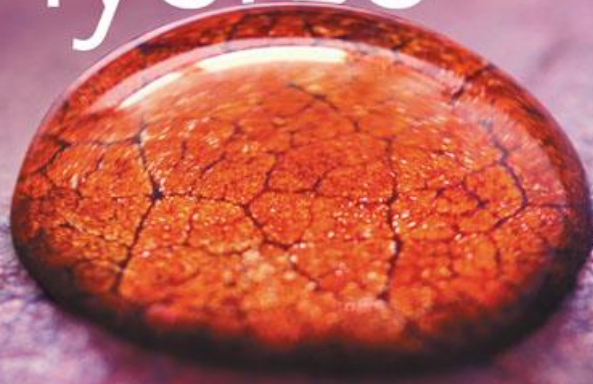


THIRD EDITION

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# Lecture Presentation

## Chapter 28

### *Quantum Physics*

# Suggested Videos for Chapter 28

- **Prelecture Videos**

- *Photons*
- *The Wave Nature of Matter*
- *Energy Levels and Quantum Jumps*

- **Class Videos**

- *The Photoelectric Effect*
- *Quantized Energy*

- **Video Tutor Solutions**

- *Quantum Physics*

# Suggested Simulations for Chapter 28

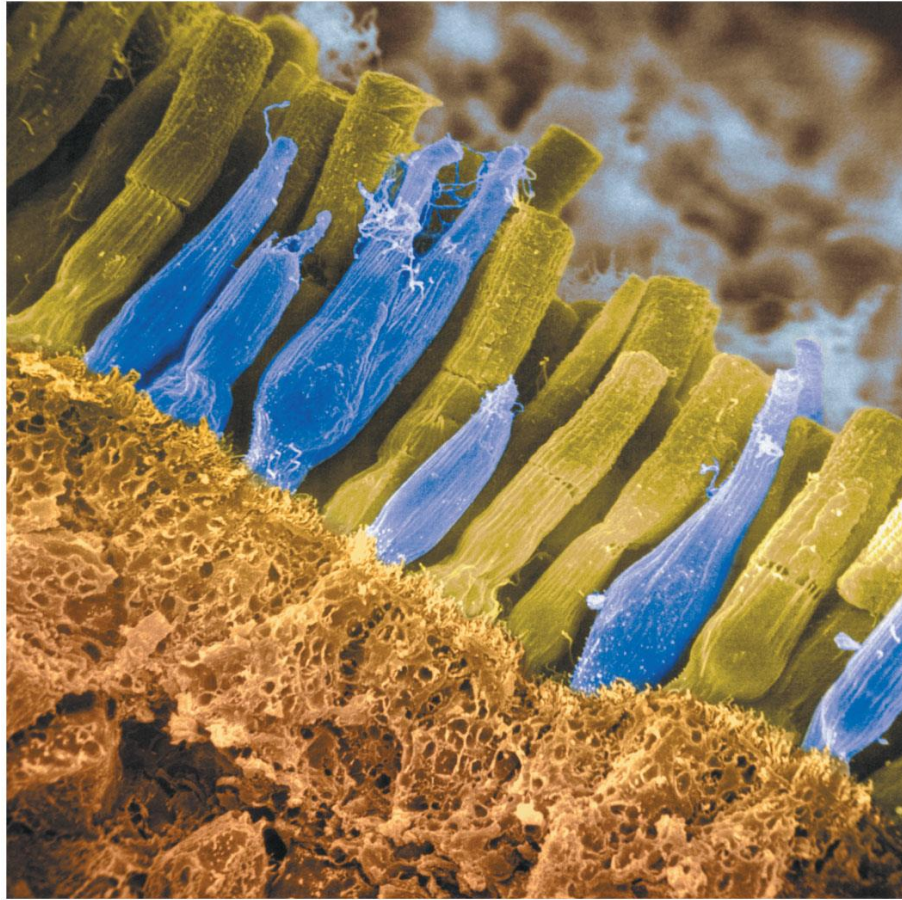
- **ActivPhysics**

- *17.3, 17.5, 17.6*

- **PhETs**

- *Photoelectric Effect*
- *Quantum Wave Interference*
- *Wave Interference*
- *Davisson-Germer: Electron Diffraction*
- *Quantum Bound States*
- *Plinko Probability*
- *Double Wells and Covalent Bonds*
- *Quantum Tunneling and Wave Packets*
- *Simplified MRI*

# Chapter 28 Quantum Physics

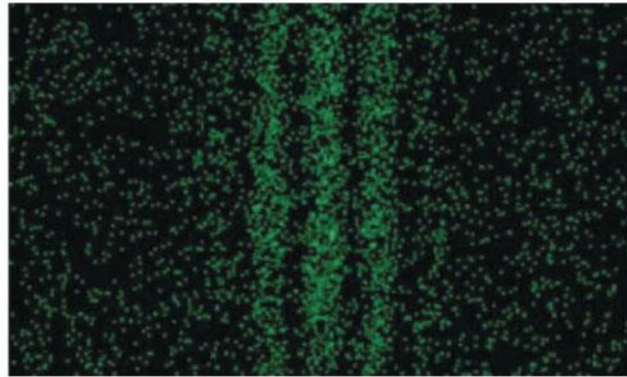


**Chapter Goal:** To understand the quantization of energy for light and matter.

# Chapter 28 Preview

## Looking Ahead: Waves Behave Like Particles

- An interference pattern made with very low intensity light shows that the light hits the screen in discrete “chunks” called **photons**.

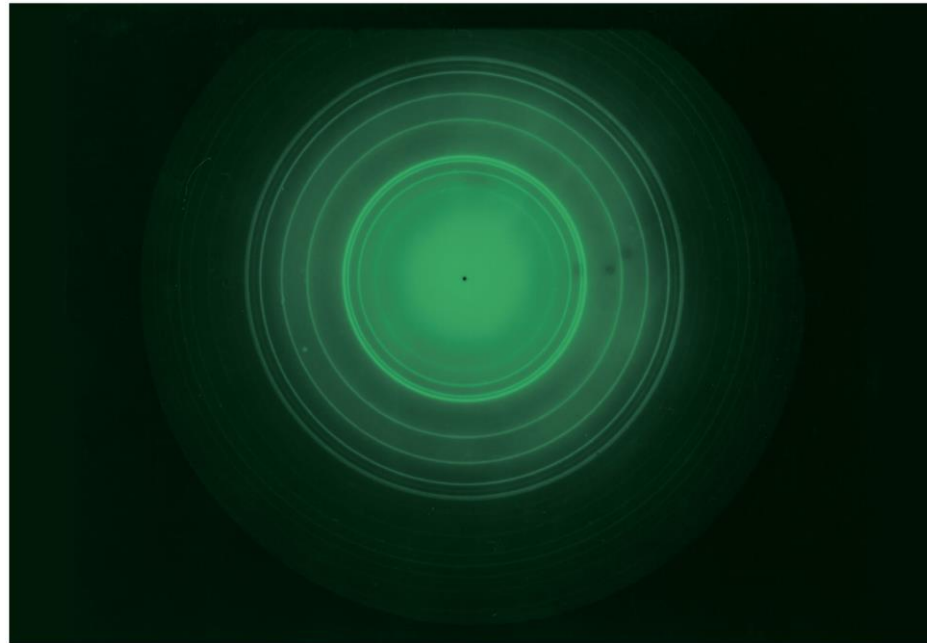


- You’ll learn how light sometimes behaves like a wave and sometimes like a particle.

# Chapter 28 Preview

## Looking Ahead: Particles Behave Like Waves

- This image of electrons diffracting from an aluminum target shows that, surprisingly, particles have a wave nature.

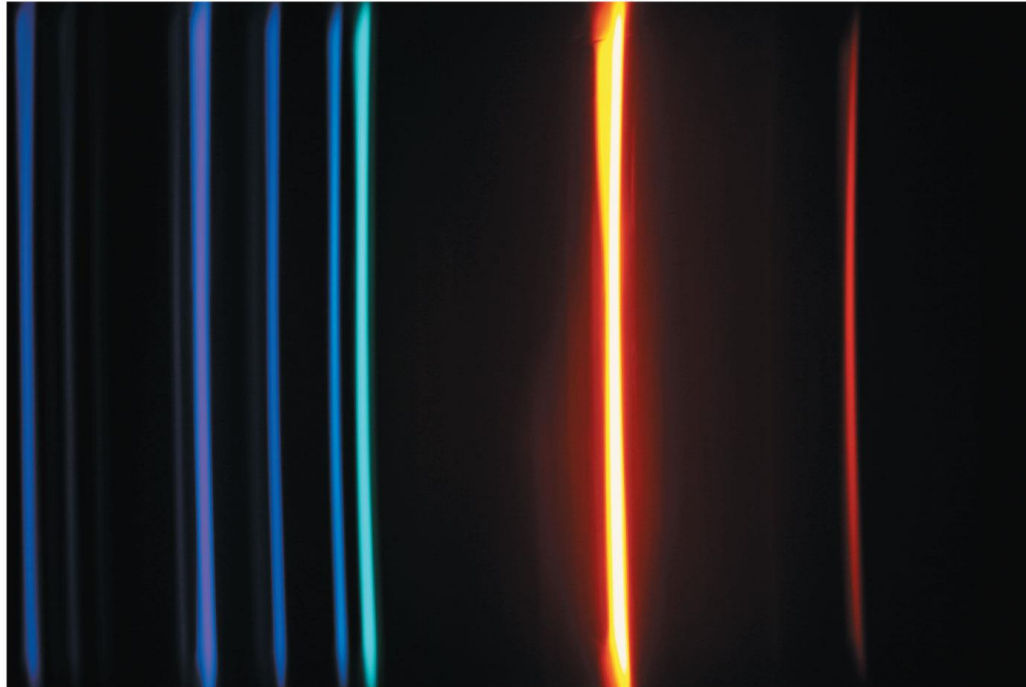


- You'll learn that the wavelength of a particle is directly related to its momentum.

# Chapter 28 Preview

## Looking Ahead: Quantization of Energy

- These discrete colors are emitted by helium atoms as their electrons “jump” between quantized **energy levels**.



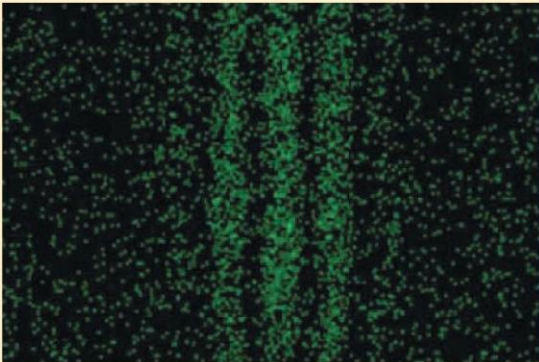
- You’ll learn how to calculate photon energies and wavelengths as quantum systems emit or absorb photons.

# Chapter 28 Preview

## Looking Ahead

### Waves Behave Like Particles

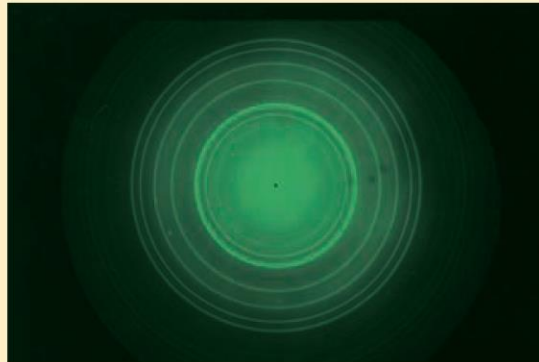
An interference pattern made with very low-intensity light shows that the light hits the screen in discrete “chunks” called **photons**.



You'll learn how light sometimes behaves like a wave and sometimes like a particle.

### Particles Behave Like Waves

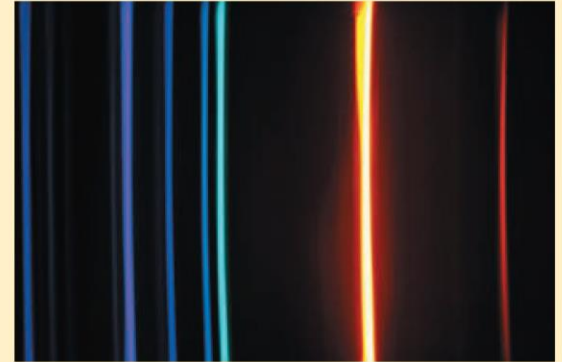
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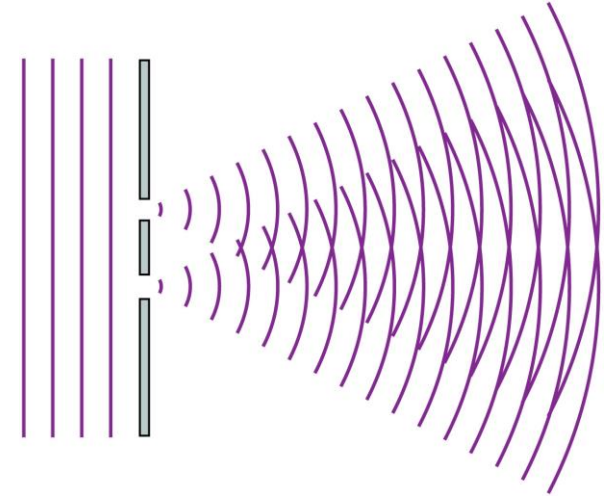
Text: p. 908



# Chapter 28 Preview

## Looking Back: Double-Slit Interference

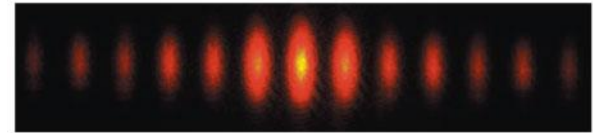
- In Section 17.2 you learned that light waves spreading out from two narrow slits interfere, producing distinct fringes on a screen. This interference is clear evidence of the wave nature of light.
- In this chapter, the double-slit interference experiment will reveal striking new properties of light and a surprising wavelike behavior of electrons.



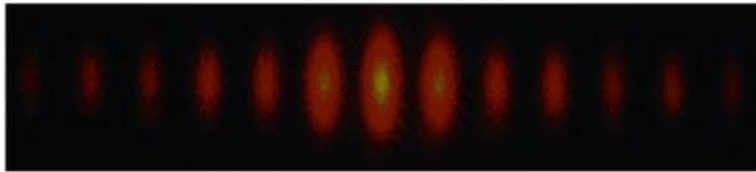
# Chapter 28 Preview

## Stop to Think

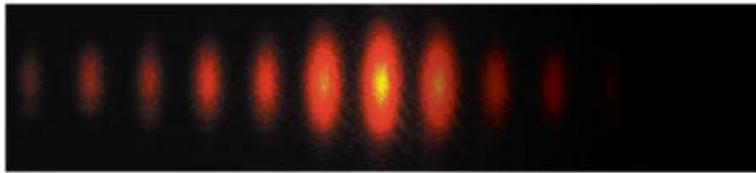
A laser illuminates two slits, leading to the interference pattern shown. After the right-hand slit is covered up, what will the pattern look like?



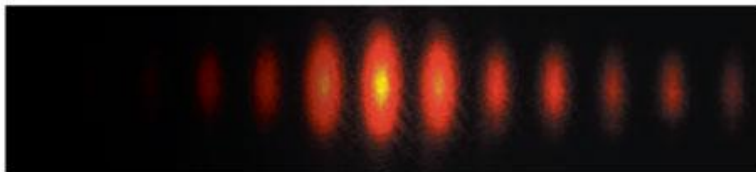
A.



B.



C.



D.



## Reading Question 28.1

The photoelectric effect tells us that

- A. Electrons have a wave nature.
- B. Light has a particle nature.
- C. A photon can be converted into an electron.
- D. Electrons are the conductors in metals.

## Reading Question 28.1

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## Reading Question 28.2

The energy of a photon depends on

- A. Its speed.
- B. Its mass.
- C. Its frequency.
- D. Its charge.

## Reading Question 28.2

The energy of a photon depends on

A. Its speed.

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## Reading Question 28.3

Which of the following wave properties are exhibited by particles?

- A. Diffraction
- B. Interference
- C. Superposition
- D. All of the above

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## Reading Question 28.4

When an electron in a quantum system drops from a higher energy level to a lower one, the system

- A. Emits a neutron.
- B. Emits a photon.
- C. Emits an electron.
- D. Emits a plasmon.

## Reading Question 28.4

When an electron in a quantum system drops from a higher energy level to a lower one, the system

- A. Emits a neutron.
-  B. Emits a photon.
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## Reading Question 28.5

If you precisely measure the position of a particle, you

- A. Destroy its wave nature.
- B. Cause it to diffract.
- C. Destroy information about its speed.
- D. Cause the particle to be annihilated.

## Reading Question 28.5

If you precisely measure the position of a particle, you

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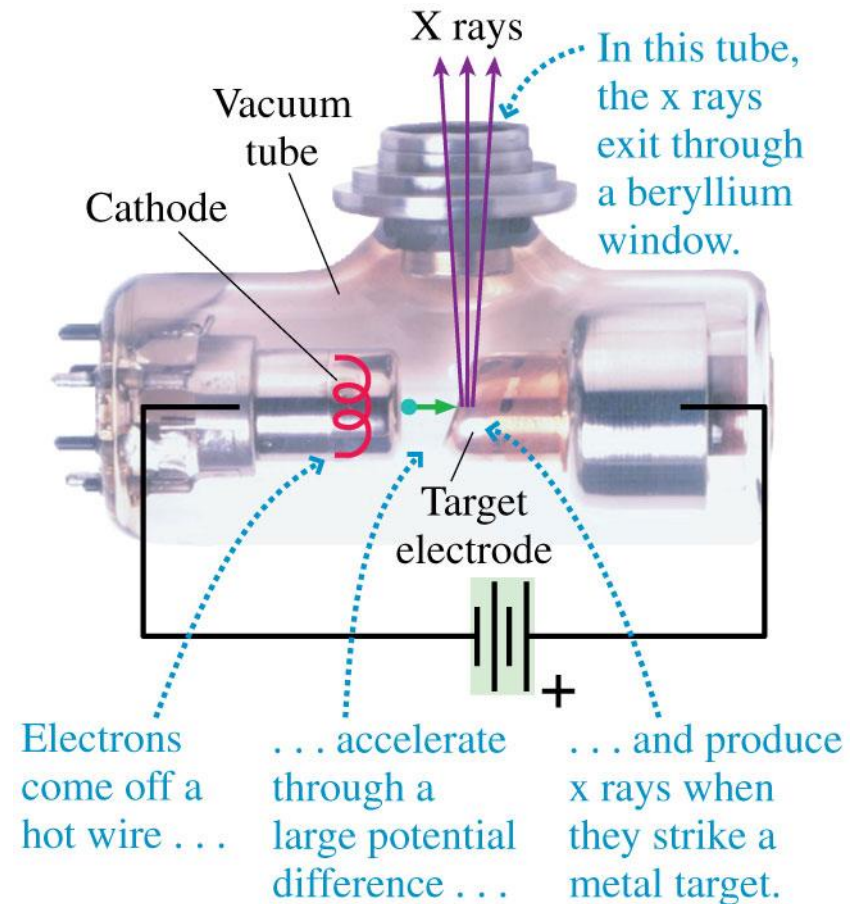
# Section 28.1 X Rays and X-Ray Diffraction

# X Rays and X-Ray Diffraction

- The rules of quantum physics apply at the scale of atoms and electrons.
- In 1895 German physicist Wilhelm Röntgen used high voltages to pull electrons from a cathode and accelerate them to very high speeds before striking a target electrode.
- One day he discovered that a sealed envelope containing film near the vacuum tube had been exposed even though it had never been removed from the envelope.
- Röntgen called whatever was coming from the vacuum tube x rays because they were unknown.
- The x rays traveled in straight lines like particles but could pass through solid materials with little absorption.

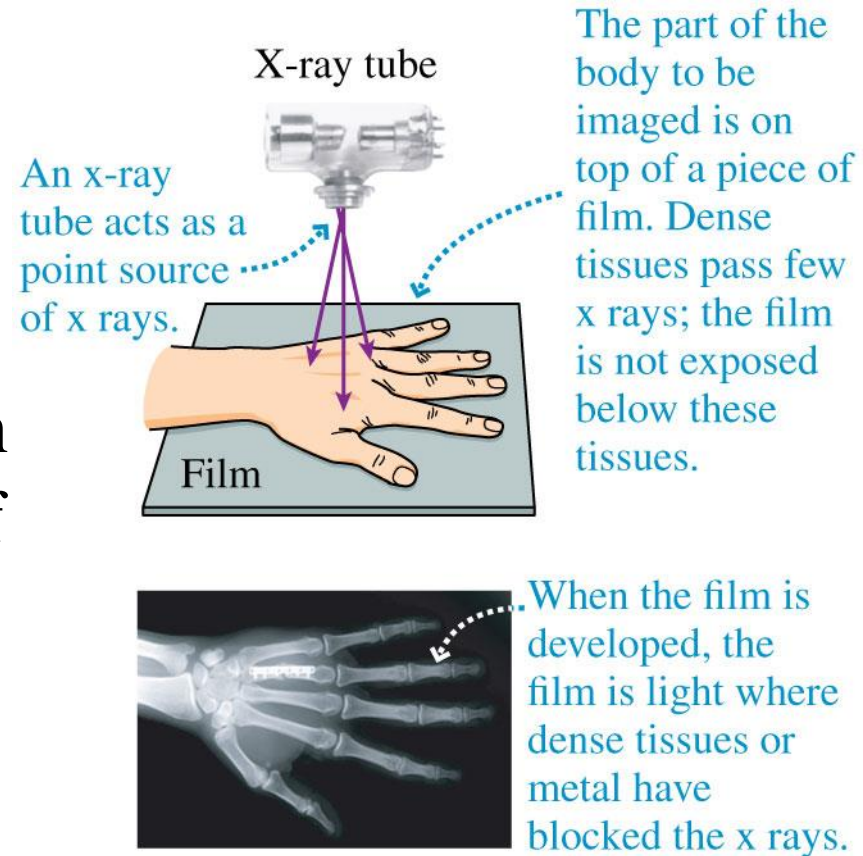
# X Rays and X-Ray Diffraction

- It was determined that the x rays were electromagnetic waves with very short wavelengths.
- X rays are still produced by colliding fast electrons with a metal target.



# X-Ray Images

- Substances with high atomic numbers, such as lead or the minerals in bone, are effective at stopping x rays.
- An x-ray image is created when x rays are sent through a part of the body lying on film. The dense tissues pass few x rays, so the film is not exposed in those regions. When the film is developed, the film is light where rays were blocked.



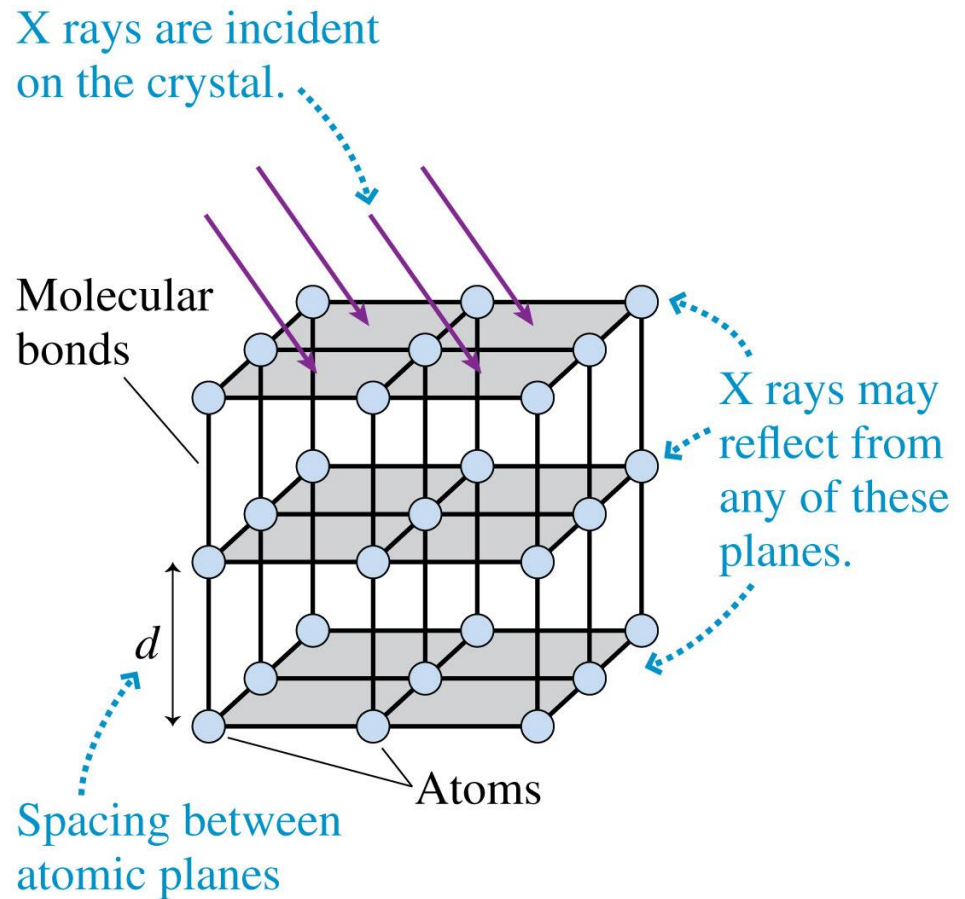


# X-Ray Diffraction

- At about the same time scientists were first concluding that x rays were electromagnetic waves, researchers were also deducing that the size of an atom is  $\sim 0.1$  nm.
- It was also suggested that solids might consist of atoms arranged in a regular crystalline *lattice*.
- German scientist Max von Laue noted that x rays passing through a crystal lattice should undergo diffraction in the same way that visible light diffracts from a diffraction grating.
- X-ray diffraction by crystals was soon confirmed experimentally, confirming that x rays are electromagnetic waves with wavelengths in the range 0.01 nm to 10 nm.

# X-Ray Diffraction

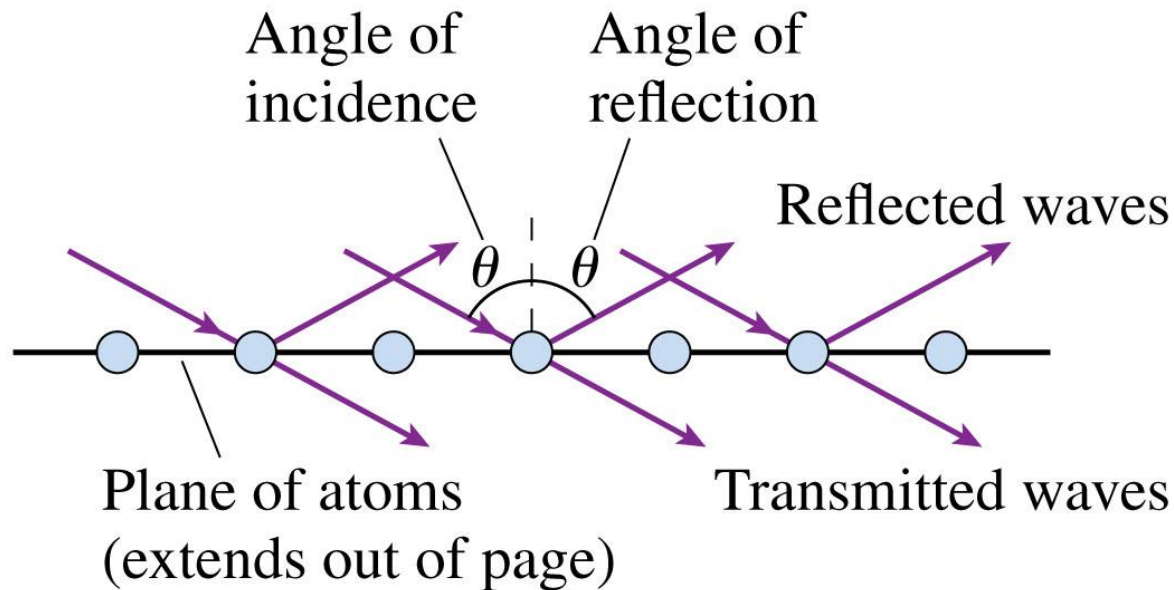
- This figure shows x rays striking a crystal with a *simple cubic lattice*. The atoms are in planes with spacing  $d$  between them.



# X-Ray Diffraction

- A side view of x rays striking a crystal shows that most x rays are transmitted through the plane, but a small fraction of the wave is reflected. The reflected wave obeys the law of reflection.

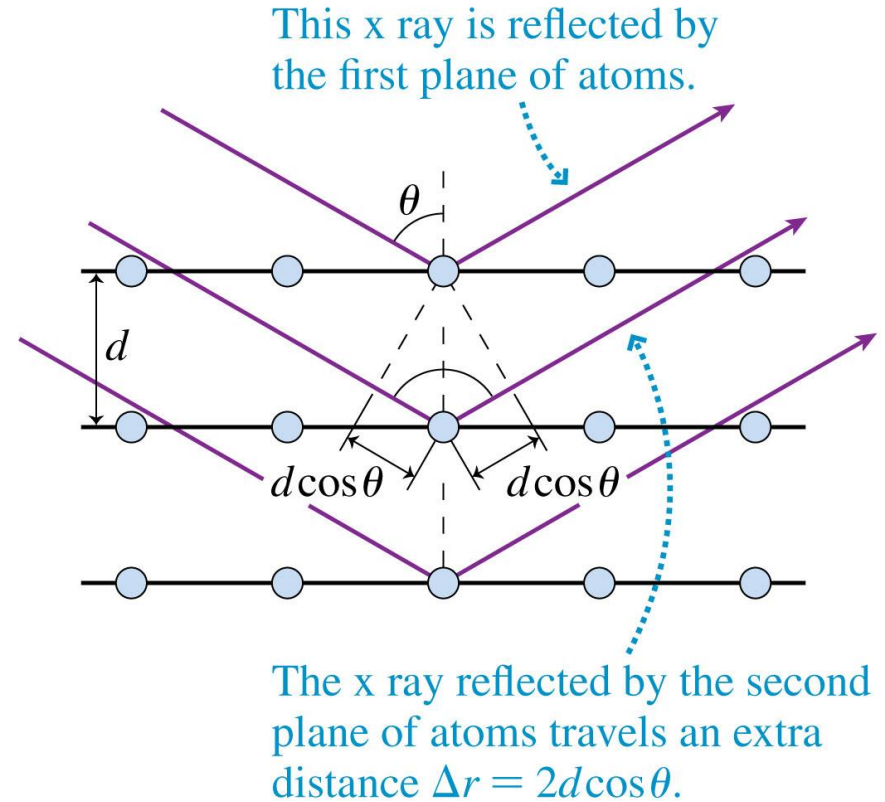
(a) X rays are transmitted and reflected at one plane of atoms.



# X-Ray Diffraction

- There are many parallel planes of atoms in a solid. A small fraction of the wave reflects from each plane.
- The *net* reflection from this solid is the *superposition* of the waves reflected by each atomic plane.

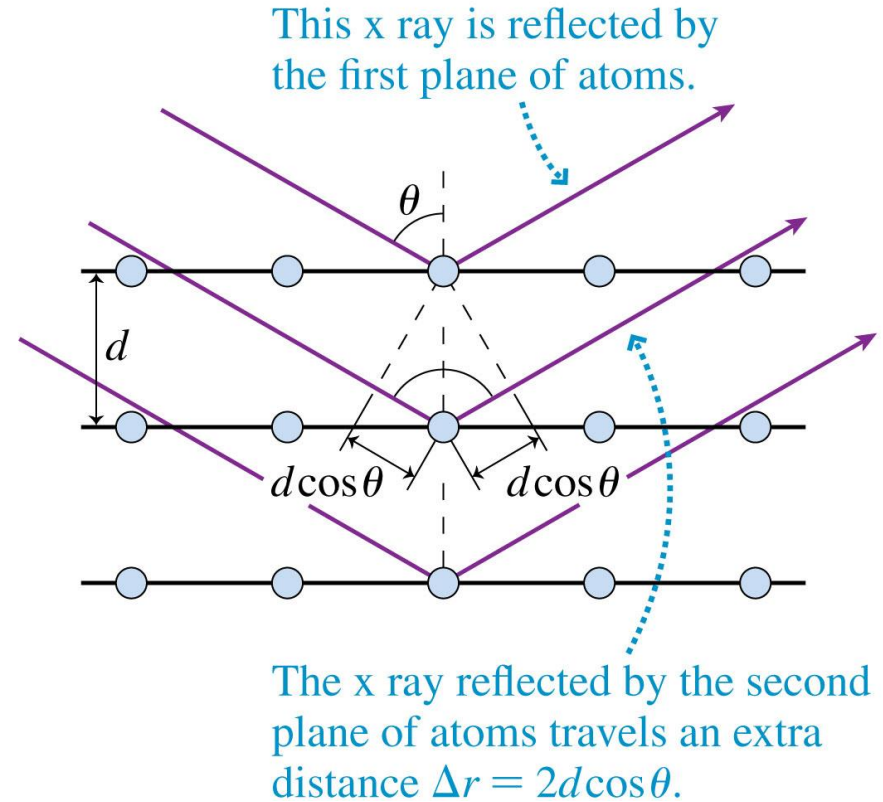
(b) The reflections from parallel planes interfere.



# X-Ray Diffraction

- For most angles of incidence, x rays reflected from the multiple atom planes of a solid are out of phase.
- At a few specific angles of incidence, the reflected waves are in phase. They interfere constructively to produce a strong reflection. This is called **x-ray diffraction**.

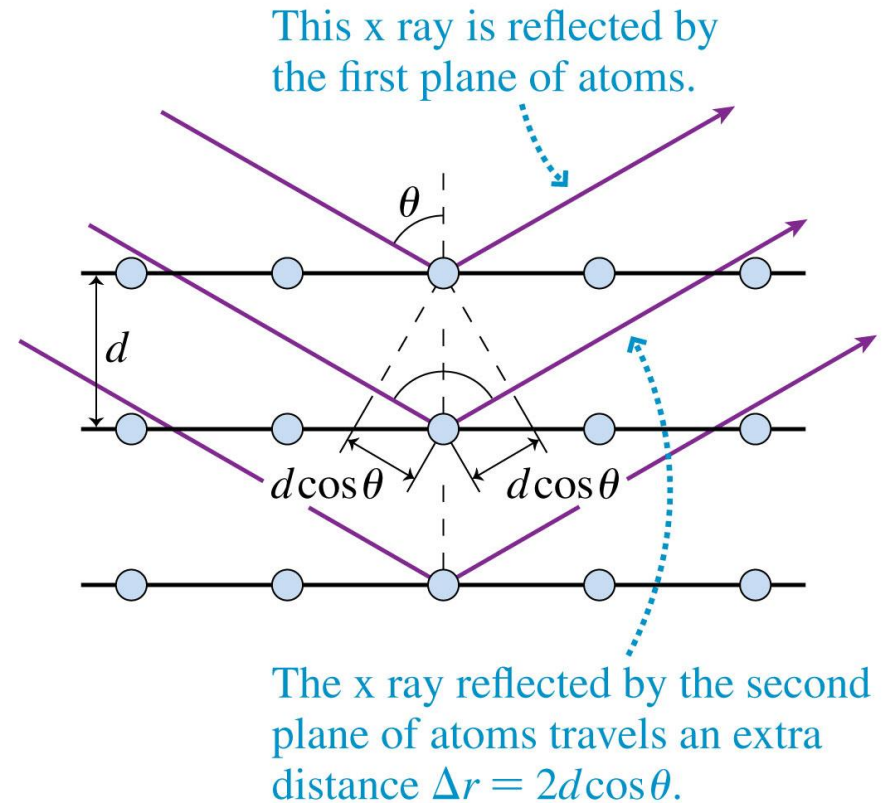
(b) The reflections from parallel planes interfere.



# X-Ray Diffraction

- Waves reflecting from any particular plane travel an extra distance  $\Delta r = 2d\cos\theta$  before combining with the reflection from the other planes.
- If  $\Delta r$  is a whole number of wavelengths, then the waves will be in phase when they recombine.

(b) The reflections from parallel planes interfere.



# X-Ray Diffraction

- X rays will reflect from the crystal when the angle of incidence  $\theta_m$  satisfies the **Bragg condition**:

$$\Delta r = 2d \cos \theta_m = m\lambda \quad m = 1, 2, 3, \dots$$

The Bragg condition for constructive interference of x rays reflected from a solid

## Example 28.1 Analyzing x-ray diffraction

X rays with a wavelength of 0.105 nm are diffracted by a crystal with a simple cubic lattice. Diffraction maxima are observed at angles  $31.6^\circ$  and  $55.4^\circ$  and at no angles between these two. What is the spacing between the atomic planes causing this diffraction?



## Example 28.1 Analyzing x-ray diffraction (cont.)

**PREPARE** The angles must satisfy the Bragg condition. We don't know the values of  $m$ , but we know that they are two consecutive integers. In Equation 28.1  $\theta_m$  *decreases* as  $m$  increases, so  $31.6^\circ$  corresponds to the larger value of  $m$ . We will assume that  $55.4^\circ$  corresponds to  $m$  and  $31.6^\circ$  to  $m + 1$ .

## Example 28.1 Analyzing x-ray diffraction (cont.)

**SOLVE** The values of  $d$  and  $\lambda$  are the same for both diffractions, so we can use the Bragg condition to find

$$\frac{m + 1}{m} = \frac{\cos 31.6^\circ}{\cos 55.4^\circ} = 1.50 = \frac{3}{2}$$

## Example 28.1 Analyzing x-ray diffraction (cont.)

Thus  $55.4^\circ$  is the second-order diffraction and  $31.6^\circ$  is the third-order diffraction. With this information we can use the Bragg condition again to find

$$d = \frac{2\lambda}{2 \cos \theta_2} = \frac{0.105 \text{ nm}}{\cos 55.4^\circ} = 0.185 \text{ nm}$$

## Example 28.1 Analyzing x-ray diffraction (cont.)

**ASSESS** We learned above that the size of atoms is  $\approx 0.1$  nm, so this is a reasonable value for the atomic spacing in a crystal.

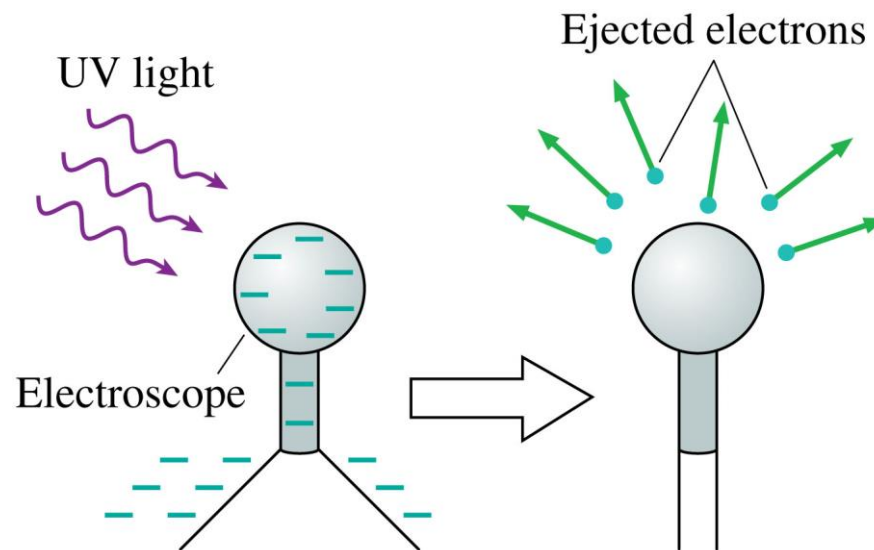
# X-Ray Diffraction

- X-ray diffraction patterns reveal details of the crystal that produced it.
- Complex crystals produce correspondingly complex patterns that can help reveal the structure of the crystals that produced them.
- X-ray diffraction is still used to decipher the three-dimensional structure of biological molecules such as proteins.

# Section 28.2 The Photoelectric Effect

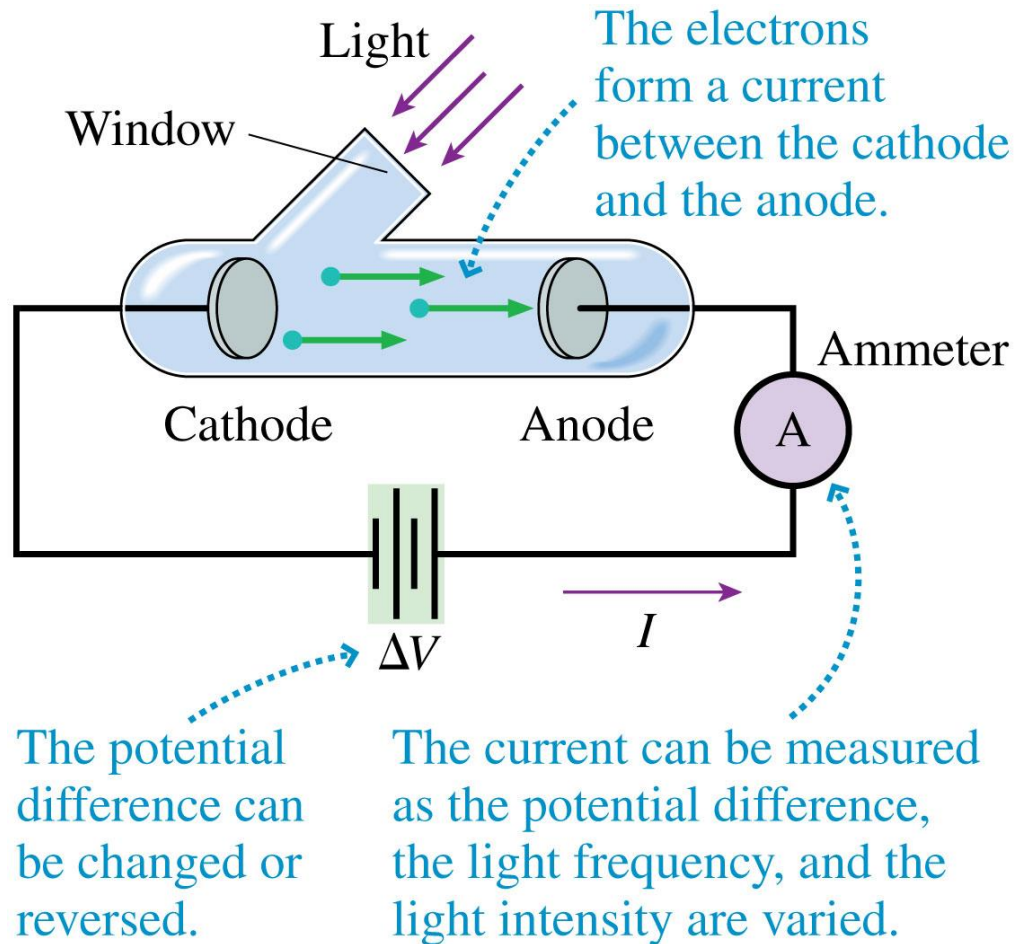
# The Photoelectric Effect

- The first hints about the photon nature of light came with discovery that a negatively charged electroscope could be discharged by shining UV light on it.
- The emission of electrons from a substance due to light striking its surface is called the **photoelectric effect**.



Ultraviolet light discharges a negatively charged electroscope by causing it to emit electrons.

# Characteristics of the Photoelectric Effect

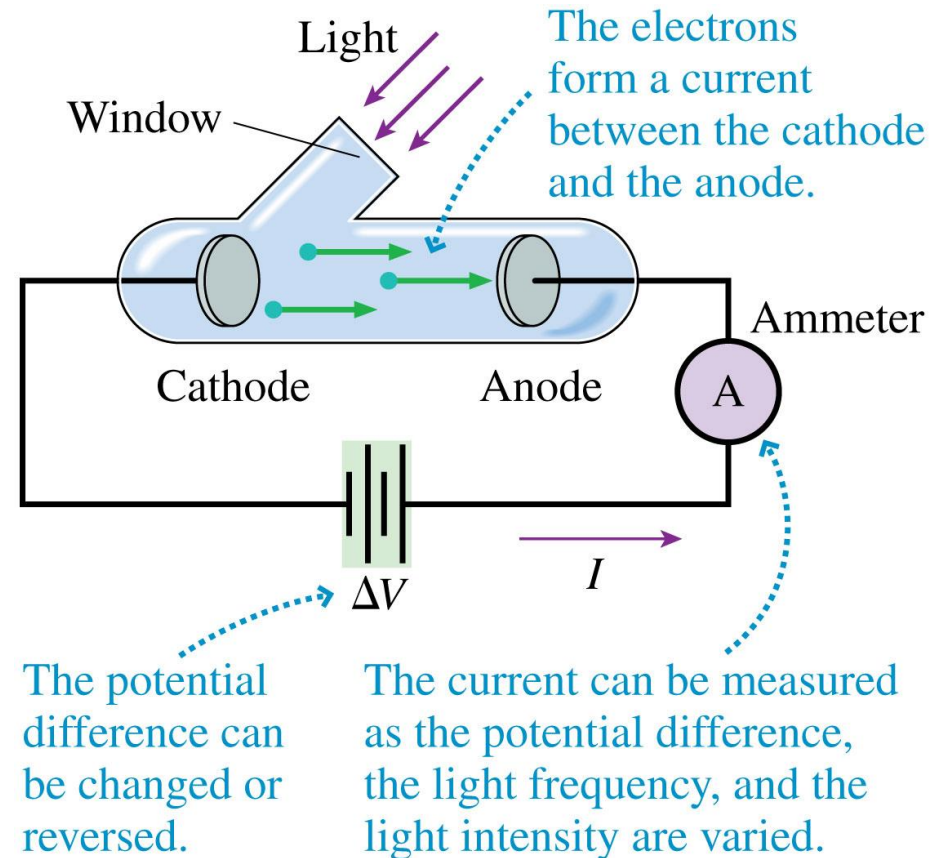


An experimental device to study the photoelectric effect.



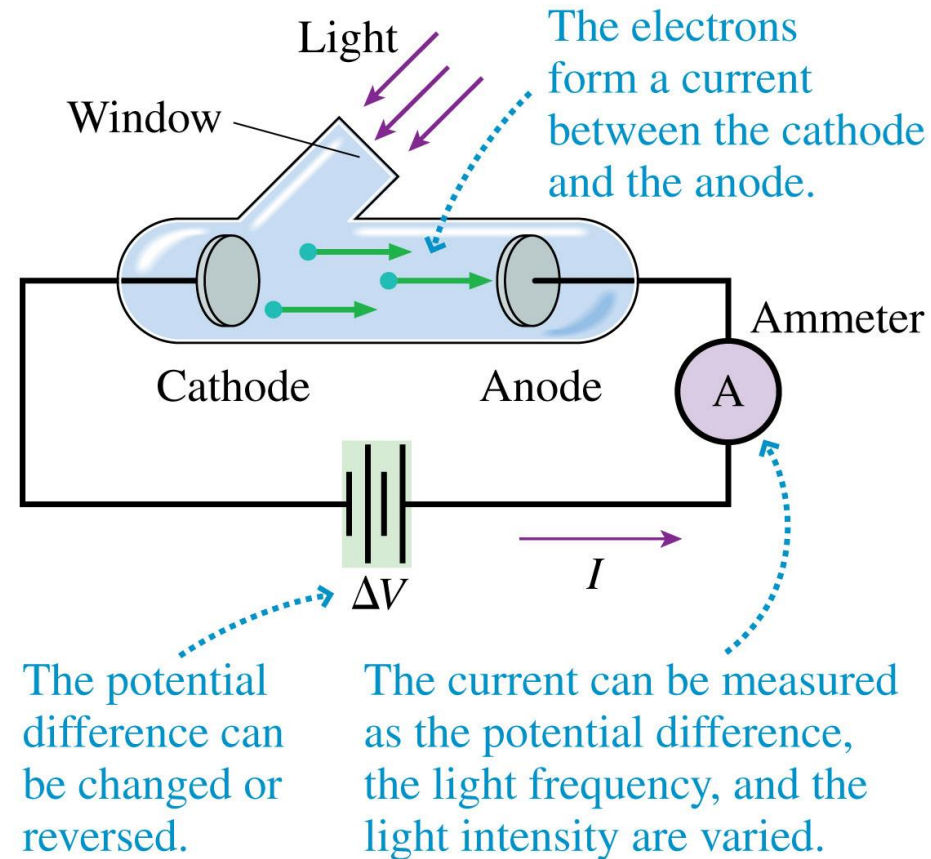
# Characteristics of the Photoelectric Effect

- When UV light shines on the cathode, a steady counterclockwise current passes through the ammeter.
- The incident light causes electrons to be ejected from the cathode at a steady rate.
- There is no current if the electrodes are in the dark, so electrons don't spontaneously leap off the cathode.



# Characteristics of the Photoelectric Effect

- The battery in this device establishes an adjustable potential difference  $\Delta V$  between the two electrodes.
- With it, we can study how the current  $I$  varies as the potential difference and the light's wavelength and intensity are changed.



# Characteristics of the Photoelectric Effect

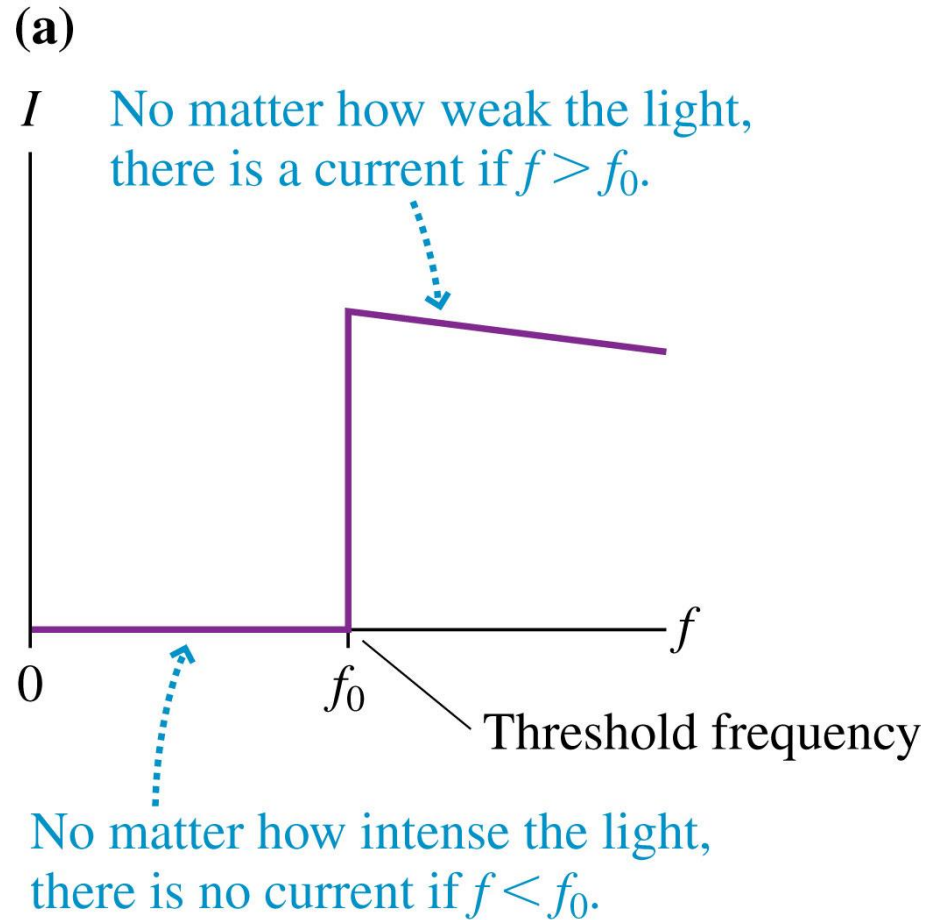
The photoelectric effect has the following characteristics:

1. The current  $I$  is directly proportional to the light intensity. If the light intensity is doubled, the current also doubles.
2. The current appears without delay when the light is applied.
3. Electrons are emitted *only* if the light frequency  $f$  exceeds a **threshold frequency**  $f_0$ .
4. The value of the threshold frequency depends on the type of metal from which the cathode is made.

# Characteristics of the Photoelectric Effect

5. If the potential difference  $\Delta V$  is more than about 1 V positive (anode positive with respect to the cathode), the current changes very little as  $\Delta V$  is increased. If  $\Delta V$  is made negative (anode negative with respect to cathode), by reversing the battery, the current decreases until at some voltage  $\Delta V = -V_{\text{stop}}$  the current reaches zero. The value of  $V_{\text{stop}}$  is called the **stopping potential**.
6. The value of  $V_{\text{stop}}$  is the same for both weak light and intense light. A more intense light causes a larger current, but in both cases the current ceases when  $\Delta V = -V_{\text{stop}}$ .

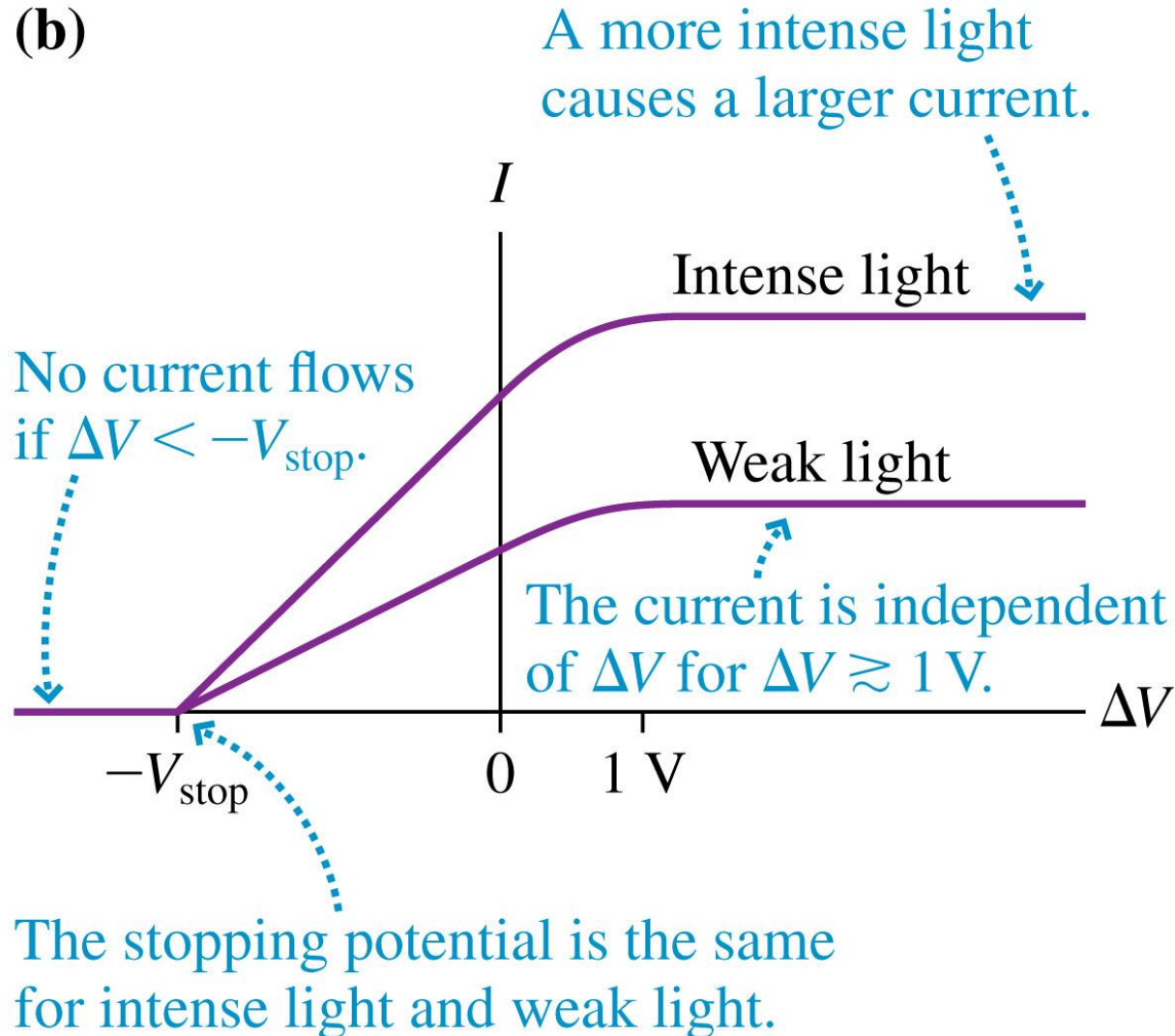
# Characteristics of the Photoelectric Effect



The photoelectric current dependence on the light frequency  $f$  and the battery potential difference  $\Delta V$ .

# Characteristics of the Photoelectric Effect

(b)



A more intense light causes a larger current.

No current flows if  $\Delta V < -V_{\text{stop}}$ .

Intense light

Weak light

The current is independent of  $\Delta V$  for  $\Delta V \gtrsim 1 \text{ V}$ .

$-V_{\text{stop}}$

$0$   $1 \text{ V}$

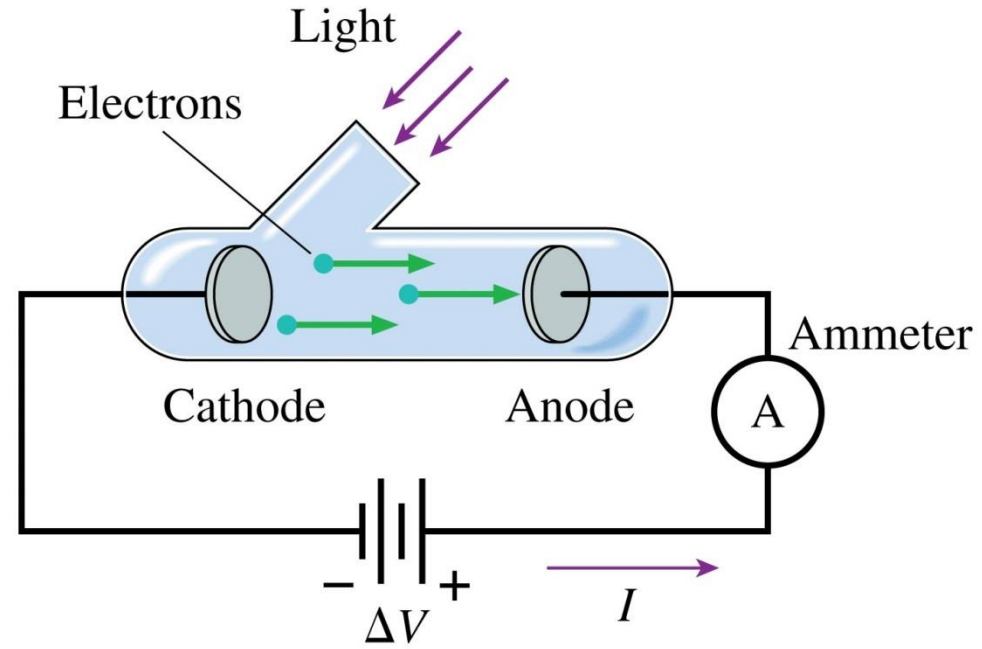
$\Delta V$

The stopping potential is the same for intense light and weak light.

## QuickCheck 28.1

In this experiment, a current is detected when ultraviolet light shines on the metal cathode. What is the source or cause of the current?

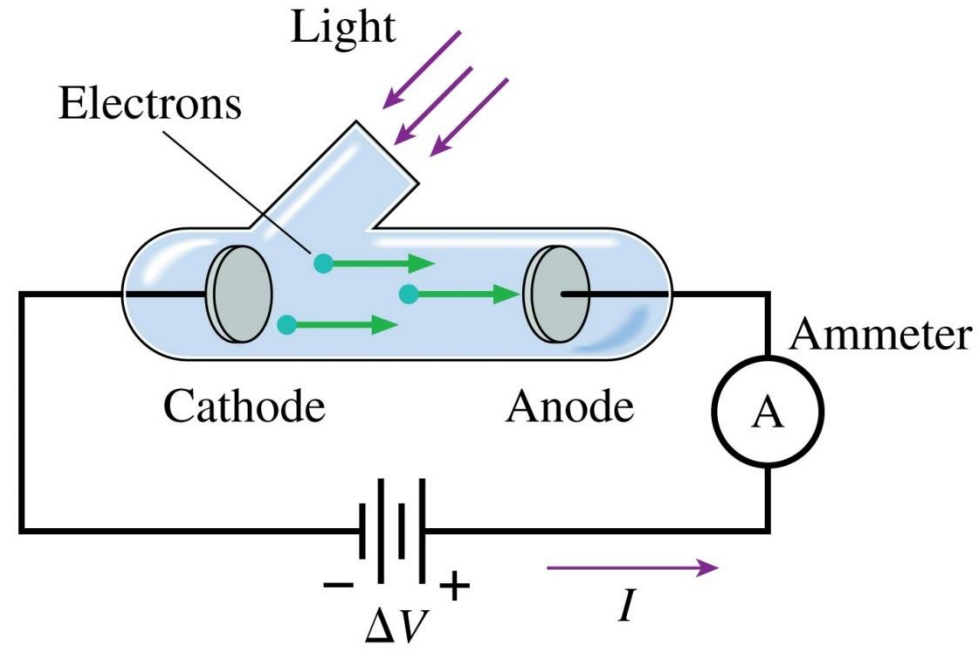
- A. The battery
- B. The light
- C. The cathode



## QuickCheck 28.1

In this experiment, a current is detected when ultraviolet light shines on the metal cathode. What is the source or cause of the current?

- A. The battery
- ✓ B. The light
- C. The cathode





# Understanding the Photoelectric Effect

- A *minimum* energy is needed to free an electron from a metal. To extract an electron, you need to increase its energy until its speed is fast enough to escape.
- The minimum energy  $E_0$  needed to free an electron is called the **work function** of the metal.
- Some electrons will require an energy greater than  $E_0$  to escape, but all will require *at least*  $E_0$ .

# Understanding the Photoelectric Effect

**TABLE 28.1** The work functions for some metals

<b>Element</b>	<b><math>E_0</math> (eV)</b>
Potassium	2.30
Sodium	2.75
Aluminum	4.28
Tungsten	4.55
Copper	4.65
Iron	4.70
Gold	5.10

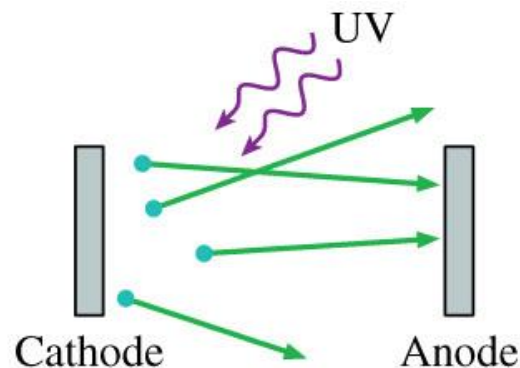
# Understanding the Photoelectric Effect

- An electron with energy  $E_{\text{elec}}$  inside a metal loses energy  $\Delta E$  as it escapes, so it emerges as an electron with kinetic energy  $K = E_{\text{elec}} - \Delta E$ .
- The work function energy  $E_0$  is the *minimum* energy needed to remove an electron, so the *maximum* possible kinetic energy of an ejected electron is

$$K_{\text{max}} = E_{\text{elec}} - E_0$$

# Understanding the Photoelectric Effect

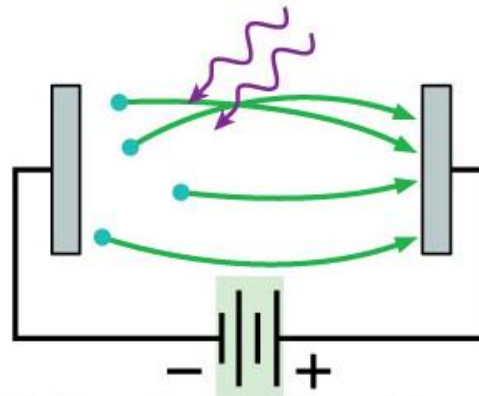
- In the experimental device we used to study the photoelectric effect, the electrons, after leaving the cathode, move out in all directions.
- If the potential difference between the cathode and anode is  $\Delta V = 0$ , there will be no electric field between the plates. Some electrons will reach the anode, creating a measurable current, but many do not.



$\Delta V = 0$ : The electrons leave the cathode in all directions. Only some reach the anode.

# Understanding the Photoelectric Effect

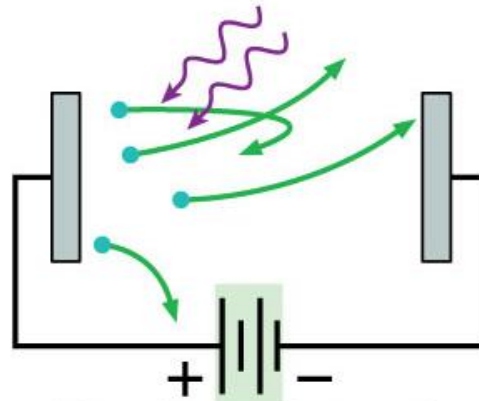
- In the photoelectric effect measuring device, if the anode is positive, it attracts *all* of the electrons to the anode. A further increase in  $\Delta V$  does not cause any more electrons to reach the anode and thus does not cause a further increase in the current  $I$ .



$\Delta V > 0$ : Making the anode positive creates an electric field that pushes all the electrons to the anode.

# Understanding the Photoelectric Effect

- In the photoelectric effect measuring device, if the anode is negative, it repels the electrons. However, an electron leaving the cathode with sufficient kinetic energy can still reach the anode. A slightly negative anode voltage turns back only the slowest electrons. The current steadily decreases as the anode voltage becomes increasingly negative until, at the stopping potential, *all* electrons are turned back and the current ceases.



$\Delta V < 0$ : Making the anode negative repels the electrons. Only the very fastest make it to the anode.

# Understanding the Photoelectric Effect

- We can use the conservation of energy to relate the maximum kinetic energy to the stopping potential. Electrons convert kinetic energy to potential energy as they slow down.

$$\Delta U = -e\Delta V = -\Delta K$$

- When  $\Delta V = -\Delta V_{\text{stop}}$ , the current ceases and the fastest electrons with  $K_{\text{max}}$  are being turned back *just* as they reach the anode. 100% of their kinetic energy is converted to potential energy, so  $eV_{\text{stop}} = K_{\text{max}}$  or

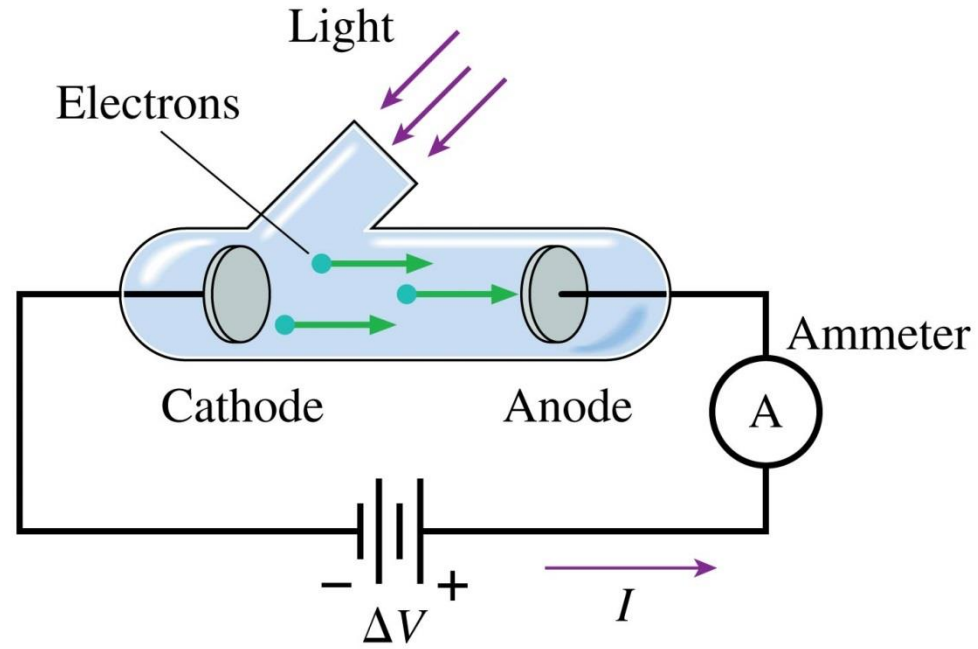
$$V_{\text{stop}} = \frac{K_{\text{max}}}{e}$$

- **Measuring the stopping potential tells us the maximum kinetic energy of the electrons.**

## QuickCheck 28.2

In this experiment, a current is detected when ultraviolet light shines on the metal cathode. What happens to the current if the battery voltage is reduced to zero?

- A. The current is unchanged.
- B. The current decreases slightly.
- C. The current becomes zero.
- D. The current goes the other direction.

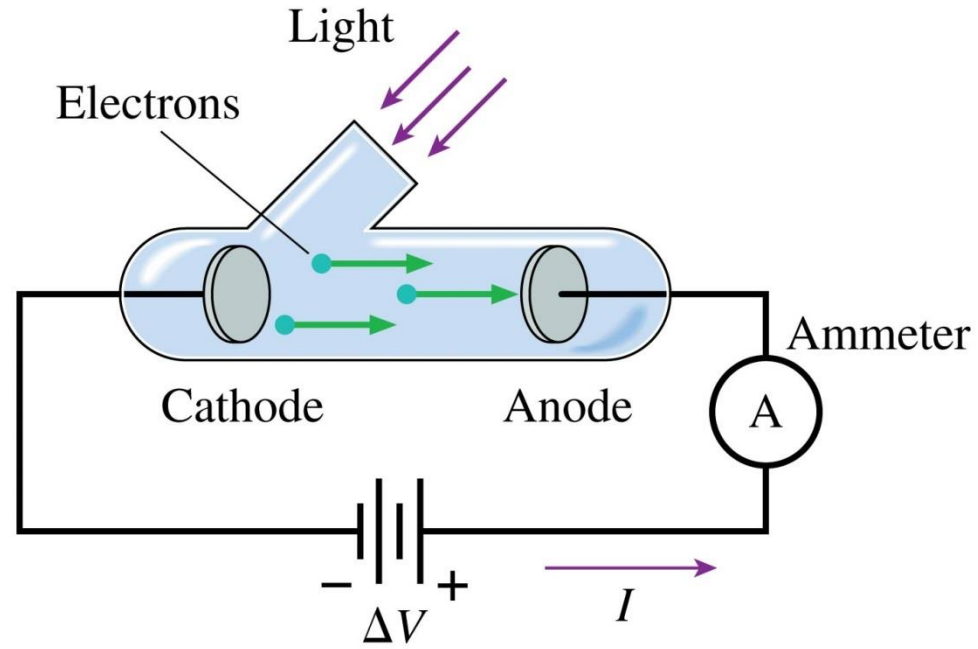




## QuickCheck 28.2

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- A. The current is unchanged.
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- D. The current goes the other direction.



# Einstein's Explanation

- Why do electrons leave the metal at all?
- The first explanation was that heating a metal could cause it to emit electrons.
- The problem with this idea was that the electrons began to leave as soon as light hit the metal surface; it did not need to be warmed for any amount of time.
- The heating hypothesis also failed to explain the threshold frequency. A strong intensity at a different frequency should be able to heat the metal, yet the experimental evidence shows a sharp frequency threshold.

# Einstein's Explanation

- Einstein explained the photoelectric effect by extending the work of German physicist Max Planck, who had found that he could explain the form of the spectrum of a glowing, incandescent object only if he assumed that the oscillating atoms inside the heated solid vibrated in a particular way.
- The energy of an atom vibrating with frequency  $f$  had to be one of the specific energies  $E = 0, hf, 2hf, 3hf\dots$  where  $h$  is a constant. The vibration energies must be **quantized**.
- $h$  is **Planck's constant**:

$$h = 6.63 \times 10^{-34} \text{ J} \cdot \text{s} = 4.14 \times 10^{-15} \text{ eV} \cdot \text{s}$$

# Einstein's Explanation

- Einstein suggested that **electromagnetic radiation itself is quantized**.
- Light is not really a continuous wave, but instead arrives in small packets or bundles of energy, called **light quantum**.
- He postulated that the energy of one light quantum is directly proportional to the frequency of the light.
- Each quantum of light, a **photon**, has energy

$$E = hf$$

The energy of a photon, a quantum of light, of frequency  $f$

- Higher-frequency light is composed of higher-frequency photons.

## QuickCheck 28.3

In the photoelectric effect experiment, why does red light not cause the emission of an electron though blue light can?

- A. The photons of red light don't have sufficient energy to eject an electron.
- B. The electric field of the red light oscillates too slowly to eject an electron.
- C. Red light contains fewer photons than blue, not enough to eject electrons.
- D. The red light doesn't penetrate far enough into the metal electrode.

## QuickCheck 28.3

In the photoelectric effect experiment, why does red light not cause the emission of an electron though blue light can?

- ✓ A. The photons of red light don't have sufficient energy to eject an electron.
- B. The electric field of the red light oscillates too slowly to eject an electron.
- C. Red light contains fewer photons than blue, not enough to eject electrons.
- D. The red light doesn't penetrate far enough into the metal electrode.

## Example 28.2 Finding the energy of ultraviolet photons

Ultraviolet light at 290 nm does 250 times as much cellular damage as an equal intensity of ultraviolet at 310 nm; there is a clear threshold for damage at about 300 nm. What is the energy, in eV, of photons with a wavelength of 300 nm?

**PREPARE** The energy of a photon is related to its frequency by  $E = hf$ .

## Example 28.2 Finding the energy of ultraviolet photons (cont.)

**SOLVE** The frequency at wavelength 300 nm is

$$f = \frac{c}{\lambda} = \frac{3.00 \times 10^8 \text{ m/s}}{300 \times 10^{-9} \text{ m}} = 1.00 \times 10^{15} \text{ Hz}$$

We can now use Equation 28.3 to calculate the energy, using the value of  $h$  in eV · s:

$$E = hf = (4.14 \times 10^{-15} \text{ eV} \cdot \text{s})(1.00 \times 10^{15} \text{ Hz}) = 4.14 \text{ eV}$$



## Example 28.2 Finding the energy of ultraviolet photons (cont.)

**ASSESS** This number seems reasonable. We saw in Chapter 25 that splitting a bond in a water molecule requires an energy of 4.7 eV. We'd expect photons with energies in this range to be able to damage the complex organic molecules in a cell. As the problem notes, there is a sharp threshold for this damage. For energies larger than about 4.1 eV, photons can disrupt the genetic material of cells. Lower energies have little effect.

# Einstein's Postulates and the Photoelectric Effect

- Einstein framed three postulates about light quanta and their interaction with matter:
  1. Light of frequency  $f$  consists of discrete quanta, each of energy  $E = hf$ . Each photon travels at the speed of light  $c$ .
  2. Light quanta are emitted or absorbed on an all-or-nothing basis. A substance can emit 1 or 2 or 3 quanta, but not 1.5. Similarly, an electron in a metal cannot absorb half a quantum but only an integer number.
  3. A light quantum, when absorbed by a metal, delivers its entire energy to *one* electron.

# Einstein's Postulates and the Photoelectric Effect

- Einstein's postulates, as applied to the photoelectric effect, say that the light shining on the metal is a torrent of photons, each of energy  $hf$ .
- Each photon is absorbed by *one* electron, giving that electron an energy  $E_{\text{elec}} = hf$ .

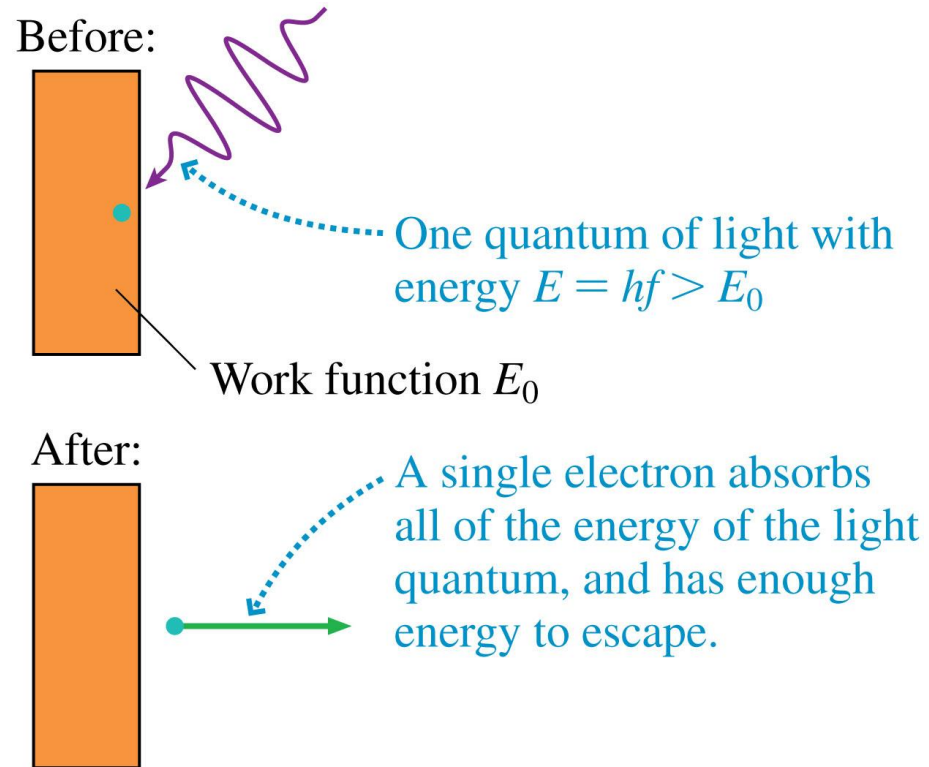
# Einstein's Postulates and the Photoelectric Effect

- An electron can escape from the metal if its energy exceeds the work function  $E_0$ , or if

$$E_{\text{elec}} = hf \geq E_0$$

- There is a *threshold frequency* for the ejection of electrons:

$$f_0 = \frac{E_0}{h}$$



# Einstein's Postulates and the Photoelectric Effect

- If the frequency  $f$  of a photon is less than  $f_0$  then none of the electrons will have sufficient energy to escape no matter the intensity of the light.
- Weak light with  $f \geq f_0$  will give a few electrons sufficient energy to escape **because each photon delivers all of its energy to one electron.**
- This threshold behavior is exactly what the data show.

# Einstein's Postulates and the Photoelectric Effect

- A more intense light delivers a larger number of photons on the surface. They eject a larger number of electrons and cause a larger current, exactly as observed.

# Einstein's Postulates and the Photoelectric Effect

- There is a distribution of kinetic energies, because different electrons require different amounts of energy to escape, but the *maximum* kinetic energy is

$$K_{\max} = E_{\text{elec}} - E_0 = hf - E_0$$

- Einstein's theory predicts that the stopping potential is related to the light frequency by

$$V_{\text{stop}} = \frac{K_{\max}}{e} = \frac{hf - E_0}{e}$$

- The stopping potential does *not* depend on the the intensity of light, which agrees with the data.

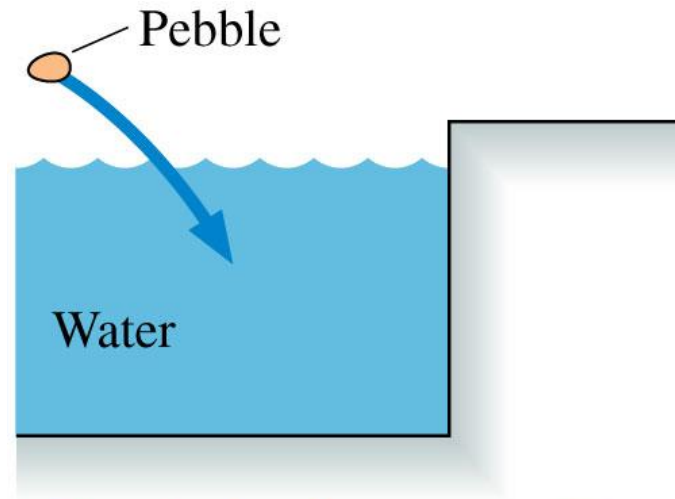
# Einstein's Postulates and the Photoelectric Effect

- If each photon transfers its energy  $hf$  to just one electron, that electron immediately has enough energy to escape. The current should begin instantly, with no delay, exactly as experiments had found.



# Einstein's Postulates and the Photoelectric Effect

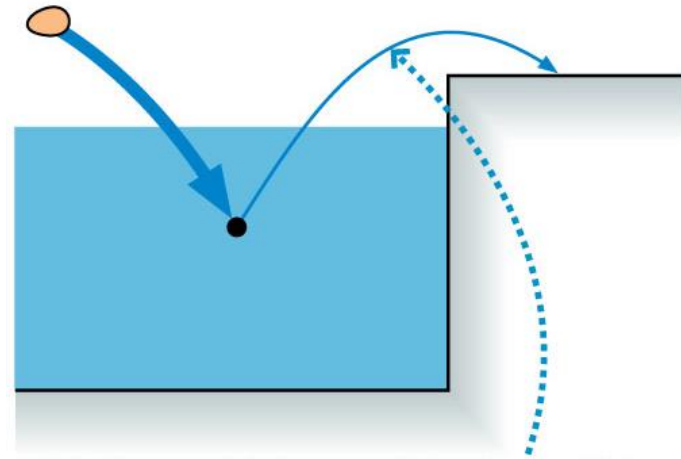
- We can use an analogy of a swimming pool to visualize the photon model of light.
- A pebble thrown in the pool increases the energy of the water, but the increase is shared among all the molecules in the pool, and creates a small ripple.



Classically, the energy of the pebble is shared by all the water molecules. One pebble causes only very small waves.

# Einstein's Postulates and the Photoelectric Effect

- Suppose *all* of the pebble's energy could go to *one drop* of water, and that drop did not have to share the energy.
- That one drop of water could easily have enough energy to leap out of the pool.



If the pebble could give *all* its energy to one drop, that drop could easily splash out of the pool.

# Einstein's Postulates and the Photoelectric Effect

- Einstein was awarded the Nobel Prize in 1921 for his explanation of the photoelectric effect.

## QuickCheck 28.4

Monochromatic light shines on the cathode in a photoelectric effect experiment, causing the emission of electrons. If the intensity of the light stays the same but the frequency of the light shining on the cathode is increased,

- A. There will be more electrons emitted.
- B. The emitted electrons will be moving at a higher speed.
- C. Both A and B are true.
- D. Neither A nor B is true.

## QuickCheck 28.4

Monochromatic light shines on the cathode in a photoelectric effect experiment, causing the emission of electrons. If the intensity of the light stays the same but the frequency of the light shining on the cathode is increased,

A. There will be more electrons emitted.

✓ B. The emitted electrons will be moving at a higher speed.

C. Both A and B are true.

D. Neither A nor B is true.

## QuickCheck 28.5

Monochromatic light shines on the cathode in a photoelectric effect experiment, causing the emission of electrons. If the frequency of the light stays the same but the intensity of the light shining on the cathode is increased,

- A. There will be more electrons emitted.
- B. The emitted electrons will be moving at a higher speed.
- C. Both A and B are true.
- D. Neither A nor B is true.

## QuickCheck 28.5

Monochromatic light shines on the cathode in a photoelectric effect experiment, causing the emission of electrons. If the frequency of the light stays the same but the intensity of the light shining on the cathode is increased,

- ✓ A. There will be more electrons emitted.
- B. The emitted electrons will be moving at a higher speed.
- C. Both A and B are true.
- D. Neither A nor B is true.

## Example 28.3 Finding the photoelectric threshold frequency

What are the threshold frequencies and wavelengths for electron emission from sodium and from aluminum?

**PREPARE** Table 28.1 gives the work function for sodium as  $E_0 = 2.75$  eV and that for aluminum as  $E_0 = 4.28$  eV.



## Example 28.3 Finding the photoelectric threshold frequency (cont.)

**SOLVE** We can use Equation 28.5, with  $h$  in units of  $\text{eV} \cdot \text{s}$ , to calculate

$$f_0 = \frac{E_0}{h} = \begin{cases} 6.64 \times 10^{14} \text{ Hz} & \text{sodium} \\ 10.34 \times 10^{14} \text{ Hz} & \text{aluminum} \end{cases}$$

These frequencies are converted to wavelengths with  $\lambda = c/f$ , giving

$$\lambda = \begin{cases} 452 \text{ nm} & \text{sodium} \\ 290 \text{ nm} & \text{aluminum} \end{cases}$$

## Example 28.3 Finding the photoelectric threshold frequency (cont.)

**ASSESS** The photoelectric effect can be observed with sodium for  $\lambda < 452$  nm. This includes blue and violet visible light but not red, orange, yellow, or green. Aluminum, with a larger work function, needs ultraviolet wavelengths,  $\lambda < 290$  nm.

## QuickCheck 28.6

Light consisting of 2.7 eV photons is incident on a piece of sodium, which has a work function of 2.3 eV. What is the maximum kinetic energy of the ejected electrons?

- A. 2.3 eV
- B. 2.7 eV
- C. 5.0 eV
- D. 0.4 eV

## QuickCheck 28.6

Light consisting of 2.7 eV photons is incident on a piece of sodium, which has a work function of 2.3 eV. What is the maximum kinetic energy of the ejected electrons?

A. 2.3 eV

B. 2.7 eV

C. 5.0 eV

 D. 0.4 eV

## Example 28.4 Determining the maximum electron speed

What are the maximum electron speed and the stopping potential if sodium is illuminated with light of wavelength 300 nm?

**PREPARE** The kinetic energy of the emitted electrons—and the potential difference necessary to stop them—depends on the energy of the incoming photons,  $E = hf$ , and the work function of the metal from which they are emitted,  $E_0 = 2.75 \text{ eV}$ .

## Example 28.4 Determining the maximum electron speed (cont.)

**SOLVE** The light frequency is  $f = c/\lambda = 1.00 \times 10^{15}$  Hz, so each light quantum has energy  $hf = 4.14$  eV. The maximum kinetic energy of an electron is

$$\begin{aligned}K_{\max} &= hf - E_0 = 4.14 \text{ eV} - 2.75 \text{ eV} = 1.39 \text{ eV} \\ &= 2.22 \times 10^{-19} \text{ J}\end{aligned}$$

## Example 28.4 Determining the maximum electron speed (cont.)

Because  $K = \frac{1}{2}mv^2$ , where  $m$  is the electron's mass, not the mass of the sodium atom, the maximum speed of an electron leaving the cathode is

$$v_{\max} = \sqrt{\frac{2K_{\max}}{m}} = 6.99 \times 10^5 \text{ m/s}$$

Note that  $K_{\max}$  must be in J, the SI unit of energy, in order to calculate a speed in m/s.

## Example 28.4 Determining the maximum electron speed (cont.)

Now that we know the maximum kinetic energy of the electrons, we can use Equation 28.7 to calculate the stopping potential:

$$V_{\text{stop}} = \frac{K_{\text{max}}}{e} = 1.39 \text{ V}$$

An anode voltage of  $-1.39 \text{ V}$  will be just sufficient to stop the fastest electrons and thus reduce the current to zero.

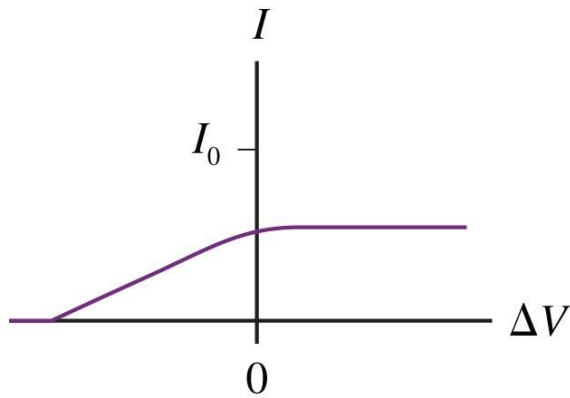
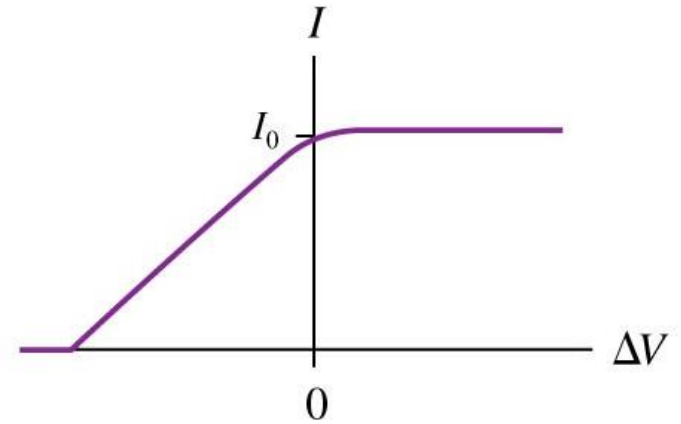


## Example 28.4 Determining the maximum electron speed (cont.)

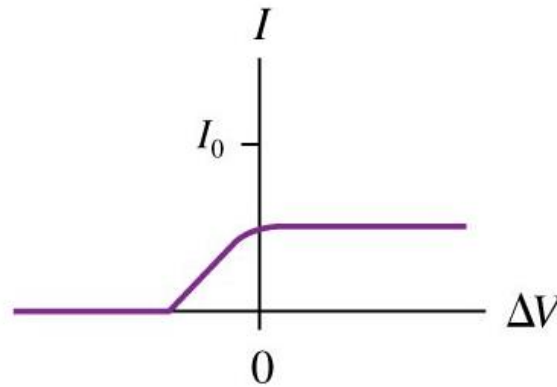
**ASSESS** The stopping potential has the *same numerical value* as  $K_{\max}$  expressed in eV, which makes sense. An electron with a kinetic energy of 1.39 eV can go “uphill” against a potential difference of 1.39 V, but no more.

## QuickCheck 28.7

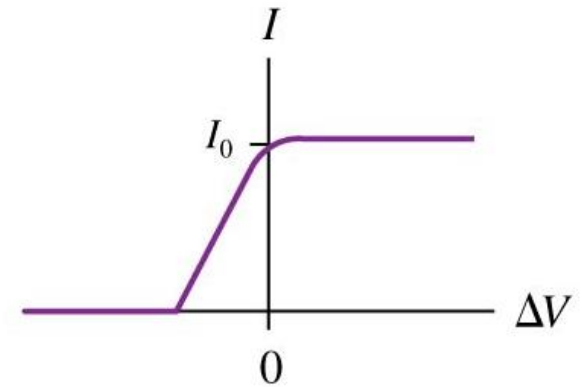
What happens to this graph of the photoelectric current if the intensity of the light is reduced by 50%?



A.



B.



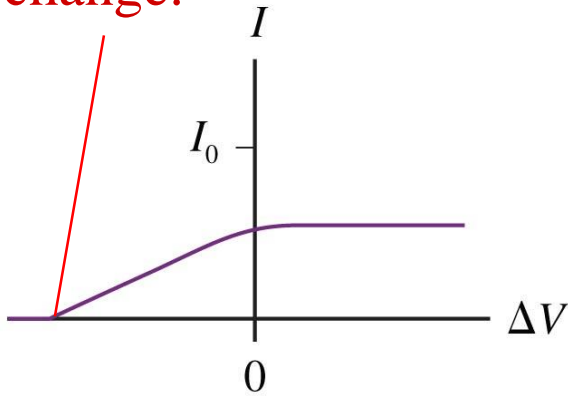
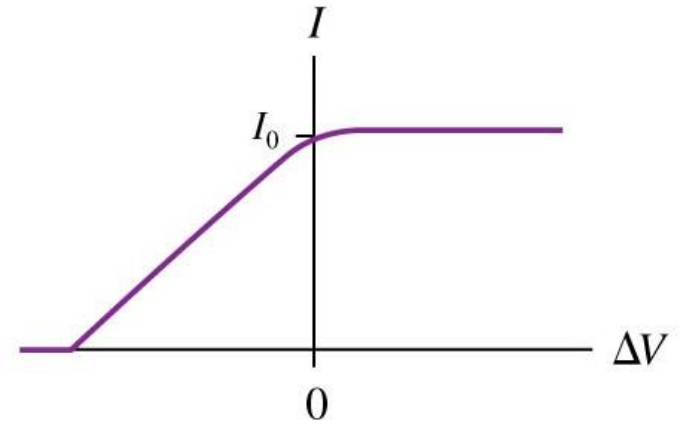
C.

D. None of these.

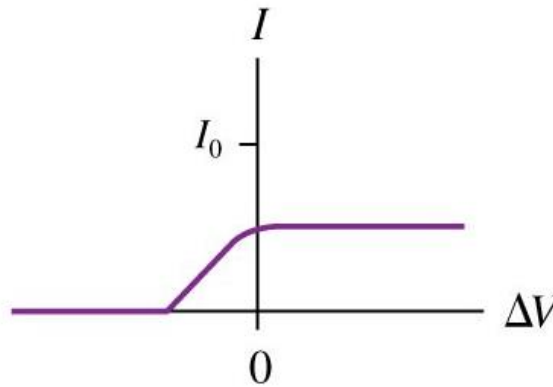
# QuickCheck 28.7

What happens to this graph of the photoelectric current if the intensity of the light is reduced by 50%?

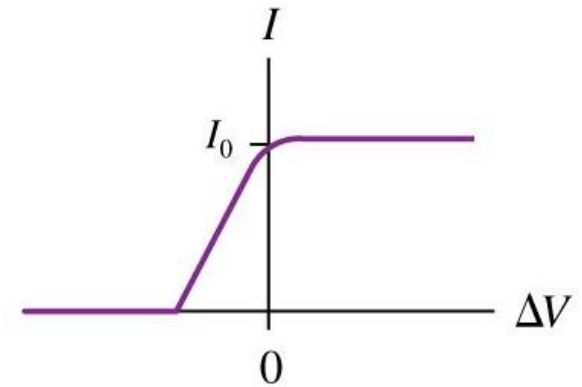
Stopping potential doesn't change.



✓ A.



B.

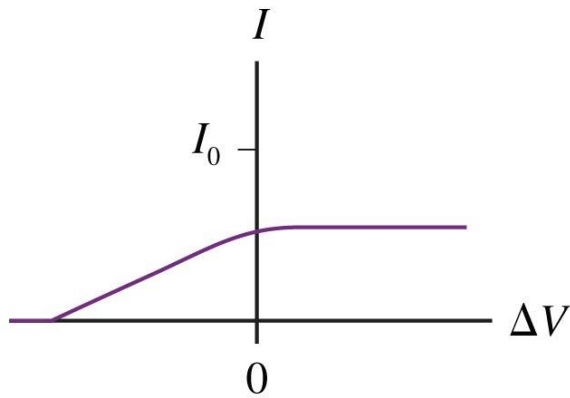
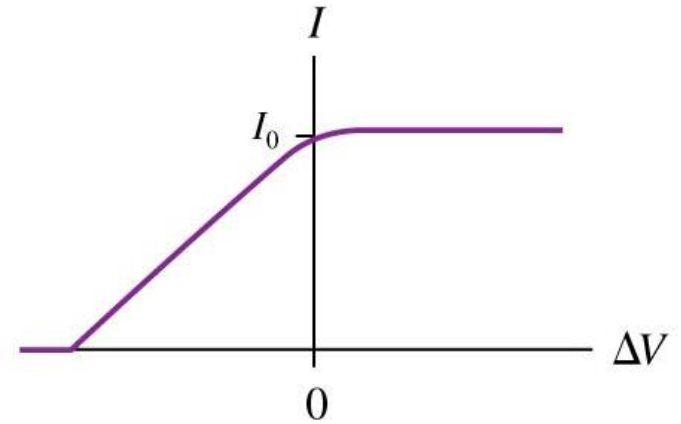


C.

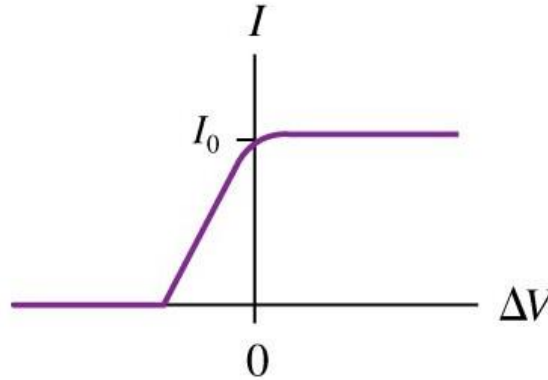
D. None of these.

## QuickCheck 28.8

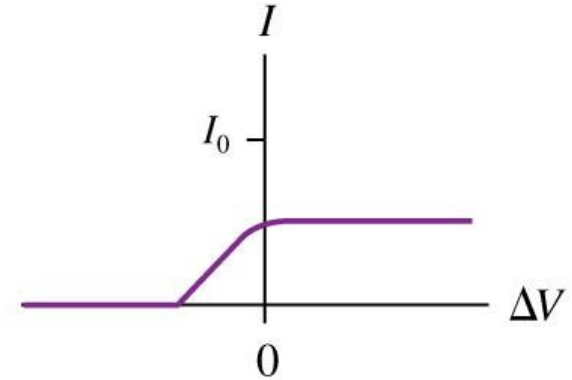
What happens to this graph of the photoelectric current if the cathode's work function is slightly increased?



A.



B.

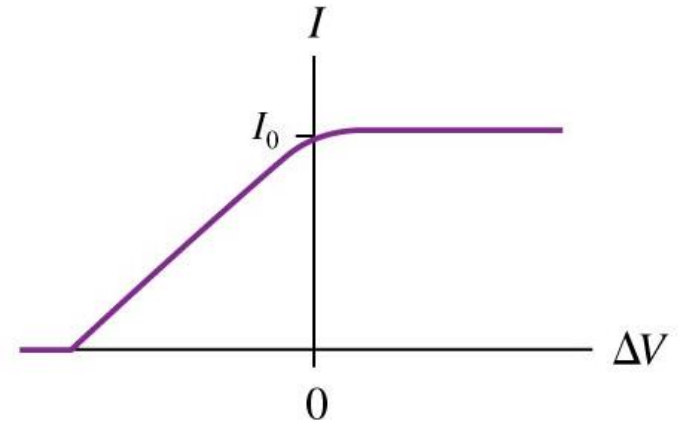


C.

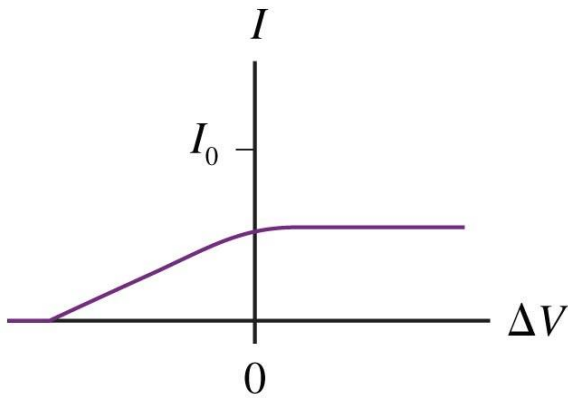
D. None of these.

# QuickCheck 28.8

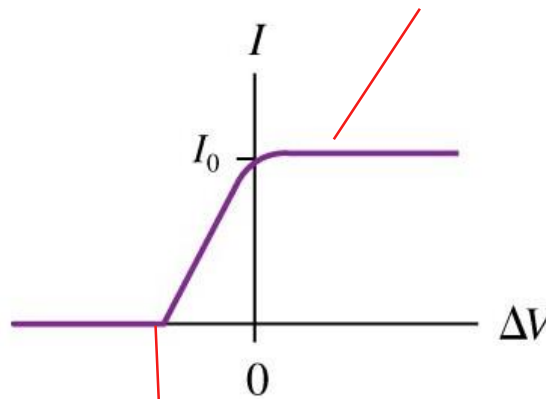
What happens to this graph of the photoelectric current if the cathode's work function is slightly increased?



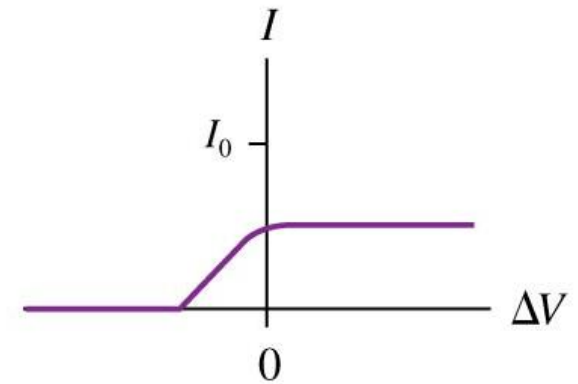
Number of photoelectrons unchanged



A.



B. ✓



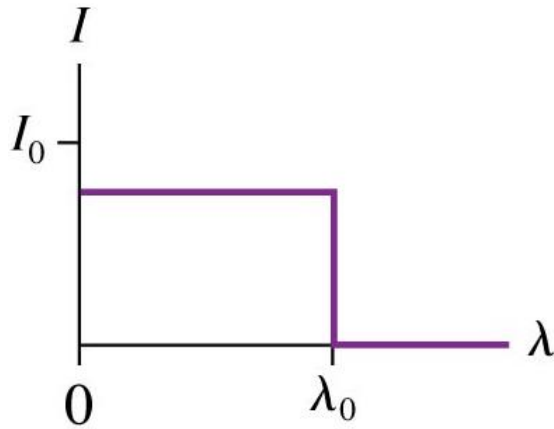
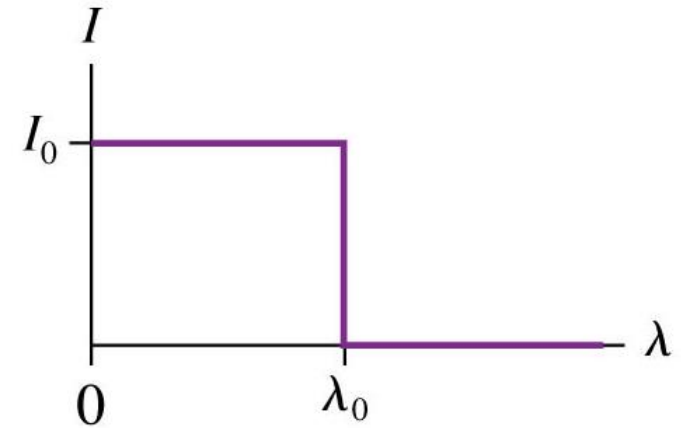
C.

D. None of these.

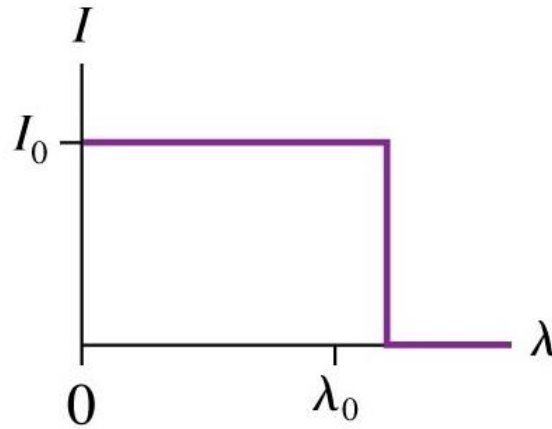
Electrons emerge slower, so stopping potential reduced.

## QuickCheck 28.9

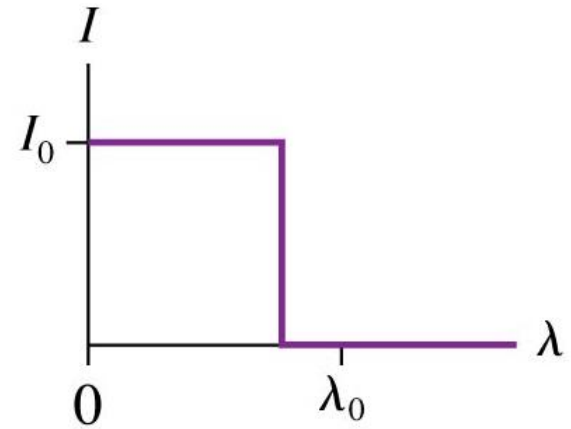
What happens to this graph of the photoelectric current if the cathode's work function is slightly increased?



A.



B.

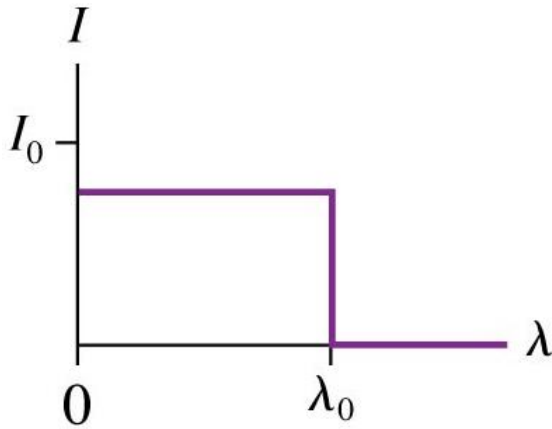
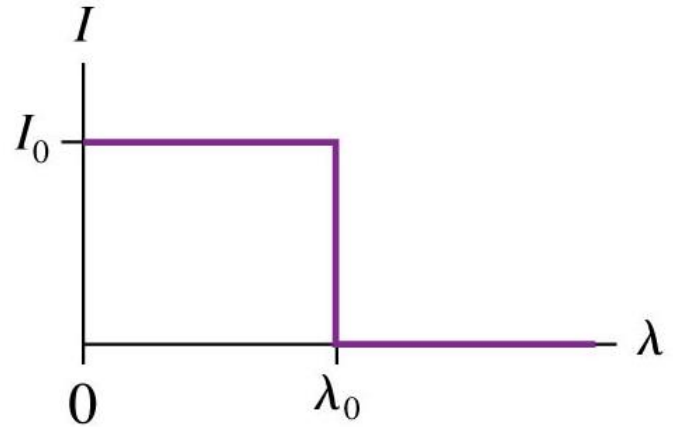


C.

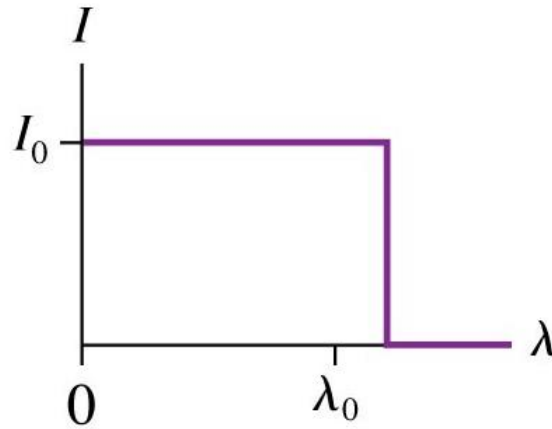
D. None of these.

## QuickCheck 28.9

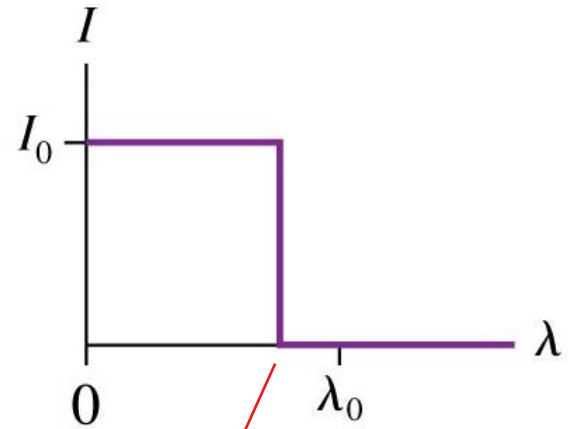
What happens to this graph of the photoelectric current if the cathode's work function is slightly increased?



A.



B.



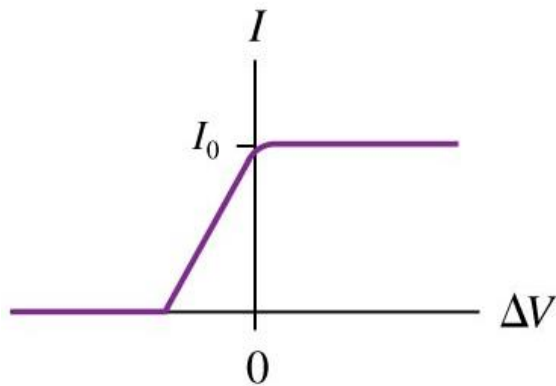
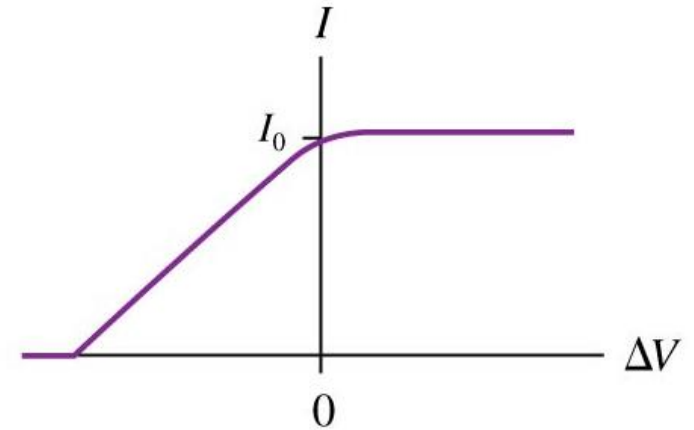
C. ✓

Threshold shifts to shorter wavelength.

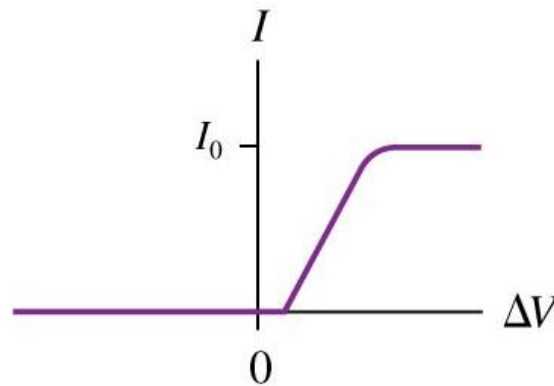
D. None of these.

## QuickCheck 28.10

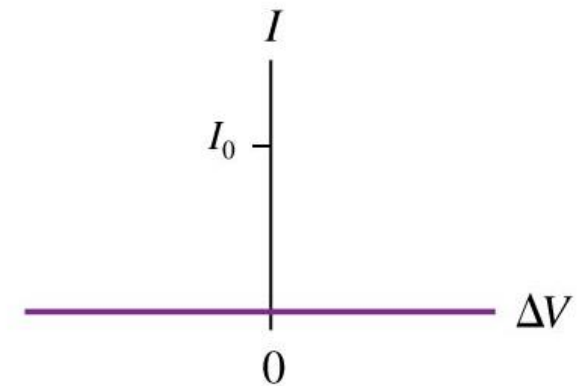
What happens to this graph of the photoelectric current if the cathode's work function is larger than the photon energy?



A.



B.



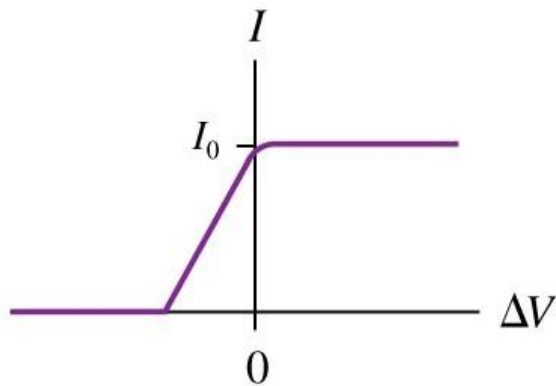
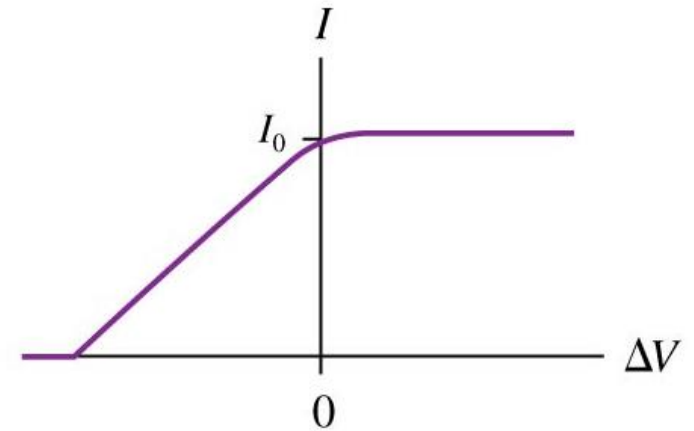
C.

D. None of these.

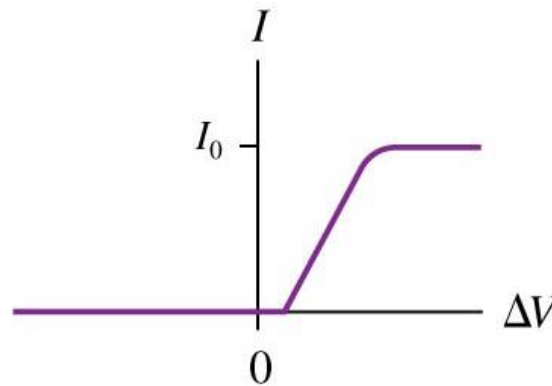


## QuickCheck 28.10

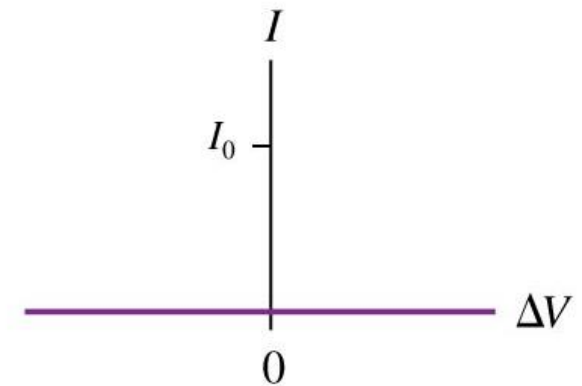
What happens to this graph of the photoelectric current if the cathode's work function is larger than the photon energy?



A.



B.



✓ C.

D. None of these.

Below threshold if  $E_{\text{photon}} < E_0$


## QuickCheck 28.11

The energy of a photon of red light is \_\_\_\_\_ the energy of a photon of blue light.

- A. Larger than
- B. The same as
- C. Smaller than

## QuickCheck 28.11

The energy of a photon of red light is \_\_\_\_\_ the energy of a photon of blue light.

- A. Larger than
- B. The same as
-  C. **Smaller than**


## QuickCheck 28.12

A beam of light has wavelength  $\lambda$ . The light's intensity is reduced without changing the wavelength. Which is true after the intensity is reduced?

- A. The photons are smaller.
- B. The photons travel slower.
- C. The photon have less energy.
- D. There are fewer photons per second.
- E. Both C and D

## QuickCheck 28.12

A beam of light has wavelength  $\lambda$ . The light's intensity is reduced without changing the wavelength. Which is true after the intensity is reduced?

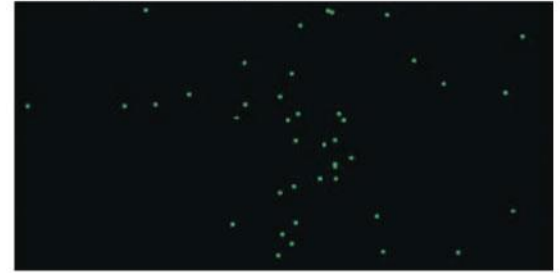
- A. The photons are smaller.
- B. The photons travel slower.
- C. The photon have less energy.
-  D. There are fewer photons per second.
- E. Both C and D

# Section 28.3 Photons

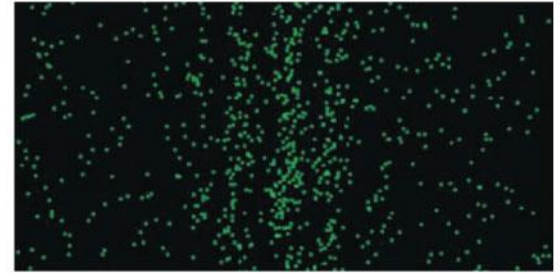
# Photons

- What *are* photons?
- If we return to the double-slit experiment, but use a much dimmer light intensity, the fringes will be too faint to see by eye.
- We can use a detector that can build up an image over time.

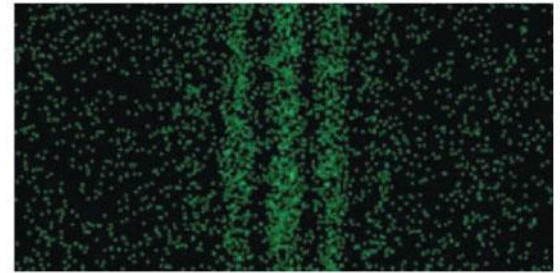
(a) Image after a very short time



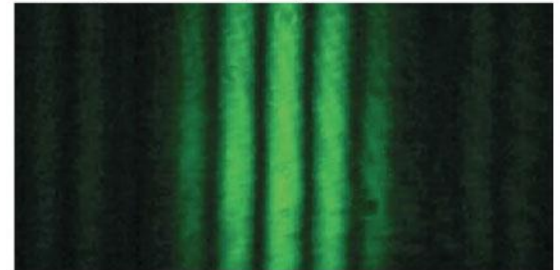
(b) Image after a slightly longer time



(c) Continuing to build up the image



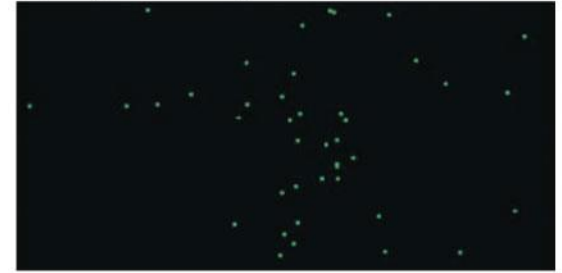
(d) Image after a very long time



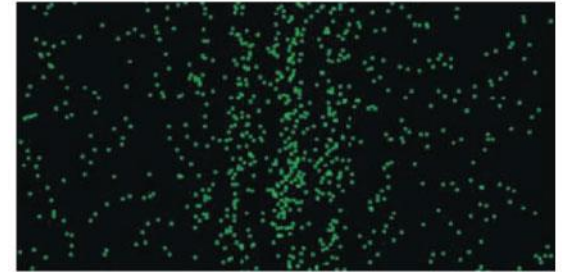
# Photons

- Individual photons pass through the double slit, and as the image builds up, the photons are grouped into bands at *exactly* the positions we expect to see the bright constructive-interference fringes.
- We see particle-like dots forming wave-like interference fringes.

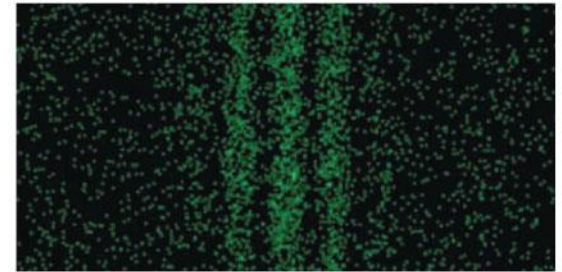
(a) Image after a very short time



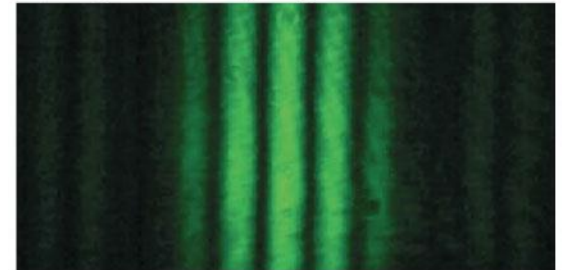
(b) Image after a slightly longer time



(c) Continuing to build up the image



(d) Image after a very long time





# Photons

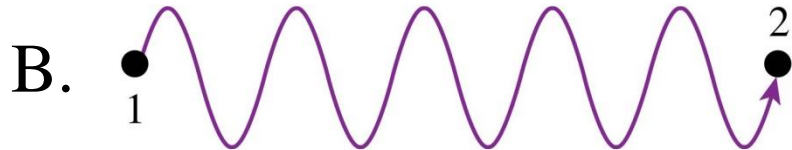
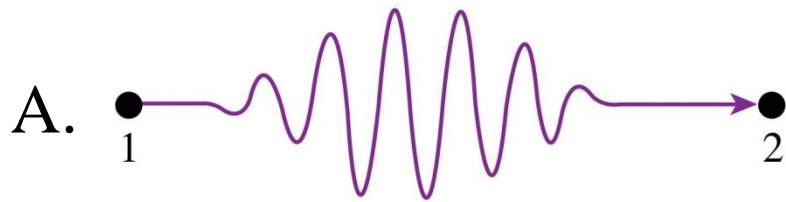
- If only one photon is passing through the apparatus at a time, it must somehow *interfere with itself* to create the wave-like pattern.
- If each photon is interfering with itself (nothing else is present), then each photon, despite the fact that it is a particle-like object, must somehow go through *both* slits, something only a wave could do.
- **Sometimes light exhibits particle-like behavior and sometimes it exhibits wave-like behavior.**

# The Photon Rate

- The photon nature of light isn't apparent in most cases. Only at extremely low intensities does the light begin to appear as a stream of individual photons.
- The light sources with which we are familiar emit such vast numbers of photons that we are only aware of their wave-like superposition, just as we notice only the roar of a heavy rain on our roof and not the individual photons.

## QuickCheck 28.13

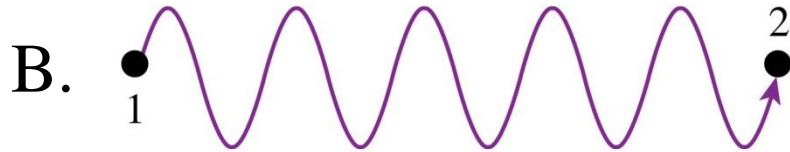
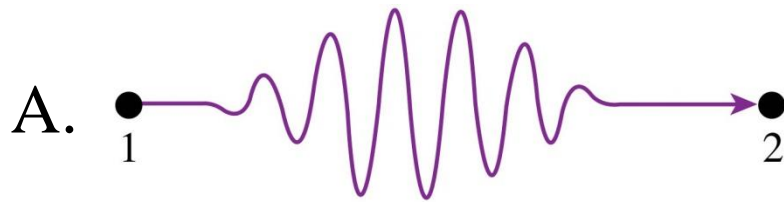
Points 1 and 2 are  $5 \mu\text{m}$  apart. Light with a wavelength of  $1 \mu\text{m}$  travels from point 1 to point 2. Which is the trajectory followed by the photons?



D. None of these.

## QuickCheck 28.13

Points 1 and 2 are  $5 \mu\text{m}$  apart. Light with a wavelength of  $1 \mu\text{m}$  travels from point 1 to point 2. Which is the trajectory followed by the photons?



Light travels in straight lines.

D. None of these.

## Example 28.5 How many photons per second does a laser emit?

The 1.0 mW light beam from a laser pointer ( $\lambda = 670 \text{ nm}$ ) shines on a screen. How many photons strike the screen each second?

**PREPARE** The power of the beam is 1.0 mW, or  $1.0 \times 10^{-3} \text{ J/s}$ . Each second,  $1.0 \times 10^{-3} \text{ J}$  of energy reaches the screen. It arrives as individual photons of energy given by Equation 28.3.

## Example 28.5 How many photons per second does a laser emit? (cont.)

**SOLVE** The frequency of the photons is  $f = c/\lambda = 4.48 \times 10^{14}$  Hz, so the energy of an individual photon is  $E = hf = (6.63 \times 10^{-34} \text{ J} \cdot \text{s})(4.48 \times 10^{14} \text{ Hz}) = 2.97 \times 10^{-19} \text{ J}$ . The number of photons reaching the screen each second is the total energy reaching the screen each second divided by the energy of an individual photon:

$$\frac{1.0 \times 10^{-3} \text{ J/s}}{2.97 \times 10^{-19} \text{ J/photon}} = 3.4 \times 10^{15} \text{ photons per second}$$

**ASSESS** Each photon carries a small amount of energy, so there must be a huge number of photons per second to produce even this modest power.

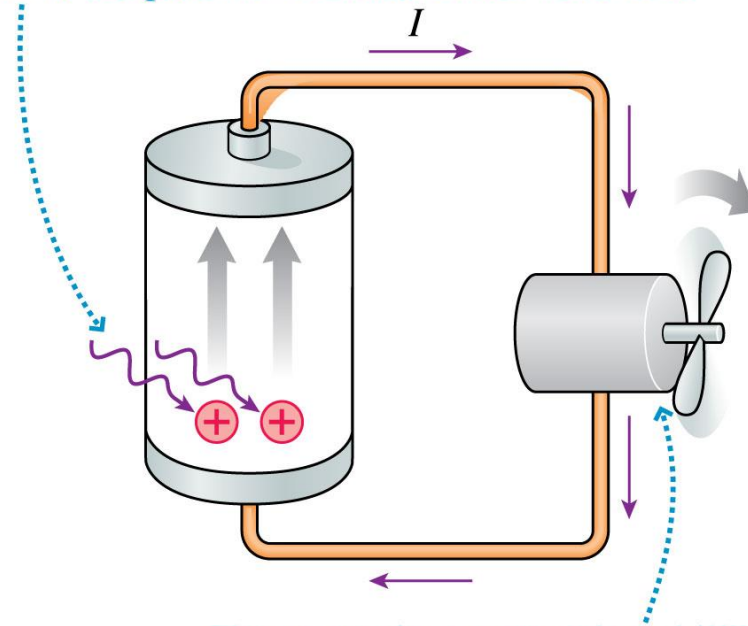
# Detecting Photons

- Early light detectors consisted of a polished metal plate in a vacuum tube. When light fell on the plate, an electron current was generated that could trigger an action, such as sounding an alarm, or could provide a measurement of the light intensity.

# Detecting Photons

- A *solar cell* works much like a battery, but the energy to lift charges to a higher potential comes from photons.

Photons with energy greater than the threshold give their energy to charge carriers, increasing their potential energy and lifting them to the positive terminal of the solar cell.



Charge carriers move “downhill” through the circuit. Their energy can be used to run useful devices.



## Example 28.7 Finding the current from a solar cell

1.0 W of monochromatic light of wavelength 550 nm illuminates a silicon solar cell, driving a current in a circuit. What is the maximum possible current this light could produce?

## Example 28.7 Finding the current from a solar cell (cont.)

**PREPARE** The wavelength is shorter than the 1200 nm threshold wavelength noted for a silicon solar cell, so the photons will have sufficient energy to cause charge carriers to flow. Each photon of the incident light will give its energy to a single charge carrier. The maximum number of charge carriers that can possibly flow in each second is thus equal to the number of photons that arrive each second.

## Example 28.7 Finding the current from a solar cell (cont.)

**SOLVE** The power of the light is  $P = 1.0 \text{ W} = 1.0 \text{ J/s}$ . The frequency of the light is  $f = c/\lambda = 5.45 \times 10^{14} \text{ Hz}$ , so the energy of individual photons is  $E = hf = 3.61 \times 10^{-19} \text{ J}$ . The number of photons arriving per second is  $(1.0 \text{ J/s}) / (3.61 \times 10^{-19} \text{ J/photon}) = 2.77 \times 10^{18}$ .

## Example 28.7 Finding the current from a solar cell (cont.)

Each photon can set at most one charge carrier into motion, so the maximum current is  $2.77 \times 10^{18}$  electrons/s. The current in amps—coulombs per second—is the electron flow rate multiplied by the charge per electron:

$$\begin{aligned} I_{\max} &= (2.77 \times 10^{18} \text{ electrons/s})(1.6 \times 10^{-19} \text{ C}) \\ &= 0.44 \text{ C/s} = 0.44 \text{ A} \end{aligned}$$

## Example 28.7 Finding the current from a solar cell (cont.)

**ASSESS** The key concept underlying the solution is that one photon gives its energy to a single charge carrier. We've calculated the current if all photons give their energy to charge carriers. The current in a real solar cell will be less than this because some photons will be reflected or otherwise “lost” and will not transfer their energy to charge carriers.

# Detecting Photons

- The *charge-coupled device* (CCD) or *complementary metal oxide semiconductor* (CMOS) detector in a digital camera consists of millions of *pixels*, each a microscopic silicon-based photodetector.
- If the frequency exceeds the threshold frequency, a photon hitting a pixel liberates one electron. The electrons are stored inside the pixel, and the total accumulated charge is directly proportional to the light intensity—the number of photons—hitting the pixel.
- After the exposure, the charge in each pixel is read and the value stored in memory. Then the pixel is reset.

## Try It Yourself: Photographing Photos

Photodetectors based on silicon can be triggered by photons with energy as low as 1.1 eV, corresponding to a wavelength in the infrared. The light-sensing chip in your digital camera can detect the infrared signal given off by a remote control. Press a button on your remote control, aim it at your digital camera, and snap a picture. The picture will clearly show the infrared emitted by the remote, though this signal is invisible to your eye. (Some cameras have infrared filters that may block most or nearly all of the signal.)

## QuickCheck 28.14

Which of the following phenomena is best explained by treating light as a wave?

- A. The threshold frequency in the photoelectric effect
- B. The emission of only certain wavelengths of light by an excited gas
- C. The limited resolution of a light microscope
- D. The quantization of energy levels for a particle in a box



## QuickCheck 28.14

Which of the following phenomena is best explained by treating light as a wave?

- A. The threshold frequency in the photoelectric effect
- B. The emission of only certain wavelengths of light by an excited gas
- C. The limited resolution of a light microscope
- ✓ D. The quantization of energy levels for a particle in a box

## QuickCheck 28.15

Which of the following phenomena is best explained by treating light as a particle?

- A. The limited resolution of a light microscope
- B. The diffraction pattern that results when x rays illuminate a crystal
- C. The threshold frequency in the photoelectric effect
- D. The quantization of energy levels for a particle in a box

## QuickCheck 28.15

Which of the following phenomena is best explained by treating light as a particle?

- A. The limited resolution of a light microscope
- B. The diffraction pattern that results when x rays illuminate a crystal
- ✓ C. The threshold frequency in the photoelectric effect
- D. The quantization of energy levels for a particle in a box

## Example Problem

Exposure to ultraviolet light can damage the skin. For this reason, there are suggested limits for exposure to ultraviolet light in work settings. These limits are wavelength-dependent. At a wavelength of 313 nm, the maximum suggested total exposure is 500 mJ/cm<sup>2</sup> of skin; for 280 nm, the limit falls to 3.4 mJ/cm<sup>2</sup> of skin.

- A. What is the photon energy corresponding to each of these wavelengths?
- B. How many total photons do each of these exposures correspond to?

## Example Problem

Port-wine birthmarks can be removed by exposure to 585 nm laser light. Pulses are strongly absorbed by oxyhemoglobin in the capillaries in the birthmark, destroying them. A typical laser pulse lasts for 1.5 ms, and contains an energy of 7.0 J.

- A. What is the power of the laser pulse?
- B. How many photons are in each pulse?

# Section 28.4 Matter Waves

# Matter Waves

- Prince Louis-Victor de Broglie reasoned by analogy with Einstein's equation  $E = hf$  for the photon and with some ideas from his theory of relativity, that all material particles should have some kind of wave-like nature.
- De Broglie determined that if a material particle of momentum  $p = mv$  has a wave-like nature, its wavelength must be given by

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

De Broglie wavelength for a moving particle

- This wavelength is called the **de Broglie wavelength**.

## Example 28.8 Calculating the de Broglie wavelength of an electron

What is the de Broglie wavelength of an electron with a kinetic energy of 1.0 eV?

**SOLVE** An electron with kinetic energy  $K = \frac{1}{2}mv^2 = 1.0 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$  has speed

$$v = \sqrt{\frac{2K}{m}} = 5.9 \times 10^5 \text{ m/s}$$



## Example 28.8 Calculating the de Broglie wavelength of an electron (cont.)

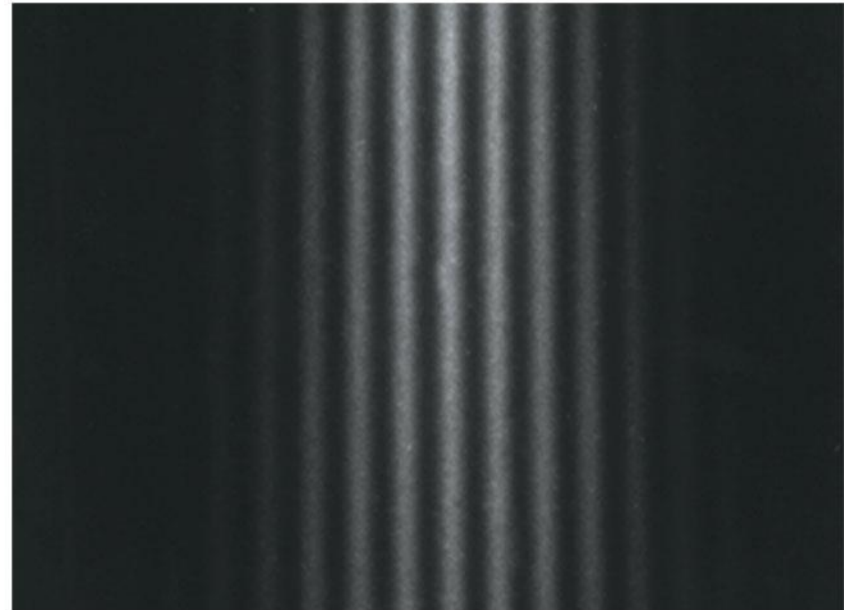
Although fast by macroscopic standards, the electron gains this speed by accelerating through a potential difference of a mere 1 V. The de Broglie wavelength is

$$\lambda = \frac{h}{mv} = 1.2 \times 10^{-9} \text{ m} = 1.2 \text{ nm}$$

**ASSESS** The electron's wavelength is small, but it is similar to the wavelengths of x rays and larger than the approximately 0.1 nm spacing of atoms in a crystal. We can observe x-ray diffraction, so if an electron has a wave nature, it should be easily observable.

# Matter Waves

- **Matter exhibits all of the properties that we associate with waves.**
- The figure shows the intensity pattern recorded after electrons passed through two narrow slits. The pattern is clearly a double-slit interference pattern and the spacing of the fringes matches the prediction for a wavelength given by de Broglie's formula.
- **The electrons are behaving like waves!**

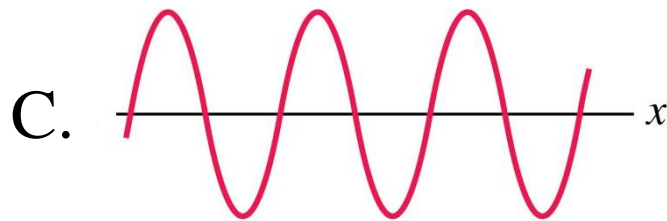
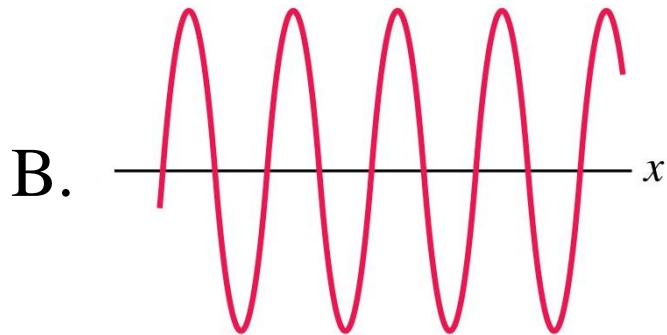
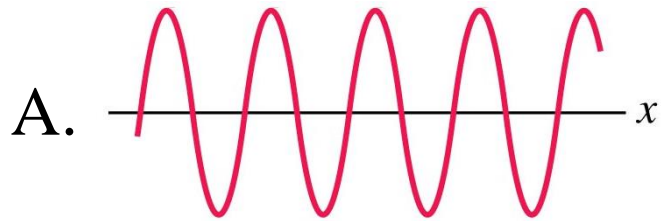


# Matter Waves

- The reason we do not see macroscopic objects, like a baseball, exhibiting wave-like behavior is because of their wavelength.
- In Chapter 17 we learned that diffraction, interference, and other wave-like phenomena are observed when the wavelength is comparable to or larger than the size of an opening a wave must pass through.
- The wave nature of macroscopic objects is unimportant and undetectable because their wavelengths are so incredibly small.

## QuickCheck 28.16

De Broglie waves are shown for three particles of equal mass. Which one or ones is moving most slowly?

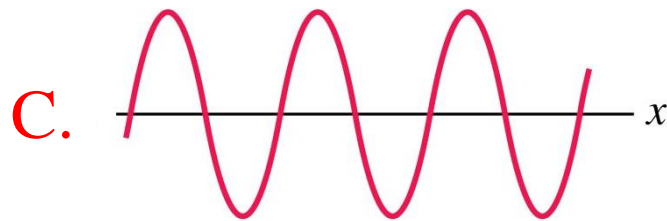
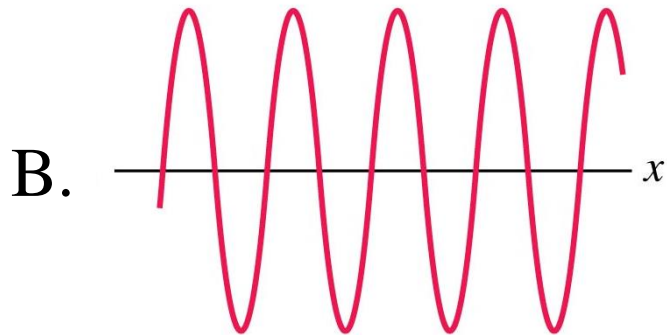
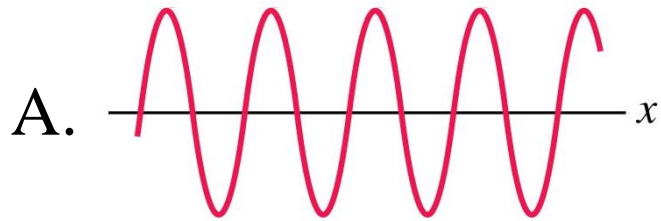


D. A and B are tied.

E. A and C are tied.

## QuickCheck 28.16

De Broglie waves are shown for three particles of equal mass. Which one or ones is moving most slowly?



D. A and B are tied.

E. A and C are tied.

Longest  
wavelength

## Example 28.9 Calculating the de Broglie wavelength of a smoke particle

One of the smallest macroscopic particles we could imagine using for an experiment would be a very small smoke or soot particle. These are  $\approx 1 \mu\text{m}$  in diameter, too small to see with the naked eye and just barely at the limits of resolution of a microscope. A particle this size has mass  $m \approx 10^{-18} \text{ kg}$ . Estimate the de Broglie wavelength for a  $1\text{-}\mu\text{m}$ -diameter particle moving at the very slow speed of  $1 \text{ mm/s}$ .

## Example 28.9 Calculating the de Broglie wavelength of a smoke particle (cont.)

**SOLVE** The particle's momentum is  $p = mv \approx 10^{-21} \text{ kg} \cdot \text{m/s}$ . The de Broglie wavelength of a particle with this momentum is

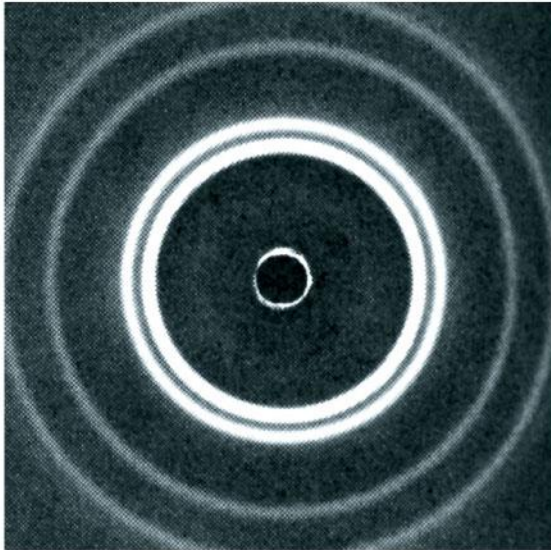
$$\lambda = \frac{h}{p} \approx 7 \times 10^{-13} \text{ m}$$

**ASSESS** The wavelength is much, much smaller than the particle itself—much smaller than an individual atom! We don't expect to see this particle exhibiting wave-like behavior.

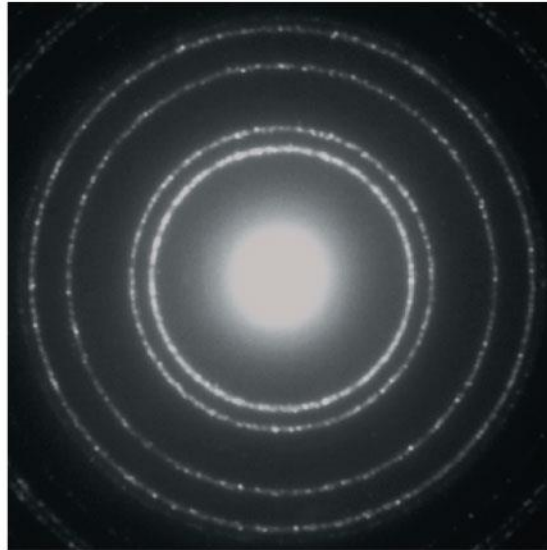
# The Interference and Diffraction of Matter

- The first evidence from experimental data for de Broglie's hypothesis came from the observation that **electrons diffract and interfere exactly like x rays.**

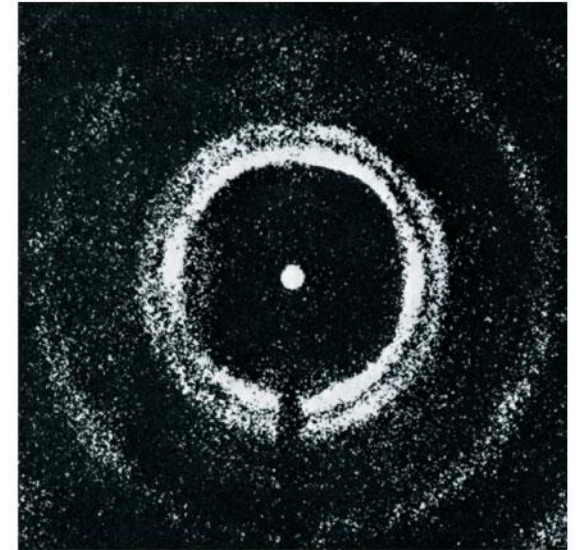
(a) X-ray diffraction pattern



(b) Electron diffraction pattern



(c) Neutron diffraction pattern



The diffraction patterns produced by x rays, electrons, and neutrons passing through an aluminum-foil target.



## QuickCheck 28.17

A beam of electrons, and then a beam of protons, are shot through a double slit with a very small slit spacing of  $1 \mu\text{m}$ . The electrons and protons travel at the same speed. Which is true?

- A. They both make interference patterns on a screen. The fringe spacing is wider for the electron interference pattern.
- B. They both make interference patterns on a screen. The fringe spacing is wider for the proton interference pattern.
- C. Only the electrons make an interference pattern on a screen.
- D. Only the protons make an interference pattern on a screen.
- E. Neither makes an interference pattern.

## QuickCheck 28.17

A beam of electrons, and then a beam of protons, are shot through a double slit with a very small slit spacing of  $1 \mu\text{m}$ . The electrons and protons travel at the same speed. Which is true?

Electrons with a longer de Broglie  $\lambda = \frac{h}{mv}$  spread out more.

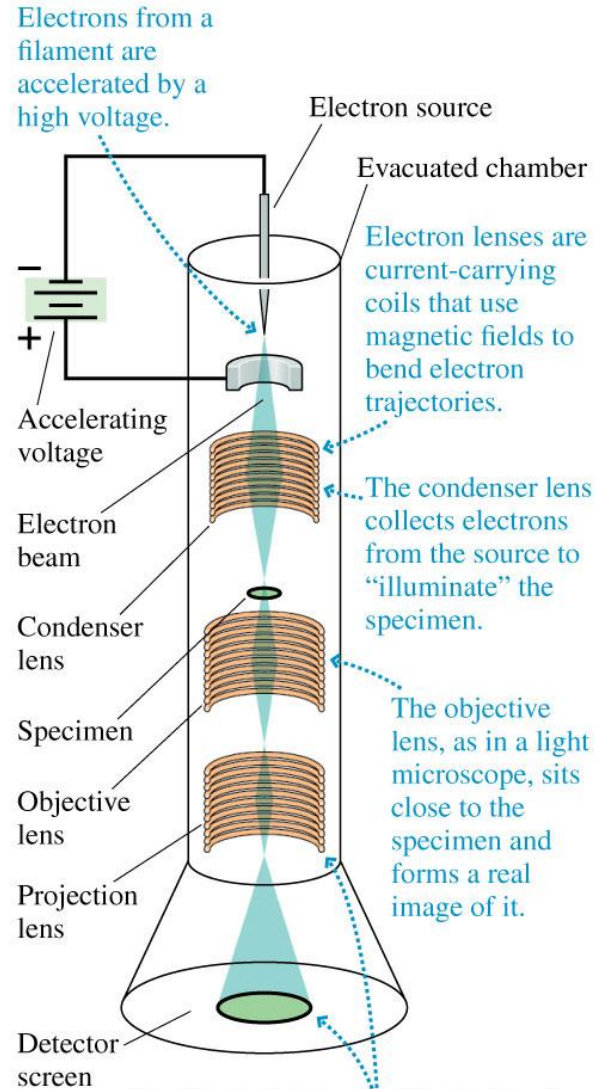
- ✓ A. They both make interference patterns on a screen. The fringe spacing is wider for the electron interference pattern.
- B. They both make interference patterns on a screen. The fringe spacing is wider for the proton interference pattern.
- C. Only the electrons make an interference pattern on a screen.
- D. Only the protons make an interference pattern on a screen.
- E. Neither makes an interference pattern.

# The Electron Microscope

- We learned that the wave nature of light limits the ultimate resolution of an optical microscope—the smallest resolvable separation between two objects—to about half a wavelength of light.
- We can see details finer than this limit using an electron microscope, which uses a beam of electrons to create an image.

# The Electron Microscope

- This figure shows a *transmission electron microscope* (TEM).



In a light microscope, the eyepiece is used to view the real image from the objective lens. An electron microscope has a projection lens that projects a magnified real image onto a detector.

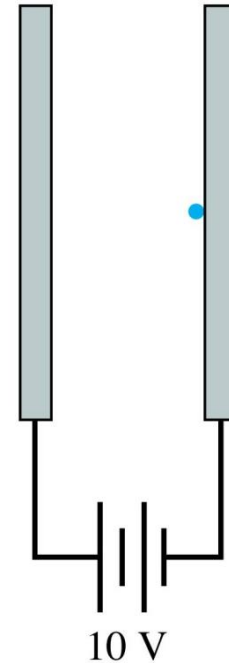
# The Electron Microscope

- Our ability to control electron trajectories allows electron microscopes to have magnifications far exceeding those of light microscopes.
- The resolution is still limited by wave effects; electrons have wave-like properties and a de Broglie wavelength  $\lambda = h/p$ .
- The resolving power is, at best, about half the electron's de Broglie wavelength. In practice, electron microscopes are limited by imperfections in the electron lenses.

## QuickCheck 28.18

An electron is released from the negative plate. Its de Broglie wavelength upon reaching the positive plate is \_\_\_\_\_ its de Broglie wavelength at the negative plate.

- A. Greater than
- B. The same as
- C. Less than

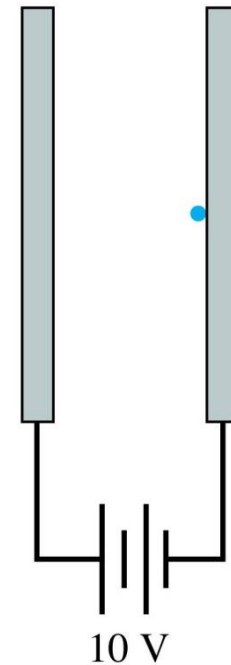


## QuickCheck 28.18

An electron is released from the negative plate. Its de Broglie wavelength upon reaching the positive plate is \_\_\_\_\_ its de Broglie wavelength at the negative plate.

- A. Greater than
- B. The same as
- ✓ C. Less than

Speeds up as it crosses, and  $\lambda = \frac{h}{mv}$



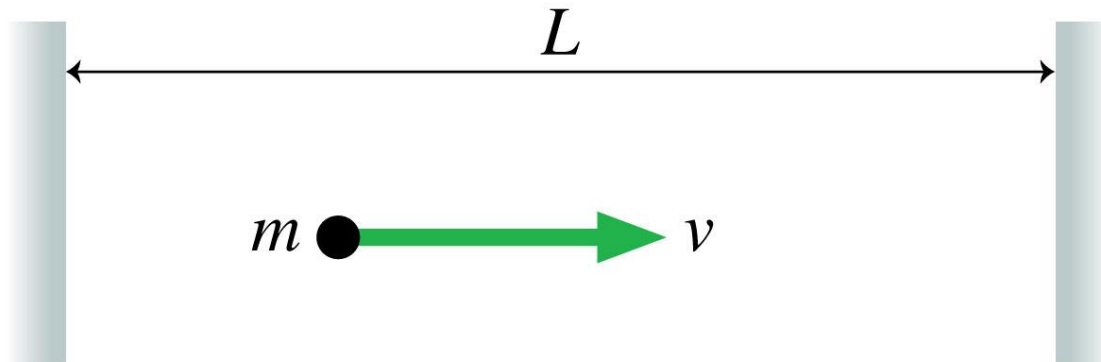
# Section 28.5 Energy is Quantized



# Energy Is Quantized

- We learned that waves on a string fixed at both ends form standing waves. Is there a “standing matter wave”?
- We consider a physical system called a “particle in a box” with only one dimensional motion. The “box” is defined by two fixed ends, and the particle bounces back and forth between the boundaries.

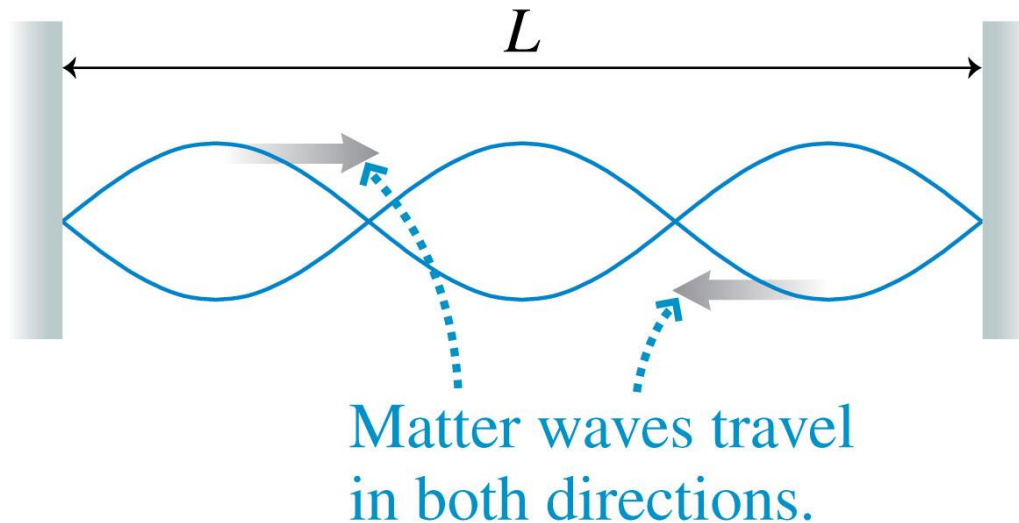
(a) A classical particle of mass  $m$  bounces back and forth between two boundaries.



# Energy Is Quantized

- Since particles have wave-like properties, we consider a *wave* reflecting back and forth from the ends of the box.
- The reflections will create a standing wave, analogous to the standing wave on a string that is tied at both ends.

(b) Matter waves moving in opposite directions create standing waves.



# Energy Is Quantized

- For a wave on a string, we saw that there were only certain possible modes. The same will be true for the particle in the box.
- The wavelength of a standing wave is related to the length  $L$  of the string by

$$\lambda_n = \frac{2L}{n} \quad n = 1, 2, 3, 4, \dots$$

- The wavelength of the particle in the box will follow the same formula, but must also satisfy the de Broglie wavelength  $\lambda = h/p$ :

$$\frac{h}{p} = \frac{2L}{n}$$

# Energy Is Quantized

- For the node of the standing wave of the particle in a box, we solve for the particle's momentum  $p$ :

$$p_n = n \left( \frac{h}{2L} \right) \quad n = 1, 2, 3, 4, \dots$$

- This tells us that the momentum of the particle can have only certain values. Other values are not possible.
- The energy of the particle is related to its momentum:

$$E = \frac{1}{2}mv^2 = \frac{p^2}{2m}$$

- The energy is therefore restricted to a specific set of values:

$$E_n = \frac{1}{2m} \left( \frac{hn}{2L} \right)^2$$

# Energy Is Quantized

- Another way to describe the specific set of values allowed for a particle's energy is

The diagram shows the equation  $E_n = \frac{h^2}{8mL^2} n^2$  with several labels and arrows pointing to its components. A label 'Energy of a particle in a box (J)' points to  $E_n$ . A label 'Planck's constant' points to  $h^2$ . A label 'Particle mass (kg)' points to  $m$ . A label 'Length of box (m)' points to  $L^2$ . A label 'The allowed energies are each labeled by an integer  $n$ . The lowest possible energy level has  $n = 1$ .' points to  $n^2$ . To the right of the equation, the text ' $n = 1, 2, 3, 4, \dots$ ' is displayed.

- Because of the wave nature of matter, **a confined particle can only have certain energies.**
- The **quantization** of energy is the result that a confined particle can have only discrete values of energy.
- The number  $n$  is the **quantum number**, and each value of  $n$  characterizes one **energy level** of the particle in the box.

# Energy Is Quantized

- The lowest possible energy a particle in a box can have is

$$E_1 = \frac{h^2}{8mL^2}$$

- In terms of the lowest possible energy, all other possible energies are

$$E_n = n^2 E_1$$

- The allowed energies are inversely proportional to both  $m$  and  $L^2$ . Both  $m$  and  $L$  must be exceedingly small before energy quantization has any significance.

# Energy Is Quantized

- An atom is more complicated than a simple one-dimensional box, but an electron is “confined” to an atom. The electron orbits are, in some sense, standing waves, and the **energy of the electrons in an atom must be quantized.**

## Example 28.11 Finding the allowed energies of a confined electron

An electron is confined to a region of space of length 0.19 nm—comparable in size to an atom. What are the first three allowed energies of the electron?

**PREPARE** We'll model this system as a particle in a box, with a box of length 0.19 nm. The possible energies are given by Equation 28.13.



## Example 28.11 Finding the allowed energies of a confined electron (cont.)

**SOLVE** The mass of an electron is  $m = 9.11 \times 10^{-31}$  kg. Thus the first allowed energy is

$$E_1 = \frac{h^2}{8mL^2} = 1.7 \times 10^{-18} \text{ J} = 10 \text{ eV}$$

This is the lowest allowed energy. The next two allowed energies are

$$E_2 = 2^2 E_1 = 40 \text{ eV}$$

$$E_3 = 3^2 E_1 = 90 \text{ eV}$$

## Example 28.11 Finding the allowed energies of a confined electron (cont.)

**ASSESS** These energies are significant;  $E_1$  is larger than the work function of any metal in Table 28.1. Confining an electron to a region the size of an atom limits its energy to states separated by significant differences in energy. Clearly, our treatment of electrons in atoms must be a quantum treatment.

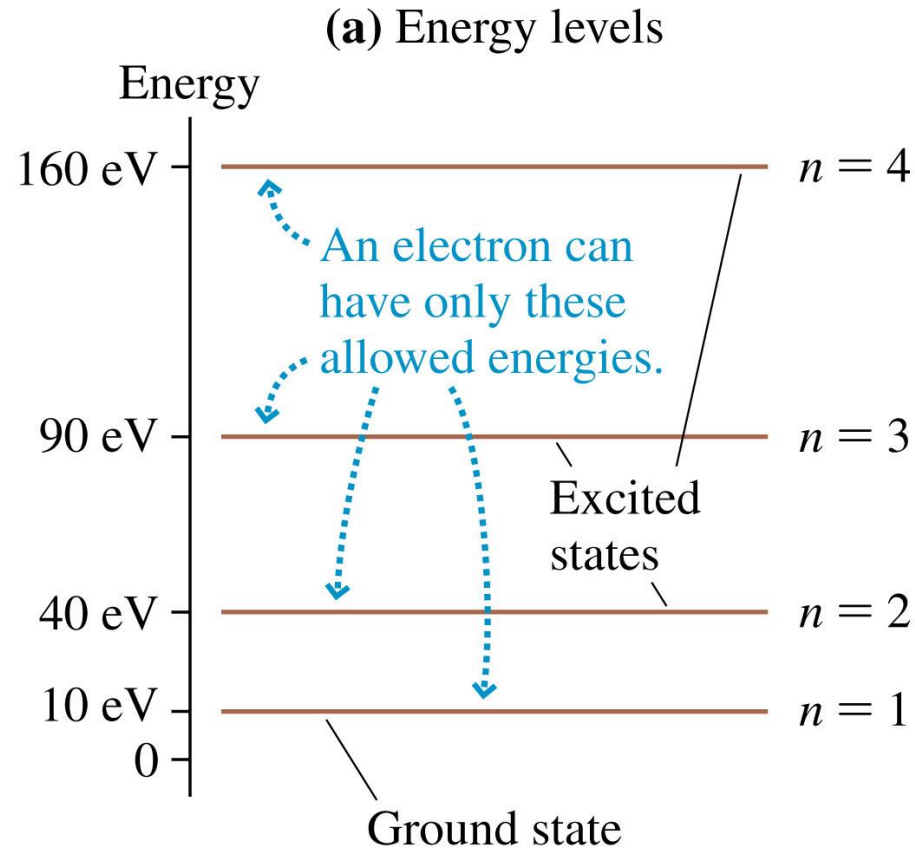
# Section 28.6 Energy Levels and Quantum Jumps

# Energy Levels and Quantum Jumps

- In 1925 Austrian physicist Erwin Schrödinger developed the first complete theory of quantum physics, called *quantum mechanics*.
- It describes how to calculate the quantized energy levels of systems from a particle in a box to electrons in atoms.
- It also describes how a quantized system gains or loses energy.

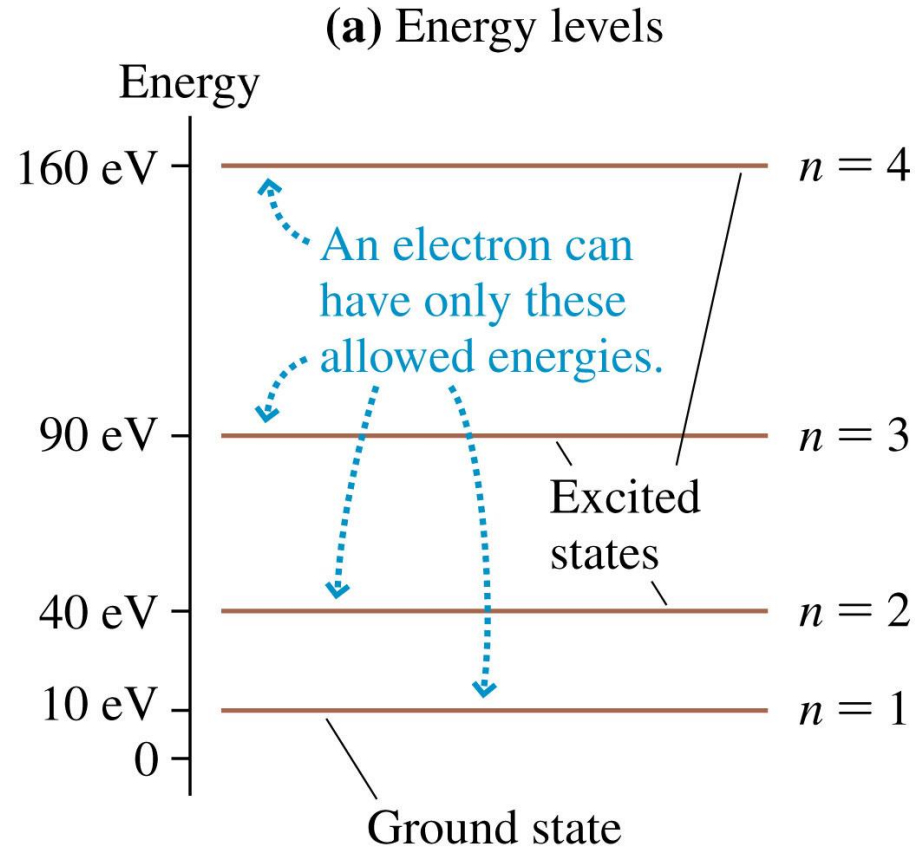
# Energy-Level Diagrams

- An **energy level diagram** is a useful visual representation of the quantized energies.
- This figure is an energy-level diagram for an electron in a 0.19-nm-long box.
- The vertical axis represents energy.
- The horizontal axis is not a scale, but can be thought of as the rungs of a ladder.



# Energy-Level Diagrams

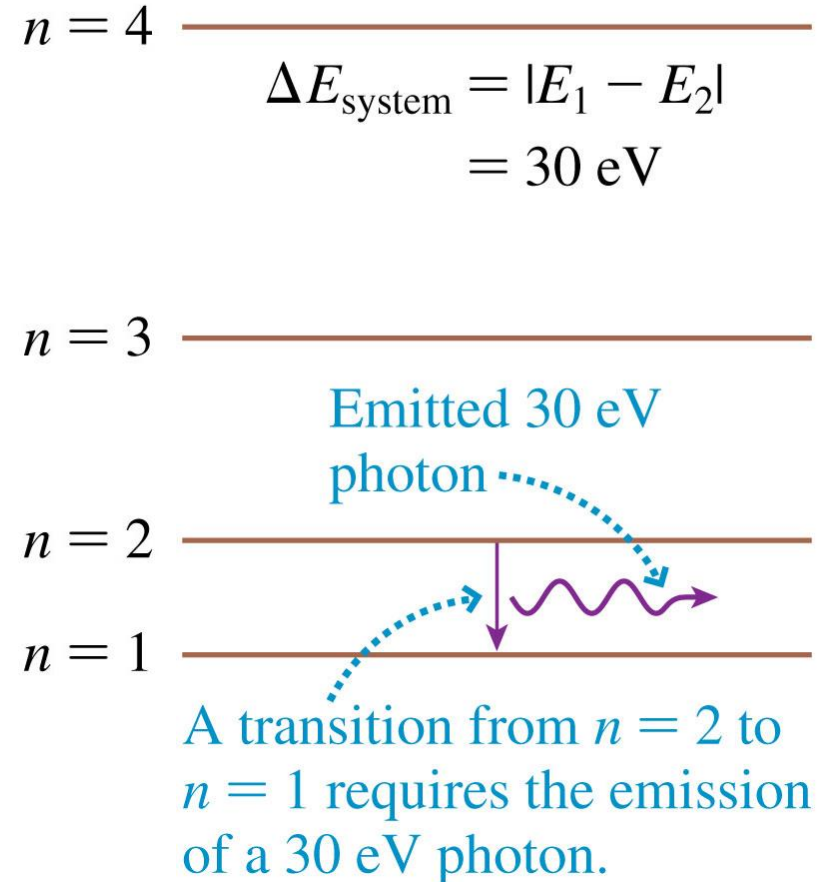
- The lowest rung on the energy-level diagram, with energy  $E_1$ , is the **ground state**.
- Higher rungs are called **excited states** and are labeled by their quantum numbers.



# Energy-Level Diagrams

- If a quantum system changes from one state to another, its energy changes, but conservation of energy still holds.
- If a system drops from a higher energy level to a lower, the excess energy  $\Delta E_{\text{system}}$  goes somewhere, generally in the form of an emitted photon.

## (b) Photon emission



# Energy-Level Diagrams

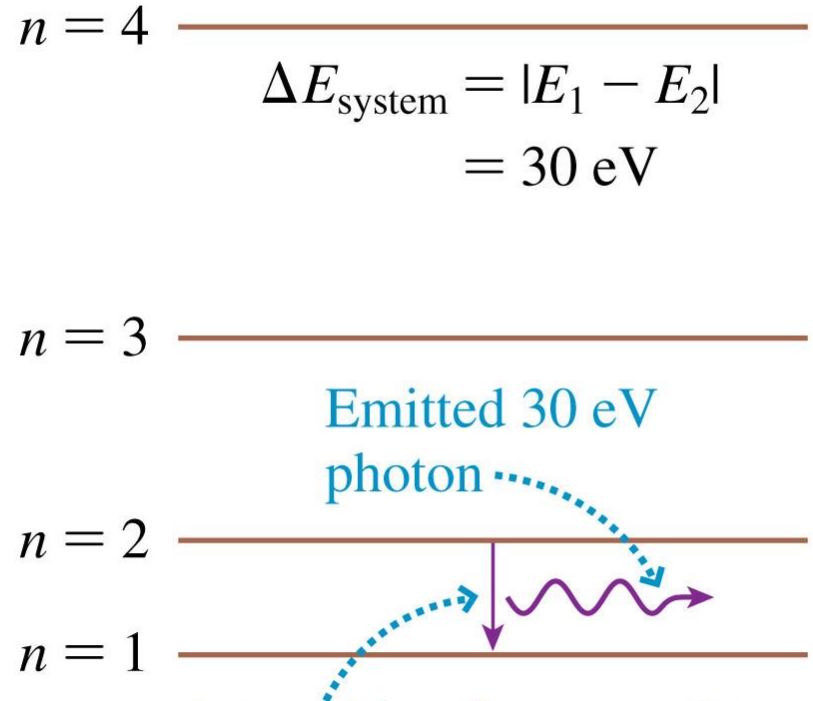
- A quantum system in energy level  $E_i$  that jumps down to energy level  $E_f$  loses an energy

$$\Delta E_{\text{system}} = |E_f - E_i|.$$

- The jump corresponds to the emission of a photon with frequency

$$f_{\text{photon}} = \frac{\Delta E_{\text{system}}}{h}$$

(b) Photon emission



$$\begin{aligned}\Delta E_{\text{system}} &= |E_1 - E_2| \\ &= 30 \text{ eV}\end{aligned}$$

A transition from  $n = 2$  to  $n = 1$  requires the emission of a 30 eV photon.



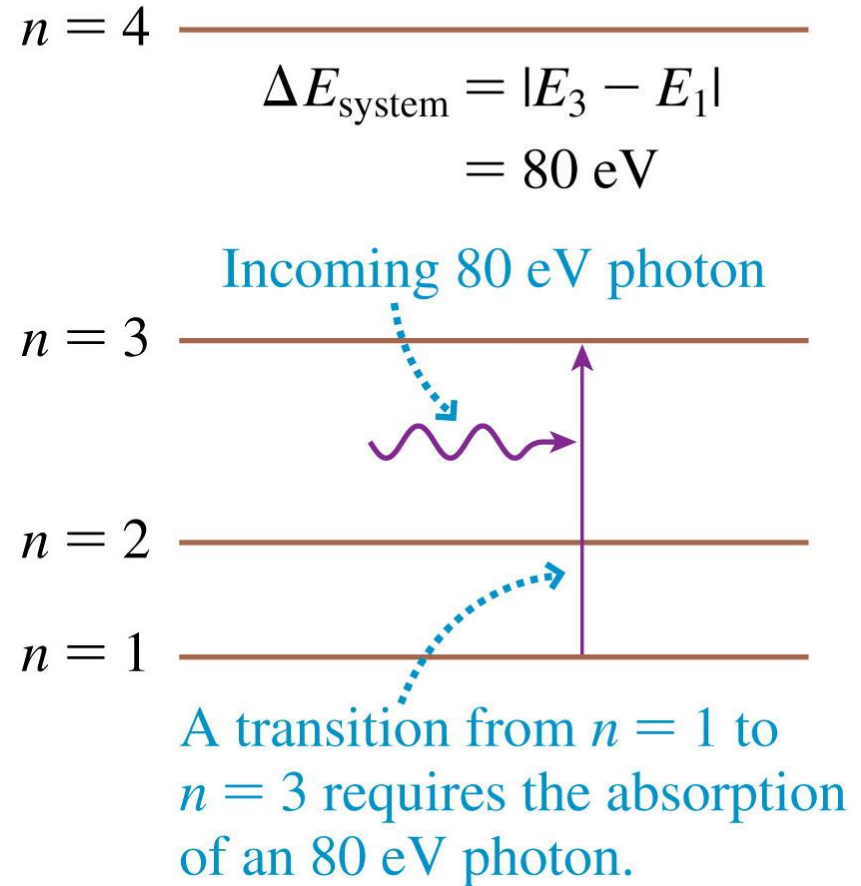
# Energy-Level Diagrams

- If the system absorbs a photon, it can “jump up” to a higher energy level.
- The frequency of the absorbed photon will have be calculated as

$$f_{\text{photon}} = \frac{\Delta E_{\text{system}}}{h}$$

- Energy jumps are called **transitions** or **quantum jumps**.

(c) Photon absorption



# Energy-Level Diagrams

- If a system jumps from an initial state with energy  $E_i$  to a final state with *lower* energy  $E_f$ , energy will be conserved if the system emits a photon with  $E_{\text{photon}} = \Delta E_{\text{system}}$ .
- The photon must have a frequency corresponding to

$$f_{\text{photon}} = \frac{\Delta E_{\text{system}}}{h}$$

- These photons form the *emission spectrum* of the quantum system.
- Similarly, a photon will not be absorbed unless it has the exact frequency required for the system to jump to a higher-energy state.

# Energy-Level Diagrams

- Let's summarize what quantum physics has to say about the properties of atomic-level systems:
  1. **The energies are quantized.** Only certain energies are allowed; all others are forbidden. This is a consequence of the wave-like properties of matter.
  2. **The ground state is stable.** Quantum systems seek the lowest possible energy state. A particle in an excited state, if left alone, will jump to lower and lower energy states until it reaches the ground state.

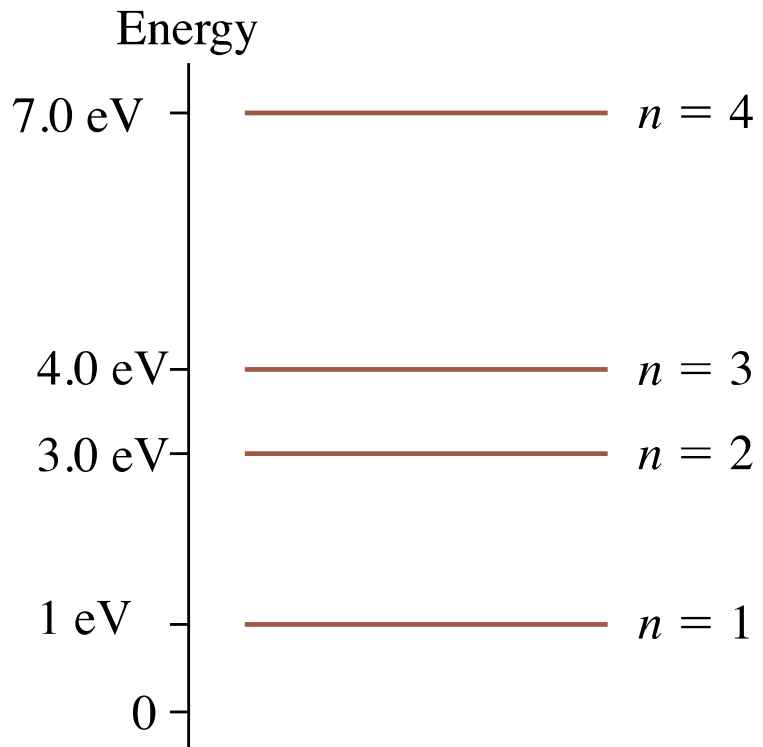
# Energy-Level Diagrams

3. **Quantum systems emit and absorb a *discrete spectrum of light*.** Only those photons whose frequencies match the energy *intervals* between the allowed energy levels can be emitted or absorbed. Photons of other frequencies cannot be emitted or absorbed without violating energy conservation.

## QuickCheck 28.19

What is the maximum photon energy that could be emitted by the quantum system with the energy level diagram shown below?

- A. 7.0 eV
- B. 6.0 eV
- C. 5.0 eV
- D. 4.0 eV



## QuickCheck 28.19

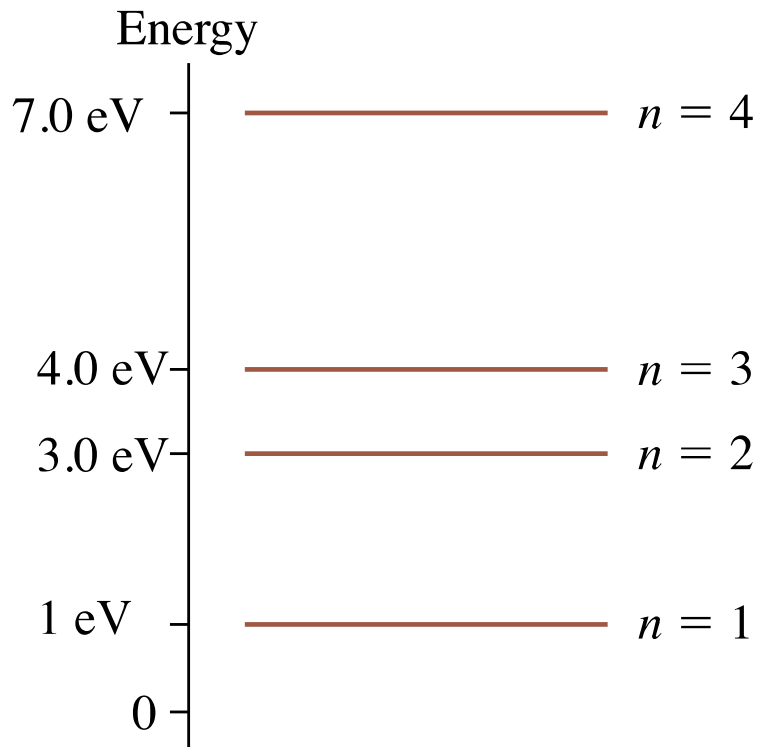
What is the maximum photon energy that could be emitted by the quantum system with the energy level diagram shown below?

A. 7.0 eV

✓ B. 6.0 eV

C. 5.0 eV

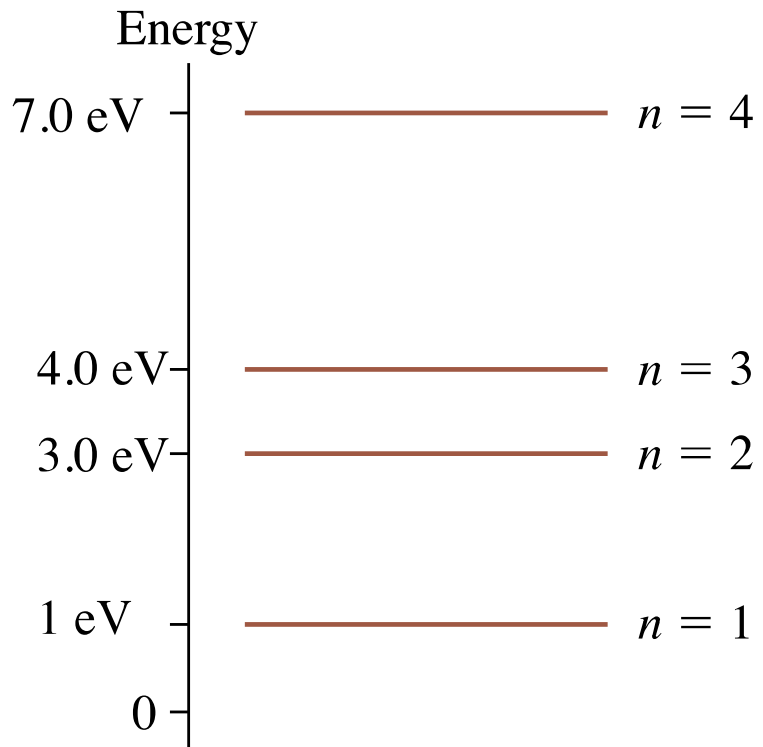
D. 4.0 eV



## QuickCheck 28.20

What is the minimum photon energy that could be emitted by the quantum system with the energy level diagram shown below?

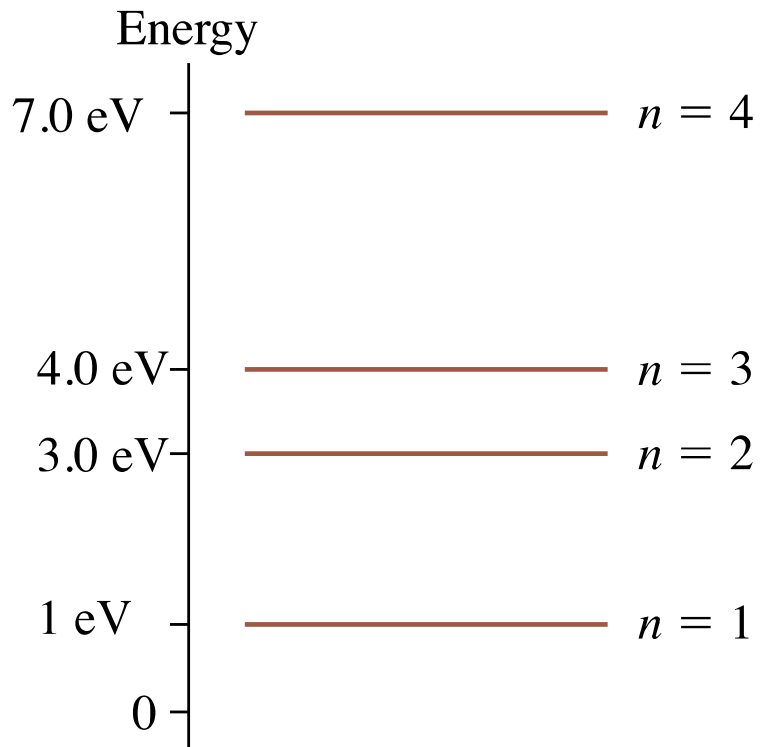
- A. 4.0 eV
- B. 3.0 eV
- C. 2.0 eV
- D. 1.0 eV



## QuickCheck 28.20

What is the minimum photon energy that could be emitted by the quantum system with the energy level diagram shown below?

- A. 4.0 eV
- B. 3.0 eV
- C. 2.0 eV
- ✓ D. 1.0 eV



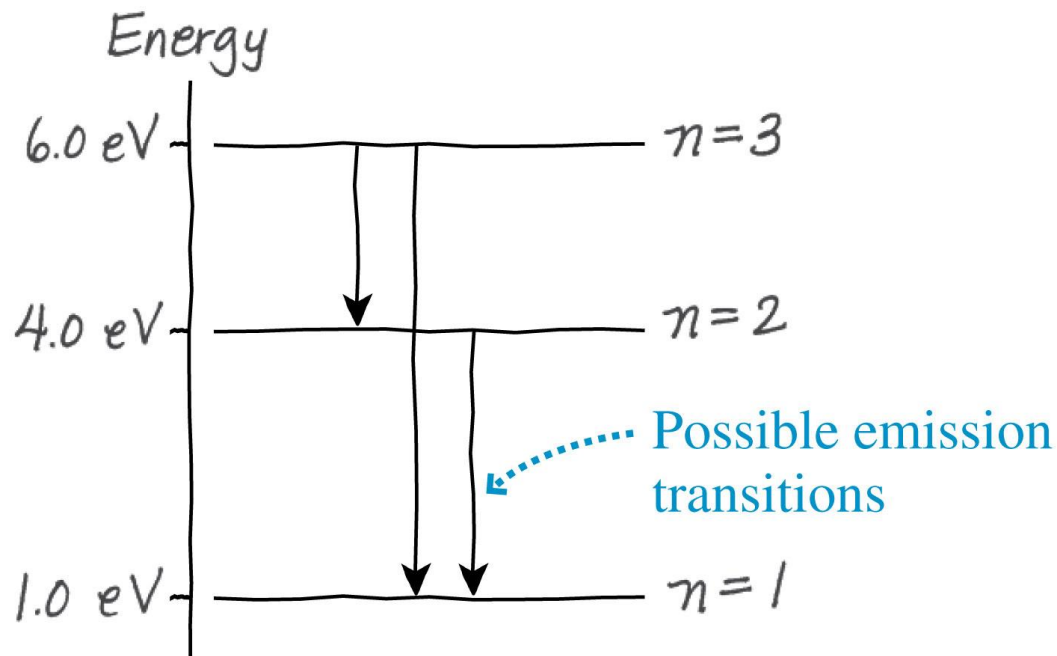


## Example 28.13 Determining an emission spectrum from quantum states

An electron in a quantum system has allowed energies  $E_1 = 1.0$  eV,  $E_2 = 4.0$  eV, and  $E_3 = 6.0$  eV. What wavelengths are observed in the emission spectrum of this system?

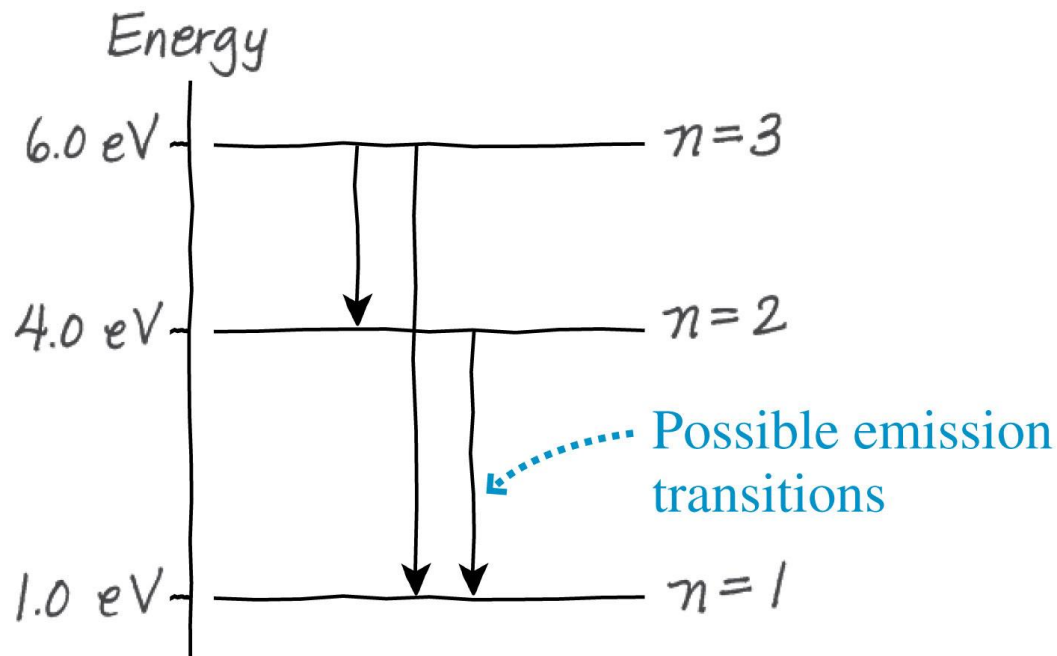
## Example 28.13 Determining an emission spectrum from quantum states (cont.)

**PREPARE** FIGURE 28.20 shows the energy-level diagram for this system. Photons are emitted when the system undergoes a quantum jump from a higher energy level to a lower energy level. There are three possible transitions.



## Example 28.13 Determining an emission spectrum from quantum states (cont.)

**SOLVE** This system will emit photons on the  $3 \rightarrow 1$ ,  $2 \rightarrow 1$ , and  $3 \rightarrow 2$  transitions, with  $\Delta E_{3 \rightarrow 1} = 5.0 \text{ eV}$ ,  $\Delta E_{2 \rightarrow 1} = 3.0 \text{ eV}$ , and  $\Delta E_{3 \rightarrow 2} = 2.0 \text{ eV}$ .



## Example 28.13 Determining an emission spectrum from quantum states (cont.)

From  $f_{\text{photon}} = \Delta E_{\text{system}}/h$  and  $\lambda = c/f$ , we find that the wavelengths in the emission spectrum are

$$3 \rightarrow 1 \quad f = 5.0 \text{ eV}/h = 1.21 \times 10^{15} \text{ Hz}$$

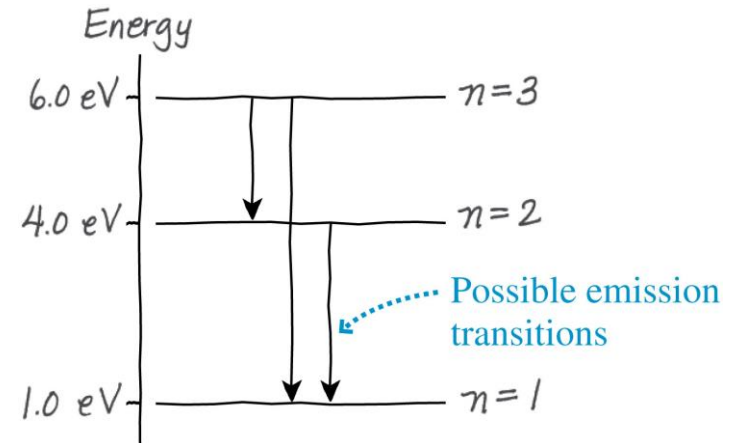
$$\lambda = 250 \text{ nm (ultraviolet)}$$

$$2 \rightarrow 1 \quad f = 3.0 \text{ eV}/h = 7.25 \times 10^{14} \text{ Hz}$$

$$\lambda = 410 \text{ nm (blue)}$$

$$3 \rightarrow 2 \quad f = 2.0 \text{ eV}/h = 4.83 \times 10^{14} \text{ Hz}$$

$$\lambda = 620 \text{ nm (orange)}$$



## Example 28.13 Determining an emission spectrum from quantum states (cont.)

**ASSESS** Transitions with a small energy difference, like  $3 \rightarrow 2$ , correspond to lower photon energies and thus longer wavelengths than transitions with a large energy difference like  $3 \rightarrow 1$ , as we would expect.

## QuickCheck 28.21

An atom has the energy levels shown. A photon with a wavelength of 620 nm has an energy of 2.0 eV. Do you expect to see a spectral line with wavelength of 620 nm in this atom's emission spectrum?

- A. Yes
- B. No
- C. There's not enough information to tell.



## QuickCheck 28.21

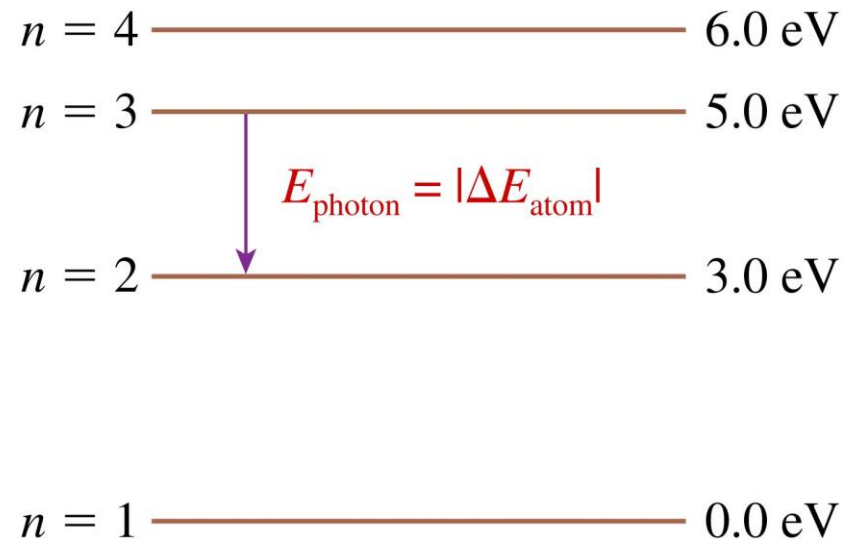
An atom has the energy levels shown. A photon with a wavelength of 620 nm has an energy of 2.0 eV. Do you expect to see a spectral line with wavelength of 620 nm in this atom's emission spectrum?



A. Yes

B. No

C. There's not enough information to tell.



## QuickCheck 28.22

An atom has the energy levels shown. How many spectral lines are in the emission spectrum?

- A. 2
- B. 3
- C. 4
- D. 5
- E. 6

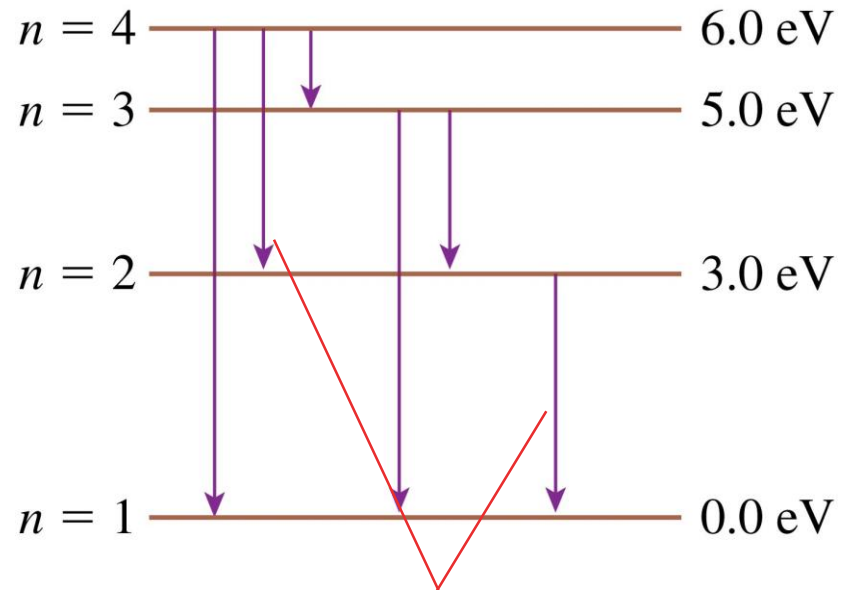




## QuickCheck 28.22

An atom has the energy levels shown. How many spectral lines are in the emission spectrum?

- A. 2
- B. 3
- C. 4
- D. 5
- E. 6



Six quantum jumps, but these two are the same wavelength so they make only one spectral line.

## Example Problem

Electrons in molecules with long chains of carbon atoms can move freely along the chain. Depending on what atoms are at the end of the chain, the electrons may well be constrained to stay in the chain and not go beyond. This means that an electron will work as a true particle in a box—it can exist between the fixed ends of the box but not beyond. The particle in a box model can be used to predict the energy levels. Such molecules will show strong absorption for photon energies corresponding to transitions between energy levels. One particular molecule has a “box” of length 1.1 nm. What is the longest wavelength photon that can excite a transition of an electron in the ground state?

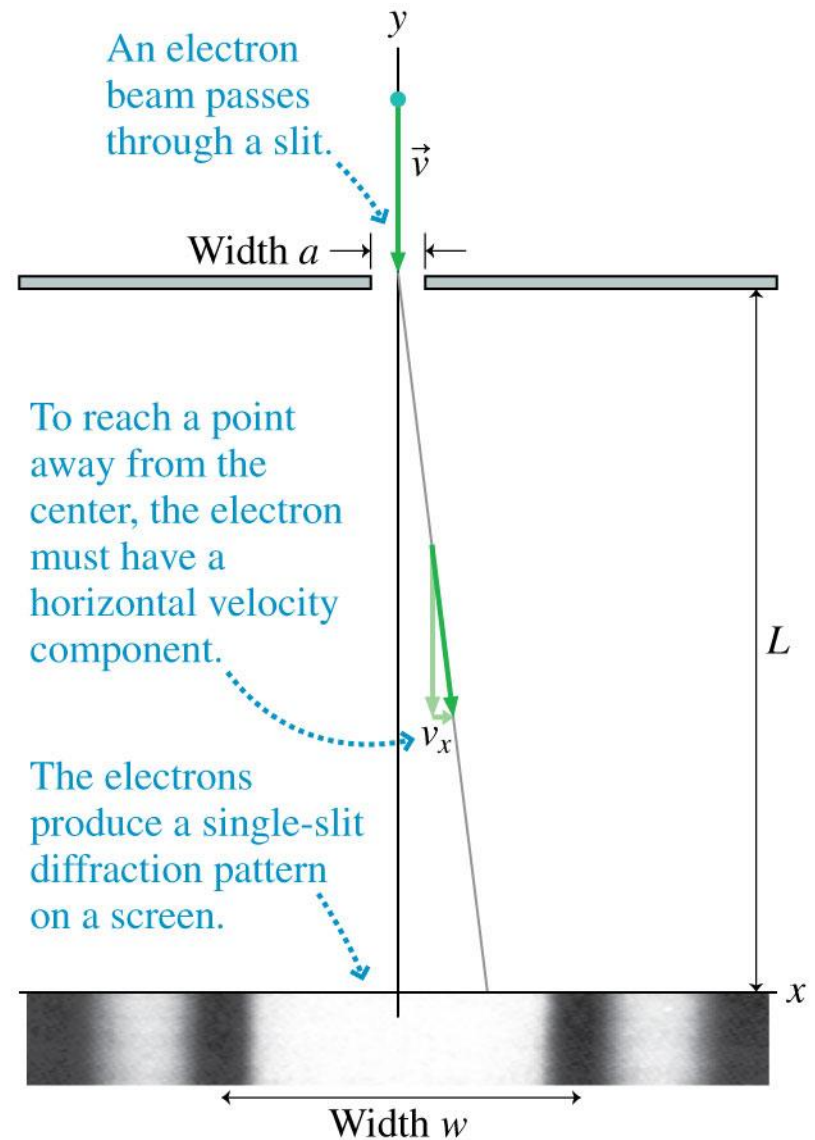
# Section 28.7 The Uncertainty Principle

# The Uncertainty Principle

- **For a particle such as an electron, if you know where it is, you cannot know exactly how fast it is moving.**
- This counterintuitive notion is a result of the wave nature of matter.

# The Uncertainty Principle

- We can design an experiment in which electrons moving along a  $y$ -axis pass through a slit of width  $a$ .
- The slit causes the electrons to spread out and produce a diffraction pattern.



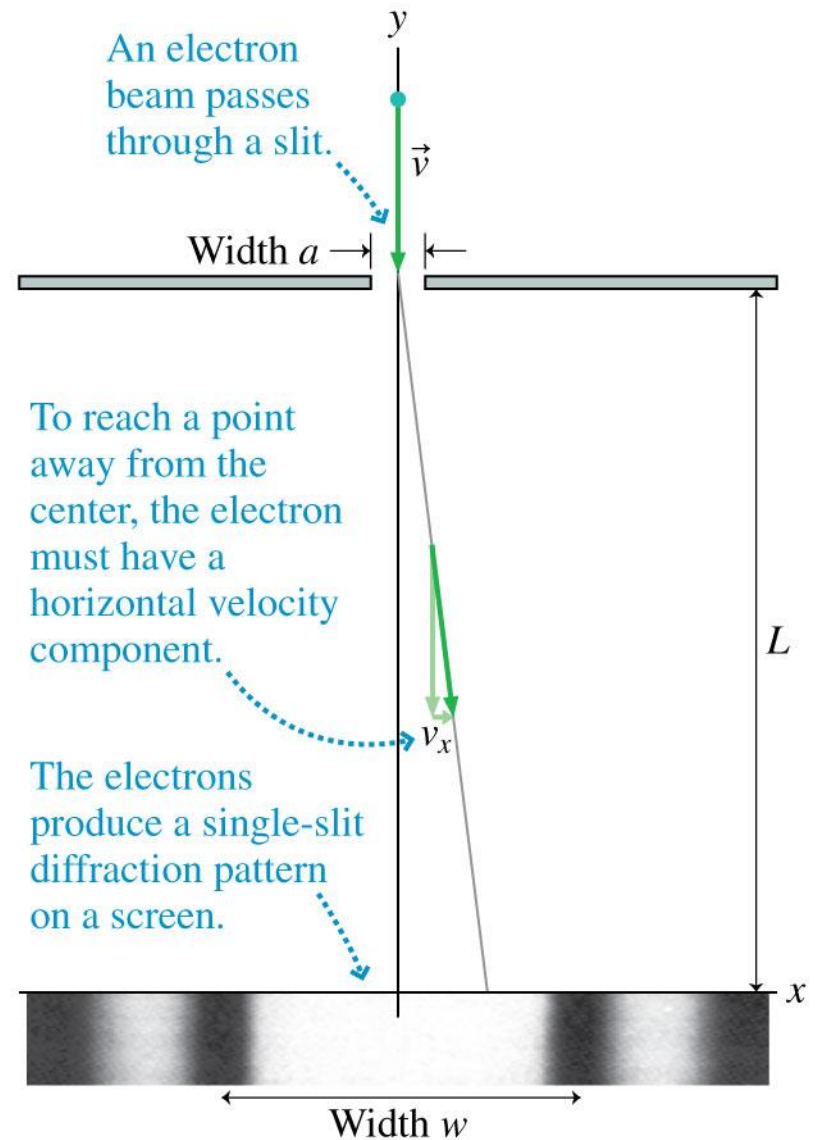
We can't predict with certainty where an electron will hit, but most land within the central maximum of the single-slit pattern.

# The Uncertainty Principle

- We can think of the experiment in a different way – as making a measurement of the position of the electrons.
- As an electron goes through the slit, we know something about its horizontal position. We can establish an *uncertainty*, a limit on our knowledge. The uncertainty in the horizontal position is  $\Delta x = a$ , the width of the slit.
- After passing through the slit, the electrons strike the screen over a range of positions. The electrons must have acquired a *horizontal component* of velocity.
- By forcing them through a slit, we gained knowledge about their *position*, but that caused uncertainty in our knowledge of the *velocity* of the electrons.

# The Uncertainty Principle

- The figure shows an experiment to illustrate the uncertainty principle.



We can't predict with certainty where an electron will hit, but most land within the central maximum of the single-slit pattern.

## Conceptual Example 28.14 Changing the uncertainty

Suppose we narrow the slit in the above experiment, allowing us to determine the electron's horizontal position more precisely. How does this affect the diffraction pattern? How does this change in the diffraction pattern affect the uncertainty in the velocity?



## Conceptual Example 28.14 Changing the uncertainty (cont.)

**REASON** We learned in Chapter 17 that the width of the central maximum of the single-slit diffraction pattern is  $w = 2\lambda L/a$ . Making the slit narrower—decreasing the value of  $a$ —increases the value of  $w$ , making the central fringe wider. If the fringe is wider, the spread of horizontal velocities must be greater, so there is a greater uncertainty in the horizontal velocity.

## Conceptual Example 28.14 Changing the uncertainty (cont.)

**ASSESS** Improving our knowledge of the position decreases our knowledge of the velocity. This is the hallmark of the *uncertainty principle*.

# The Uncertainty Principle

- In 1927, German physicist Werner Heisenberg proved that, for any particle, the product of the uncertainty  $\Delta x$  in its position and the uncertainty  $\Delta p_x$  in its  $x$ -momentum has a lower limit fixed by the expression

$$\Delta x \Delta p_x \geq \frac{h}{4\pi}$$

Heisenberg uncertainty principle for position and momentum

- A decreased uncertainty in position—knowing more precisely where a particle is—comes at the expense of an increased uncertainty in velocity and thus momentum.
- Knowing a particle's velocity or momentum more precisely requires an increase in the uncertainty about its position.

# The Uncertainty Principle

- No matter how good your experiment, you *cannot* measure both  $x$  and  $p_x$  simultaneously with good precision.
- **The position and momentum of a particle are *inherently* uncertain.**
- This is because of the wave-like nature of matter. The “particle” is spread out in space, so there simply is not a precise value of its position  $x$ .

## Example 28.15 Determining Uncertainties

- a. What range of velocities might an electron have if confined to a 0.30-nm-wide region, about the size of an atom?
- b. A 1.0- $\mu\text{m}$ -diameter dust particle ( $m \approx 10^{-15}$  kg) is confined within a 5- $\mu\text{m}$ -long box. Can we know with certainty if the particle is at rest? If not, within what range is its velocity likely to be found?

## Example 28.15 Determining Uncertainties (cont.)

**PREPARE** Localizing a particle means specifying its position with some accuracy—so there must be an uncertainty in the velocity. We can estimate the uncertainty by using Heisenberg's uncertainty principle.

## Example 28.15 Determining Uncertainties (cont.)

**SOLVE** a. We aren't given the exact position of the particle, only that it is within a 0.30-nm-wide region. This means that we have specified the electron's position within a range  $\Delta x = 3.0 \times 10^{-10}$  m.

## Example 28.15 Determining Uncertainties (cont.)

The uncertainty principle thus specifies that the least possible uncertainty in the momentum is

$$\Delta p_x = \frac{h}{4\pi \Delta x}$$

The uncertainty in the velocity is thus approximately

$$\Delta v_x = \frac{\Delta p_x}{m} \approx \frac{h}{4\pi m \Delta x} \approx 2 \times 10^5 \text{ m/s}$$



## Example 28.15 Determining Uncertainties (cont.)

Because the *average* velocity is zero, (the particle is equally likely to be moving right or left) the best we can do is to say that the electron's velocity is somewhere in the interval  $-1 \times 10^5 \text{ m/s} \leq v_x \leq 1 \times 10^5 \text{ m/s}$ . **It is simply not possible to specify the electron's velocity more precisely than this.**

## Example 28.15 Determining Uncertainties (cont.)

- b. We know the particle is somewhere in the box, so the uncertainty in our knowledge of its position is at most  $\Delta x = L = 5 \mu\text{m}$ . With a finite  $\Delta x$ , the uncertainty  $\Delta p_x$  *cannot* be zero. **We cannot know with certainty if the particle is at rest inside the box.**

## Example 28.15 Determining Uncertainties (cont.)

No matter how hard we try to bring the particle to rest, the uncertainty in our knowledge of the particle's momentum will be approximately  $\Delta p_x \approx h/(4\pi \Delta x) = h/(4\pi L)$ . Consequently, the range of possible velocities is

$$\Delta v_x = \frac{\Delta p_x}{m} \approx \frac{h}{4\pi mL} \approx 1.0 \times 10^{-14} \text{ m/s}$$

## Example 28.15 Determining Uncertainties (cont.)

This range of possible velocities will be centered on  $v_x = 0$  m/s if we have done our best to have the particle be at rest. Therefore all we can know with certainty is that the particle's velocity is somewhere within the interval  $-5 \times 10^{-15}$  m/s  $\leq v_x \leq 5 \times 10^{-15}$  m/s.

## Example 28.15 Determining Uncertainties (cont.)

**ASSESS** Our uncertainty about the electron's velocity is enormous. For an electron confined to a region of this size, the best we can do is to state that its speed is less than one million miles per hour! The uncertainty principle clearly sets real, practical limits on our ability to describe electrons.

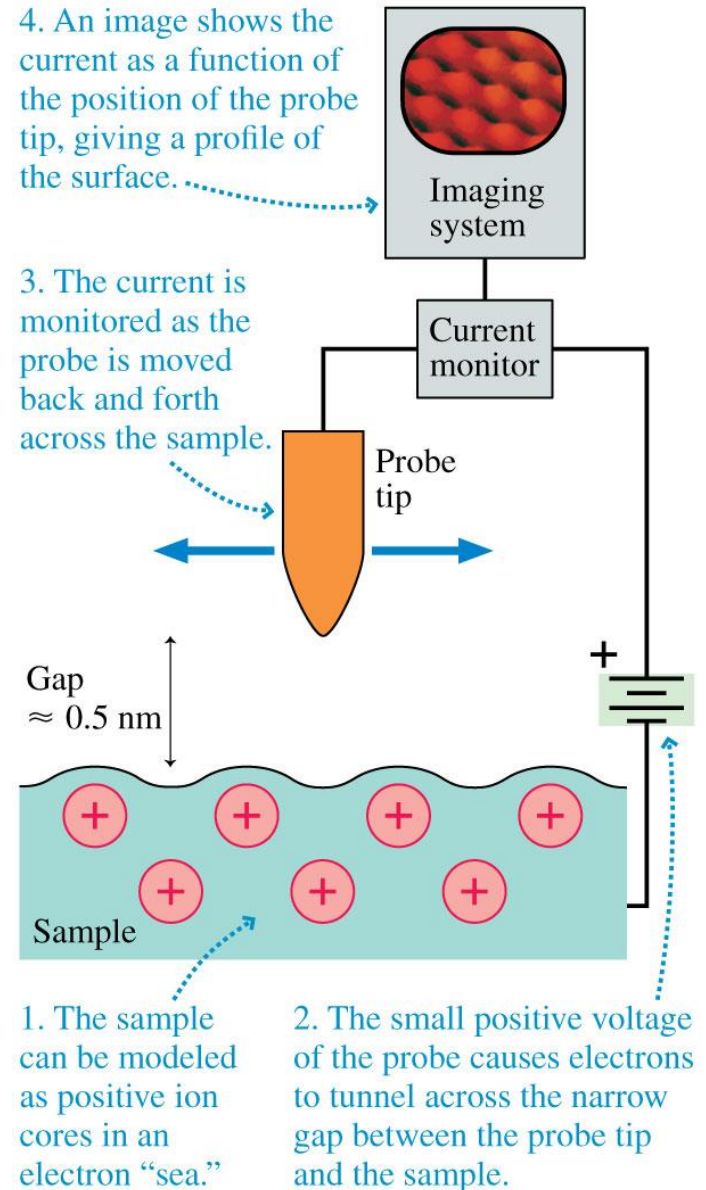
## Example 28.15 Determining Uncertainties (cont.)

The situation for the dust particle is different. We can't say for certain that the particle is absolutely at rest. But knowing that its speed is less than  $5 \times 10^{-15}$  m/s means that the particle is at rest for all practical purposes. At this speed, the dust particle would require nearly 6 hours to travel the width of one atom! Again we see that the quantum view has profound implications at the atomic scale but need not affect the way we think of macroscopic objects.

# Section 28.8 Applications and Implications of Quantum Theory

# Tunneling and the Scanning Tunneling Microscope

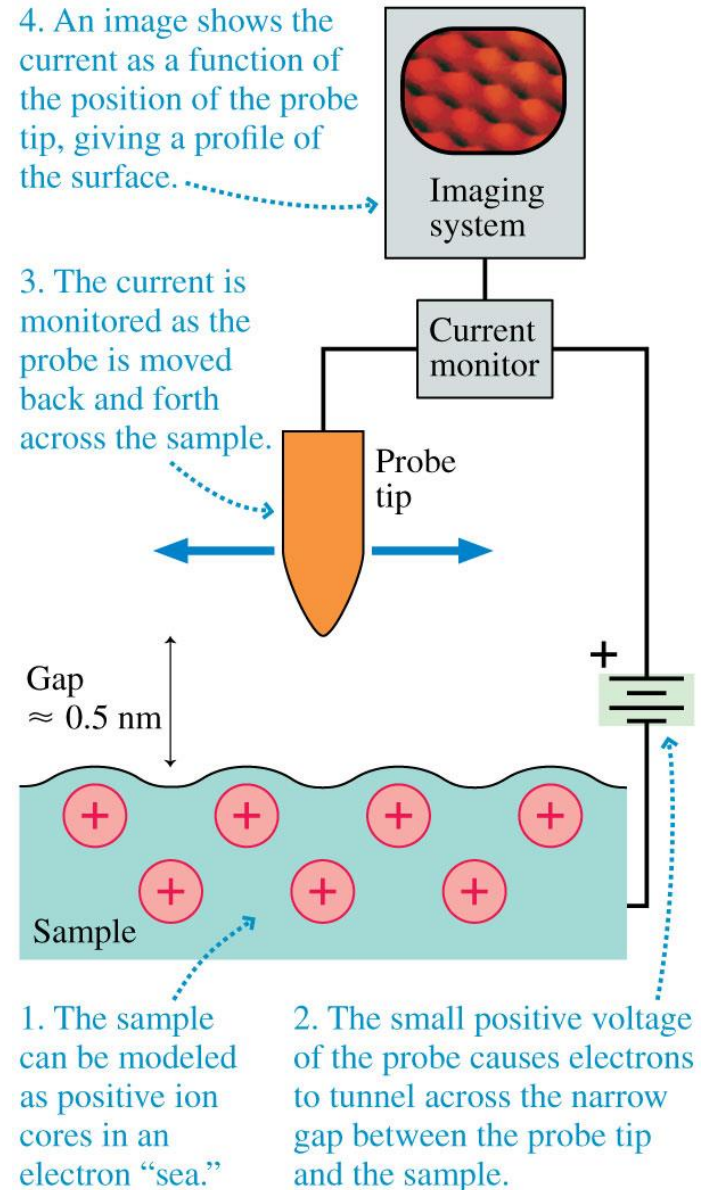
- A *scanning tunneling microscope* (STM) does not work like other microscopes; it builds an image of a solid surface by scanning a probe near the surface.





# Tunneling and the Scanning Tunneling Microscope

- In this figure we see a very thin metal needle called the *probe tip*. The space between the tip and the surface is only a few atomic diameters.
- Electrons in the sample are attracted to the positive probe tip, but no current should flow because electrons cannot cross the gap; it is an incomplete circuit.



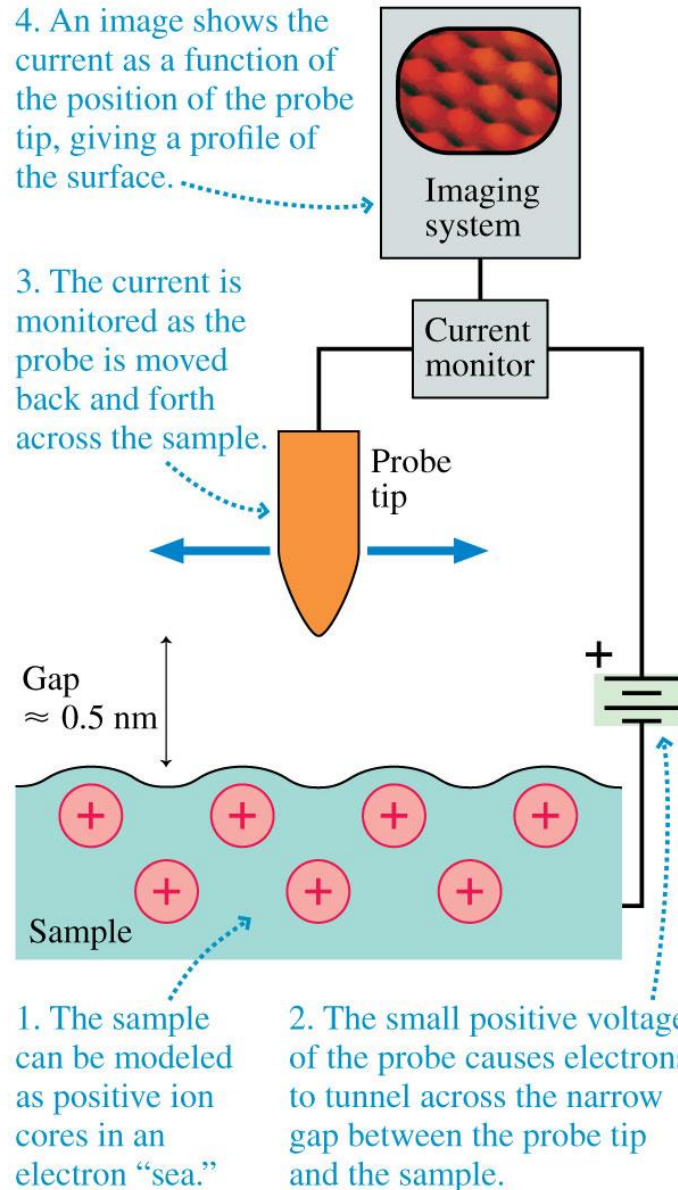
# Tunneling and the Scanning Tunneling Microscope

- Electrons are not classical particles; they have a wave nature.
- When the probe in the STM comes close enough to the surface, close enough to poke into an electron's wave function, an electron can suddenly find itself in the probe tip, causing current to flow in the circuit.
- This process is called **tunneling** because it is like tunneling through an impassable mountain barrier: The electron should not be able to cross the gap.
- Tunneling is completely forbidden by the laws of classical physics.

# Tunneling and the Scanning Tunneling Microscope

- The probability that an electron will tunnel across the gap in the STM is very sensitive to the size of the gap. This is what makes the **scanning tunneling microscope** possible.
- When the probe tip of the STM passes over an atom or an atomic-level bump on the surface, the gap narrows and the current increases as more electrons are able to tunnel across.

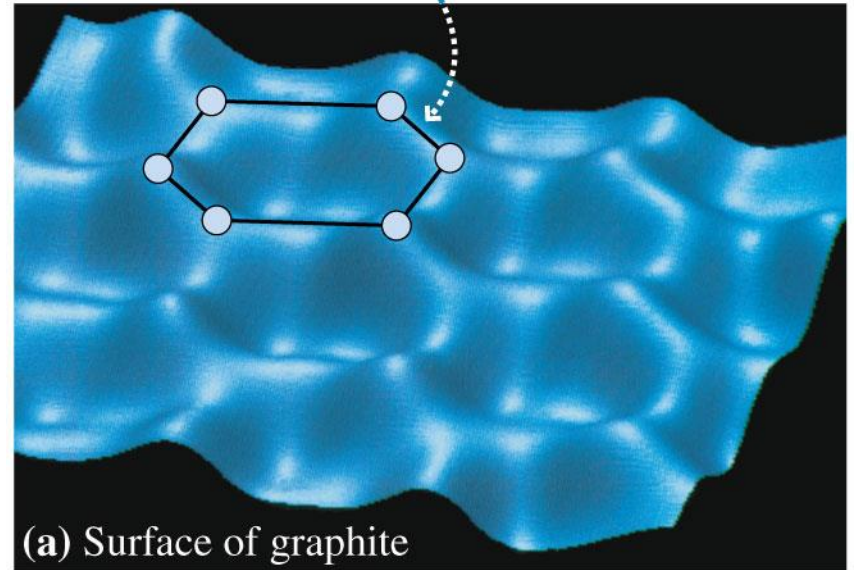
# Tunneling and the Scanning Tunneling Microscope



# Tunneling and the Scanning Tunneling Microscope

- The STM was the first technology that allowed imaging of individual atoms.
- It helped jump-start the current interest in nanotechnology.
- This image shows the hexagonal arrangement of individual atoms on the surface of pyrolytic graphite.

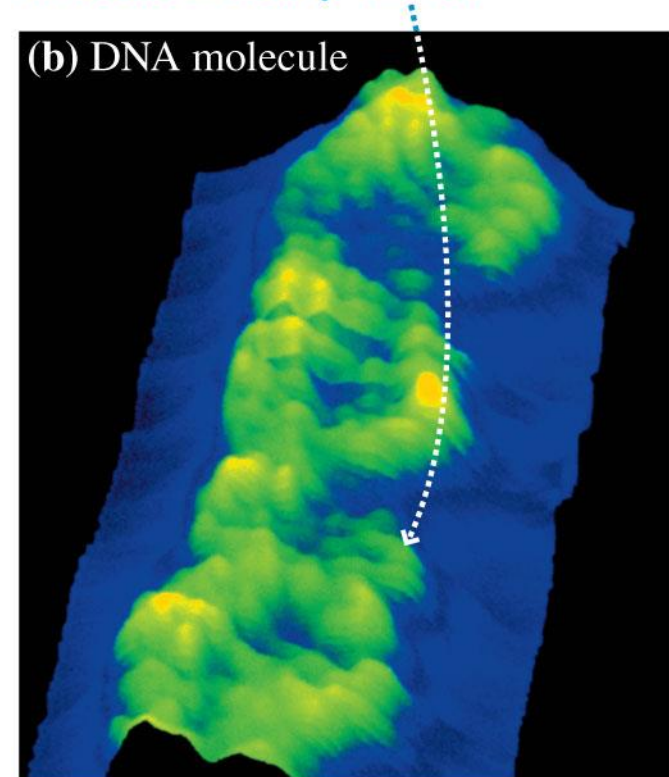
The hexagonal arrangement of atoms is clearly visible.



# Tunneling and the Scanning Tunneling Microscope

- This image is of a DNA molecule and shows the actual twists of the double-helix structure.
- Current research efforts aim to develop methods for sequencing DNA with STM to directly “read” a single strand of DNA.

The double-helix structure of the DNA molecule is clearly visible.



# Wave-Particle Duality

- The various objects of classical physics are *either* waves *or* particles.
- You may think that light and matter are *both* a wave *and* a particle, but the basic definitions of particleness and waviness are mutually exclusive.
- It is more profitable to conclude that light and matter are *neither* a wave *nor* a particle. At the microscopic scale of atoms and their constituents, the classical concepts of particles and waves turn out to be simply too limited to explain the subtleties of nature.

# Wave-Particle Duality

- Although matter and light have both wave-like aspects and particle-like aspects, they show us only one face at a time.
- If we arrange an experiment to measure a wave-like property, such as interference, we find photons and electrons acting like waves, not particles.
- An experiment to look for particles will find photons and electrons acting like particles, not waves.
- Neither the wave nor the particle model alone provides an adequate picture of light or matter, they must be taken together.
- This two-sided point of view is called *wave-particle duality*.



# Summary: General Principles

## Light has particle-like properties

- The energy of a light wave comes in discrete packets (light quanta) we call **photons**.
- For light of frequency  $f$ , the energy of each photon is  $E = hf$ , where  $h = 6.63 \times 10^{-34} \text{ J} \cdot \text{s}$  is **Planck's constant**.
- When light strikes a metal surface, all of the energy of a single photon is given to a single electron.

Text: p. 932

## Summary: General Principles

### Matter has wave-like properties

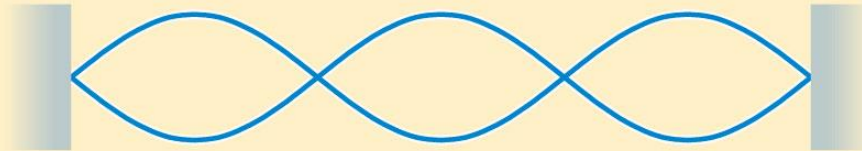
- The de Broglie wavelength of a particle of mass  $m$  is  $\lambda = h/mv$ .
- The wave-like nature of matter is seen in the interference patterns of electrons, protons, and other particles.

Text: p. 932

# Summary: General Principles

## Quantization of energy

When a particle is confined, it sets up a de Broglie standing wave.



The fact that standing waves can have only certain allowed wavelengths leads to the conclusion that a confined particle can have only certain allowed energies.

Text: p. 932

# Summary: General Principles

## Wave-particle duality

- Experiments designed to measure wave properties will show the wave nature of light and matter.
- Experiments designed to measure particle properties will show the particle nature of light and matter.

Text: p. 932

# Summary: General Principles

## Heisenberg uncertainty principle

A particle with wave-like characteristics does not have a precise value of position  $x$  or a precise value of momentum  $p_x$ . Both are uncertain. The position uncertainty  $\Delta x$  and momentum uncertainty  $\Delta p_x$  are related by

$$\Delta x \Delta p_x \geq \frac{h}{4\pi}$$

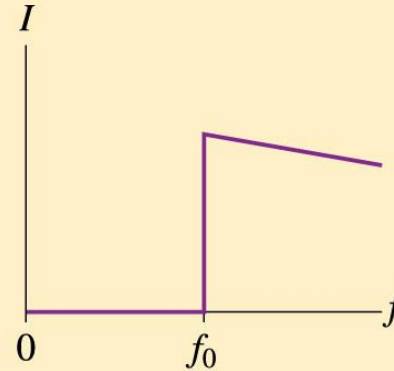
The more you pin down the value of one, the less precisely the other can be known.

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# Summary: Important Concepts

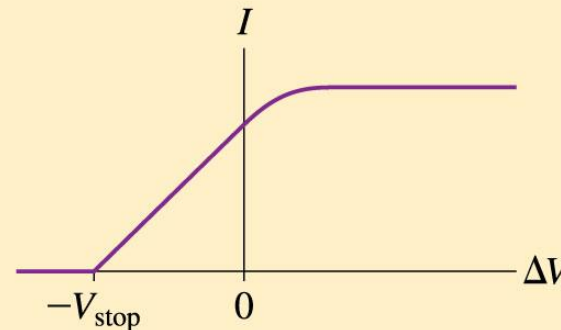
## Photoelectric effect

Light with frequency  $f$  can eject electrons from a metal only if  $f \geq f_0 = E_0/h$ , where  $E_0$  is the metal's **work function**. Electrons will be ejected even if the intensity of the light is very small.



The **stopping potential** that stops even the fastest electrons is

$$V_{\text{stop}} = \frac{K_{\text{max}}}{e} = \frac{hf - E_0}{e}$$



The details of the photoelectric effect could not be explained with classical physics. New models were needed.

Text: p. 932

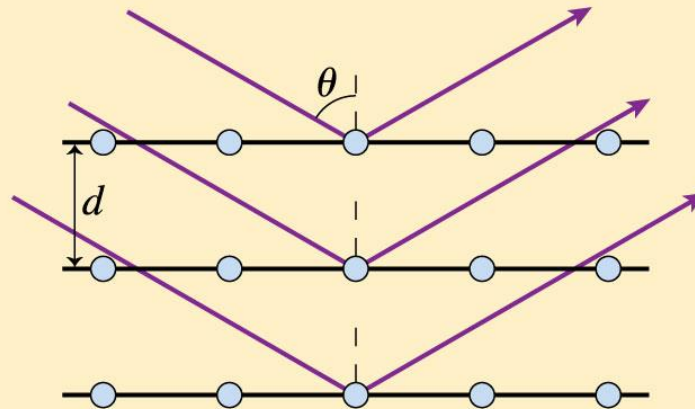
# Summary: Important Concepts

## X-ray diffraction

X rays with wavelength  $\lambda$  undergo strong reflections from atomic planes spaced by  $d$  when the angle of incidence satisfies the **Bragg condition**:

$$2d \cos \theta = m\lambda$$

$$m = 1, 2, 3, \dots$$

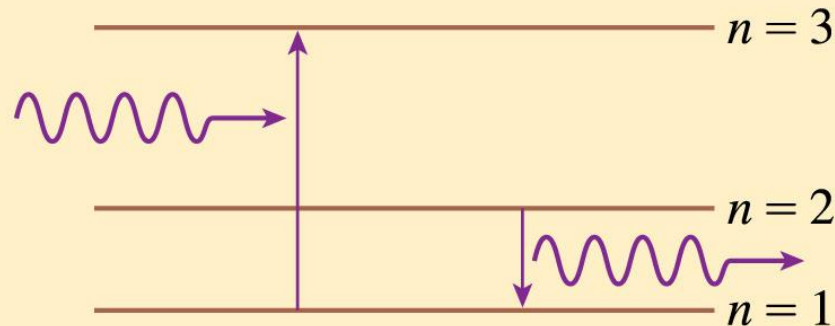


Text: p. 932

# Summary: Important Concepts

## Energy levels and quantum jumps

The localization of electrons leads to quantized energy levels. An electron can exist only in certain energy states. An electron can jump to a higher level if a photon is absorbed, or to a lower level if a photon is emitted. The energy difference between the levels equals the photon energy.

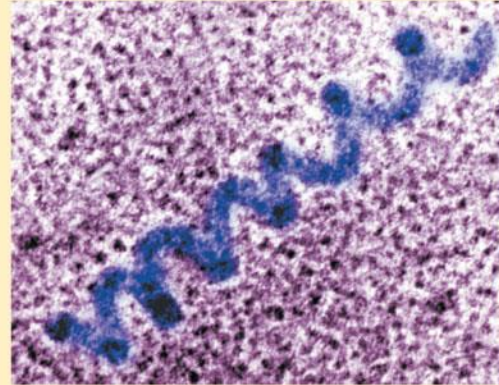


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# Summary: Applications

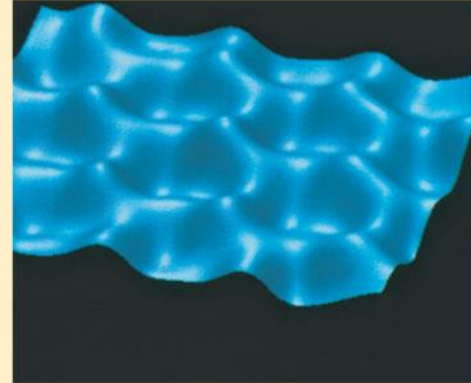
The wave nature of light limits the resolution of a light microscope. A more detailed image may be made with an **electron microscope** because of the very small de Broglie wavelength of fast electrons.



Text: p. 932

# Summary: Applications

The wave nature of electrons allows them to **tunnel** across an insulating gap to the tip of a **scanning tunneling microscope**, revealing details of the atoms on a surface.



Text: p. 932

# Summary

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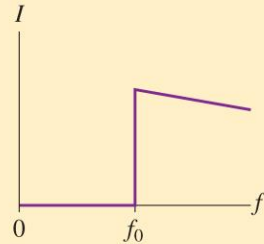
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# Summary

## IMPORTANT CONCEPTS

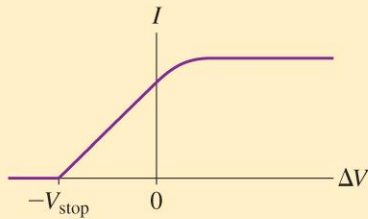
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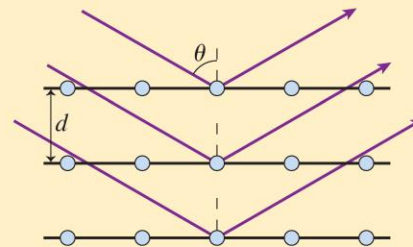
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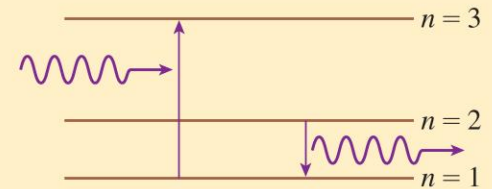
$$2d \cos \theta = m\lambda$$

$$m = 1, 2, 3, \dots$$



### Energy levels and quantum jumps

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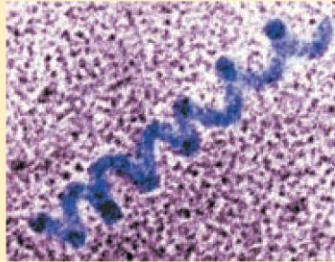


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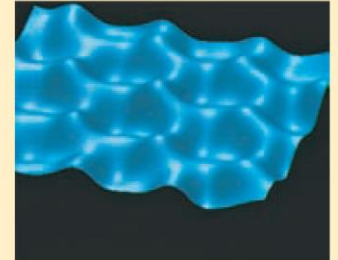
# Summary

## APPLICATIONS

The wave nature of light limits the resolution of a light microscope. A more detailed image may be made with an **electron microscope** because of the very small de Broglie wavelength of fast electrons.



The wave nature of electrons allows them to **tunnel** across an insulating gap to the tip of a **scanning tunneling microscope**, revealing details of the atoms on a surface.



Text: p. 932