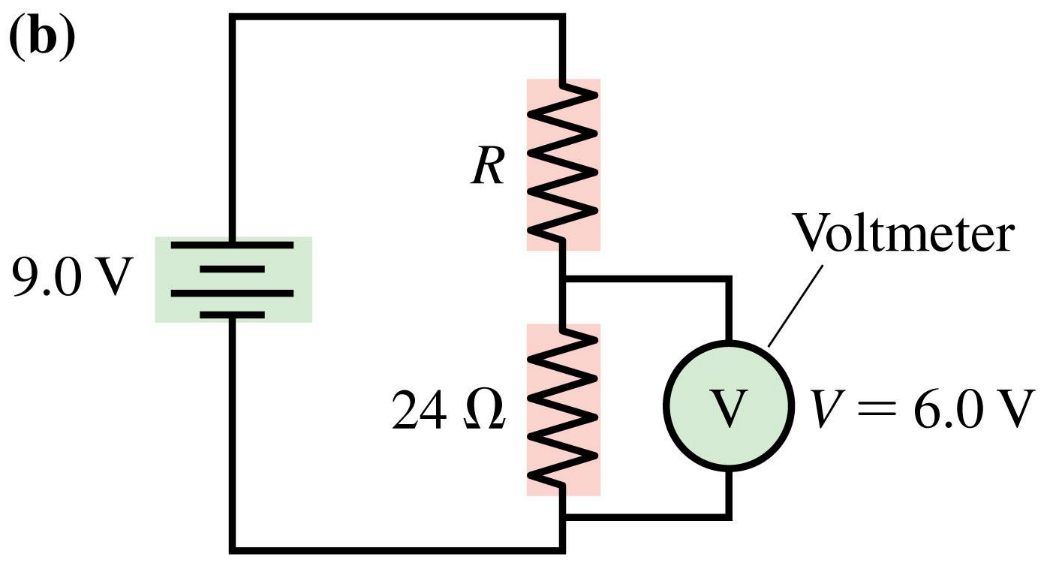
- **An ideal voltmeter has infinite resistance** so that it can *measure* the voltage without *changing* the voltage.
- Because it is in parallel with the resistor, the voltmeter's resistance must be very large so that it draws very little current.



Measuring Voltage and Current

- The voltmeter finds the potential difference across the 24 Ω resistor to be 6.0 V.
- The current through the resistor is:

$$I = \frac{\Delta V}{R} = \frac{6.0\ \text{V}}{24\ \Omega} = 0.25\ \text{A}$$



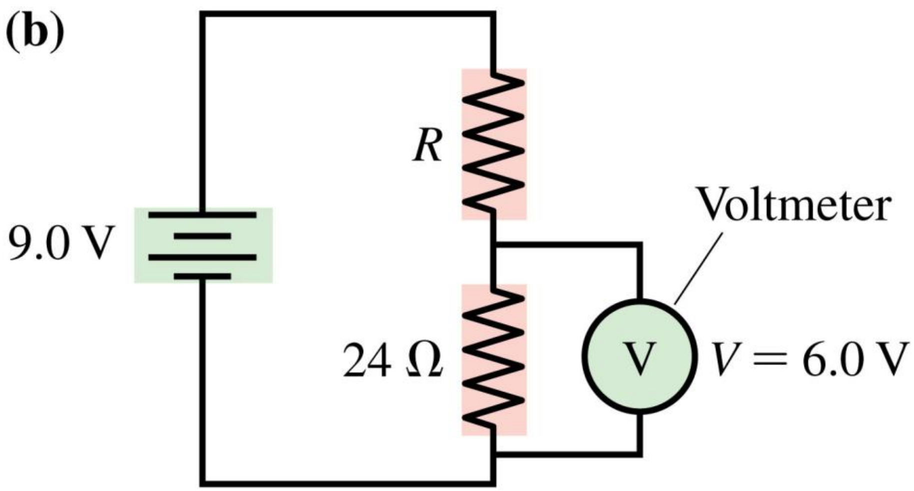
Measuring Voltage and Current

- Kirchhoff's law gives the potential difference across the unknown resistor:

$$\sum_i \Delta V_i = 9.0\text{ V} + \Delta V_R - 6.0\text{ V} = 0$$

- $\Delta V_R = -3.0\text{V}$

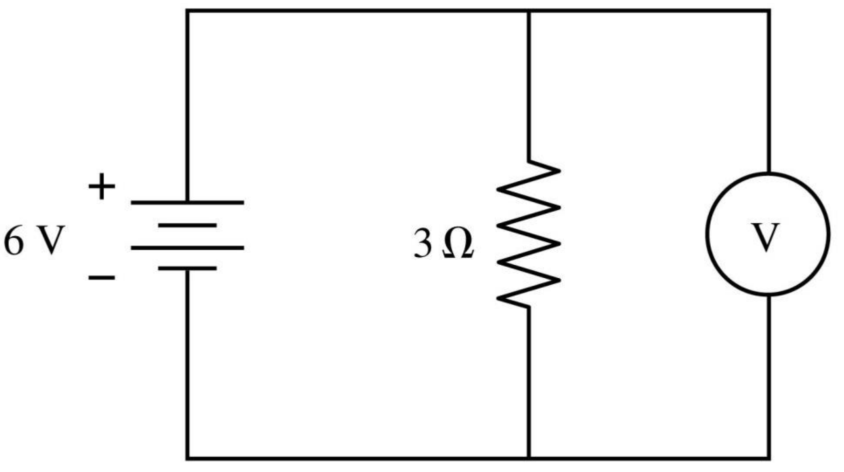
$$R = \frac{-\Delta V_R}{I} = -\frac{(-3.0\text{ V})}{0.25\text{ A}} = 12\ \Omega$$



QuickCheck 23.19

What does the voltmeter read?

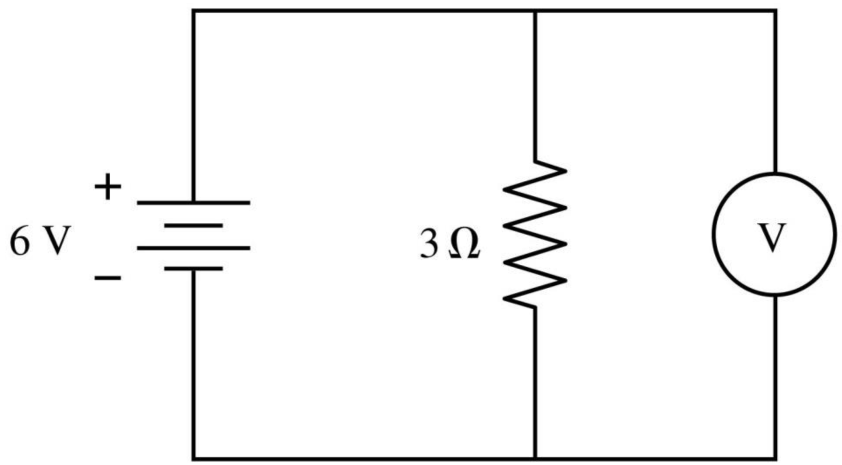
A. 6 V
B. 3 V
C. 2 V
D. Some other value
E. Nothing because this will fry the meter.



QuickCheck 23.19

What does the voltmeter read?

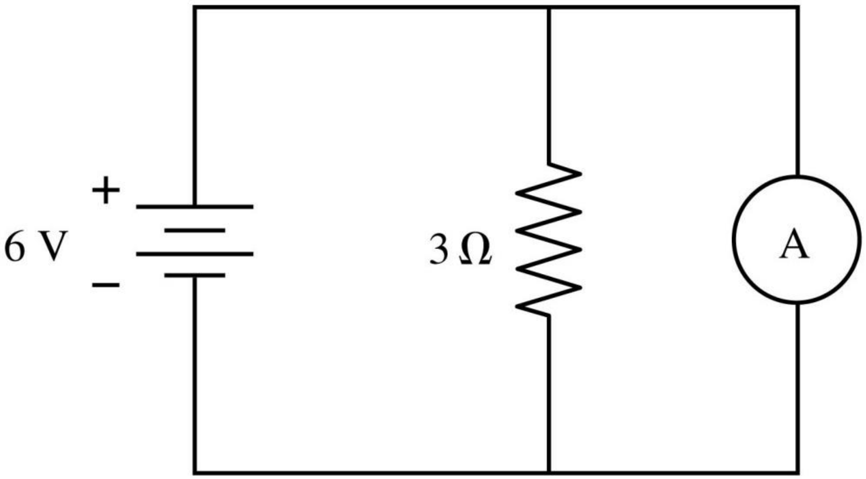
A. 6 V
B. 3 V
C. 2 V
D. Some other value
E. Nothing because this will fry the meter.



QuickCheck 23.20

What does the ammeter read?

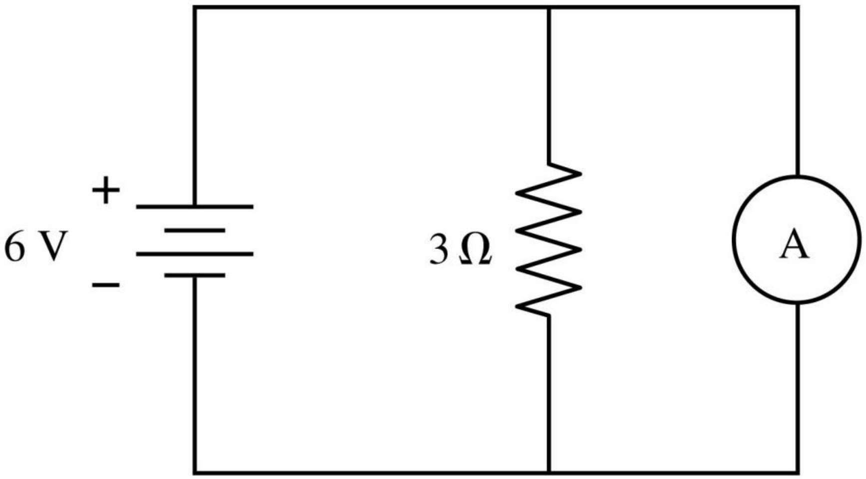
A. 6 A
B. 3 A
C. 2 A
D. Some other value
E. Nothing because this will fry the meter.



QuickCheck 23.20

What does the ammeter read?

A. 6 A
B. 3 A
C. 2 A
D. Some other value
✔ E. Nothing because this will fry the meter.



Section 23.5 More Complex Circuits

More Complex Circuits

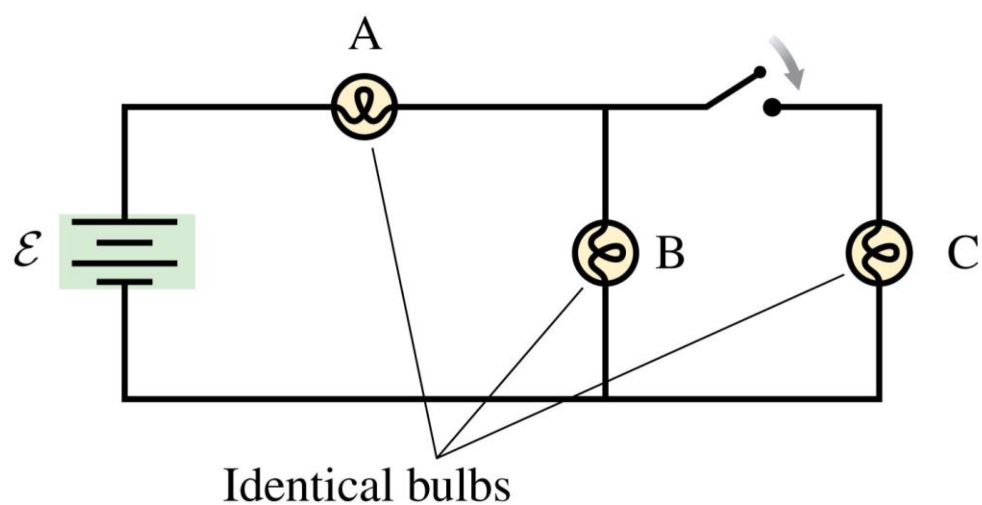
- Combinations of resistors can often be reduced to a single equivalent resistance through a step-by-step application of the series and parallel rules.
- Two special cases:

  Two identical resistors in series: $R_{eq} = 2R$

  Two identical resistors in parallel: $R_{eq} = R/2$

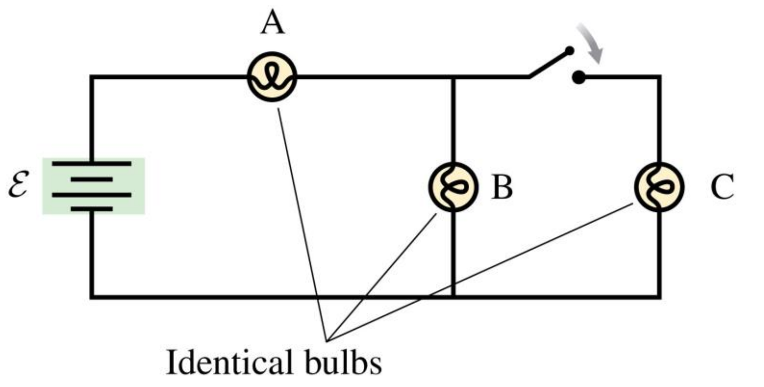
Example 23.8 How does the brightness change?

Initially the switch in **FIGURE 23.28** is open. Bulbs A and B are equally bright, and bulb C is not glowing. What happens to the brightness of A and B when the switch is closed? And how does the brightness of C then compare to that of A and B? Assume that all bulbs are identical.



Example 23.8 How does the brightness change? (cont.)

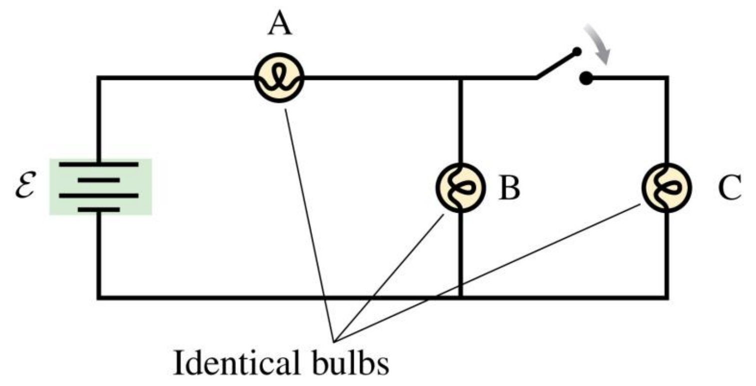
SOLVE Suppose the resistance of each bulb is R. Initially, before the switch is closed, bulbs A and B are in series; bulb C is not part of the circuit. A and B are identical resistors in series, so their equivalent resistance is $2R$ and the current from the battery is



$$I_{\text{before}} = \frac{\mathcal{E}}{R_{\text{eq}}} = \frac{\mathcal{E}}{2R} = \frac{1}{2}\frac{\mathcal{E}}{R}$$

This is the initial current in bulbs A and B, so they are equally bright.

Example 23.8 How does the brightness change? (cont.)

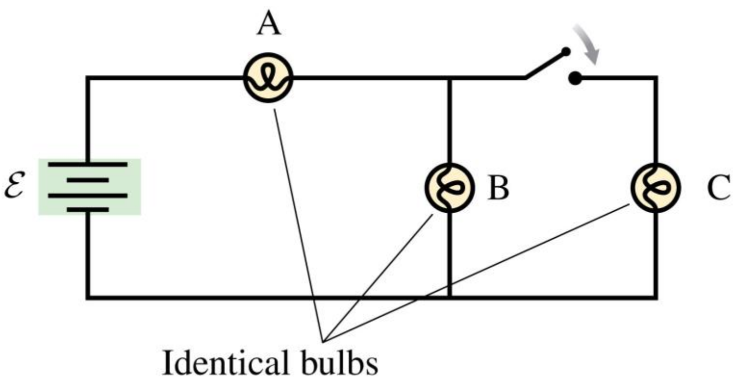


Closing the switch places bulbs B and C in parallel with each other. The equivalent resistance of the two identical resistors in parallel is $R_{B+C} = R/2$. This equivalent resistance of B and C is in series with bulb A; hence the total resistance of the circuit is $R_{eq} = R + \frac{1}{2}R = \frac{3}{2}R$, and the current leaving the battery is

$$I_{after} = \frac{\mathcal{E}}{R_{eq}} = \frac{\mathcal{E}}{3R/2} = \frac{2}{3}\frac{\mathcal{E}}{R} > I_{before}$$

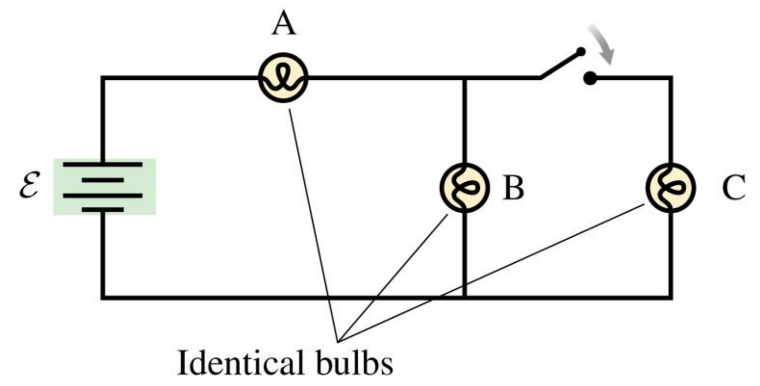
Closing the switch decreases the total circuit resistance and thus *increases* the current leaving the battery.

Example 23.8 How does the brightness change? (cont.)



All the current from the battery passes through bulb A, so A *increases* in brightness when the switch is closed. The current I_{after} then splits at the junction. Bulbs B and C have equal resistance, so the current divides equally. The current in B is $\frac{1}{3}(\mathcal{E}/R)$, which is *less* than I_{before}. Thus B *decreases* in brightness when the switch is closed. With the switch closed, bulbs B and C are in parallel, so bulb C has the same brightness as bulb B.

Example 23.8 How does the brightness change? (cont.)



ASSESS Our final results make sense. Initially, bulbs A and B are in series, and all of the current that goes through bulb A goes through bulb B. But when we add bulb C, the current has another option—it can go through bulb C. This will increase the total current, and all that current must go through bulb A, so we expect a brighter bulb A. But now the current through bulb A can go through bulbs B and C. The current splits, so we'd expect that bulb B will be dimmer than before.

Analyzing Complex Circuits

PROBLEM-SOLVING STRATEGY 23.1 **Resistor circuits**

We can analyze any resistor circuit by sequentially reducing parallel and series resistor combinations to their equivalent resistors until only the battery and a single equivalent resistor are left.

PREPARE Draw a circuit diagram. Label all known and unknown quantities.

Text: p. 739

Analyzing Complex Circuits

SOLVE Base your mathematical analysis on Kirchhoff's laws and on the rules for series and parallel resistors:

- Step by step, reduce the circuit to the smallest possible number of equivalent resistors.
- Determine the current through and potential difference across the equivalent resistors.
- Rebuild the circuit, using the facts that the current is the same through all resistors in series and the potential difference is the same across all parallel resistors.

Text: p. 739

Analyzing Complex Circuits

ASSESS Use two important checks as you rebuild the circuit.

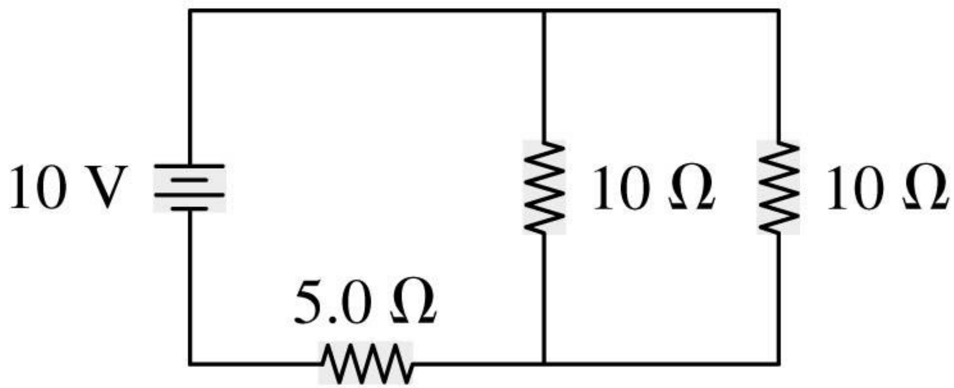
- Verify that the sum of the potential differences across series resistors matches ΔV for the equivalent resistor.
- Verify that the sum of the currents through parallel resistors matches I for the equivalent resistor.

Exercise 23

Text: p. 739

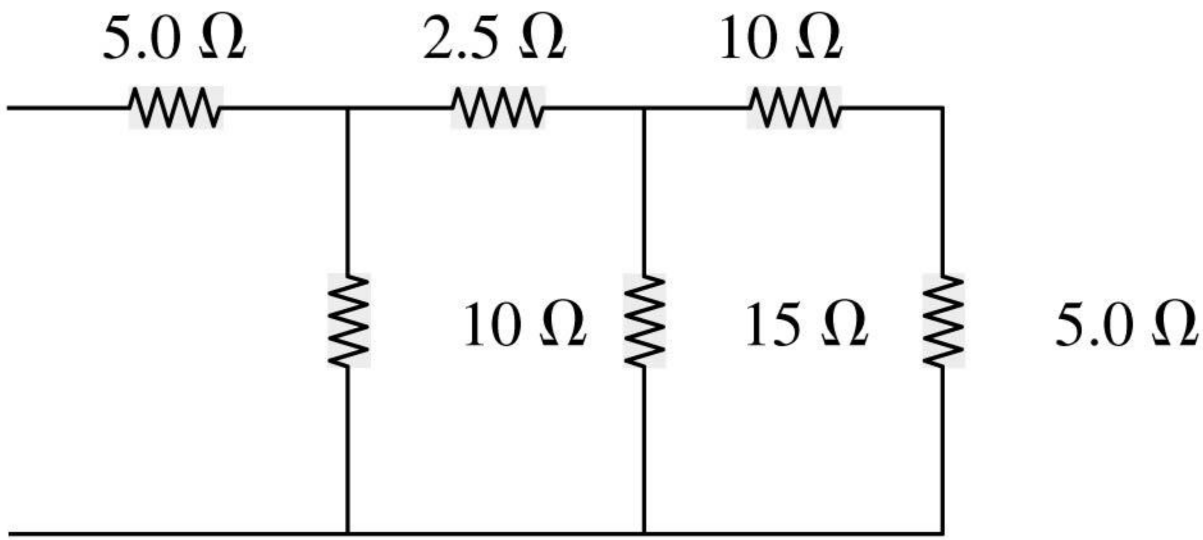
Example Problem

What is the current supplied by the battery in the following circuit? What is the current through each resistor in the circuit? What is the potential difference across each resistor?



Example Problem

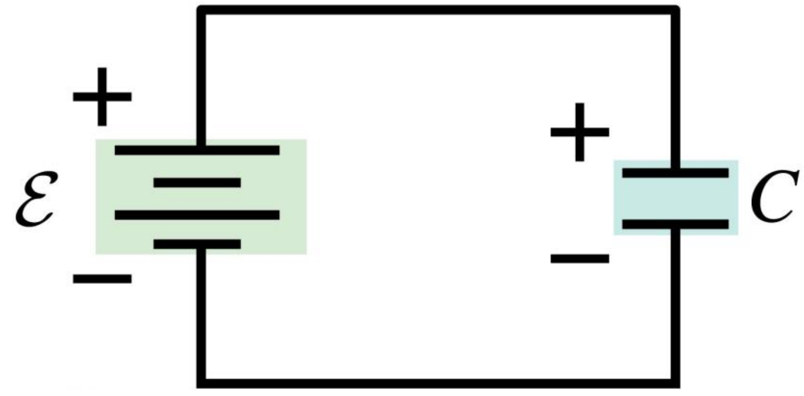
What is the equivalent resistance of the following circuit?



Section 23.6 Capacitors in Parallel and Series

Capacitors in Parallel and Series

- A *capacitor* is a circuit element made of two conductors separated by an insulating layer.



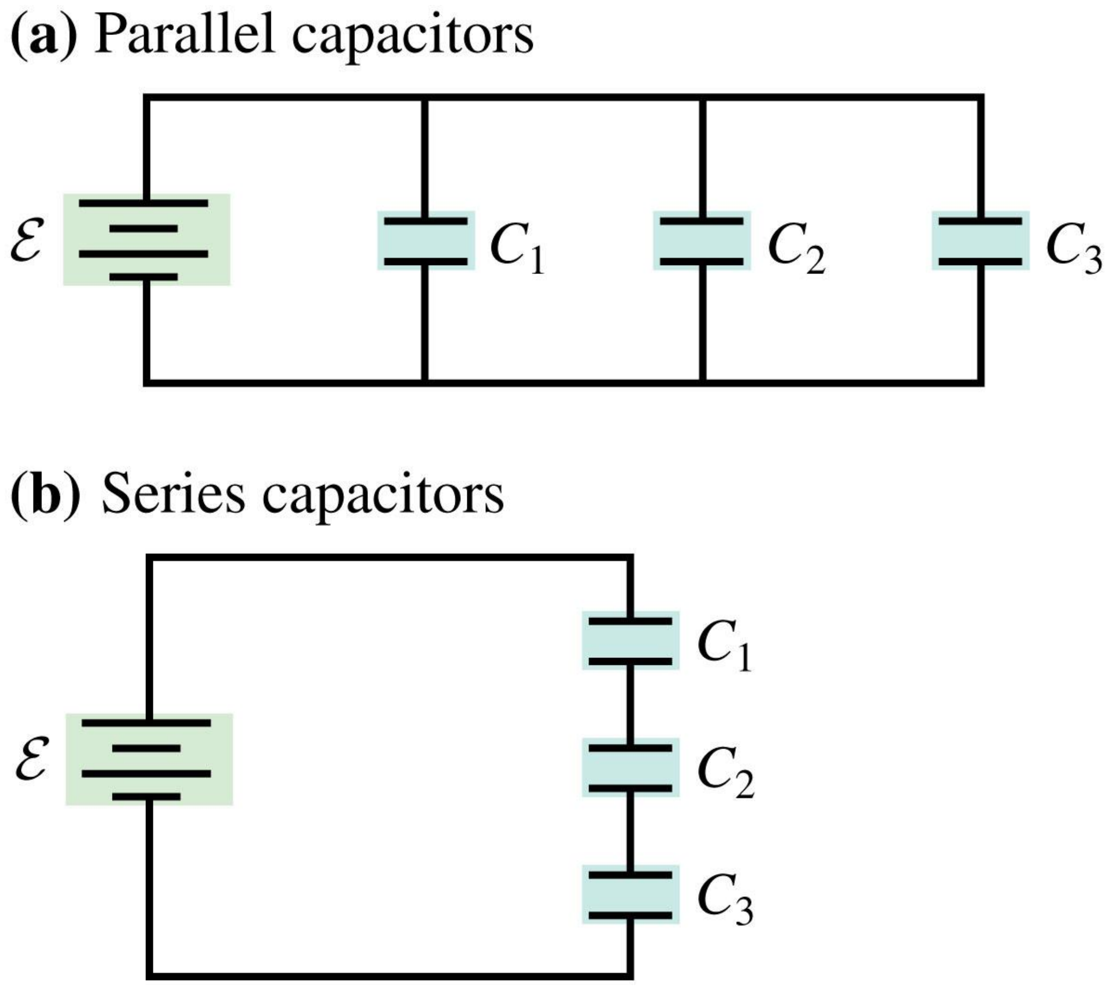
- When the capacitor is connected to the battery, charge will flow to the capacitor, increasing its potential difference until

$$\Delta V_C = \mathcal{E}$$

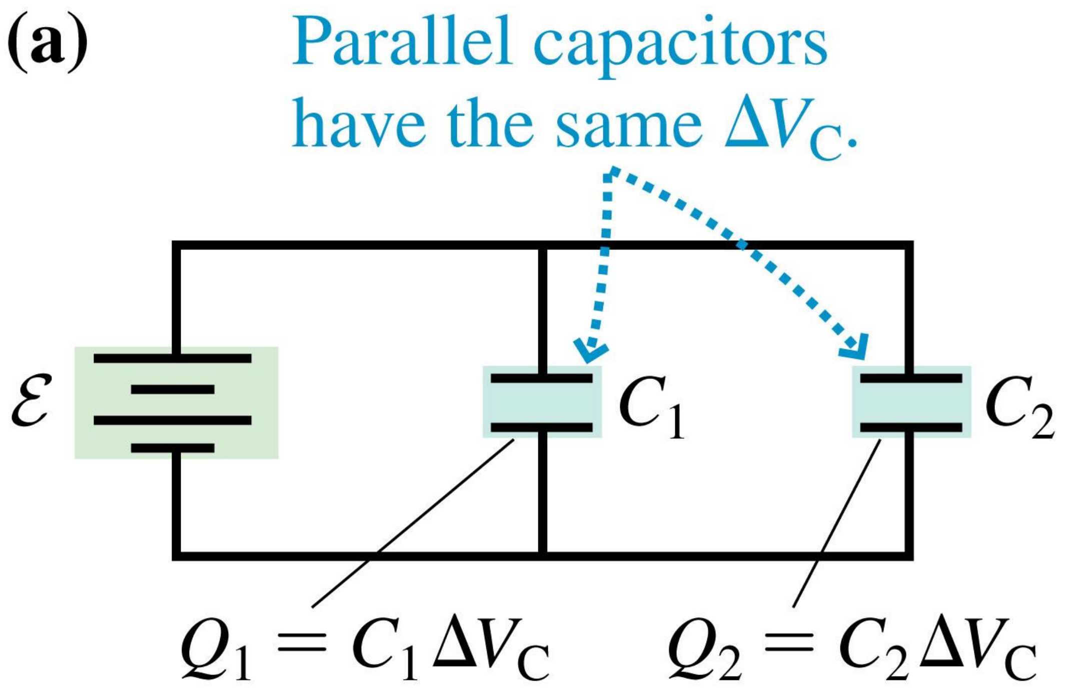
- Once the capacitor is fully charged, there will be no further current.

Capacitors in Parallel and Series

- Parallel or series capacitors can be represented by a single **equivalent capacitance.**



Capacitors in Parallel and Series



- The total charge Q on the two capacitors is

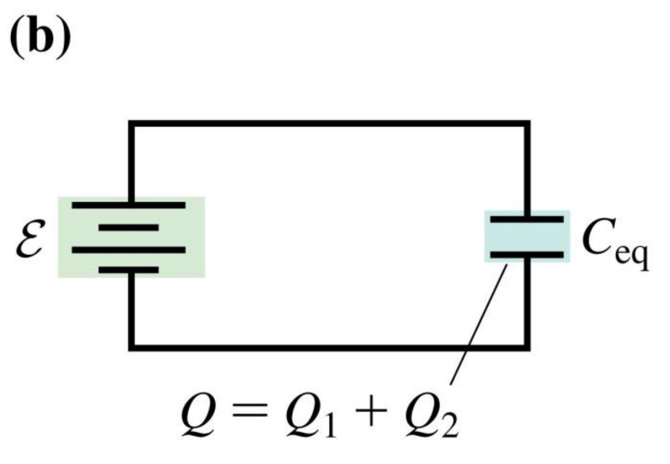
$$Q = Q_1 + Q_2 = C_1 \Delta V_C + C_2 \Delta V_C = (C_1 + C_2)\Delta V_C$$

Capacitors in Parallel and Series

- We can replace two capacitors in parallel by a single equivalent capacitance C_{eq}:

$$C_{eq} = \frac{Q}{\Delta V_C} = \frac{(C_1 + C_2)\Delta V_C}{\Delta V_C} = C_1 + C_2$$

- The equivalent capacitance has the same ΔV_C but a greater charge.



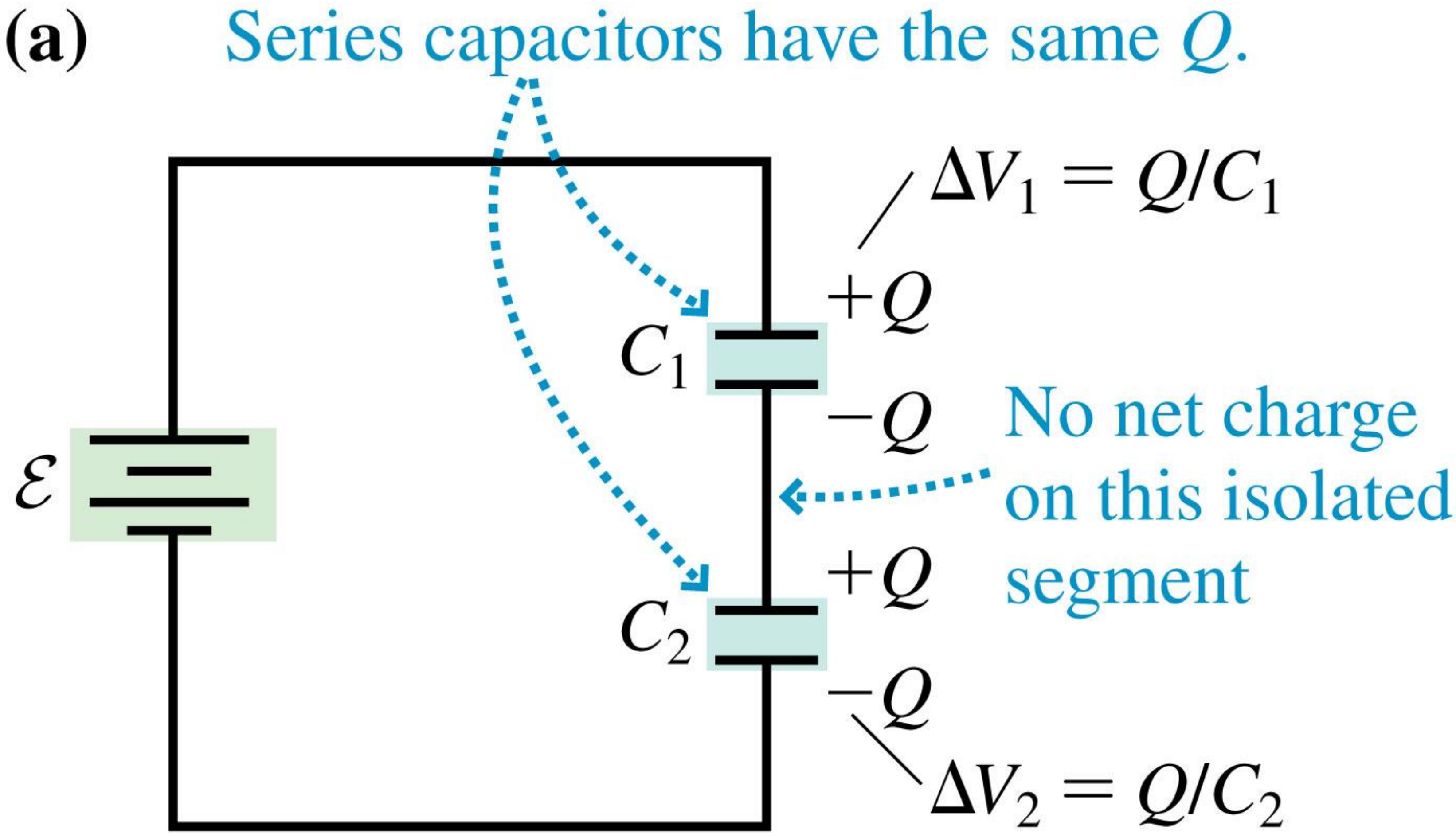
Capacitors in Parallel and Series

- If N capacitors are in parallel, their equivalent capacitance is the sum of the individual capacitances:

$$C_{eq} = C_1 + C_2 + C_3 + \cdots + C_N$$

Equivalent capacitance of N parallel capacitors

Capacitors in Parallel and Series



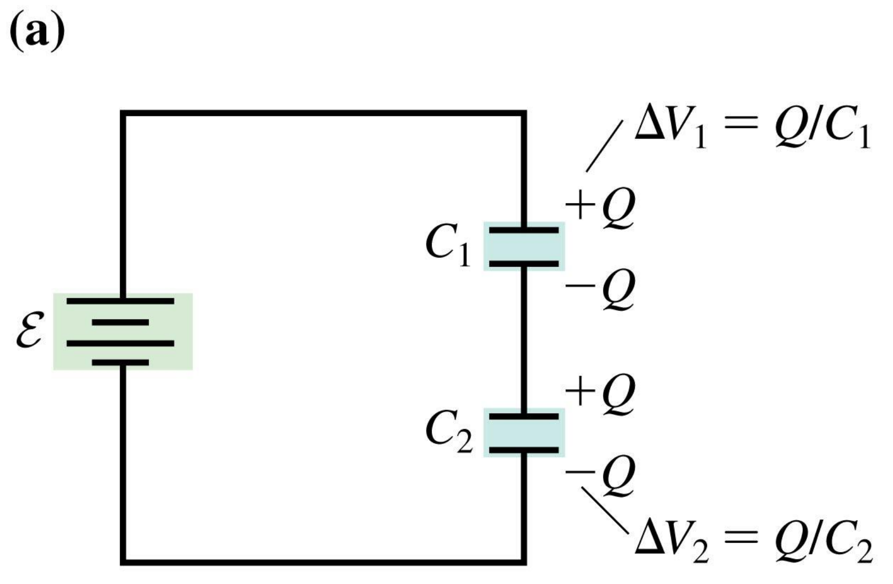
Capacitors in Parallel and Series

- The potential differences across two capacitors in series are

$$\Delta V_1 = Q/C_1 \text{ and } \Delta V_2 = Q/C_2$$

- The total potential difference across both capacitors is

$$\Delta V_C = \Delta V_1 + \Delta V_2$$

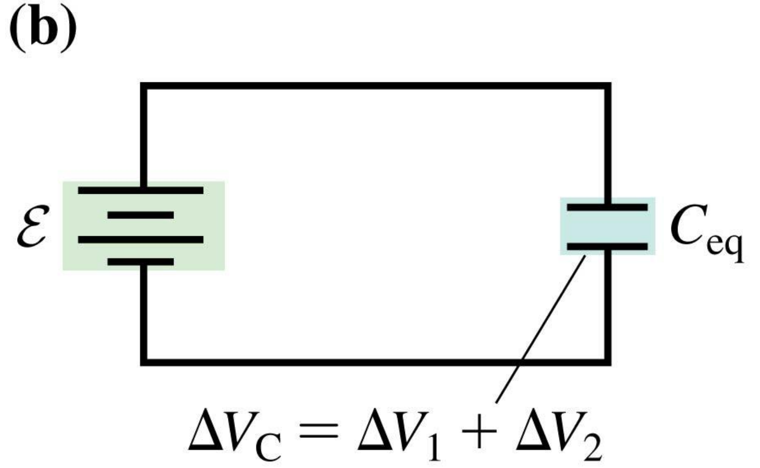


Capacitors in Parallel and Series

- If we replace the two capacitors with a single capacitor having charge Q and potential difference $\Delta V_{C,}$ then the inverse of the capacitance of this equivalent capacitor is

$$\frac{1}{C_{eq}} = \frac{\Delta V_C}{Q} = \frac{\Delta V_1 + \Delta V_2}{Q} = \frac{\Delta V_1}{Q} + \frac{\Delta V_2}{Q} = \frac{1}{C_1} + \frac{1}{C_2}$$

- This analysis hinges on the fact that **series capacitors each have the same charge *Q*.**



Capacitors in Parallel and Series

- If N capacitors are in series, their equivalent capacitance is the inverse of the sum of the inverses of the individual capacitances:

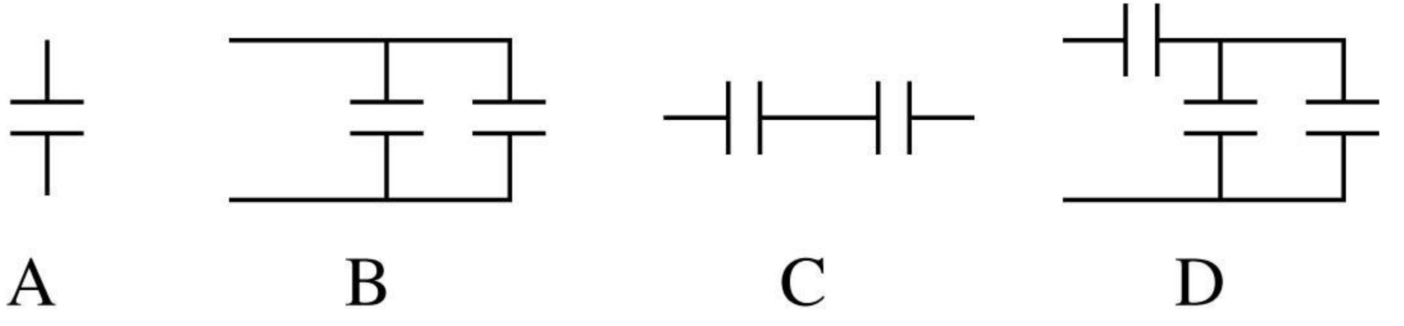
$$C_{\text{eq}} = \left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots + \frac{1}{C_N} \right)^{-1}$$

Equivalent capacitance of N series capacitors

- For series capacitors the equivalent capacitance is less than that of the individual capacitors.

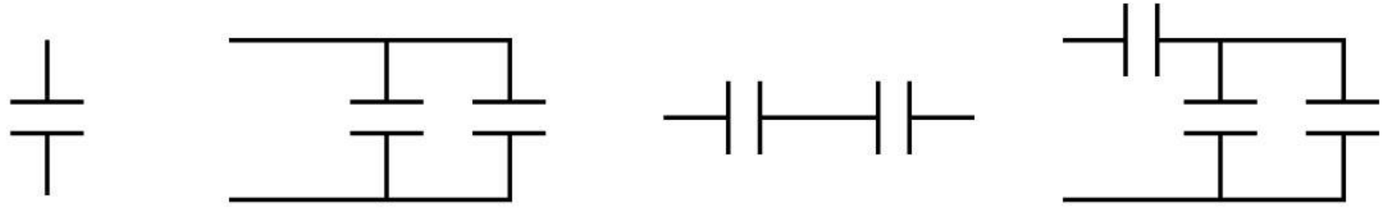
QuickCheck 23.21

Which of the following combinations of capacitors has the highest capacitance?



QuickCheck 23.21

Which of the following combinations of capacitors has the highest capacitance?

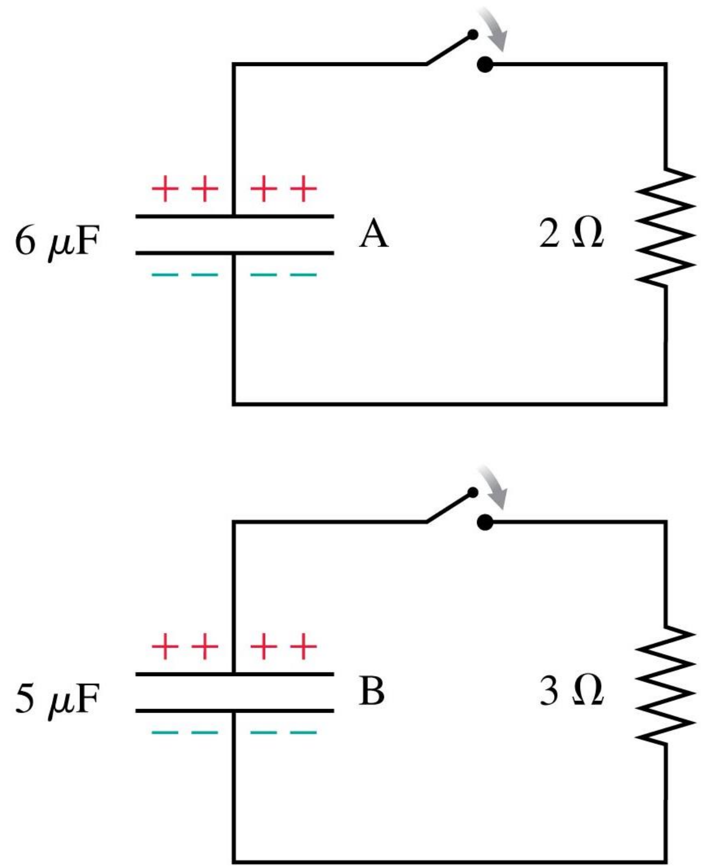


A ✔ B C D

QuickCheck 23.22

Which capacitor discharges more quickly after the switch is closed?

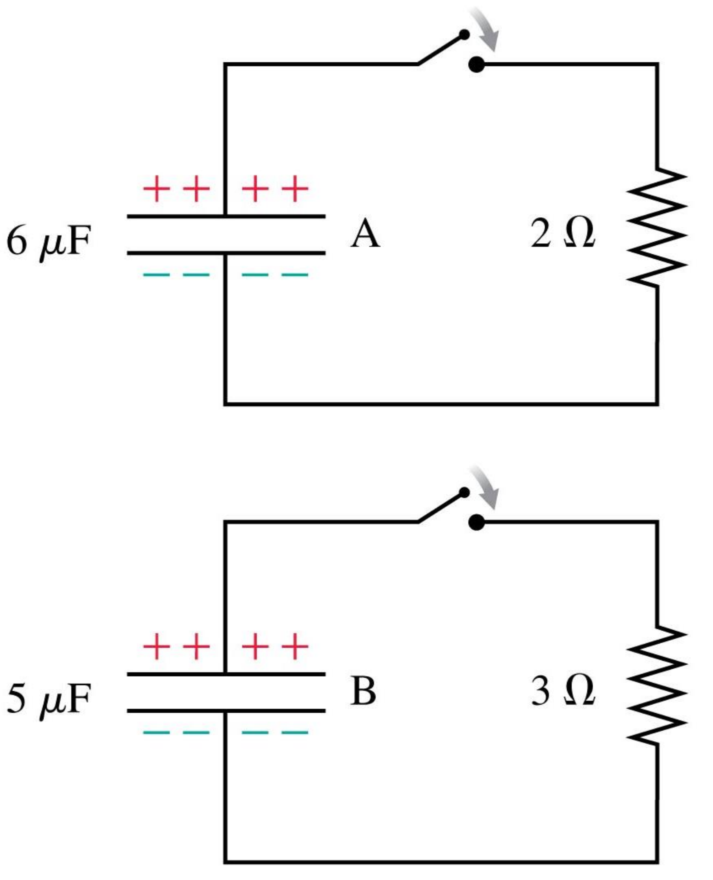
A. Capacitor A
B. Capacitor B
C. They discharge at the same rate.
D. We can't say without knowing the initial amount of charge.



QuickCheck 23.22

Which capacitor discharges more quickly after the switch is closed?

Smaller time constant $\tau = RC$

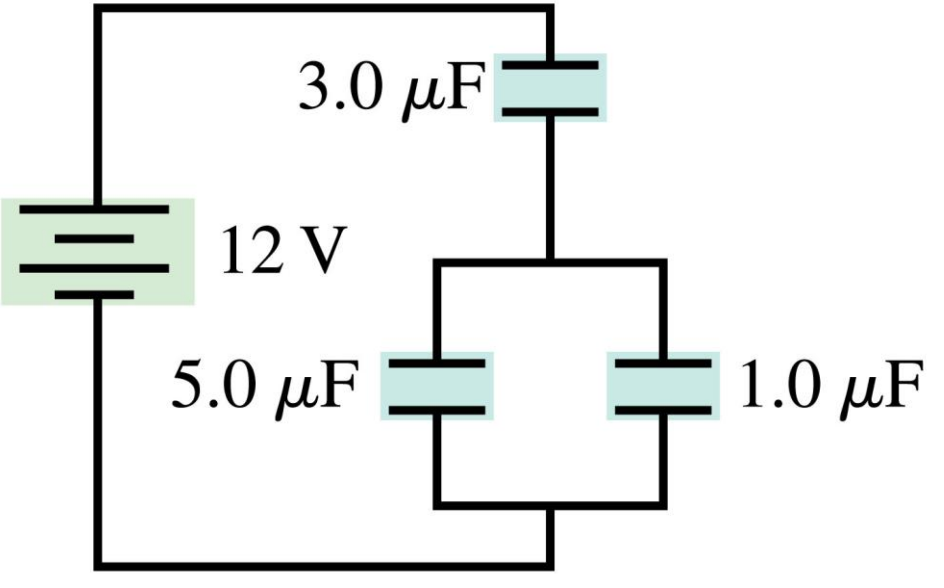


A. Capacitor A
B. Capacitor B
C. They discharge at the same rate.
D. We can't say without knowing the initial amount of charge.

Example 23.10 Analyzing a capacitor circuit

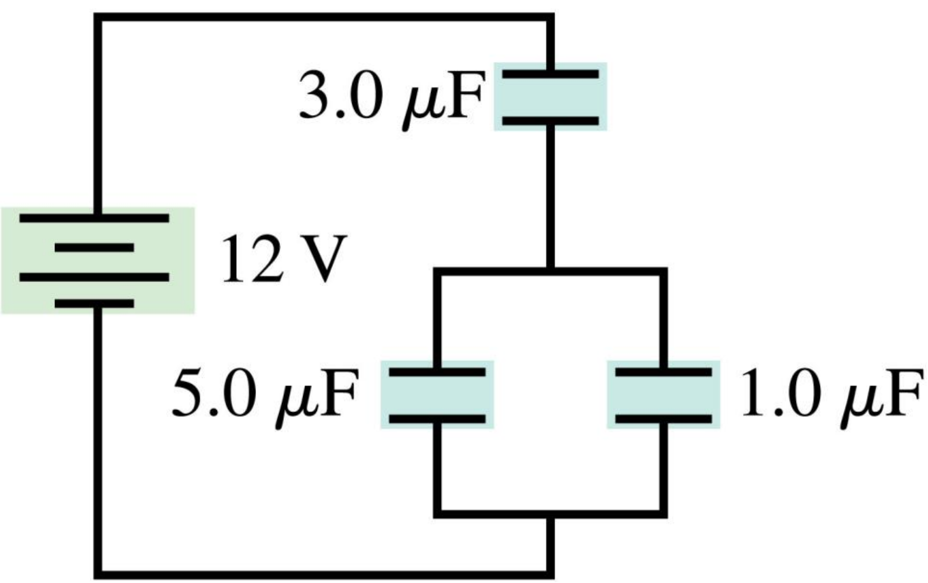
a. Find the equivalent capacitance of the combination of capacitors in the circuit of **FIGURE 23.35**.

b. What charge flows through the battery as the capacitors are being charged?

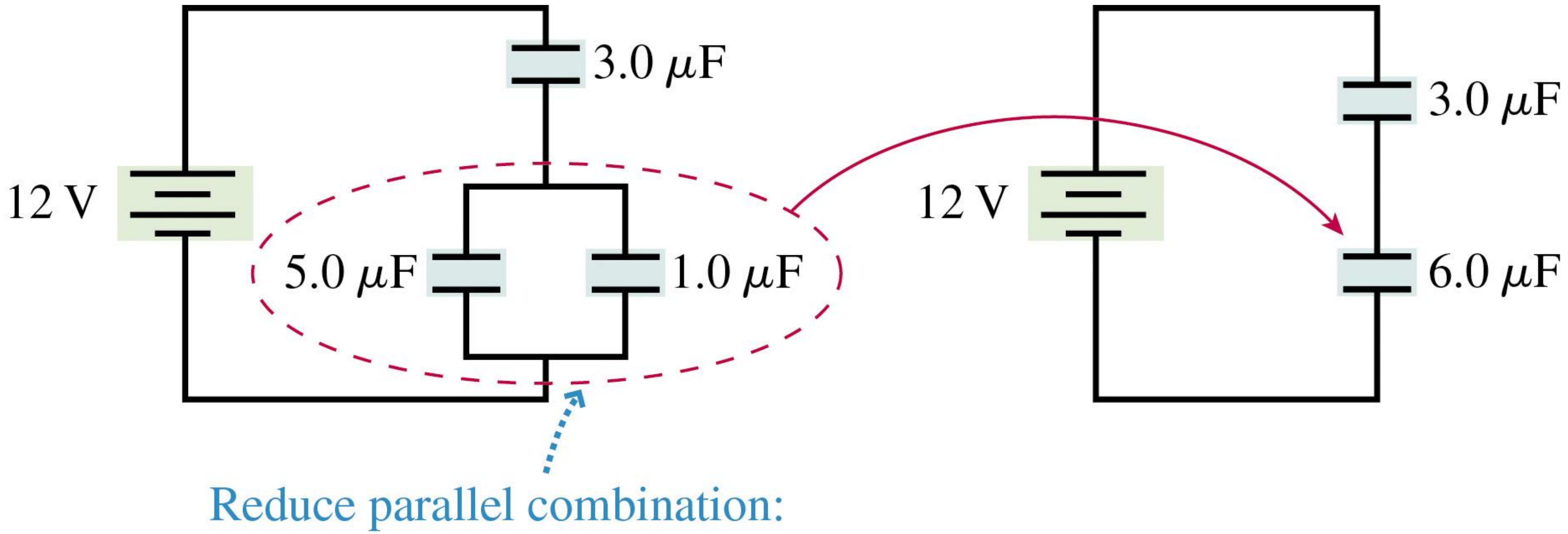


Example 23.10 Analyzing a capacitor circuit (cont.)

PREPARE We can use the relationships for parallel and series capacitors to reduce the capacitors to a single equivalent capacitance, much as we did for resistor circuits. We can then compute the charge through the battery using this value of capacitance.

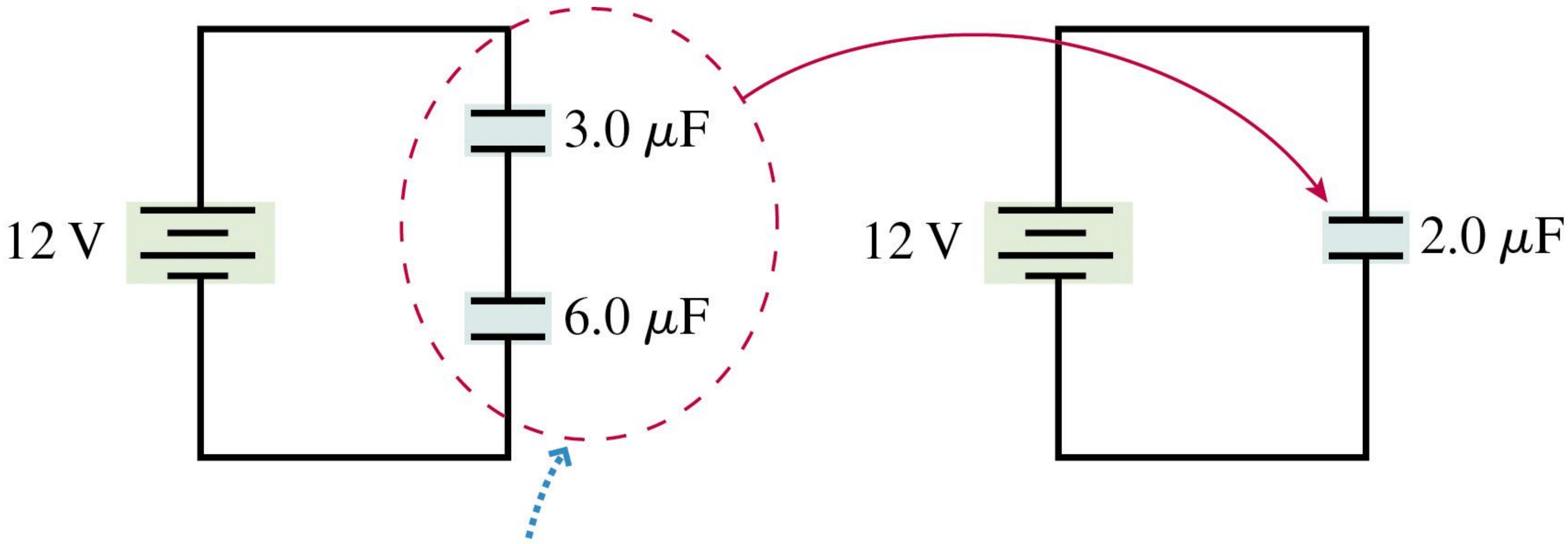


Example 23.10 Analyzing a capacitor circuit (cont.)



$C_{eq} = 5.0\ \mu F + 1.0\ \mu F = 6.0\ \mu F$

Example 23.10 Analyzing a capacitor circuit (cont.)



Reduce series combination:

$$C_{eq} = \left(\frac{1}{3.0\ \mu F} + \frac{1}{6.0\ \mu F}\right)^{-1} = 2.0\ \mu F$$

Example 23.10 Analyzing a capacitor circuit (cont.)

The battery sees a capacitance of 2.0 μF. To establish a potential difference of 12 V, the charge that must flow is

$$Q = C_{\text{eq}}\,\Delta V_{\text{C}} = (2.0 \times 10^{-6}\ \text{F})(12\ \text{V}) = 2.4 \times 10^{-5}\ \text{C}$$

ASSESS We solve capacitor circuit problems in a manner very similar to what we followed for resistor circuits.

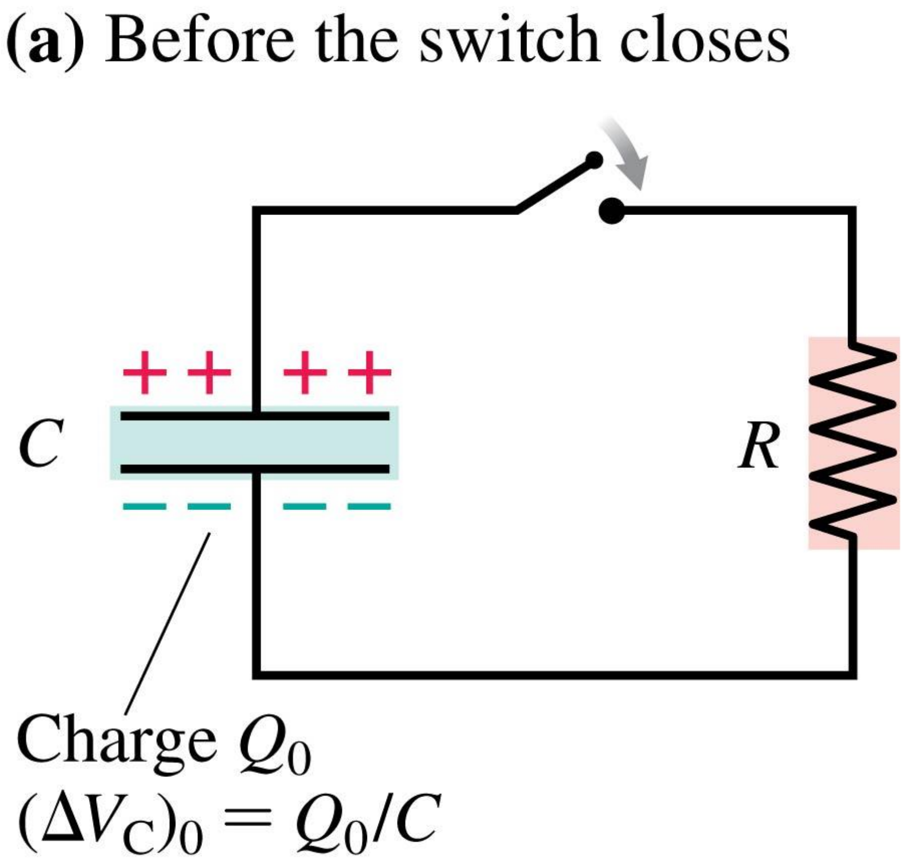
Section 23.7 *RC* Circuits

RC Circuits

- ***RC* circuits** are circuits containing resistors and capacitors.
- In *RC* circuits, the current varies with time.
- The values of the resistance and the capacitance in an *RC* circuit determine the *time* it takes the capacitor to charge or discharge.

RC Circuits

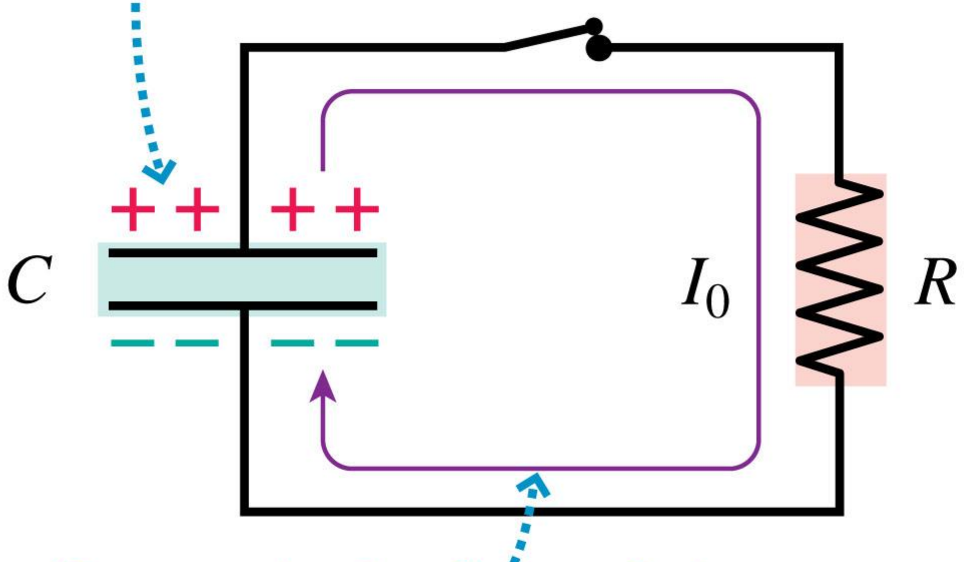
- The figure shows an *RC* circuit consisting of a charged capacitor, an open switch, and a resistor before the switch closes. The switch will close at $t = 0$.



RC Circuits

(b) Immediately after the switch closes

The charge separation on the capacitor produces a potential difference, which causes a current.



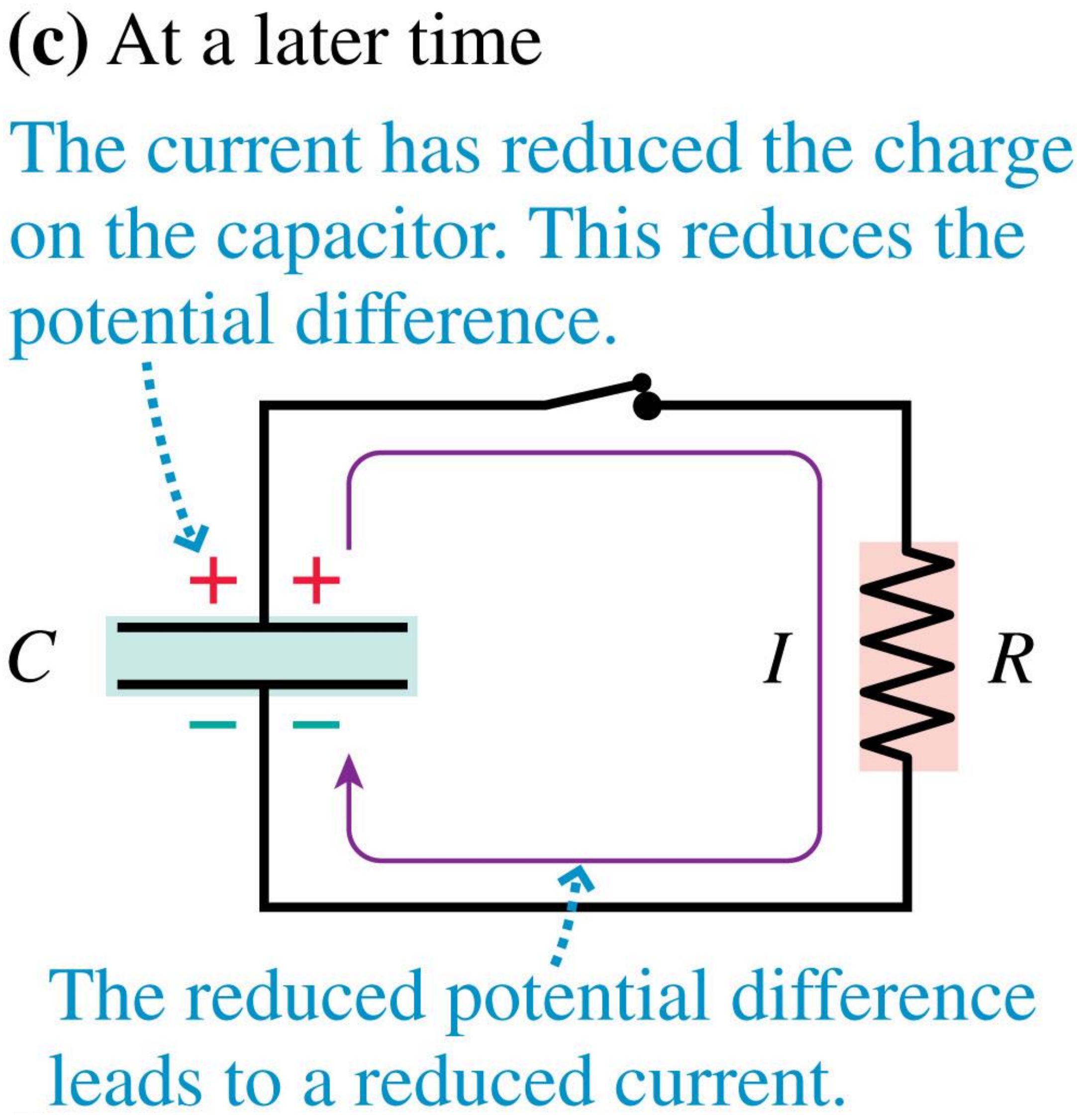
Current is the flow of charge, so the current discharges the capacitor.

RC Circuits

- The initial potential difference is $(\Delta V_C)_0 = Q_0/C$.
- The *initial* current—the initial rate at which the capacitor begins to discharge—is

$$I_0 = \frac{(\Delta V_C)_0}{R}$$

RC Circuits



(c) At a later time
The current has reduced the charge on the capacitor. This reduces the potential difference.
+ +
C
− −
I
R
The reduced potential difference leads to a reduced current.

RC Circuits

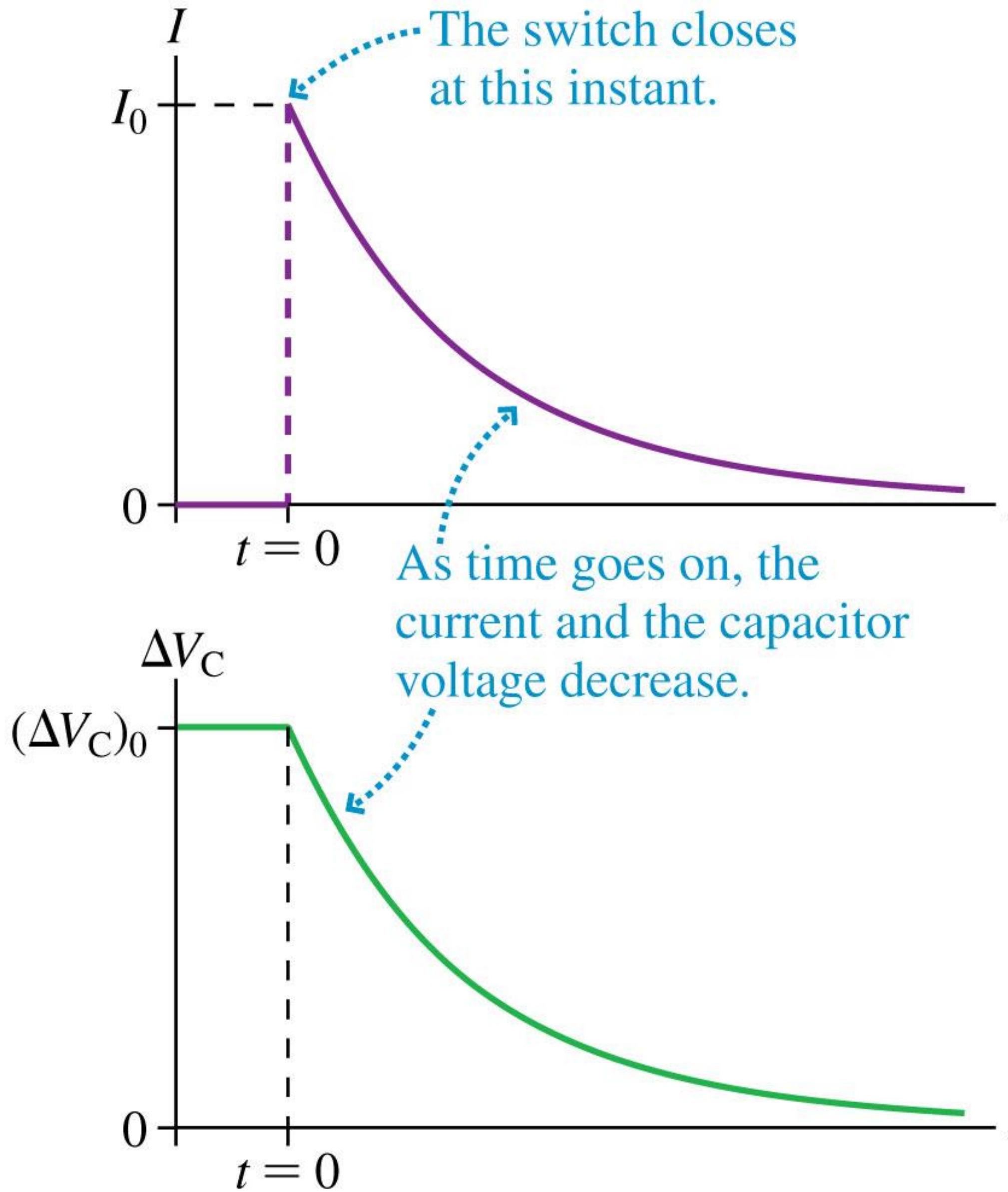
- After some time, both the charge on the capacitor (and thus the potential difference) and the current in the circuit have decreased.
- When the capacitor voltage has decreased to ΔV_C, the current has decreased to

$$I = \frac{\Delta V_C}{R}$$

- The current and the voltage decrease until the capacitor is fully discharged and the current is zero.

RC Circuits

- The current and the capacitor voltage decay to zero after the switch closes, but *not* linearly.



RC Circuits

- The decays of the voltage and the current are *exponential decays*:

$$I = I_0 e^{-t/RC}$$

$$\Delta V_C = (\Delta V_C)_0 e^{-t/RC}$$

Current and voltage during a capacitor discharge

RC Circuits

- The **time constant** τ is a *characteristic time* for a circuit. A long time constant implies a slow decay; a short time constant, a rapid decay:

$$\tau = RC$$

- In terms of the time constant, the current and voltage equations are

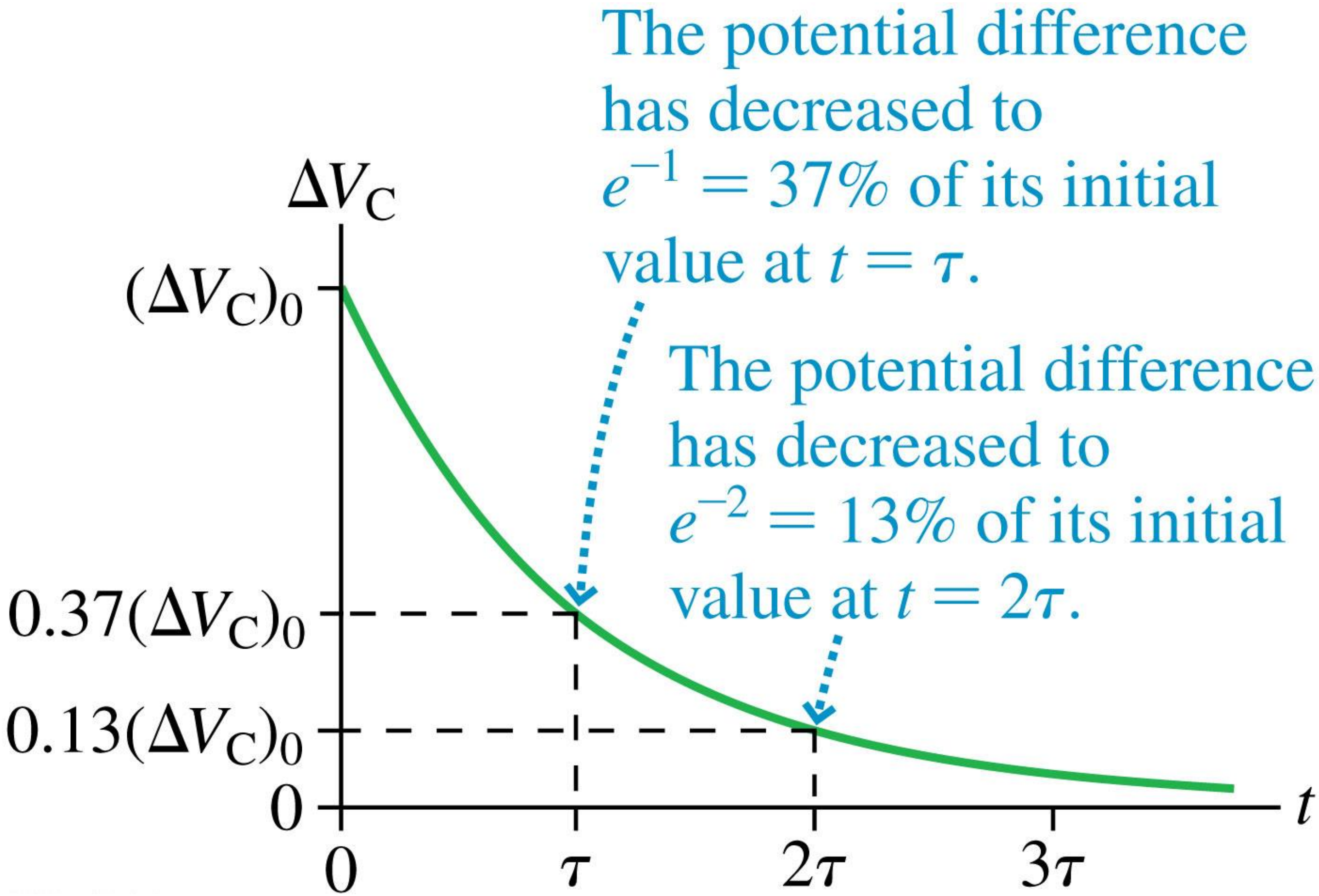
$$I = I_0 e^{-t/\tau}$$

$$\Delta V_C = (\Delta V_C)_0 e^{-t/\tau}$$

RC Circuits

- The current and voltage in a circuit do not drop to zero after one time constant. Each increase in time by one time constant causes the voltage and current to decrease by a factor of $e^{-1} = 0.37$.

RC Circuits



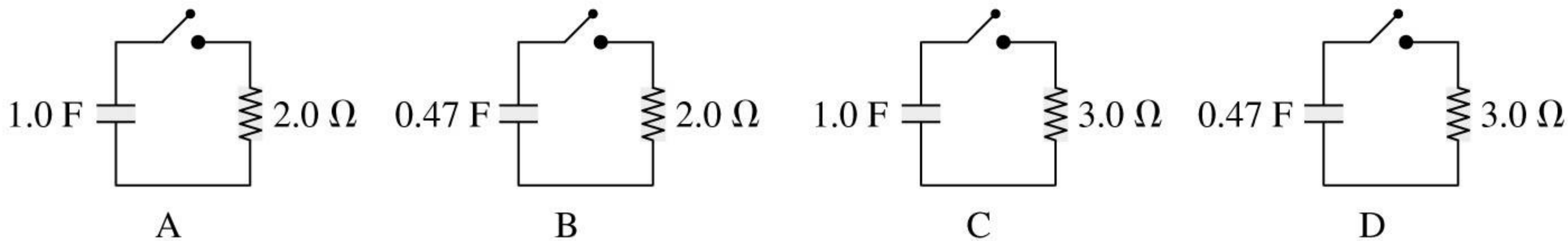
ΔV_C
$(\Delta V_C)_0$
$0.37(\Delta V_C)_0$
$0.13(\Delta V_C)_0$
0
0
τ
2τ
3τ
t
The potential difference has decreased to $e^{-1} = 37\%$ of its initial value at $t = \tau$.
The potential difference has decreased to $e^{-2} = 13\%$ of its initial value at $t = 2\tau$.

RC Circuits

- The time constant has the form $\tau = RC$.
- A large resistance opposes the flow of charge, so increasing *R* increases the decay time.
- A larger capacitance stores more charge, so increasing *C* also increases the decay time.

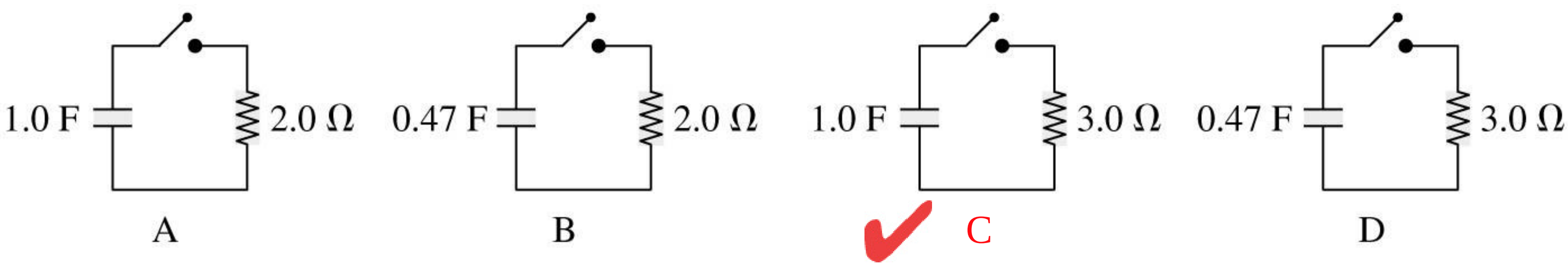
QuickCheck 23.23

The following circuits contain capacitors that are charged to 5.0 V. All of the switches are closed at the same time. After 1 second has passed, which capacitor is charged to the highest voltage?



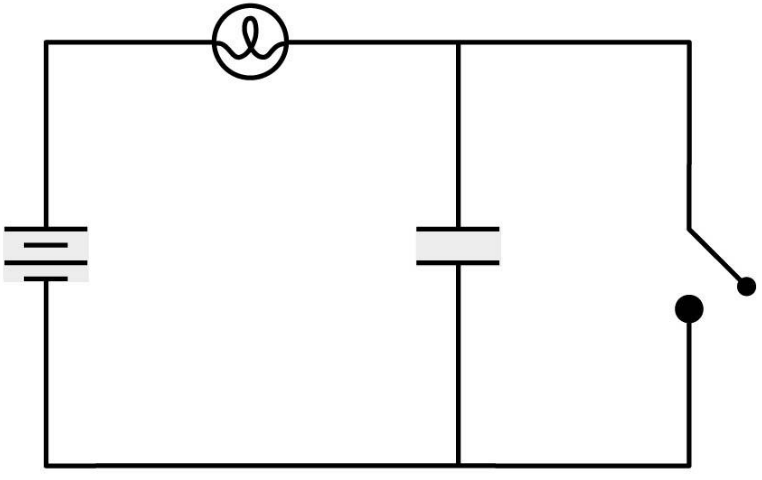
QuickCheck 23.23

The following circuits contain capacitors that are charged to 5.0 V. All of the switches are closed at the same time. After 1 second has passed, which capacitor is charged to the highest voltage?



QuickCheck 23.24

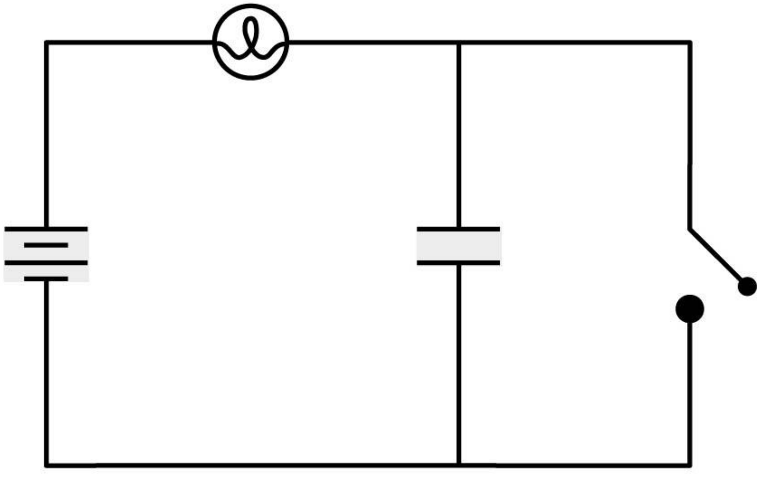
In the following circuit, the switch is initially closed and the bulb glows brightly. When the switch is opened, what happens to the brightness of the bulb?



A. The brightness of the bulb is not affected.
B. The bulb starts at the same brightness and then gradually dims.
C. The bulb starts and the same brightness and then gradually brightens.
D. The bulb initially brightens, then gradually dims.
E. The bulb initially dims, then gradually brightens.

QuickCheck 23.24

In the following circuit, the switch is initially closed and the bulb glows brightly. When the switch is opened, what happens to the brightness of the bulb?



A. The brightness of the bulb is not affected.

✔ B. The bulb starts at the same brightness and then gradually dims.

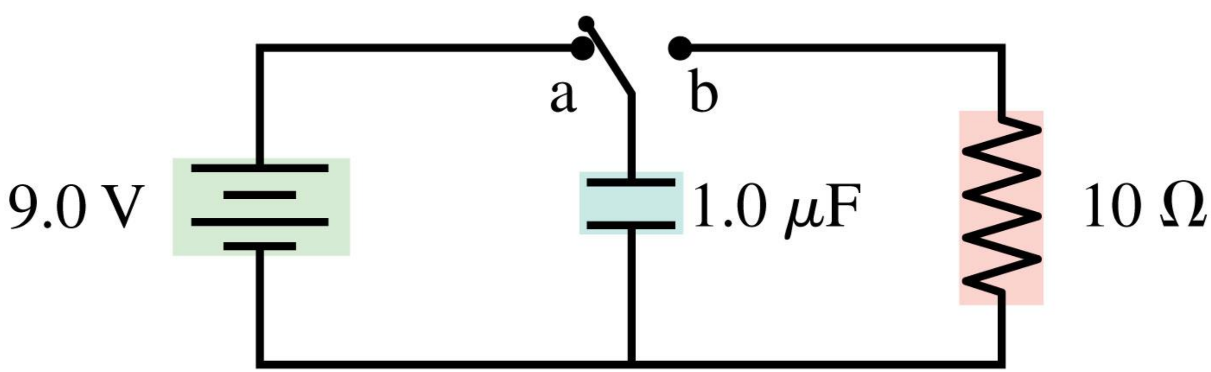
C. The bulb starts and the same brightness and then gradually brightens.

D. The bulb initially brightens, then gradually dims.

E. The bulb initially dims, then gradually brightens.

Example 23.11 Finding the current in an RC circuit

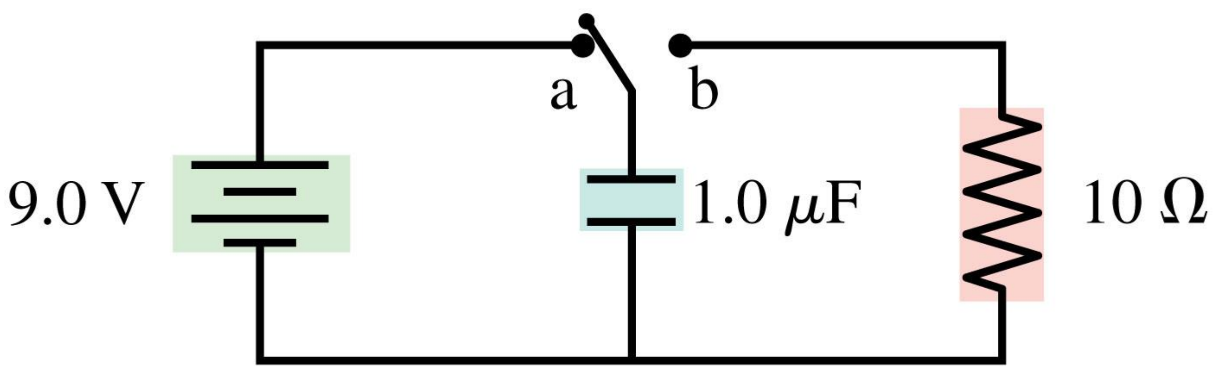
The switch in the circuit of **FIGURE 23.40** has been in position a for a long time, so the capacitor is fully charged. The switch is changed to position b at $t = 0$. What is the current in the circuit immediately after the switch is closed? What is the current in the circuit 25 μs later?



Example 23.11 Finding the current in an RC circuit (cont.)

SOLVE The capacitor is connected across the battery terminals, so initially it is charged to $(\Delta V_C)_0 = 9.0$ V. When the switch is closed, the initial current is given by Equation 23.20:

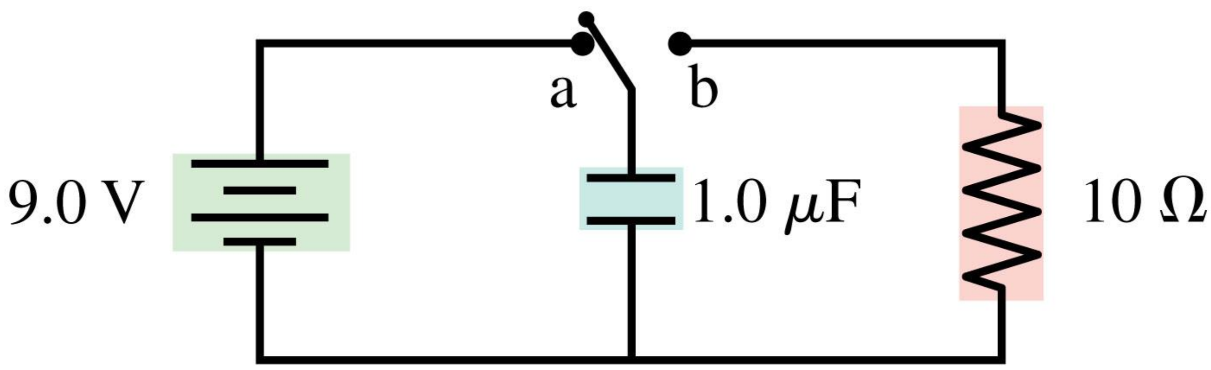
$$I_0 = \frac{(\Delta V_C)_0}{R} = \frac{9.0\ \text{V}}{10\ \Omega} = 0.90\ \text{A}$$



Example 23.11 Finding the current in an RC circuit (cont.)

As charge flows, the capacitor discharges. The time constant for the decay is given by Equation 23.23:

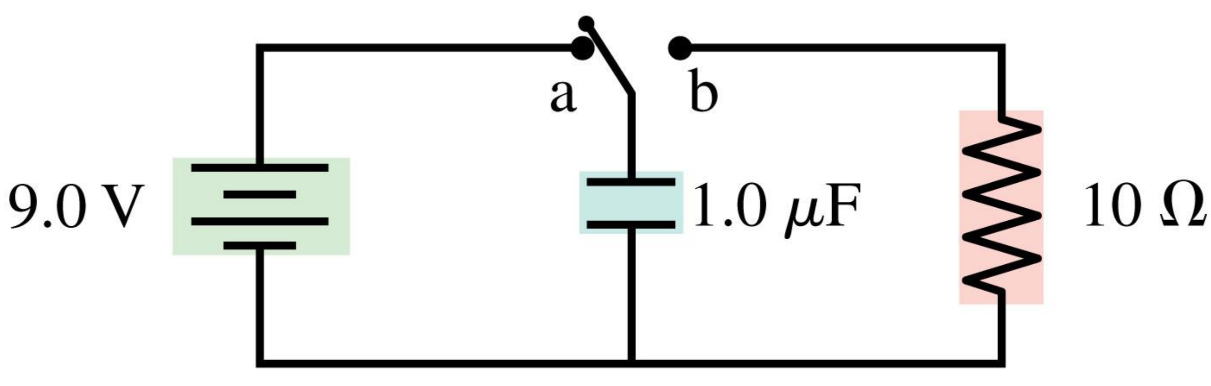
$$\tau = (10\ \Omega)(1.0 \times 10^{-6}\ \text{F}) = 1.0 \times 10^{-5}\ \text{s} = 10\ \mu\text{s}$$



Example 23.11 Finding the current in an RC circuit (cont.)

The current in the circuit as a function of time is given by Equation 23.22. 25 μs after the switch is closed, the current is

$$\tau = (10\ \Omega)(1.0 \times 10^{-6}\ \text{F}) = 1.0 \times 10^{-5}\ \text{s} = 10\ \mu\text{s}$$

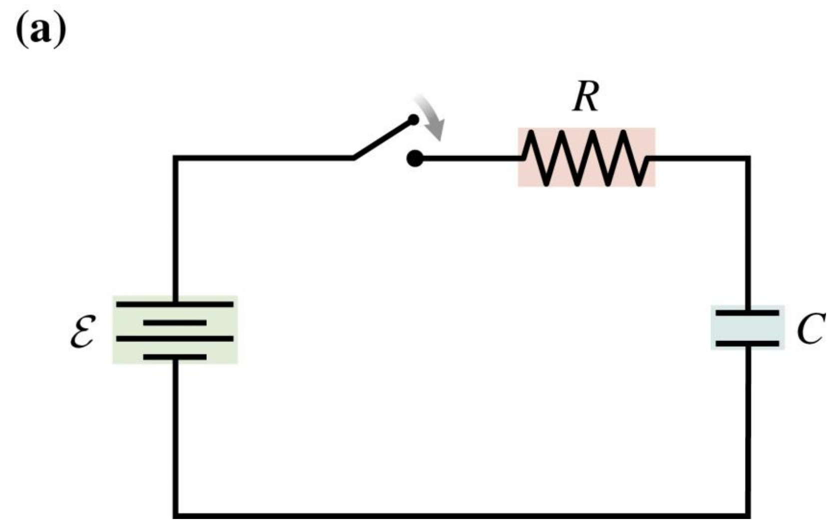


Example 23.11 Finding the current in an RC circuit (cont.)

ASSESS This result makes sense. 25 μs after the switch has closed is 2.5 time constants, so we expect the current to decrease to a small fraction of the initial current. Notice that we left times in units of μs; this is one of the rare cases where we needn't convert to SI units. Because the exponent is $-t/\tau$, which involves a ratio of two times, we need only be certain that both t and τ are in the same units.

Charging a Capacitor

- In a circuit that charges a capacitor, once the switch is closed, the potential difference of the battery causes a current in the circuit, and the capacitor begins to charge.



- As the capacitor charges, it develops a potential difference that opposes the current, so the current decreases, and so does the rate of charging.
- The capacitor charges until $\Delta V_C = \mathcal{E}$.

Charging a Capacitor

- The equations that describe the capacitor voltage and the current as a function of time are

$$I = I_0 e^{-t/RC}$$

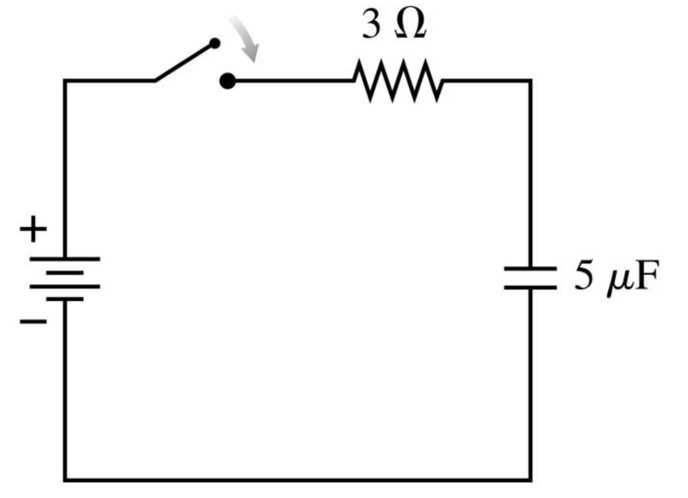
$$\Delta V_C = \mathcal{E}(1 - e^{-t/RC})$$

Current and voltage while charging a capacitor

QuickCheck 23.25

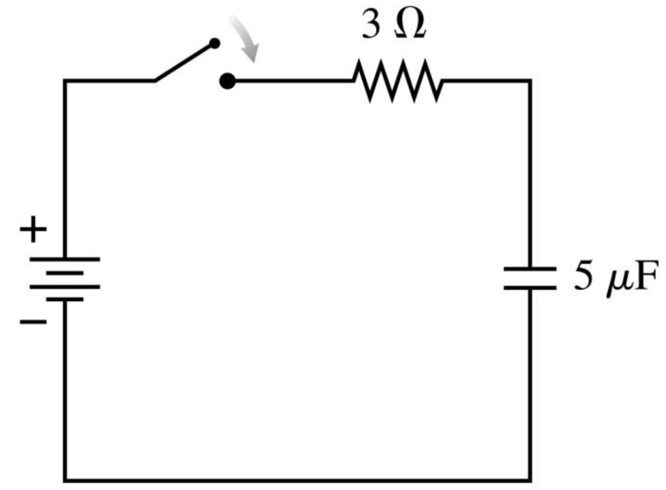
The capacitor is initially unchanged. Immediately after the switch closes, the capacitor voltage is

A. 0 V
B. Somewhere between 0 V and 6 V
C. 6 V
D. Undefined.



QuickCheck 23.25

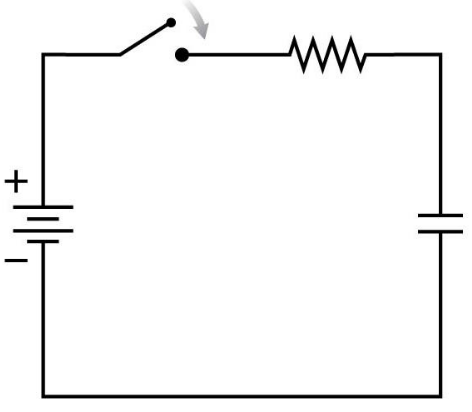
The capacitor is initially unchanged. Immediately after the switch closes, the capacitor voltage is

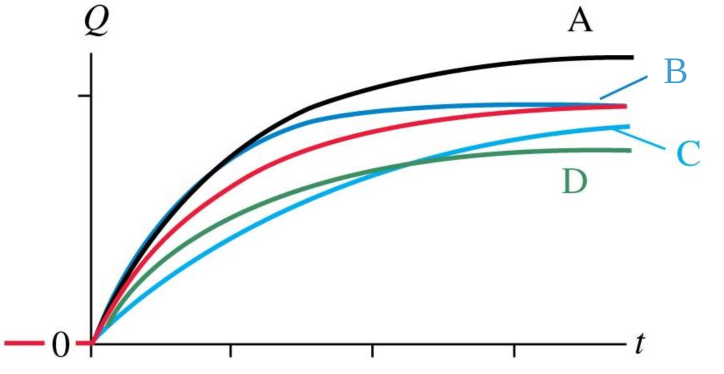


- ✔ A. 0 V
- B. Somewhere between 0 V and 6 V
- C. 6 V
- D. Undefined.

QuickCheck 23.26

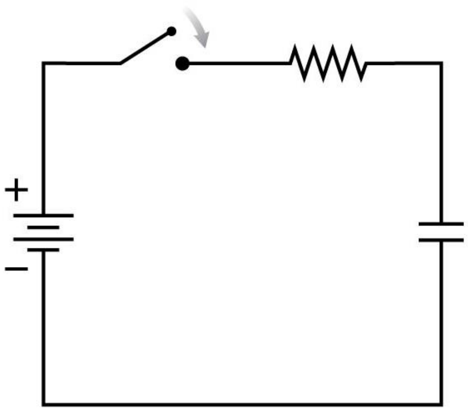
The red curve shows how the capacitor charges after the switch is closed at $t = 0$. Which curve shows the capacitor charging if the value of the resistor is reduced?

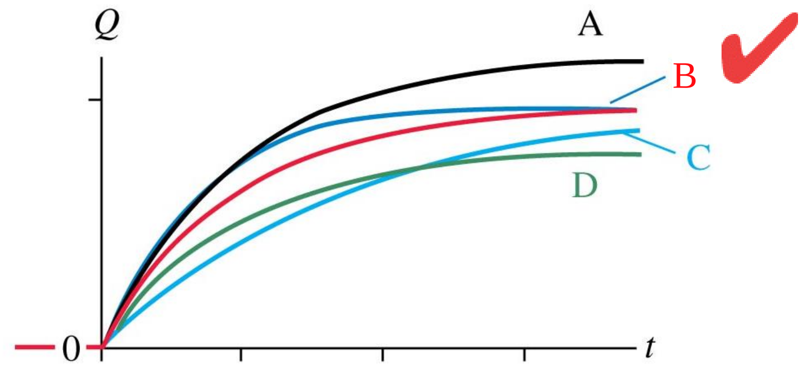




QuickCheck 23.26

The red curve shows how the capacitor charges after the switch is closed at $t = 0$. Which curve shows the capacitor charging if the value of the resistor is reduced?

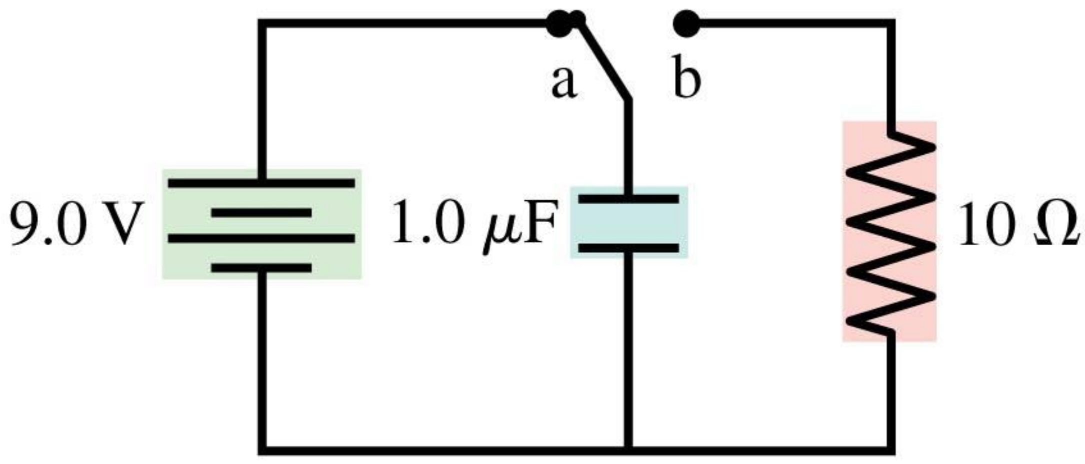




Smaller time constant.
Same ultimate amount of charge.

Example Problem

The switch in the circuit shown has been in position a for a long time, so the capacitor is fully charged. The switch is changed to position b at $t = 0$. What is the current in the circuit immediately after the switch is closed? What is the current in the circuit 25 μs later?



Section 23.8 Electricity in the Nervous System

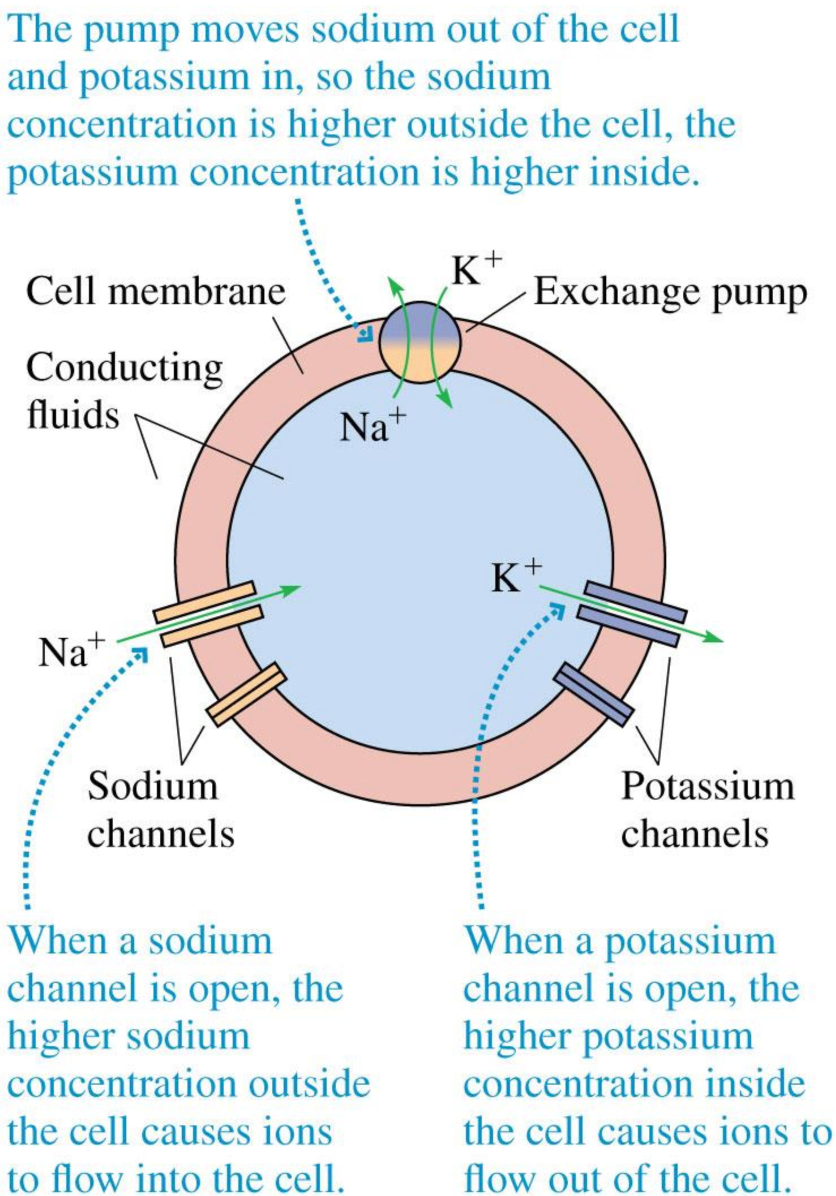
Electricity in the Nervous System

- In the late 1700s, the scientist Galvini and others revealed that electrical signals can animate muscle cells, and a small potential applied to the *axon* of a nerve cell can produce a signal that propagates down its length.

The Electrical Nature of Nerve Cells

- A simple *model* of a nerve cell begins with a *cell membrane*, an insulating layer of lipids approximately 7 nm thick that separates regions of conducting fluid inside and outside the cell.
- Ions, rather than electrons, are the charge carriers of the cell.
- Our simple model will only consider the transport of two positive ions, sodium and potassium, although other ions are important as well.

The Electrical Nature of Nerve Cells



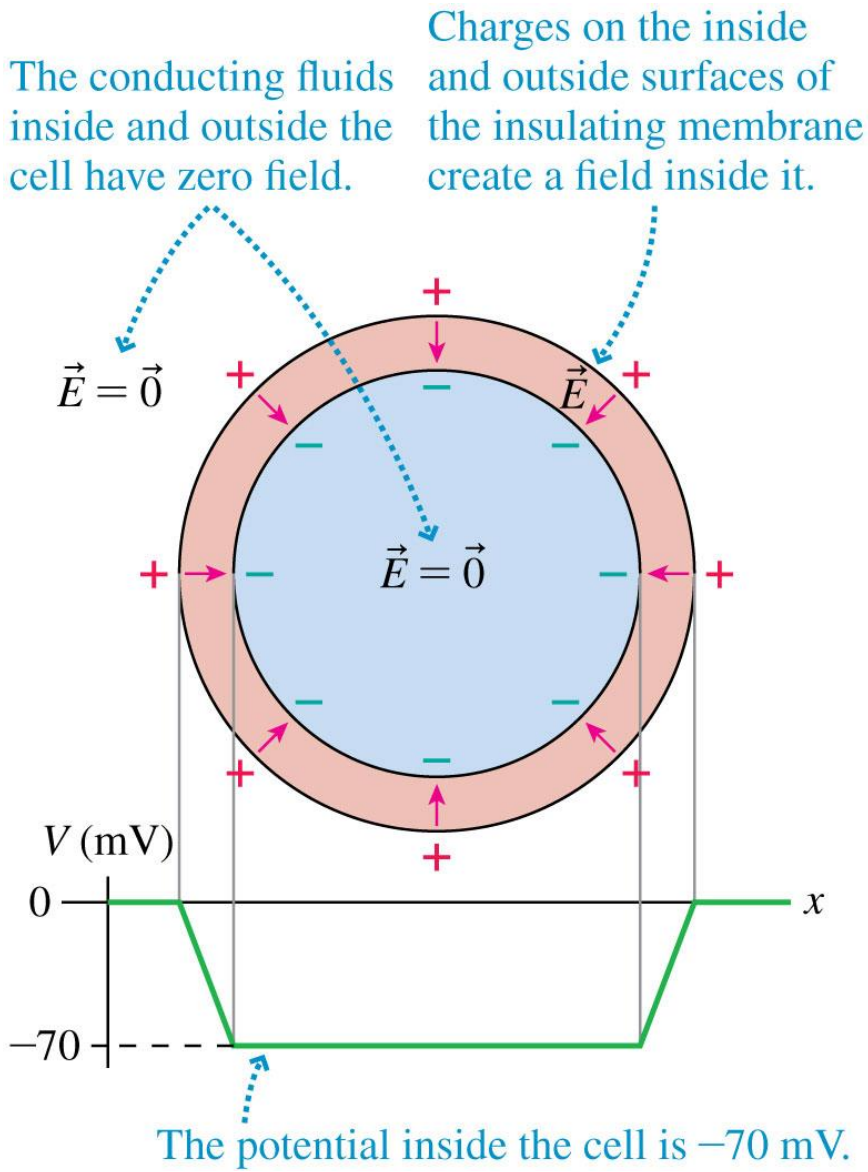
The Electrical Nature of Nerve Cells

- The ion exchange pumps act much like the charge escalator of a battery, using chemical energy to separate charge by transporting ions.
- **A living cell generates an emf.**
- The charge separation produces an electric field inside the cell membrane and results in a potential difference between the inside and outside of the cell.

The Electrical Nature of Nerve Cells

- The potential inside a nerve cell is less than the outside of the cell. It is called the cell's *resting potential*.
- Because the potential difference is produced by a charge separation across the membrane, we say the membrane is *polarized.*
- Because the potential difference is entirely across the membrane, we call this potential difference the *membrane potential*.

The Electrical Nature of Nerve Cells



Example 23.12 Electric field in a cell membrane

The thickness of a typical nerve cell membrane is 7.0 nm. What is the electric field inside the membrane of a resting nerve cell?

PREPARE The potential difference across the membrane of a resting nerve cell is –70 mV. The inner and outer surfaces of the membrane are equipotentials. We learned in Chapter 21 that the electric field is perpendicular to the equipotentials and is related to the potential difference by $E = \Delta V/d$.

Example 23.12 Electric field in a cell membrane (cont.)

SOLVE The magnitude of the potential difference between the inside and the outside of the cell is 70 mV. The field strength is thus

$$E = \frac{\Delta V}{d} = \frac{70 \times 10^{-3}\ \text{V}}{7.0 \times 10^{-9}\ \text{m}} = 1.0 \times 10^{7}\ \text{V/m}$$

The field points from positive to negative, so the electric field is

$$\vec{E} = (1.0 \times 10^{7}\ \text{V/m, inward})$$

Example 23.12 Electric field in a cell membrane (cont.)

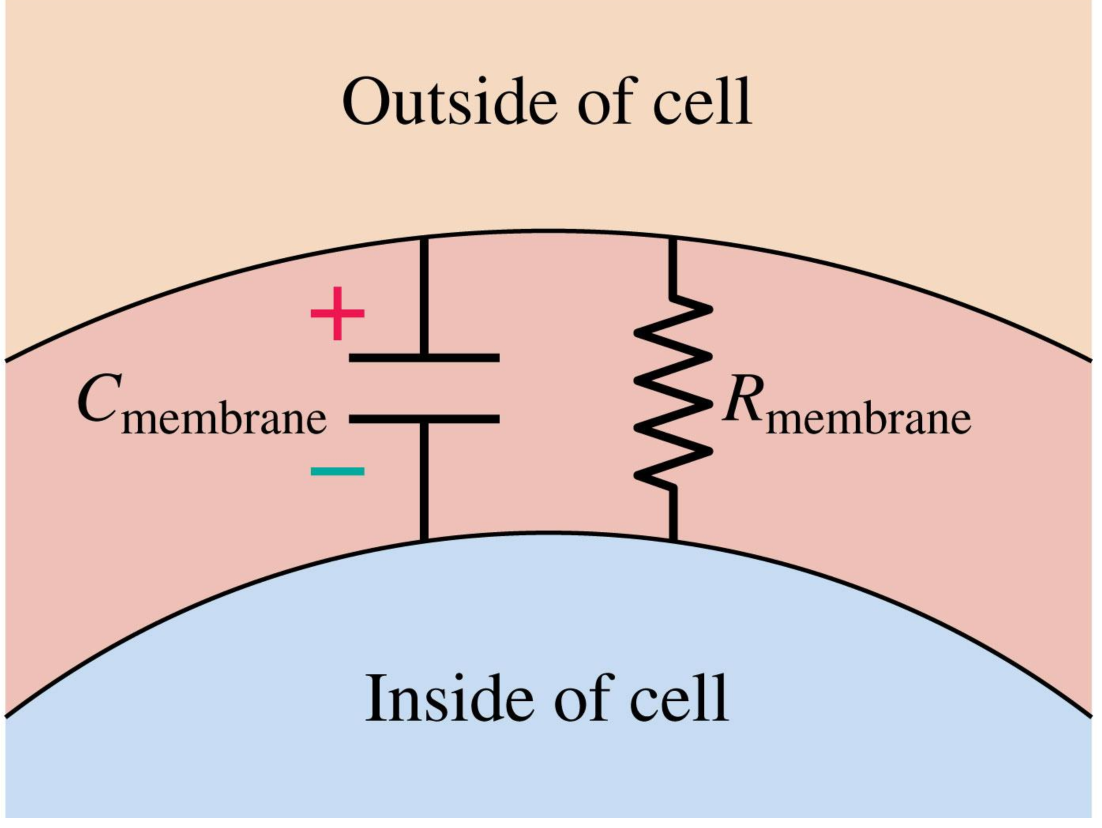
ASSESS This is a very large electric field; in air it would be large enough to cause a spark! But we expect the fields to be large to explain the cell's strong electrical character.

The Electrical Nature of Nerve Cells

- The fluids inside and outside of the membrane are good conductors, but the cell membrane is not. Charges therefore accumulate on the inside and outside surfaces of the membrane.
- A cell thus looks like two charged conductors separated by an insulator—a capacitor.
- The membrane is not a perfect insulator because charges can move across it. The cell membrane will have a certain resistance.

The Electrical Nature of Nerve Cells

- Because the cell membrane has both resistance and capacitance, it can be modeled as an *RC* circuit.
- Like any *RC* circuit, the membrane has a time constant.

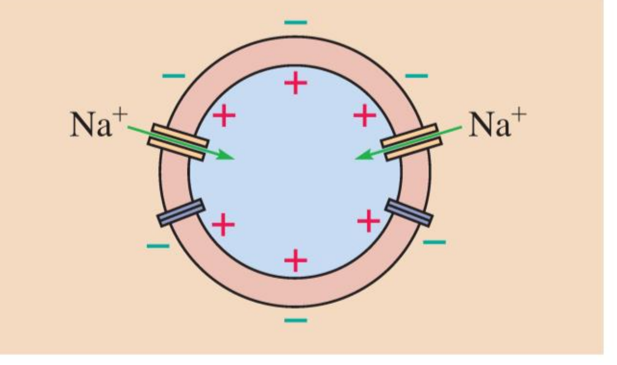


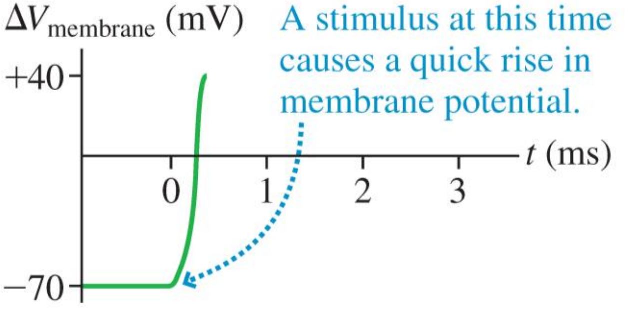
The Action Potential

- The membrane potential can change drastically in response to a *stimulus*.
- *Neurons*—nerve cells—can be stimulated by neurotransmitter chemicals released at synapse junctions.
- A neuron can also be stimulated by a changing potential.
- The result of the stimulus is a rapid change called an *action potential*.

The Action Potential

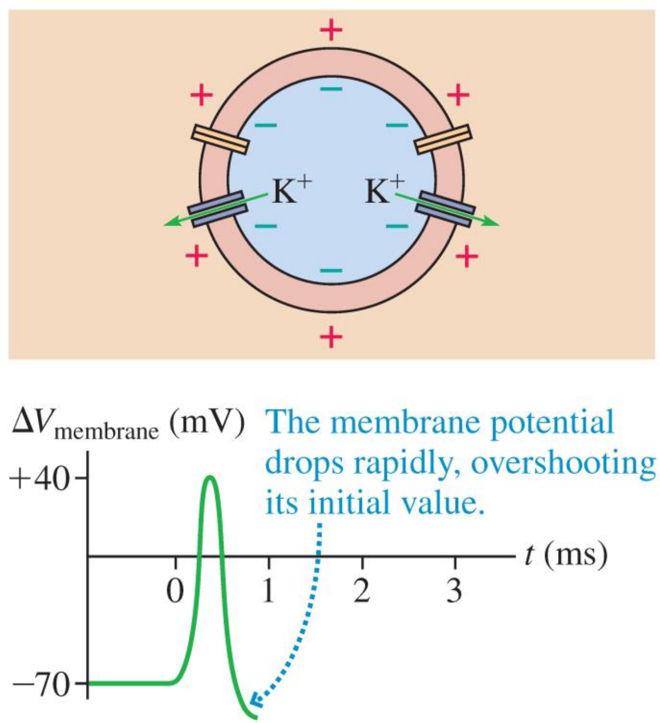
- A cell *depolarizes* when a stimulus causes the opening of the sodium channels. The concentration of sodium ions is much higher outside the cell, so positive sodium ions flow into the cell, rapidly raising its potential to 40 mV, at which point the sodium channels close.





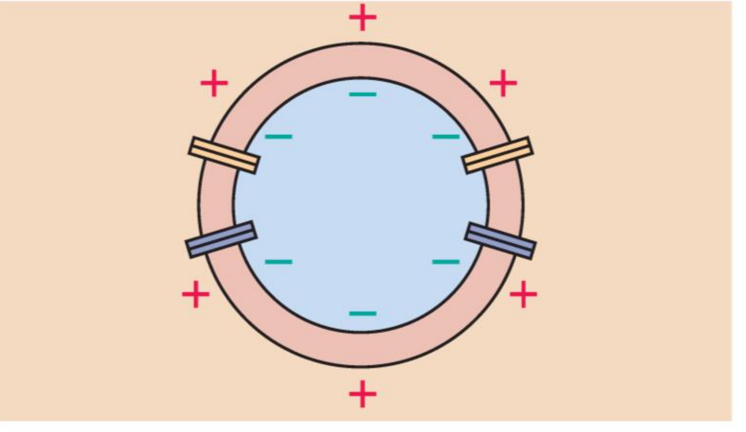
The Action Potential

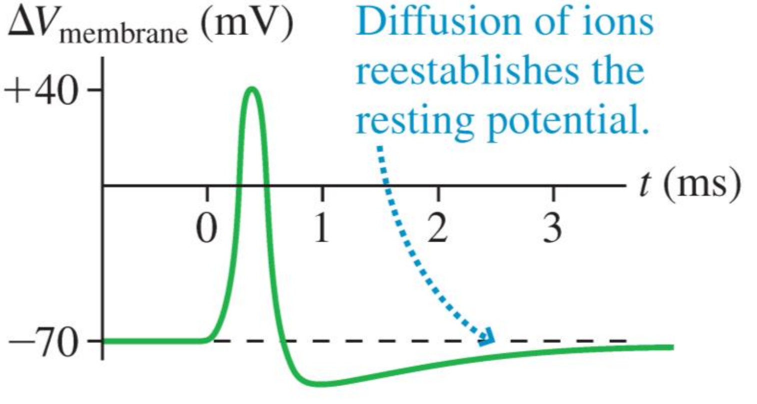
- The cell *repolarizes* as the potassium channels open. The higher potassium concentration inside the cell drives these ions out of the cell. The potassium channels close when the membrane potential reaches about –80 mV, slightly *less* than the resting potential.



The Action Potential

- The reestablishment of the resting potential after the sodium and potassium channels close is a relatively slow process controlled by the motion of ions across the membrane.





The Action Potential

- After the action potential is complete, there is a brief resting period, after which the cell is ready to be triggered again.
- The action potential is driven by ionic conduction through sodium and potassium channels, so the potential changes are rapid.
- Muscles also undergo a similar cycle of polarization.

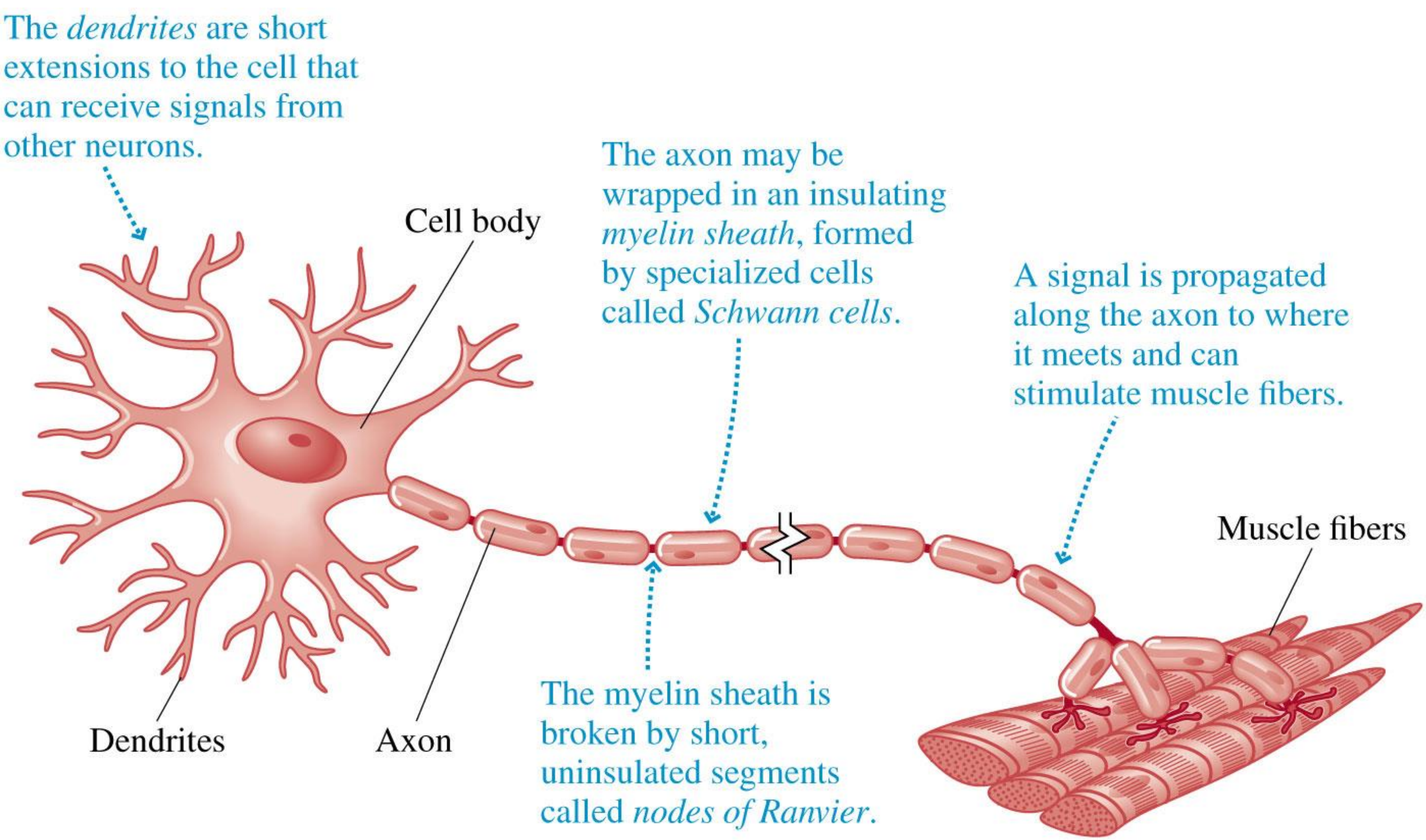
The Propagation of Nerve Impulses

- How is a signal transmitted from the brain to a muscle in the hand?
- The primary cells of the nervous system responsible for signal transmission are known as *neurons*.
- The transmission of a signal to a muscle is the function of a *motor neuron*.
- The transmission of signals takes place along the *axon* of the neuron, a long fiber that connects the cell body to a muscle fiber.

The Propagation of Nerve Impulses

- The signal is transmitted along an axon because the axon is long enough that it may have different potentials at different points. When one point is stimulated, the membrane will depolarize at that point.
- The resulting action potential may trigger depolarization in adjacent parts of the membrane. This *wave* of action potential—a nerve impulse—travels along the axon.

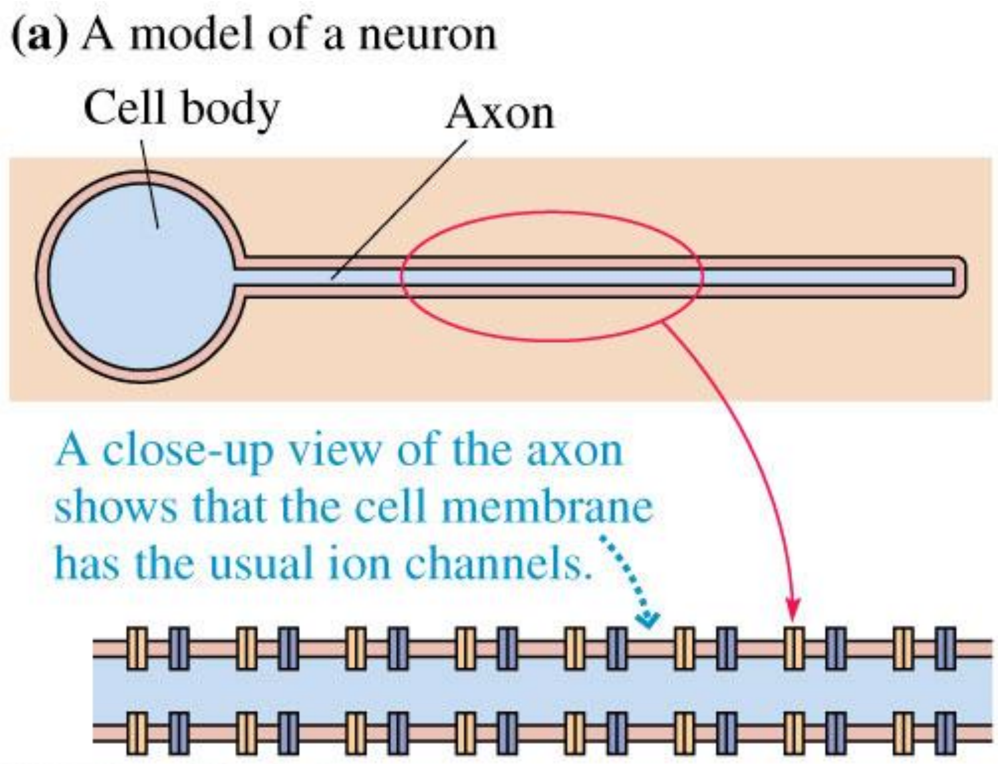
The Propagation of Nerve Impulses



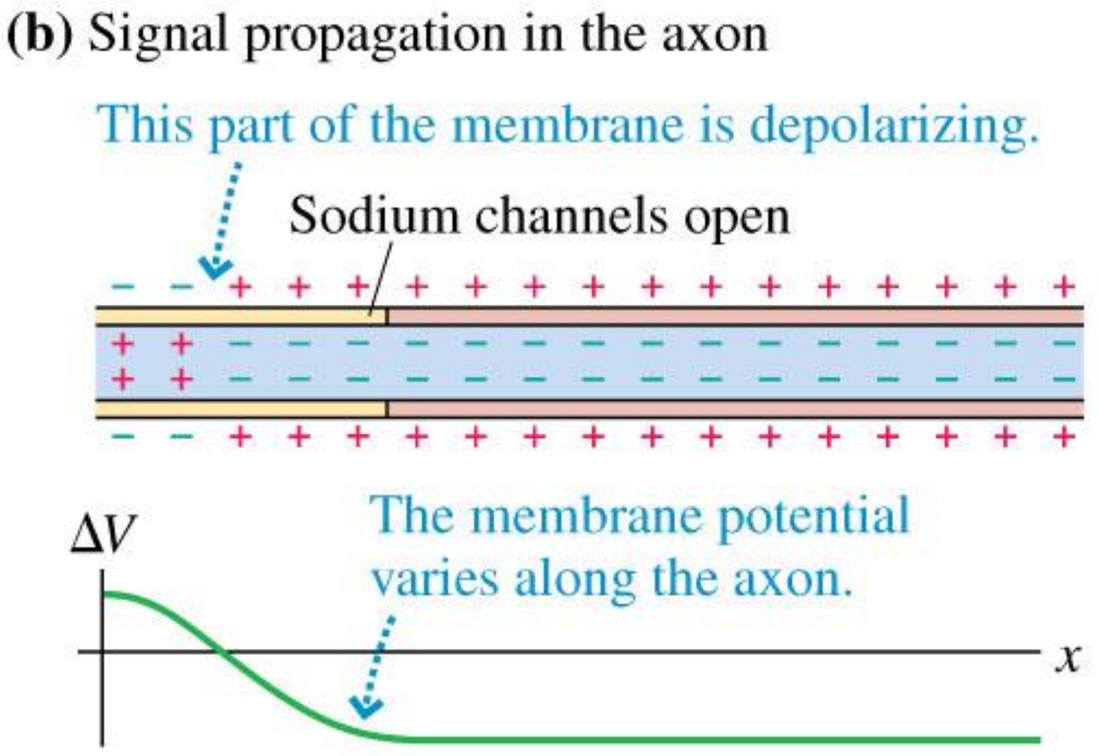
The Propagation of Nerve Impulses

- **A small increase in the potential difference across the membrane of a cell causes the sodium channels to open, triggering a large action potential response.**

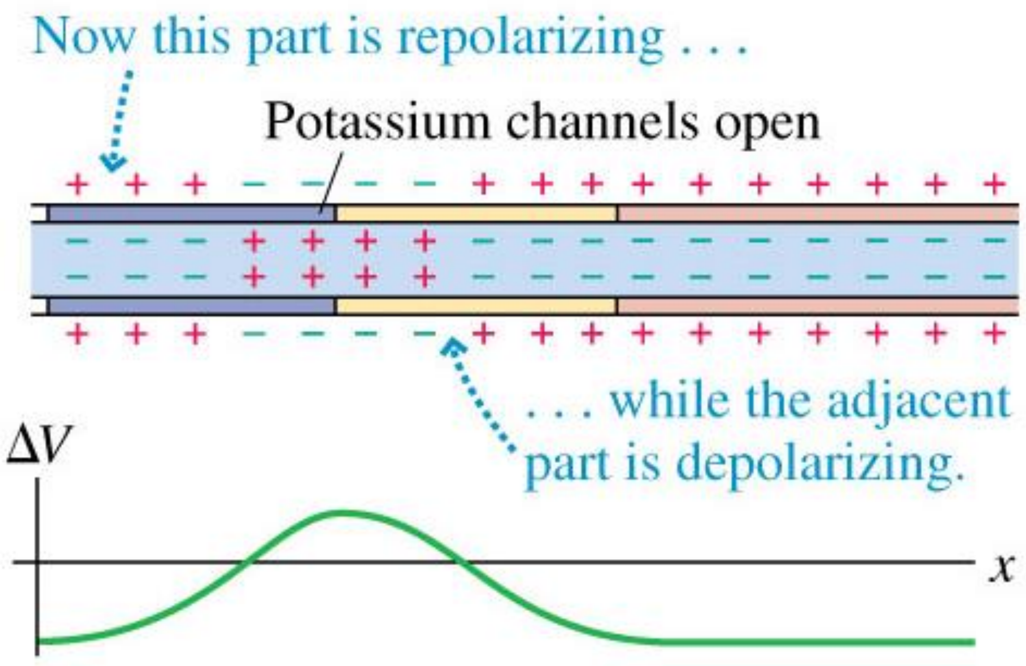
The Propagation of Nerve Impulses



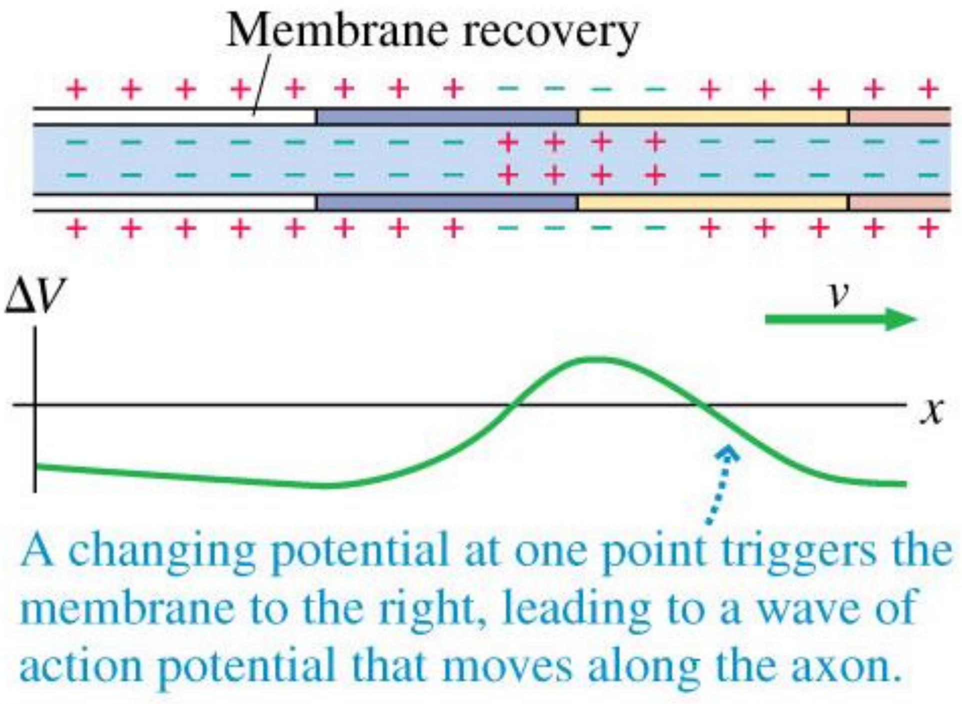
The Propagation of Nerve Impulses



The Propagation of Nerve Impulses



The Propagation of Nerve Impulses

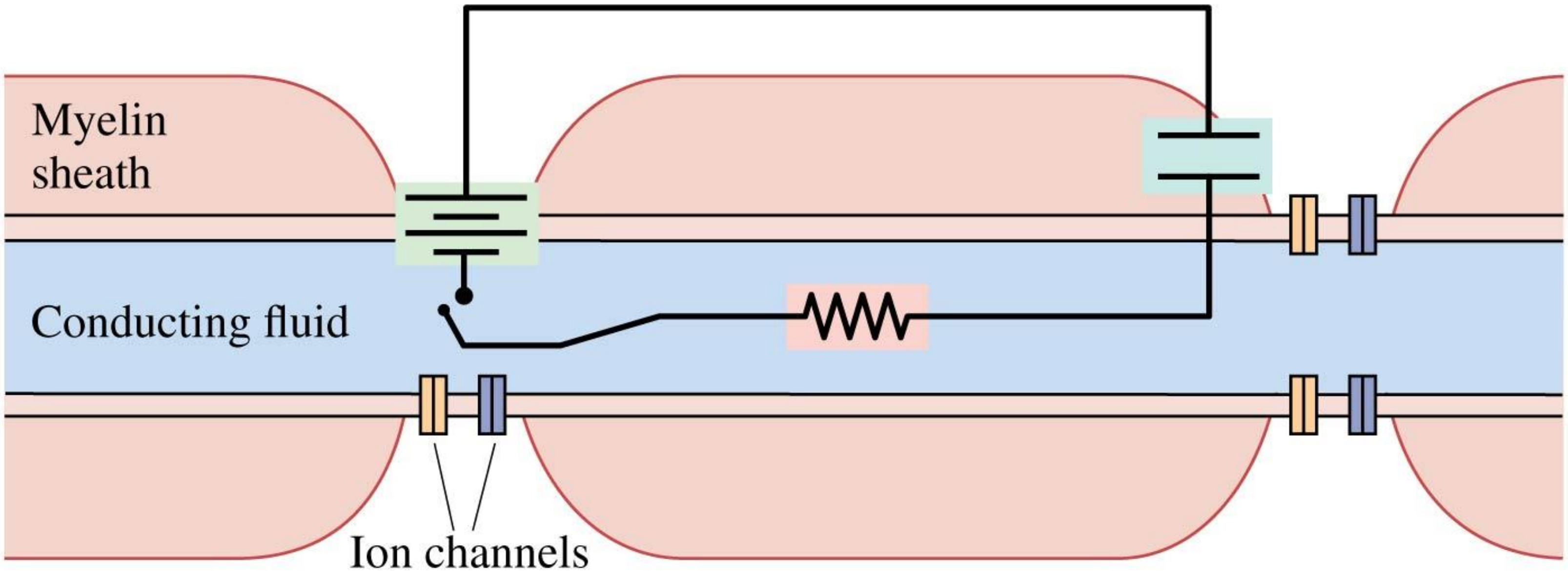


Increasing Speed by Insulation

- The axons of motor neurons and most other neurons can transmit signals at very high speeds because they are insulated with a myelin sheath.
- Schwann cells wrap the axon with myelin, insulating it electrically and chemically, with breaks at the node of Ranvier.
- The ion channels are concentrated in these nodes because this is the only place where the extracellular fluid is in contact with the cell membrane.
- In an insulated axis, a signal propagates by jumping from one node to the next in a process called *saltatory conduction.*

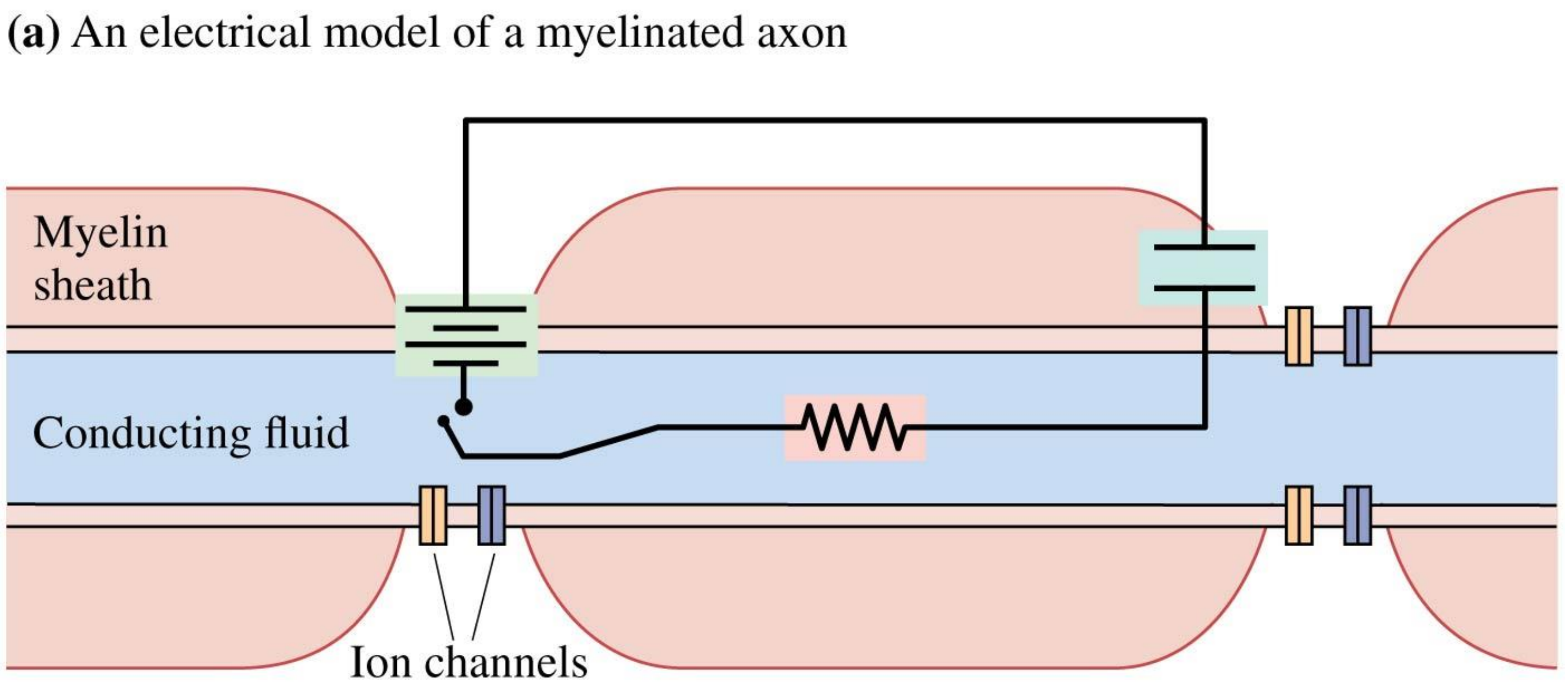
Increasing Speed by Insulation

(a) An electrical model of a myelinated axon



- The ion channels are located at the nodes. When they are triggered, the potential at the node changes rapidly to +40 mV. Thus this section of the axon acts like a battery with a switch.

Increasing Speed by Insulation

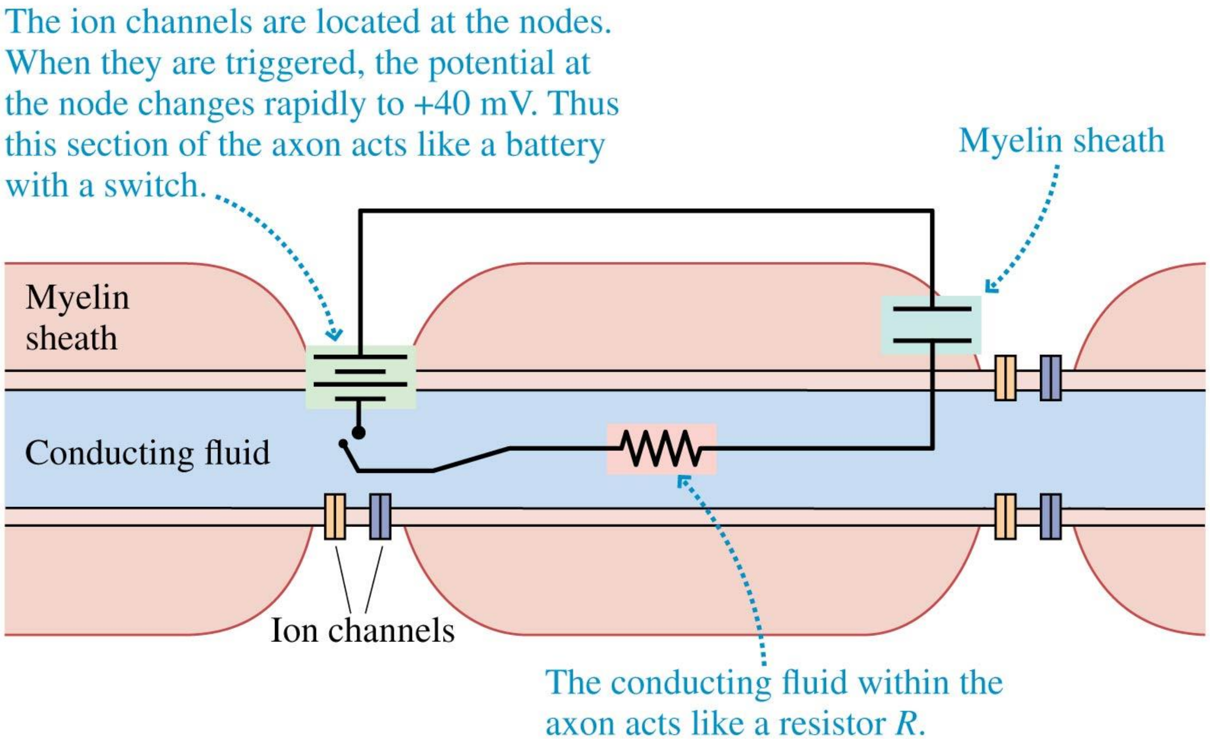


- The conduction fluid within the axon acts like the resistor R.

Increasing Speed by Insulation

A circuit model of nerve-impulse propagation along myelinated axons.

(a) An electrical model of a myelinated axon

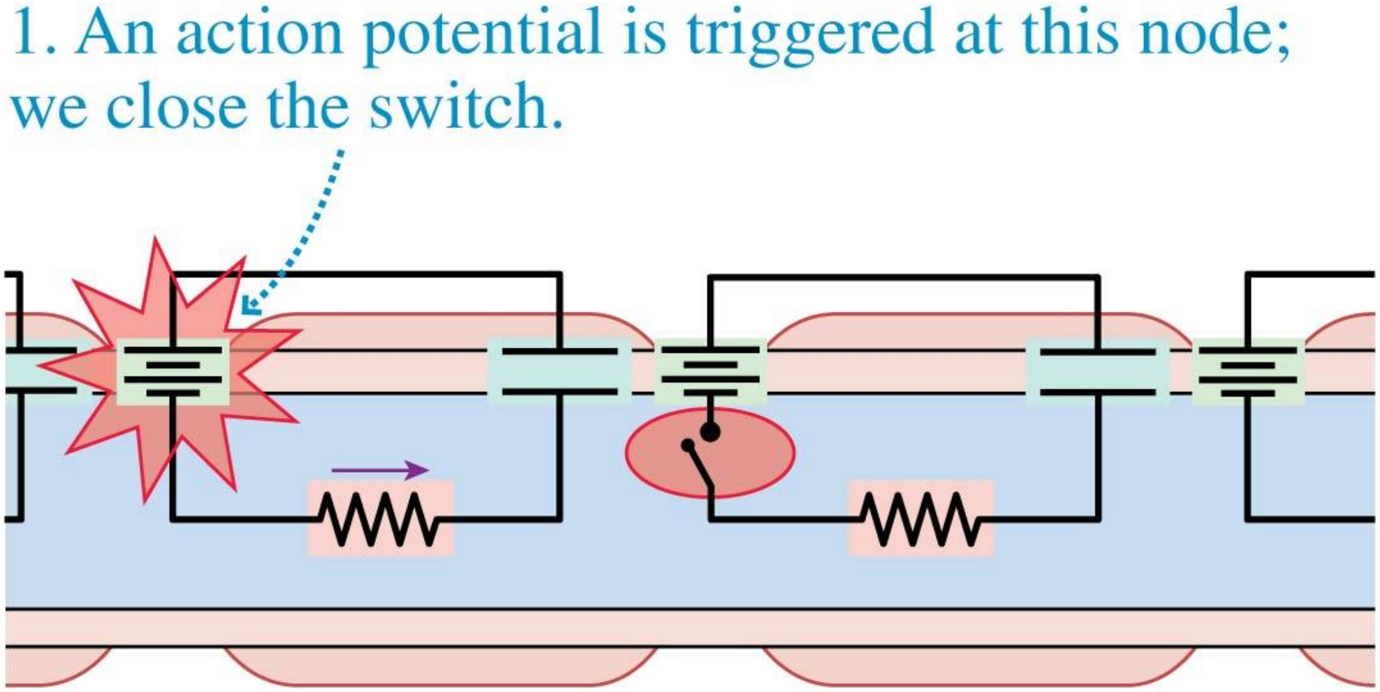


- The myelin sheath acts like the dielectric of a capacitor *C* between the conducting fluids inside and outside the axon.

Increasing Speed by Insulation

A circuit model of nerve-impulse propagation along myelinated axons.

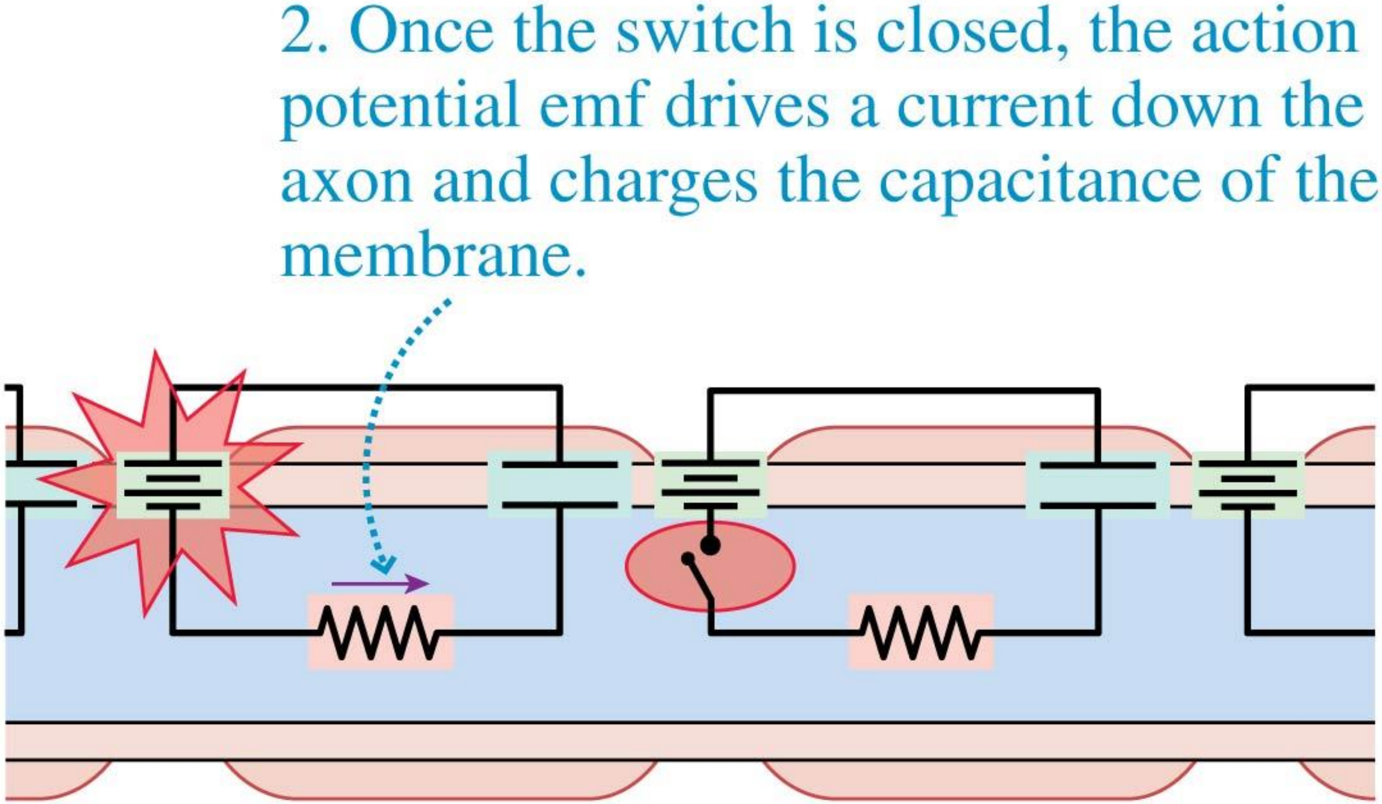
(b) Signal propagation in the myelinated axon



Increasing Speed by Insulation

A circuit model of nerve-impulse propagation along myelinated axons.

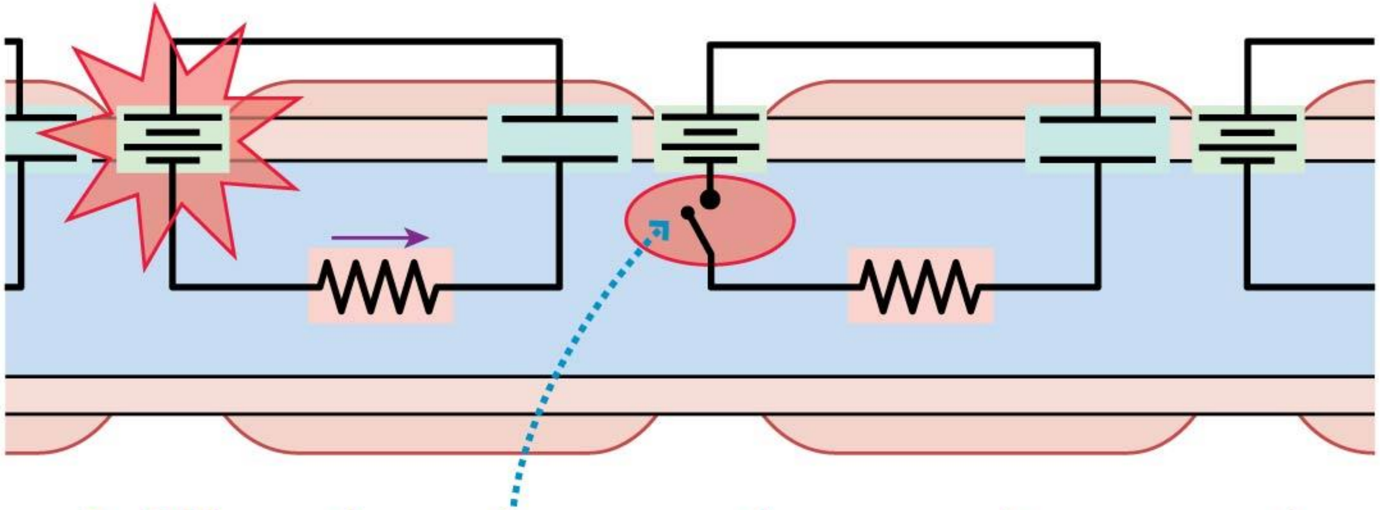
(b) Signal propagation in the myelinated axon



Increasing Speed by Insulation

A circuit model of nerve-impulse propagation along myelinated axons.

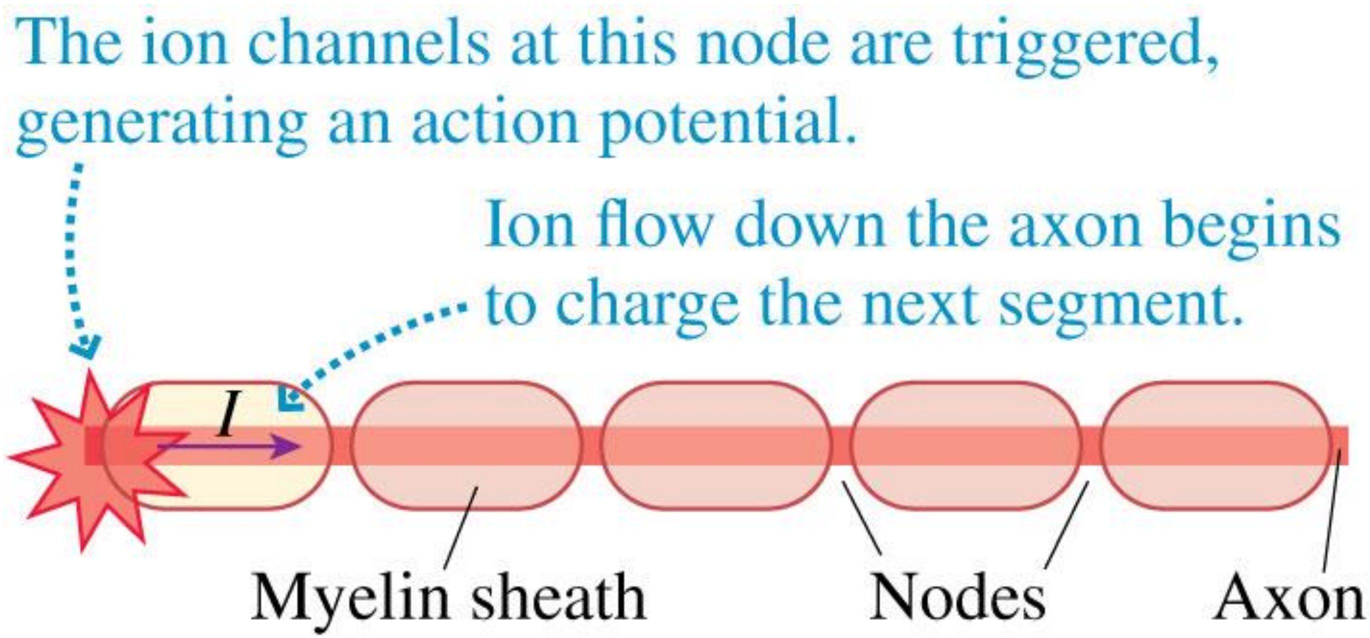
(b) Signal propagation in the myelinated axon



3. When the voltage on the capacitor exceeds a threshold, it triggers an action potential at this node—the next switch is closed.

Increasing Speed by Insulation

- Nerve propagation along a myelinated axon:



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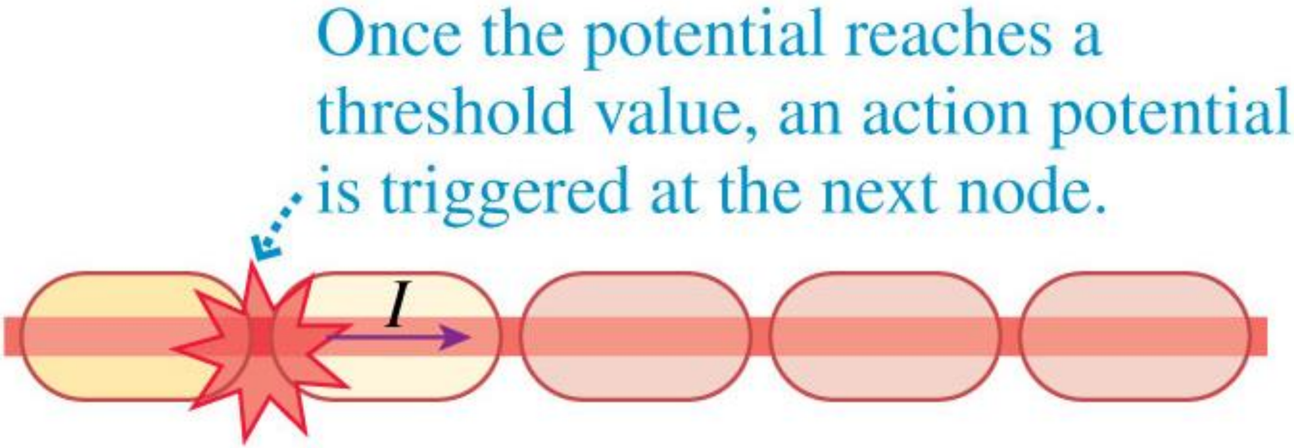

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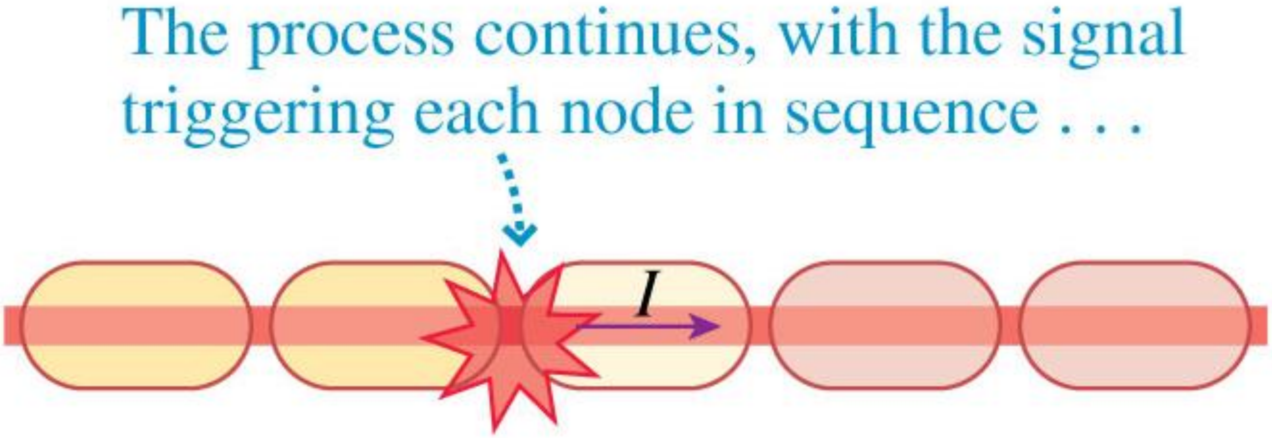

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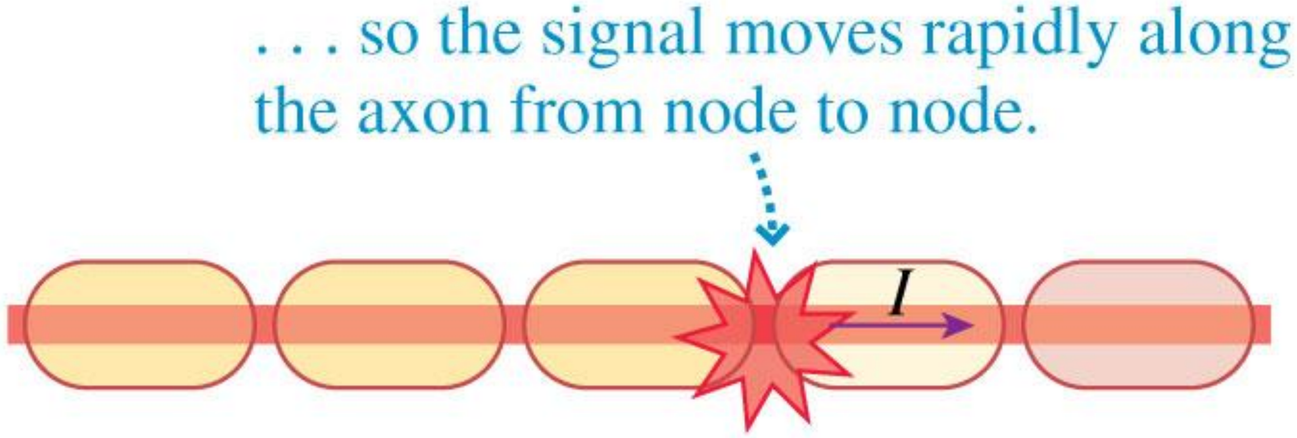


Increasing Speed by Insulation

- How rapidly does a pulse move down a myelinated axon?
- The critical time for propagation is the time constant *RC*.
- The resistance of an axon between nodes is 25 MΩ.
- The capacitance of the membrane is 1.6 pF per segment.

$$\tau = R_{\text{axon}} C_{\text{membrane}} = (25 \times 10^{6}\ \Omega)(1.6 \times 10^{-12}\ \text{F}) = 40\ \mu\text{s}$$

- Because the nodes of Ranvier are spaced about 1 mm apart, the speed at which the nerve impulse travels down the axon is approximately

$$v = \frac{L_{\text{node}}}{\tau} = \frac{1.0 \times 10^{-3}\ \text{m}}{40 \times 10^{-6}\ \text{s}} = 25\ \text{m/s}$$

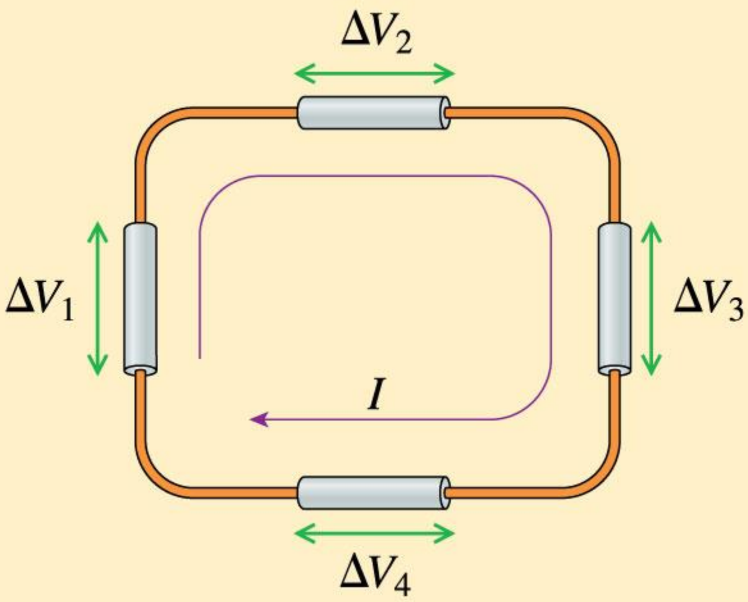
Summary: General Principles

Kirchhoff's loop law

For a closed loop:

- Assign a direction to the current.
- Add potential differences around the loop:

$$\sum_i \Delta V_i = 0$$



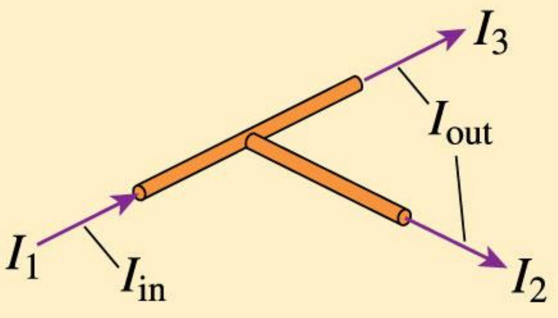
Text: p. 753

Summary: General Principles

Kirchhoff's junction law

For a junction:

$$\sum I_{\text{in}} = \sum I_{\text{out}}$$



Text: p. 753

Summary: General Principles

Analyzing Circuits

PREPARE Draw a circuit diagram.

SOLVE *Break the circuit down:*

- Reduce the circuit to the smallest possible number of equivalent resistors.
- Find the current and potential difference.

Rebuild the circuit:

- Find current and potential difference for each resistor.

ASSESS Verify that

- The sum of the potential differences across series resistors matches that for the equivalent resistor.
- The sum of the currents through parallel resistors matches that for the equivalent resistor.

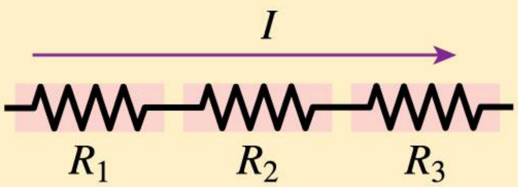
Text: p. 753

Summary: Important Concepts

Series elements

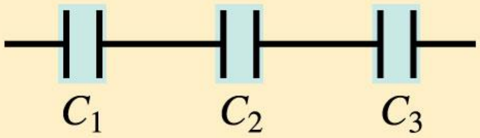
A series connection has no junction.
The current in each element is the same.

Resistors in series can be reduced to an equivalent resistance:



$$R_{eq} = R_1 + R_2 + R_3 + \cdots$$

Capacitors in series can be reduced to an equivalent capacitance:



$$C_{eq} = \left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots\right)^{-1}$$

Text: p. 753

Summary: Important Concepts

Parallel elements

Elements connected in parallel are connected by wires at both ends. The potential difference across each element is the same.

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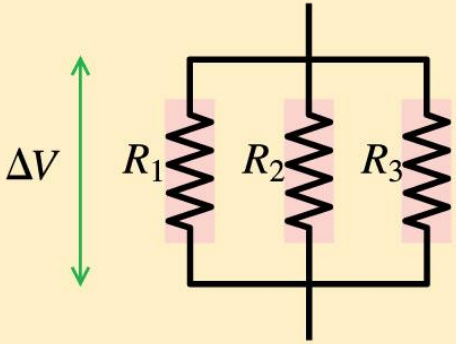

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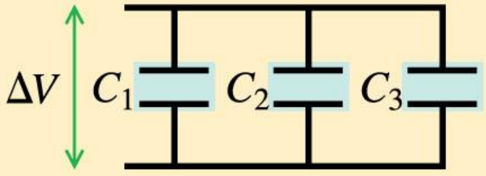

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Summary: Applications

RC circuits

The discharge of a capacitor through a resistor is an exponential decay:

$$\Delta V_C = (\Delta V_C)_0 e^{-t/RC}$$

The **time constant** for the decay is

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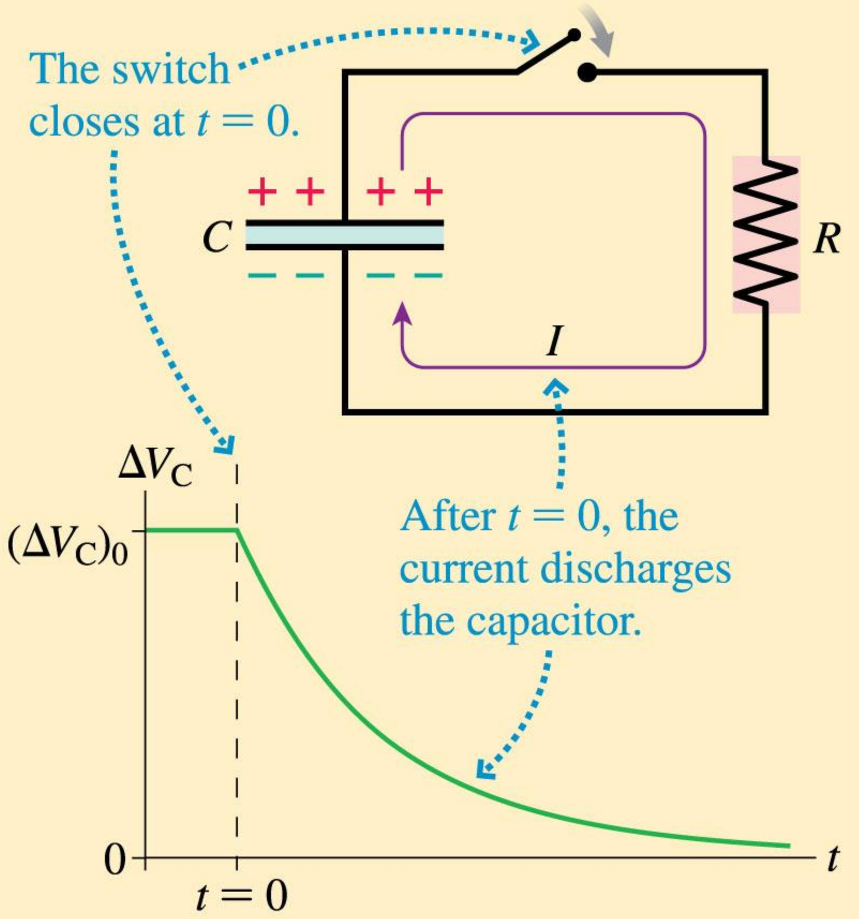


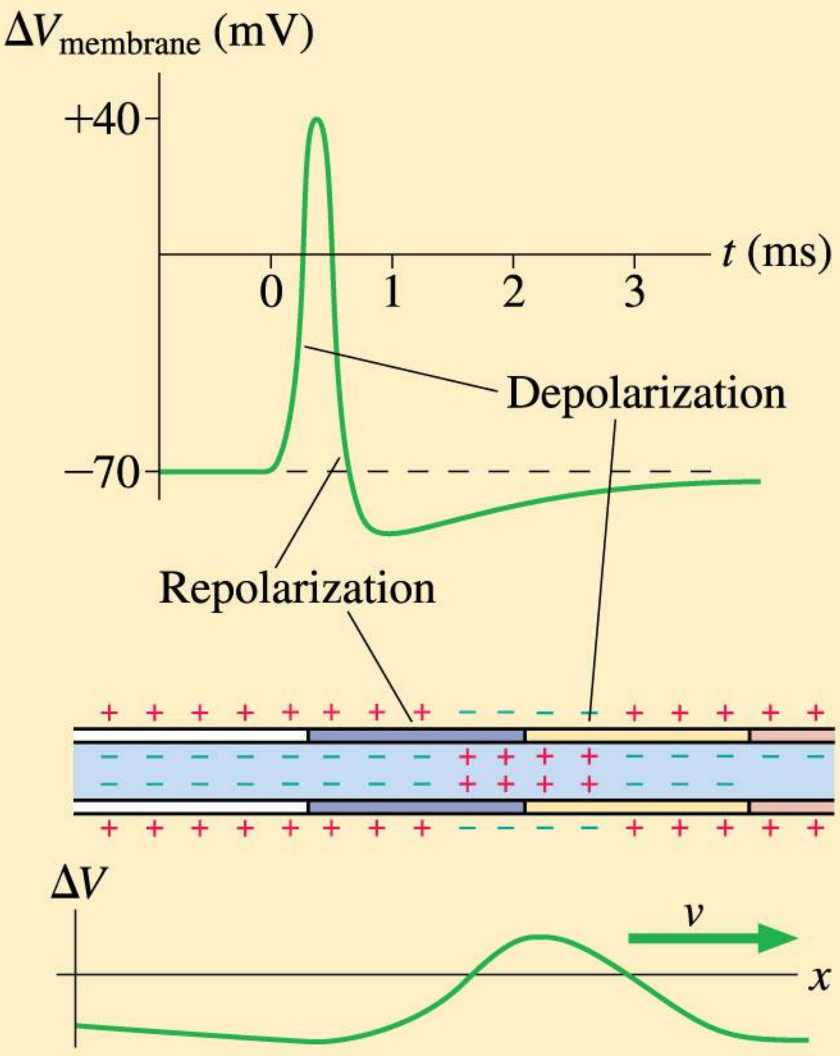
Text: p. 753

Summary: Applications

Electricity in the nervous system

Cells in the nervous system maintain a negative potential inside the cell membrane. When triggered, the membrane depolarizes and generates an *action potential.*

An action potential travels as a wave along the axon of a neuron. More rapid saltatory conduction can be achieved by insulating the axon with myelin, causing the action potential to jump from node to node.



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Summary

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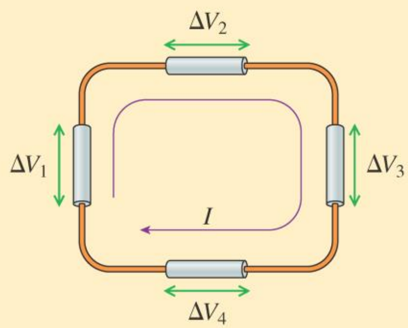

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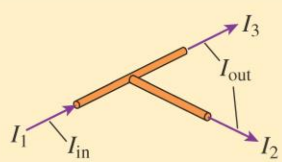

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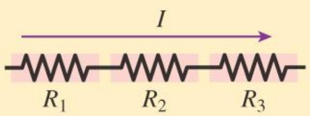

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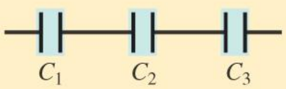

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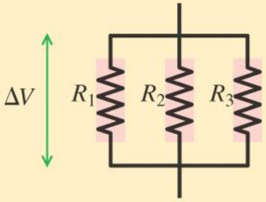

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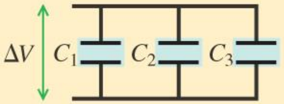

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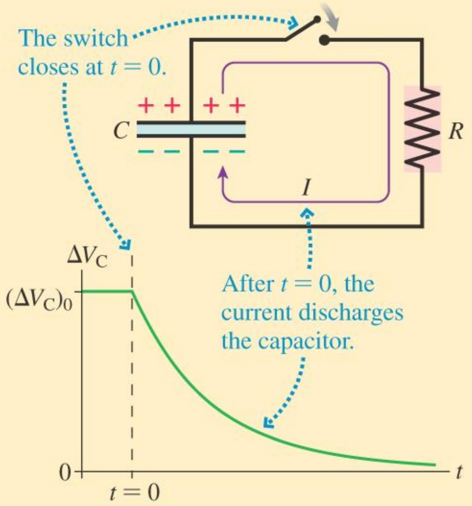

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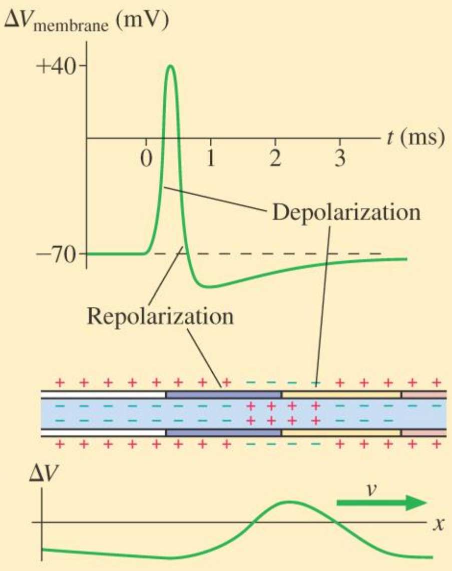

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