

## Lecture Presentation

## Chapter 21

## Electric Potential

## Suggested Videos for Chapter 21

- Prelecture Videos
- Electric Potential
- Connecting Field and Potential
- Capacitors and Capacitance
- Class Videos
- Electric Potential
- Sparks in the Air
- Energy Changes and Energy Units
- Video Tutor Solutions
- Electric Potential
- Video Tutor Demos
- Charged Conductor with Teardrop Shape


## Suggested Simulations for Chapter 21

- ActivPhysics
- 11.11-11.13
- 12.6
- PhETs
- Charges and Fields
- Battery Voltage


## Chapter 21 Electric Potential



Chapter Goal: To calculate and use the electric potential and electric potential energy.

## Chapter 21 Preview Looking Ahead: Electric Potential

- The voltage of a battery is the difference in electric potential between its two terminals.

- You'll learn how an electric potential is created when positive and negative charges are separated.


## Chapter 21 Preview Looking Ahead: Capacitors

- The capacitors on this circuit board store charge and electric potential energy.

- You'll learn how the energy stored in a capacitor depends on its charge.


## Chapter 21 Preview Looking Ahead: Potential and Field

- There is an intimate connection between the electric potential and the electric field.

- You'll learn how to move back and forth between field and potential representations.


## Chapter 21 Preview Looking Ahead

## Electric Potential

The voltage of a battery is the difference in electric potential between its two terminals.


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The capacitors on this circuit board store charge and electric potential energy.


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## Potential and Field

There is an intimate connection between the electric potential and the electric field.


You'll learn how to move back and forth between field and potential representations.

Text: p. 665

## Chapter 21 Preview Looking Back: Work and Potential Energy

- In Section 10.4 you learned that it is possible to store potential energy in a system of interacting objects. In this chapter, we'll learn about a new form of potential energy, electric potential energy.
- This roller coaster is pulled to the top of the first hill by a chain. The tension in the chain does work on the coaster, increasing its gravitational potential energy.



## Chapter 21 Preview Stop to Think

You lift a book at a constant speed. Which statement is true about the work $W$ done by your hand the change in the book's gravitational potential energy $\Delta U_{g}$ ?

A. $W>\Delta U_{g}>0$
B. $W<\Delta U_{g}<0$
C. $W=\Delta U_{g}>0$
D. $W=\Delta U_{g}<0$


## Reading Question 21.1

What are the units of potential difference?
A. C
B. J
C. $\Omega$
D. V
E. F

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E. F

## Reading Question 21.2

New units of the electric field were introduced in this chapter. They are which of the following?
A. V/C
B. $\mathrm{N} / \mathrm{C}$
C. V/m
D. $\mathrm{J} / \mathrm{m}^{2}$
E. $\Omega / \mathrm{m}$
F. J/C

## Reading Question 21.2

New units of the electric field were introduced in this chapter. They are which of the following?
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C. V/m
D. $\mathrm{J} / \mathrm{m}^{2}$
E. $\Omega / \mathrm{m}$
F. J/C

## Reading Question 21.3

The electron volt is a unit of
A. Potential difference.
B. Voltage.
C. Charge.
D. Energy.
E. Power.
F. Capacitance.

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## Reading Question 21.4

The electric potential inside a parallel-plate capacitor
A. Is constant.
B. Increases linearly from the negative to the positive plate.
C. Decreases linearly from the negative to the positive plate.
D. Decreases inversely with distance from the negative plate.
E. Decreases inversely with the square of the distance from the negative plate.

## Reading Question 21.4

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D. Decreases inversely with distance from the negative plate.
E. Decreases inversely with the square of the distance from the negative plate.

## Reading Question 21.5

The electric field
A. Is always perpendicular to an equipotential surface.
B. Is always tangent to an equipotential surface.
C. Always bisects an equipotential surface.
D. Makes an angle to an equipotential surface that depends on the amount of charge.

## Reading Question 21.5

The electric field
A. Is always perpendicular to an equipotential surface.
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D. Makes an angle to an equipotential surface that depends on the amount of charge.

## Section 21.1 Electric Potential Energy and Electric Potential

## Electric Potential Energy and Electric Potential

- Conservation of energy was a powerful tool for understanding the motion of mechanical systems.
- As the force of the hand does work $W$ on the book,
(a) . . . the book gains gravitational potential energy $U_{\mathrm{g}}$.



## Electric Potential Energy and Electric Potential

- As the force of the hand does work $W$ by compressing the spring,
(b) . . . the spring gains elastic potential energy $U_{\mathrm{s}}$.



## Electric Potential Energy and Electric Potential

- A charge $q$ is repelled by stationary source charges. A hand must push on the charge $q$ in order to move it closer to the source charges.
- The hand does work, transferring energy into the system of charges.


$$
\Delta U_{\mathrm{elec}}=W
$$

## Electric Potential Energy and Electric Potential

- The energy is electric potential energy $U_{\text {elec }}$.
- We can determine the electric potential energy of a charge when it's at a particular position by computing how much work it took to move the charge to that position.


$$
\Delta U_{\mathrm{elec}}=W
$$

## QuickCheck 21.1

In physics, what is meant by the term "work"?
A. Force $\times$ distance.
B. Energy transformed from one kind to another.
C. Energy transferred into a system by pushing on it.
D. Potential energy gained or lost.

## QuickCheck 21.1

In physics, what is meant by the term "work"?
A. Force $\times$ distance.
B. Energy transformed from one kind to another.
C. Energy transferred into a system by pushing on it.
D. Potential energy gained or lost.

## Electric Potential

- Source charges alter the space around them, creating an electric field.

> The electric field tells us what the force would be if a charge were placed at this point.


Source charges


Is there a quantity associated with each point around the source charges that would tell us the electric potential energy of a charge placed at that point?

## Electric Potential

- In order to know if we could determine what the electric potential energy would be at a point near a source charge, we must understand how to find the electric potential energy of a charge $q$.


## Electric Potential

- To better understand electric potential energy, we take a charge $q=10 \mathrm{nC}$ and set $U_{\text {elec }}=0$ at a point A.
- To find $q$ 's electric potential energy at any other point, we must measure the amount of work it takes to move the charge from point A to that other point.
(a) The electric potential energy of a 10 nC charge at A is zero. What is its potential energy at point B or C ?

$$
\begin{gathered}
q=10 \mathrm{nC} \\
\oplus+\mathrm{A}
\end{gathered}
$$

- B

$$
\left(U_{\text {elec }}\right)_{\mathrm{A}}=0
$$


-C

## Electric Potential

- It takes the hand $4 \mu \mathrm{~J}$ of work to move the charge $q$ from point A to point B, thus its electric potential energy at B is $\left(U_{\text {elec }}\right)_{\mathrm{B}}=4 \mu \mathrm{~J}$. Similarly, $\left(U_{\text {elec }}\right)_{\mathrm{C}}=6 \mu \mathrm{~J}$.
(b) The charge's electric potential energy at any point is equal to the amount of work done in moving it there from point A .



## Electric Potential

- If we were to have a charge $q=20 \mathrm{nC}$, then according to Coulomb's law, the electric force on the charge would be twice that of a charge with $q=10 \mathrm{nC}$. A hand would have to do twice as much work to move the charge.
- A charged particle's potential energy is proportional to its charge.


## Electric Potential

- If the electric potential energy for a charge $q=10 \mathrm{nC}$ is $4 \mu \mathrm{~J}$ at a given point, the the electric potential energy is $8 \mu \mathrm{~J}$ for a charge $q=20 \mathrm{nC}$ at the same point, and $2 \mu \mathrm{~J}$ for $q=5 \mathrm{nC}$ :

$$
\frac{U_{(\text {elec }) \mathrm{B}}}{q}=\frac{2 \mu \mathrm{~J}}{5 \mathrm{nC}}=\frac{4 \mu \mathrm{~J}}{10 \mathrm{nC}}=\frac{8 \mu \mathrm{~J}}{20 \mathrm{nC}}=400 \frac{\mathrm{~J}}{\hat{\mathrm{C}}}
$$

- The expression for the electric potential energy that any charge $q$ would have if placed at that same point is

$$
\left(U_{\text {elec }}\right)_{\mathrm{B}}=\left(400 \frac{\mathrm{~J}}{\mathrm{C}}\right) q_{\mathrm{E}}
$$

This number is associated * This part depends on the with point B. charge we place at B.

## Electric Potential

- The electric potential $V$ is the potential for creating an electric potential energy if a charge is placed at a given point.

$$
U_{\mathrm{elec}}=q \mathrm{~V}
$$

## Relationship between electric potential and electric potential energy

- The electric field tells us how a source will exert a force on $q$; the electric potential tells us how the source charges would provide $q$ with potential energy.
- The unit of potential energy is the joule per coulomb, or volt V :

$$
1 \text { volt }=1 \mathrm{~V}=1 \mathrm{~J} / \mathrm{C}
$$

## Electric Potential

(b) The charge's electric potential energy at any point is equal to the amount of work done in moving it there from point A .

(c) The electric potential is created by the source charges. It exists at every point in space, not only at $A, B$, and $C$.

$$
V_{\mathrm{B}}=400 \mathrm{~J} / \mathrm{C}=400 \mathrm{~V}
$$

$$
V_{\mathrm{C}}=600 \mathrm{~J} / \mathrm{C}=600 \mathrm{~V}
$$

## QuickCheck 21.2

As a positive charge is moved from position i to position f , is its change in electric potential energy positive (+), negative $(-)$, or zero (0)?


|  | 1 | 2 | 3 | 4 |
| :--- | :--- | :--- | :--- | :--- |
| A | 0 | + | + | - |
| B | 0 | - | + | - |
| C | - | + | - | + |
| D | + | + | - | 0 |

## QuickCheck 21.2

As a positive charge is moved from position i to position f , is its change in electric potential energy positive (+), negative $(-)$, or zero (0)?


## QuickCheck 21.3

Two charges are brought separately into the vicinity of a fixed charge $+Q$.

- First, $+q$ is brought to point A, a distance $r$ away.
- Second, $+q$ is removed and $-q$ is brought to the same point A.

Situation 1

Situation 2
The electric potential at point A is:
Fixed source charge $+Q$

## QuickCheck 21.3

Two charges are brought separately into the vicinity of a fixed charge $+Q$.

- First, $+q$ is brought to point A, a distance $r$ away.
- Second, $+q$ is removed and $-q$ is brought to the same point A.

Situation 1

Situation 2
The electric potential at point A is:
Fixed source charge $+Q$

## QuickCheck 21.4

Two charges are brought separately into the vicinity of a fixed charge $+Q$.

- First, $+q$ is brought to point A, a distance $r$ away.
- Second, $+q$ is removed and $-q$ is brought to the same point A.
The electric potential is:

A. Greater for the $+q$ charge in situation 1 .
B. Greater for the $-q$ charge in situation 2 .
C. The same for both.


## QuickCheck 21.4

Two charges are brought separately into the vicinity of a fixed charge $+Q$.

- First, $+q$ is brought to point A, a distance $r$ away.
- Second, $+q$ is removed and $-q$ is brought to the same point A.
The electric potential is:

A. Greater for the $+q$ charge in situation 1 .
B. Greater for the $-q$ charge in situation 2 .
C. The same for both.


## Electric Potential

TABLE 21.1 Typical electric potentials
Source of potential
Approximate
potential
Brain activity at scalp $\quad 10-100 \mu \mathrm{~V}$ (EEG)
Cells in human body $\quad 100 \mathrm{mV}$
Battery
Household electricity
Static electricity
Transmission lines

## Example 21.1 Finding the change in a charge's electric potential energy

A 15 nC charged particle moves from point A , where the electric potential is 300 V , to point B , where the electric potential is -200 V . By how much does the electric potential change? By how much does the particle's electric potential energy change? How would your answers differ if the particle's charge were -15 nC ?

## Example 21.1 Finding the change in a charge's electric potential energy (cont.)

PREPARE The change in the electric potential $\Delta \mathrm{V}$ is the potential at the final point B minus the potential at the initial point A. From Equation 21.1, we can find the change in the electric potential energy by noting that $\Delta U_{\text {elec }}=\left(U_{\text {elec }}\right)_{\mathrm{B}}-$ $\left(U_{\text {elec }}\right)_{\mathrm{A}}=q\left(V_{\mathrm{B}}-V_{\mathrm{A}}\right)=q \Delta V$.

## Example 21.1 Finding the change in a charge's electric potential energy (cont.)

Solve We have

$$
\Delta V=V_{\mathrm{B}}-V_{\mathrm{A}}=(-200 \mathrm{~V})-(300 \mathrm{~V})=-500 \mathrm{~V}
$$

This change is independent of the charge $q$ because the electric potential is created by source charges.

The change in the particle's electric potential energy is

$$
\Delta U_{\text {elec }}=q \Delta V=\left(15 \times 10^{-9} \mathrm{C}\right)(-500 \mathrm{~V})=-7.5 \mu \mathrm{~J}
$$

A -15 nC charge would have $\Delta U_{\text {elec }}+7.5 \mu \mathrm{~J}$ because $q$ changes sign while $\Delta V$ remains unchanged.

## Example 21.1 Finding the change in a charge's electric potential energy (cont.)

ASSESS Because the electric potential at B is lower than that at A, the positive $(+15 \mathrm{nC})$ charge will lose electric potential energy, while the negative $(-15 \mathrm{nC})$ charge will gain energy.

## Section 21.2 Sources of Electric Potential

## Sources of Electric Potential

- How is an electric potential created?
(a) The force on charge $q$ is zero. No work is needed to move it from $A$ to $B$, so there is no potential difference between A and B .



## Sources of Electric Potential

- If electrons were transferred from the right side of a capacitor to the left side of a capacitor, giving the left electrode charge $-Q$ and a right electrode charge $+Q$, the capacitor would have no net charge, but the charge would be separated.


## Sources of Electric Potential

(b) The capacitor still has no net charge, but charge has been separated to give the plates charges $+Q$ and $-Q$.


Now, because $q$ is repelled from the positive plate and attracted to the negative plate, the hand must do work on $q$ to push it from A to B, so there must be an electric potential difference .$\Delta V$ between A and B.

- A potential difference is created by separating positive charge from negative charge.


## QuickCheck 21.19

Metal wires are attached to the terminals of a 3 V battery. What is the potential difference between points 1 and 2 ?
A. 6 V
B. 3 V
C. 0 V
D. Undefined
E. Not enough information to tell

## QuickCheck 21.19

Metal wires are attached to the terminals of a 3 V battery. What is the potential difference between points 1 and 2 ?
A. 6 V
B. 3 V
C. 0 V
D. Undefined
E. Not enough information to tell

Every point on this conductor is at the same potential as the positive terminal of the battery.


Every point on this conductor is at the same potential as the negative terminal of the battery.

## QuickCheck 21.20

Metal spheres 1 and 2 are connected by a metal wire. What quantities do spheres 1 and 2 have in common?
A. Same potential
B. Same electric field
C. Same charge
D. Both A and B
E. Both A and C

## QuickCheck 21.20

Metal spheres 1 and 2 are connected by a metal wire. What quantities do spheres 1 and 2 have in common?

## A. Same potential

B. Same electric field
C. Same charge
D. Both A and B
E. Both A and C

## Sources of Electric Potential

- As you shuffle your feet across the carpet, friction between your feet and the carpet transfers charge to your body, causing a potential difference between your body and a nearby doorknob.


## Sources of Electric Potential

- Lightning is the result of charge separation that occurs in clouds. Small ice particles in the clouds collide and become charged by frictional rubbing.
- The heavier particles fall to the
 bottom of the cloud and gain negative charge; light particles move to the top and gain positive charge. The negative charge at the bottom of the cloud causes a positive charge to accumulate on the ground below.
- A lightning strike occurs when the potential difference between the cloud and the ground becomes too large for the air to sustain.


## Sources of Electric Potential

- A battery creates a fixed potential difference using chemical processes.
- All batteries use chemical reactions to create internal charge separation.
- In biology, chemicals produce a potential difference of about 70 mV between the inside and the outside of a cell, with inside the cell more negative than outside.


## Sources of Electric Potential

- The membrane potential of a cell is caused by an imbalance of potassium and sodium ions.



## Measuring Electric Potential

- The potential at a given point depends on where we choose $V$ to be zero, but the difference is independent of any choices. The potential difference is measured for practical purposes.

- A voltmeter, the basic instrument for measuring potential differences, always has two inputs.


## Measuring Electric Potential

- This image is a micrograph of a nerve cell whose membrane potential is being measured.
- A small glass pipette filled with conductive fluid is inserted through the cell's membrane.
- The second probe is immersed in the conducting fluid that surrounds the cell.



## Section 21.3 Electric Potential and Conservation of Energy

## Electric Potential and Conservation of Energy

TABLE 21.2 Distinguishing electric potential and potential energy

The electric potential is created by the source charges. The electric potential is present whether or not a charged particle is there to experience it. Potential is measured in $\mathrm{J} / \mathrm{C}$, or V.
The electric potential energy is the interaction energy of a charged particle with the source charges. Potential energy is measured in J .

## Electric Potential and Conservation of Energy

- The conservation of energy equation for a charged particle is

$$
K_{\mathrm{f}}+\left(U_{\text {elec }}\right)_{\mathrm{f}}=K_{\mathrm{i}}+\left(U_{\text {elec }}\right)_{\mathrm{i}}
$$

- In terms of electric potential $V$ the equation is

$$
K_{\mathrm{f}}+q V_{\mathrm{f}}=K_{\mathrm{i}}+q V_{\mathrm{i}}
$$

Conservation of energy for a charged particle moving in an electric potential $V$

## Electric Potential and Conservation of Energy

- The motion of the charges can be written

$$
\Delta K=-q \Delta V
$$

- When $\Delta K$ is positive, the particle speeds up as it moves from higher to lower potential.
- When $\Delta K$ is negative, the particle slows down.


## Electric Potential and Conservation of Energy

(a) A positive charge speeds up $(\Delta K>0)$ as it moves from higher to lower potential $(\Delta V<0)$. Electric potential energy is transformed into kinetic energy.


$$
\Delta V=-100 \mathrm{~V}<0
$$

$\because$ Dashed green lines are lines of constant electric potential. The potential is 500 V at all points on this line.

## Electric Potential and Conservation of Energy

(b) A positive charge slows down $(\Delta K<0)$ as it moves from lower to higher potential ( $\Delta V>0$ ). Kinetic energy is transformed into electric potential energy.

500 V
After:


$$
\Delta V=+100 \mathrm{~V}>0
$$

A double-headed green arrow is used to represent a potential difference.

## Electric Potential and Conservation of Energy

## PROBLEM-SOLVING STRATEGY 21.1 <br> Conservation of energy in charge interactions

We use the principle of conservation of energy for electric charges in exactly the same way as we did for mechanical systems. The only difference is that we now need to consider electric potential energy.
prepare Draw a before-and-after visual overview. Define symbols that will be used in the problem, list known values, and identify what you're trying to find. solve The mathematical representation is based on the law of conservation of mechanical energy:

$$
K_{\mathrm{f}}+q V_{\mathrm{f}}=K_{\mathrm{i}}+q V_{\mathrm{i}}
$$

## Electric Potential and Conservation of Energy

## PROBLEM-SOLVING STRATEGY 21.1 <br> Conservation of energy in charge interactions

- Find the electric potential at both the initial and final positions. You may need to calculate it from a known expression for the potential, such as that of a point charge.
- $K_{\mathrm{i}}$ and $K_{\mathrm{f}}$ are the total kinetic energies of all moving particles.
- Some problems may need additional conservation laws, such as conservation of charge or conservation of momentum.

Assess Check that your result has the correct units, is reasonable, and answers the question.

## Electric Potential and Conservation of Energy

- Electric energy can be transformed into other types of energy in addition to kinetic energy.
- When a battery is connected to a lightbulb, the electric potential energy is transformed into thermal energy in the bulb, making the bulb hot enough to glow.
- As charges move from the high- to low-potential terminals of an elevator motor, their electric potential is transformed into gravitational potential energy.


## QuickCheck 21.5

A positive charge moves as shown. Its kinetic energy
A. Increases.
B. Remains constant.
C. Decreases.


## QuickCheck 21.5

A positive charge moves as shown. Its kinetic energy
A. Increases.
B. Remains constant.
C. Decreases.


Increasing PE Decreasing KE

## QuickCheck 21.6

A positive and a negative charge are
 released from rest in vacuum. They move toward each other. As they do:
A. A positive potential energy becomes more positive.
B. A positive potential energy becomes less positive.
C. A negative potential energy becomes more negative.
D. A negative potential energy becomes less negative.
E. A positive potential energy becomes a negative potential energy.

## QuickCheck 21.6

A positive and a negative charge are
 released from rest in vacuum. They move toward each other. As they do:
A. A positive potential energy becomes more positive.
B. A positive potential energy becomes less positive.
C. A negative potential energy becomes more negative.
D. A negative potential energy becomes less negative.
E. A positive potential energy becomes a negative potential energy.

$$
U_{\text {elec }}=\frac{K q_{1} q_{2}}{r} \text { Opposite signs, so } U \text { is Negative. } \quad U \text { increases in magnitude as } r \text { decreases. }
$$

## QuickCheck 21.7

A proton is released from rest at the dot. Afterward, the proton

-     -         -             -                 -                     -                         -                             -                                 -                                     -                                         -                                             - +50 V
A. Remains at the dot.
B. Moves upward with steady speed.

$$
-----\cdot \bullet----\quad 0 \text { V }
$$

$$
\text { - - - - - - - - - - - - - - } 50 \text { V }
$$

C. Moves upward with an increasing speed.
D. Moves downward with a steady speed.
E. Moves downward with an increasing speed.

## QuickCheck 21.7

A proton is released from rest at the dot. Afterward, the proton
A. Remains at the dot.
B. Moves upward with steady speed.
C. Moves upward with an increasing speed.
D. Moves downward with


Decreasing PE
Increasing KE a steady speed.
E. Moves downward with an increasing speed.

## QuickCheck 21.8

If a positive charge is released from rest, it moves in the direction of
A. A stronger electric field.
B. A weaker electric field.
C. Higher electric potential.
D. Lower electric potential.
E. Both B and D.

## QuickCheck 21.8

If a positive charge is released from rest, it moves in the direction of
A. A stronger electric field.
B. A weaker electric field.
C. Higher electric potential.
D. Lower electric potential.
E. Both B and D.

## The Electron Volt

- An electron $(q=e)$ accelerating through a potential difference $\Delta V=1 \mathrm{~V}$ gains kinetic energy:
$\Delta K=-q \Delta V=e \Delta V=\left(1.60 \times 10^{-19} \mathrm{C}\right)(1 \mathrm{~V})=1.60 \times 10^{-19} \mathrm{~J}$
- The electron volt is a unit of energy.
- 1 electron volt $=1 \mathrm{eV}=1 \times 10^{-19} \mathrm{~J}$
- 1 electron volt is the kinetic energy gained by an electron (or proton) as it accelerates through a potential difference of 1 volt.


## The Electron Volt

- A proton or electron that accelerates through a potential difference of $V$ volts gains $V \mathrm{eV}$ of kinetic energy.
- A proton or electron the decelerates through a potential difference of $V$ volts loses $V \mathrm{eV}$ of kinetic energy.


## Example 21.3 The speed of a proton

Atomic particles are often characterized by their kinetic energy in MeV . What is the speed of an 8.7 MeV proton? SOLVE The kinetic energy of this particle is $8.7 \times 10^{6} \mathrm{eV}$. First, we convert the energy to joules:

$$
K=8.7 \times 10^{6} \mathrm{eV} \times \frac{1.60 \times 10^{-19} \mathrm{~J}}{1.0 \mathrm{eV}}=1.39 \times 10^{-12} \mathrm{~J}
$$

## Example 21.3 The speed of a proton (cont.)

Now we can find the speed from

$$
K=\frac{1}{2} m v^{2}
$$

which gives

$$
v=\sqrt{\frac{2 K}{m}}=\sqrt{\frac{2\left(1.39 \times 10^{-12} \mathrm{~J}\right)}{1.67 \times 10^{-27} \mathrm{~kg}}}=4.1 \times 10^{7} \mathrm{~m} / \mathrm{s}
$$

## Example Problem

A proton has a speed of $3.50 \times 10^{5} \mathrm{~m} / \mathrm{s}$ when at a point where the potential is +100 V . Later, it's at a point where the potential is -150 V . What's its speed at this later point?

## QuickCheck 21.9

Two protons, one after the other, are launched from point 1 with the same speed. They follow the two trajectories shown. The protons' speeds at points 2 and
 3 are related by
A. $v_{2}>v_{3}$
B. $v_{2}=v_{3}$
C. $v_{2}<v_{3}$
D. Not enough information to compare their speeds

## QuickCheck 21.9

Two protons, one after the other, are launched from point 1 with the same speed. They follow the two trajectories shown. The protons' speeds at points 2 and
 3 are related by
A. $v_{2}>v_{3}$
B. $v_{2}=v_{3}$ Energy conservation
C. $v_{2}<v_{3}$
D. Not enough information to compare their speeds

## QuickCheck 21.13

An electron follows the trajectory shown from point 1 to point 2. At point 2,
$\begin{array}{ll}\text { A. } & v_{2}>v_{1} \\ \text { B. } & v_{2}=v_{1} \\ \text { C. } & v_{2}<v_{1}\end{array}$

D. Not enough information to compare the speeds at these points

## QuickCheck 21.13

An electron follows the trajectory shown from point 1 to point 2. At point 2,
A. $v_{2}>v_{1}$
B. $v_{2}=v_{1}$
C. $v_{2}<v_{1}$

D. Not enough information to compare the speeds at these points

Increasing PE (becoming less negative) so decreasing KE

## Section 21.4 Calculating the Electric Potential

## Calculating the Electric Potential

- The electric potential can be written

$$
V=\frac{U_{\mathrm{elec}}}{q}
$$

- To find the potential at a point in space, we calculate the electric potential energy of a charge $q$ at a point. Then we can solve for the electric potential.


## The Electric Potential Inside a Parallel-Plate Capacitor

- The parallel-plate capacitor creates a uniform electric field by placing equal but opposite charges on two parallel conducting plates.


## The Electric Potential Inside a Parallel-Plate

 Capacitor

The hand does work on $q$ to move it "uphill" against the field, thus giving the charge electric potential energy.

## The Electric Potential Inside a Parallel-Plate Capacitor

- For a parallel-plate capacitor we are free to choose the point of zero potential energy where it is convenient, so we choose $U_{\text {elec }}=0$ when the mobile charge $q$ is at the negative plate.
- The charge's potential energy at any other point is then the amount of external work required to move the charge from the negative plate to that position.


## The Electric Potential Inside a Parallel-Plate Capacitor

- To move a charge to the right at a constant speed in an electric field pointing to the left, the external force $\vec{F}_{\text {net }}=\overrightarrow{0}$ must push hard to the right with a force of the same magnitude: $F_{\text {hand }}=q E$
- The work to move the charge to position $x$ is

$$
W=\text { force } \times \text { displacement }=F_{\text {hand }} x=q E x
$$

- The electric potential energy is is

$$
U_{\mathrm{elec}}=W=q E x
$$

## The Electric Potential Inside a Parallel-Plate Capacitor

- The electric potential of a parallel-plate capacitor at a position $x$, measured from the negative plate, is

$$
V=\frac{U_{\mathrm{elec}}}{q}=E x=\frac{Q}{\epsilon_{0} A} x
$$

- The electric potential increases linearly from the negative plate $(x=0)$ to the positive plate at $x=d$.
- The potential difference $\Delta V_{\mathrm{C}}$ between the two capacitor plates is

$$
\Delta V_{\mathrm{C}}=V_{+}-V_{-}=E d
$$

## The Electric Potential Inside a Parallel-Plate Capacitor

- In many cases, the capacitor voltage is fixed at some value $\Delta V_{\mathrm{C}}$ by connecting its plates to a battery with a known voltage. In this case, the electric field strength inside the capacitor is

$$
E=\frac{\Delta V_{\mathrm{C}}}{d}
$$

- This means we can establish an electric field of known strength by applying a voltage across a capacitor whose plate spacing is known.


## The Electric Potential Inside a Parallel-Plate Capacitor

- The electric potential at position $x$ inside a capacitor is

$$
V=\frac{x}{d} \Delta V_{\mathrm{C}}
$$

- The potential increases linearly from $V=0$ at $x=0$ (the negative plate) to $V=\Delta V_{\mathrm{C}}$ at $x=d$ (the positive plate).


## The Electric Potential Inside a Parallel-Plate Capacitor

- Below are graphical representations of the electric potential inside a capacitor.



## The Electric Potential Inside a Parallel-Plate Capacitor

- A graph of potential versus $x$. You can see the potential increasing from 0 V at the negative plate to 1.5 V at the positive plate.



## The Electric Potential Inside a Parallel-Plate Capacitor

- A three-dimensional view showing equipotential surfaces. These are mathematical surfaces, not physical surfaces, that have the same value of $V$ at every point. The equipotential surfaces of
 a capacitor are planes parallel to the capacitor plates. The capacitor plates are also equipotential surfaces.


## The Electric Potential Inside a Parallel-Plate Capacitor

- A two-dimensional equipotential map. The green dashed lines represent slices through the equipotential surfaces, so $V$ has the same value everywhere along such a line. We call these lines of constant potential equipotential lines or simply equipotentials.



## Example 21.4 A proton in a capacitor

A parallel-plate capacitor is constructed of two disks spaced 2.00 mm apart. It is charged to a potential difference of 500 V . A proton is shot through a small hole in the negative plate with a speed of $2.0 \times 10^{5} \mathrm{~m} / \mathrm{s}$. What is the farthest distance from the negative plate that the proton reaches?


## Example 21.4 A proton in a capacitor (cont.)

PREPARE Energy is conserved. The proton's potential energy inside the capacitor can be found from the capacitor's electric potential. FIGURE 21.10 is a before-and-after visual overview of the proton in the capacitor.


## Example 21.4 A proton in a capacitor (cont.)

SOLVE The proton starts at the negative plate, where $x_{\mathrm{i}}=0$ mm . Let the final point, where $v_{\mathrm{f}}=0 \mathrm{~m} / \mathrm{s}$, be at $x_{\mathrm{f}}$. The potential inside the capacitor is given by $V=\Delta V_{\mathrm{C}} x / d$ with $d=0.0020 \mathrm{~m}$ and $\Delta V_{\mathrm{C}}=500 \mathrm{~V}$.


## Example 21.4 A proton in a capacitor (cont.)

Conservation of energy requires
$K_{\mathrm{f}}+e V_{\mathrm{f}}=K_{\mathrm{i}}+e V_{\mathrm{i}}$. This is

$$
0+e \Delta V_{\mathrm{C}} \frac{x_{\mathrm{f}}}{d}=\frac{1}{2} m v_{\mathrm{i}}^{2}+0
$$

where we used $V_{\mathrm{i}}=0 \mathrm{~V}$ at the
 negative plate $\left(x_{\mathrm{i}}=0\right)$ and $K_{\mathrm{f}}=0$

$$
x_{\mathrm{i}}=0 \mathrm{~mm} \quad x_{\mathrm{f}} \quad d=2.00 \mathrm{~mm}
$$

$$
x_{\mathrm{f}}=\frac{m d v_{\mathrm{i}}^{2}}{2 e \Delta V_{\mathrm{C}}}=0.84 \mathrm{~mm}
$$

## Example 21.4 A proton in a capacitor (cont.)

The proton travels 0.84 mm , less than halfway across, before stopping and being turned back.

ASSESS We were able to use the electric potential inside the capacitor to determine the proton's potential energy.


## The Electric Potential Inside a Parallel-Plate Capacitor

## sYnthesis 21.1 The parallel-plate capacitor: Potential and electric field

There are several related expressions for the electric potential and field inside a parallel-plate capacitor.
The variables used in describing the parallel-plate capacitor


Text: p. 677

## The Electric Potential Inside a Parallel-Plate Capacitor

## synthesis 21.1 The parallel-plate capacitor: Potential and electric field

There are several related expressions for the electric potential and field inside a parallel-plate capacitor.
Three equivalent expressions for the electric potential are related by two expressions for the electric field inside the capacitor:
$\begin{array}{lc}\text { Potential } V \quad V=E \quad E \quad \frac{Q}{\epsilon_{0} A} x \quad V=\frac{x}{d} \sqrt{\Delta V_{\mathrm{C}}} \\ \text { Field } \boldsymbol{E} \quad & \quad E=\frac{Q}{\epsilon_{0} A} \quad{ }^{2} \quad E=\frac{\Delta V_{\mathrm{C}}}{d}\end{array}$

## The Potential of a Point Change

- To find the electric potential due to a single fixed charge $q$, we first find the electric potential energy when a second charge, $q^{\prime}$, is a distance $r$ away from $q$.
- We find the electric potential energy by determining the work done to move $q^{\prime}$ from the point where $U_{\text {elec }}=0$ to a point with a distance $r$ from $q$.
- We choose $U_{\text {elec }}=0$ to be at a point infinitely far from $q$ for convenience, since the influence of a point charge goes to zero infinitely far from the charge.


## The Potential of a Point Change

- We cannot determine the work with the simple expression $W=F d$ for a moving charge $q^{\prime}$ because the force isn't constant.
- From Coulomb's law, we know the force on $q^{\prime}$ gets larger as it approaches $q$.


## The Potential of a Point Change



## QuickCheck 21.11

What is the ratio $V_{\mathrm{B}} / V_{\mathrm{A}}$ of the electric potentials at the two points?
A. 9
B. 3
C. $1 / 3$
D. $1 / 9$
E. Undefined without knowing the charge


## QuickCheck 21.11

What is the ratio $V_{\mathrm{B}} / V_{\mathrm{A}}$ of the electric potentials at the two points?
A. 9
B. 3
C. $1 / 3$ Potential of a point charge decreases

1 mm
D. $1 / 9$ inversely with distance.
E. Undefined without knowing the charge

## The Potential of a Point Change

- The electric potential energy of two point charges is:

$$
U_{\mathrm{elec}}=K \frac{q q^{\prime}}{r}=\frac{1}{4 \pi \epsilon_{0}} \frac{q q^{\prime}}{r}
$$

Electric potential energy of two charges $q$ and $q^{\prime}$ separated by distance $r$

## The Potential of a Point Change

- In the case where $q$ and $q^{\prime}$ are opposite charges, the potential energy of the charges is negative. $q^{\prime}$ accelerates toward fixed charge $q$.



## The Potential of a Point Change

- For a charge $q^{\prime}$ a distance $r$ from a charge $q$, the electric potential is related to the potential energy by $V=U_{\text {elec }} / q^{\prime}$. Thus the electric potential of charge $q$ is

$$
V=K \frac{q}{r}=\frac{1}{4 \pi \epsilon_{0}} \frac{q}{r}
$$

## Electric potential at distance r from a point charge $q$

- Only the source appears in this expression. The source charge creates the electric potential around it.


## The Potential of a Point Change

- Three graphical representations of the electric potential of a positive point charge:



## Example 21.6 Calculating the potential of a point charge

What is the electric potential 1.0 cm from a 1.0 nC charge? What is the potential difference between a point 1.0 cm away and a second point 3.0 cm away?

Prepare We can use Equation 21.10 to find the potential at the two distances from the charge.

## Example 21.6 Calculating the potential of a point charge (cont)

SOLVE The potential at $r=1.0 \mathrm{~cm}$ is

$$
V_{1 \mathrm{~cm}}=K \frac{q}{r}=\left(9.0 \times 10^{9} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{C}^{2}\right)\left(\frac{1.0 \times 10^{-9} \mathrm{C}}{0.010 \mathrm{~m}}\right)=900 \mathrm{~V}
$$

We can similarly calculate $V_{3 \mathrm{~cm}}=300 \mathrm{~V}$. Thus the potential difference between these two points is $\Delta V=V_{1 \mathrm{~cm}}-V_{3 \mathrm{~cm}}=$ 600 V .

## Example 21.6 Calculating the potential of a point charge (cont)

ASSESS 1 nC is typical of the electrostatic charge produced by rubbing, and you can see that such a charge creates a fairly large potential nearby. Why aren't we shocked and injured when working with the "high voltages" of such charges? As we'll learn in Chapter 26, the sensation of being shocked is a result of current, not potential. Some high-potential sources simply do not have the ability to generate much current.

## The Electric Potential of a Charged Sphere

- The electric potential outside a charged sphere is the same as the electric potential outside a point charge:

$$
V=K \frac{Q}{r}=\frac{1}{4 \pi \epsilon_{0}} \frac{Q}{r}
$$

Electric potential at a distance $r>R$ from the center of a sphere of radius $R$ and with charge $Q$

## The Electric Potential of a Charged Sphere

- It is common to charge a metal object, such as a sphere, "to" a certain potential, for instance using a battery.
- The potential $V_{0}$ is the potential at the surface of the sphere.

$$
V_{0}=V(\text { at } r=R)=\frac{Q}{4 \pi \epsilon_{0} R}
$$

- The charge for a sphere of radius $R$ is therefore

$$
Q=4 \pi \epsilon_{0} R V_{0}
$$

- The potential outside a sphere charged to potential $V_{0}$ is

$$
V=\frac{R}{r} V_{0}
$$

- The potential decreases inversely with distance from center.


## The Electric Potential of a Charged Sphere

## SYNTHESIS 21.2 Potential of a point charge and a charged sphere

The electric potential $V$ on or outside a charged sphere is the same as for a point charge: It depends only on the distance and the charge on the sphere.

## Point charge $q$




Text: p. 680

## The Electric Potential of a Charged Sphere

## SYNTHESIS 21.2 Potential of a point charge and a charged sphere

Sphere with total charge $\boldsymbol{Q} \quad$ We want to $\quad$ When the total charge $Q$
calculate $V$ here.

on the sphere is known:

$$
V=K \frac{Q}{r}=\frac{1}{4 \pi \epsilon_{0}} \frac{Q}{r}
$$

When the potential $V_{0}$ at the surface of the sphere is known:

$$
V=\frac{R}{r} V_{0}
$$

These expressions are valid only on or outside the sphere, so $r \geq R$.
Text: p. 680

## QuickCheck 21.12

What is the electric potential at the surface of the sphere?
A. 15 V
B. 30 V
C. 60 V
D. 90 V
E. 120 V


## QuickCheck 21.12

What is the electric potential at the surface of the sphere?
A. 15 V
B. $\quad 30 \mathrm{~V}$
C. 60 V
D. 90 V
E. 120 V


## Example Problem

A proton is fired from far away toward the nucleus of an iron atom, which we can model as a sphere containing 26 protons. The diameter of the nucleus is $9.0 \times 10^{-15} \mathrm{~m}$. What initial speed does the proton need to just reach the surface of the nucleus? Assume the nucleus remains at rest.

## The Electric Potential of Many Charges

- Suppose there are many source charges, $q_{1}, q_{2} \ldots$ The electric potential $V$ at a point in space is the sum of the potentials due to each charge.

$$
V=\sum_{i} K \frac{q_{i}}{r_{i}}=\sum_{i} \frac{1}{4 \pi \epsilon_{0}} \frac{q_{i}}{r_{i}}
$$

- $r_{i}$ is the distance from charge $q_{i}$ to the point in space where the potential is being calculated.


## The Electric Potential of Many Charges

- The potential of an electric dipole is the sum of the potentials of the positive and negative charges.


Equipotential lines

## QuickCheck 21.15

At which point or points is the electric potential zero?

E. More than one of these

## QuickCheck 21.15

At which point or points is the electric potential zero?

$\checkmark$ E. More than one of these

## QuickCheck 21.16

Four charges lie on the corners of a square with sides of length $a$. What is the electric potential at the center of the square?

$$
\begin{aligned}
& \text { A. } 0 \\
& \text { B. } \frac{4 K q}{a} \\
& \text { C. } \frac{4 K q}{\sqrt{a}} \\
& \text { D. } \frac{4 K q}{\sqrt{2} a} \\
& \text { E. } \frac{K q}{\sqrt{2} a}
\end{aligned}
$$



## QuickCheck 21.16

Four charges lie on the corners of a square with sides of length $a$. What is the electric potential at the center of the square?

$$
\begin{aligned}
& \text { A. } 0 \\
& \text { B. } \frac{4 K q}{a} \\
& \text { C. } \frac{4 K q}{\sqrt{a}} \\
& \text { D. } \frac{4 K q}{\sqrt{2} a} \\
& \text { E. } \frac{K q}{\sqrt{2} a}
\end{aligned}
$$



## Example 21.8 Finding the potential of two charges

What is the electric potential at the point indicated in FIGURE 21.17?

PREPARE The potential is the sum of the potentials due to each charge.


## Example 21.8 Finding the potential of two charges (cont.)

SOLVE The potential at the indicated point is

$$
\begin{aligned}
V & =\frac{K q_{1}}{r_{1}}+\frac{K q_{2}}{r_{2}} \\
& =\left(9.0 \times 10^{9} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{C}^{2}\right)\left(\frac{2.0 \times 10^{-9} \mathrm{C}}{0.050 \mathrm{~m}}+\frac{-1.0 \times 10^{-9} \mathrm{C}}{0.040 \mathrm{~m}}\right) \\
& =140 \mathrm{~V}
\end{aligned}
$$

## Example 21.8 Finding the potential of two charges (cont.)

ASSESS As noted, the potential is a scalar, so we found the net potential by adding two scalars. We don't need any angles or components to calculate the potential.


## Section 21.5 Connecting Potential and Field

## Connecting Potential and Field

- The electric potential and electric field are not two distinct entities but, instead, two different perspectives or two different mathematical representations of how source charges alter the space around them.


## Connecting Potential and Field



- The electric field at a point is perpendicular to the equipotential surface at that point.


## Connecting Potential and Field



- The electric field points in the direction of decreasing potential.


## Connecting Potential and Field

- The work required to move a charge, at a constant speed, in a direction opposite the electric field is

$$
W=\Delta U_{\text {elec }}=q \Delta V
$$

- Because the charge moves at a constant speed, the force of a hand moving the charge is equal to that of the electric force:

$$
W=F_{\text {hand }} d=q E d
$$

- Comparing these equations, we find that the strength of the electric field is

$$
E=\frac{\Delta V}{d}
$$

Electric field strength in terms of the potential difference $\Delta V$ between two equipotential surfaces a distance $d$ apart

## Connecting Potential and Field

Higher potential $-\cdots \int / \vec{F}_{\text {hand }}$

## Lower

 potentia
## QuickCheck 21.10

A proton starts from rest at point A . It then accelerates past point B.
The proton's kinetic energy at Point $B$ is
A. 250 eV
B. 200 eV
C. 150 eV
D. 100 eV

## QuickCheck 21.10

A proton starts from rest at point A . It then accelerates past point B.
The proton's kinetic energy at Point $B$ is
A. 250 eV
B. 200 eV
C. 150 eV
D. 100 eV

## Connecting Potential and Field



## Connecting Potential and Field

Point charge
Electric dipole
Parallel-plate capacitor


For the capacitor, the field is uniform and so the equipotential

spacing is constant.

## QuickCheck 21.14

At the midpoint between these two equal but opposite charges,
A. $\vec{E}=\overrightarrow{0} ; V=0$
B. $\vec{E}=\overrightarrow{0} ; V>0$
C. $\vec{E}=\overrightarrow{0} ; V<0$
D. $\vec{E}$ points right; $V=0$
E. $\vec{E}$ points left; $V=0$

## QuickCheck 21.14

At the midpoint between these two equal but opposite charges,
A. $\vec{E}=\overrightarrow{0} ; V=0$
B. $\vec{E}=\overrightarrow{0} ; V>0$
C. $\vec{E}=\overrightarrow{0} ; V<0$
D. $\vec{E}$ points right; $V=0$
E. $\vec{E}$ points left; $V=0$

## QuickCheck 21.17

A particle follows the trajectory shown from initial position i to final position f . The potential difference $\Delta V$ is
A. 100 V
B. 50 V
C. 0 V
D. -50 V
E. -100 V


## QuickCheck 21.17

A particle follows the trajectory shown from initial position i to final position f . The potential difference $\Delta V$ is
A. 100 V
B. 50 V
C. 0 V

D. $-50 \mathrm{~V} \quad \Delta V=V_{\text {final }}-V_{\text {initial }}$, independent of the path
E. -100 V

## QuickCheck 21.18

Which set of equipotential surfaces matches this electric field?


## QuickCheck 21.18

Which set of equipotential surfaces matches this electric field?


## Example Problem

What are the magnitude and direction of the electric field at the dot?


## A Conductor in Electrostatic Equilibrium

- The four important properties about conductors in electrostatic equilibrium we already knew:

1. Any excess charge is on the surface.
2. The electric field inside is zero.
3. The exterior electric field is perpendicular to the surface.
4. The field strength is largest at sharp corners.

- The fifth important property we can now add:

5. The entire conductor is at the same potential, and thus the surface is an equipotential surface.

## A Conductor in Electrostatic Equilibrium

- Inside a conductor, the electric field is zero. Therefore no work can be done on a charge and so there can be no difference in potential between two points.
- Any two points inside a conductor in electrostatic equilibrium are at the same potential.



## A Conductor in Electrostatic Equilibrium

- Properties of a conductor in electrostatic equilibrium:



## Section 21.6 The Electrocardiogram

## The Electrocardiogram

- The electrical activity of cardiac muscle cells makes the beating heart an electric dipole.
- A resting nerve cell is polarized; the outside is positive and the inside negative.
- Initially, all muscle cells in the heart are polarized, until an electrical impulse from the heart triggers the cells to depolarize, moving ions through the cell wall until the outside becomes negative.
- This causes the muscle to contract.
- The depolarization of one cell triggers a "wave" of depolarization to spread across the tissues of the heart.


## The Electrocardiogram

- At any instant, a boundary divides the negative charges of depolarized cells from the positive charges of cells that have not yet depolarized in the heart.
- This separation of charges creates an electric dipole and produces a dipole electric field and potential.


## The Electrocardiogram

(a)


## The Electrocardiogram



## The Electrocardiogram

- A measurement of the electric potential of the heart is an invaluable diagnostic tool.
- The potential difference in a patient is measured between several pairs of electrodes.
- A chart of the potential differences is the electrocardiogram, also called an ECG or an EKG.



## The Electrocardiogram

- With each heart beat, the wave of depolarization moves across the heart muscle.
- The dipole moment of the heart changes magnitude and direction.


## The Electrocardiogram

## Position a.



Atrial depolarization

$V_{2}$ is positive, $V_{1}$ negative.

The record of the potential difference between the two electrodes is the electrocardiogram.

$$
\Delta V=V_{2}-V_{1}
$$



The potential differences at $\mathrm{a}, \mathrm{b}$, and c correspond to those measured in the three stages shown to the left.

## The Electrocardiogram

## Position b.


$V_{2}$ is negative, $V_{1}$ positive.

The record of the potential difference between the two electrodes is the
electrocardiogram.

$$
\Delta V=V_{2}-V_{1}
$$



The potential differences at $\mathrm{a}, \mathrm{b}$, and c correspond to those measured in the three stages shown to the left.

## The Electrocardiogram

## Position c.



The record of the potential difference between the two electrodes is the


The potential differences at a, b, and c correspond to those measured in the three stages shown to the left.

## Section 21.7 Capacitance and Capacitors

## Capacitance and Capacitors

- A capacitor is formed by two conductors with equal but opposite charge.
- The two conductors are the electrodes or plates.
- Capacitors can be used to store charge, making them invaluable in all kinds of electronic circuits.


## Capacitance and Capacitors

- In a capacitor, the electric field strength $E$ and the potential difference $\Delta V_{\mathrm{C}}$ increase as the charge on each electrode increases.


## Capacitance and Capacitors



## Capacitance and Capacitors

- The potential difference between the electrodes is directly proportional to their charge.
- Stated another way, the charge of a capacitor is directly proportional to the potential difference between its electrodes.

$$
Q=C \Delta V_{\mathrm{C}}
$$

## Charge on a capacitor with potential difference $\Delta V_{\mathrm{C}}$

- The constant of proportionality, $C$, is the capacitance.
- The SI unit of capacitance is the farad.
- 1 farad $=1 \mathrm{~F}=1$ coulomb/volt $=1 \mathrm{C} / \mathrm{V}$.


## Capacitance and Capacitors

- Capacitance depends on the shape, size, and spacing of the two electrodes.
- A capacitor with a large capacitance holds more charge for a given potential difference than one with a small capacitance.


## Charging a Capacitor

- To "charge" a capacitor, we need to move charge from one electrode to the other.
- The simplest way to do this is to use a source of potential difference such as a battery.
- A battery uses its internal chemistry to maintain a fixed potential difference between its terminals.


## Charging a Capacitor

(a)

Direction of charge motion


## Charging a Capacitor

(b) The movement of the charge stops when $\Delta V_{\mathrm{C}}$ is equal to the battery voltage. The capacitor is then fully charged.


## Charging a Capacitor

## (c)

If the battery is removed, the capacitor remains charged, with $\Delta V_{\mathrm{C}}$ still equal to the battery voltage.


- A capacitor can be used to store charge.


## QuickCheck 21.21

## What is the capacitance of these two electrodes?

A. 8 nF
B. 4 nF
C. 2 nF
D. 1 nF
E. Some other value


## QuickCheck 21.21

## What is the capacitance of these two electrodes?

A. 8 nF
B. 4 nF
C. $2 \mathrm{nF} \quad C=\frac{Q}{\Delta V}$
D. 1 nF
E. Some other value


## Example 21.10 Charging a capacitor

A $1.3 \mu \mathrm{~F}$ capacitor is connected to a 1.5 V battery. What is the charge on the capacitor?
PREPARE Charge flows through the battery from one capacitor electrode to the other until the potential difference $\Delta V_{\mathrm{C}}$ between the electrodes equals that of the battery, or 1.5 V .

## Example 21.10 Charging a capacitor (cont.)

SOLVE The charge on the capacitor is given by Equation 21.18:

$$
Q=C \Delta V_{\mathrm{C}}=\left(1.3 \times 10^{-6} \mathrm{~F}\right)(1.5 \mathrm{~V})=2.0 \times 10^{-6} \mathrm{C}
$$

ASSESS This is the charge on the positive electrode; the other electrode has a charge of $-2.0 \times 10^{-6} \mathrm{C}$.

## The Parallel-Plate Capacitor

- A parallel-plate capacitor is important because it creates a uniform electric field between its flat electrodes.
- The electric field of a parallelplate capacitor is
$\vec{E}=\left(\frac{Q}{\epsilon_{0} A}\right.$, from positive to negative $)$
- $A$ is the surface area of the electrodes, and $Q$ is the charge
 on the capacitor.


## The Parallel-Plate Capacitor

- The electric field strength of a parallel-plate capacitor is related to the potential difference $\Delta V$ and plate spacing $d$ by

$$
E=\frac{\Delta V_{\mathrm{C}}}{d}
$$

- Comparing the different equations describing the electric field of a parallel-plate capacitor we find

$$
\frac{Q}{\epsilon_{0} A}=\frac{\Delta V_{\mathrm{C}}}{d}
$$

- Or equivalently,

$$
Q=\frac{\epsilon_{0} A}{d} \Delta V_{\mathrm{C}}
$$

## The Parallel-Plate Capacitor

$$
\begin{aligned}
& C=\frac{\epsilon_{0} A}{d} \\
& \text { Capacitance of a parallel-plate capacitor } \\
& \text { with plate area } A \text { and separation } d
\end{aligned}
$$

## QuickCheck 21.22

A capacitor has a charge $Q$. The plates are then pulled apart so that the distance between them is larger.


After the plates are pulled apart,
A. The charge increases and the electric field decreases.
B. The charge decreases and the electric field increases.
C. Both the charge and the field increase.
D. Both the charge and the field decrease.
E. The charge and the field remain constant.

## QuickCheck 21.22

A capacitor has a charge $Q$. The plates are then pulled apart so that the distance between them is larger.


After the plates are pulled apart,
A. The charge increases and the electric field decreases.
B. The charge decreases and the electric field increases.
C. Both the charge and the field increase.
D. Both the charge and the field decrease.
E. The charge and the field remain constant.

## QuickCheck 21.23

A capacitor is attached to a battery. The plates are then pulled apart so that the distance between them is larger.


After the plates are pulled apart,
A. The charge increases and the electric field decreases.
B. The charge decreases and the electric field increases.
C. Both the charge and the field increase.
D. Both the charge and the field decrease.
E. The charge and the field remain constant.

## QuickCheck 21.23

A capacitor is attached to a battery. The plates are then pulled apart so that the distance between them is larger.


After the plates are pulled apart,
A. The charge increases and the electric field decreases.
B. The charge decreases and the electric field increases.
C. Both the charge and the field increase.
D. Both the charge and the field decrease.
E. The charge and the field remain constant.

## Example Problem

A parallel-plate capacitor is constructed of two square plates, 1 m on each side, separated by a 1.0 mm gap. What is the capacitance of this capacitor? If it were charged to 100 V, how much charge would be on the capacitor?

## Example 21.11 Charging a parallel-plate capacitor

The spacing between the plates of a $1.0 \mu \mathrm{~F}$ parallel-plate capacitor is 0.070 mm .
a. What is the surface area of the plates?
b. How much charge is on the plates if this capacitor is attached to a 1.5 V battery?

## Example 21.11 Charging a parallel-plate capacitor (cont.)

SOLVE a. From the definition of capacitance,

$$
A=\frac{d C}{\epsilon_{0}}=\frac{\left(0.070 \times 10^{-3} \mathrm{~m}\right)\left(1.0 \times 10^{-6} \mathrm{~F}\right)}{8.85 \times 10^{-12} \mathrm{~F} / \mathrm{m}}=7.9 \mathrm{~m}^{2}
$$

b. The charge is $Q=C \Delta V_{\mathrm{C}}=\left(1.0 \times 10^{-6} \mathrm{~F}\right)(1.5 \mathrm{~V})=$ $1.5 \times 10^{-6} \mathrm{C}=1.5 \mathrm{mC}$.

## Example 21.11 Charging a parallel-plate capacitor (cont.)

ASSESS The surface area needed to construct a $1.0 \mu \mathrm{~F}$ capacitor (a fairly typical value) is enormous and hardly practical. We'll see in the next section that real capacitors can be reduced to a more manageable size by placing an insulator between the capacitor plates.

## Dielectrics and Capacitors

- An insulator consists of vast numbers of atoms. When an insulator is placed in an electric field, each atom polarizes.
- Polarization occurs when an atom's negative electron cloud and positive nucleus shift very slightly in opposite directions in response to an applied electric field.
- An induced positive charge builds up on one surface of the insulator, and an induced negative charge builds up on the other surface.


## Dielectrics and Capacitors

The positive nuclei shift very slightly in the direction of the field.

The negative electron cloud shifts very slightly in the direction opposite the field.

## Applied field

Because of polarization, . . . and this surface this surface has an excess has an excess of of positive charge . . . negative charge.

## Dielectrics and Capacitors

- The induced charge on an insulator will create a uniform electric field, like in a parallel-plate capacitor, but one that is directed opposite to the applied electric field.
- A dielectric is an insulator placed between the plates of a capacitor.
- A capacitor's electric field polarizes the dielectric; the dielectric creates an electric field of its own opposite the capacitor's field.
- The two fields add to give a net field in the same direction as the applied field, but smaller. Thus the electric field between the capacitor plates is smaller with a dielectric.


## Dielectrics and Capacitors

(a)

Applied field from charge on capacitor


Field due to induced charges on dielectric
(b)


## Dielectrics and Capacitors

- When a dielectric is inserted, the electric field between the plates decreases, which implies the potential difference decreases as well. The charge remains the same.
- The capacitance $C=Q / \Delta V_{\mathrm{C}}$ increases.
- The presence of a dielectric results in an increased capacitance.
- The dielectric constant $\kappa$ of the material determines the factor by which the capacitance is increased:

$$
C=\kappa C_{0}
$$

## Capacitance of a parallel-plate capacitor with a dielectric of dielectric constant $\kappa$

- $C_{0}$ is the capacitance without a dielectric present.


## Dielectrics and Capacitors

TABLE 21.3 Dielectric constants of some materials at $20^{\circ} \mathrm{C}$

| Material | Dielectric constant $\boldsymbol{\kappa}$ |
| :--- | :---: |
| Vacuum | 1 (exactly) |
| Air | $1.00054^{*}$ |
| Teflon | 2.0 |
| Paper | 3.0 |
| Pyrex glass | 4.8 |
| Cell membrane | 9.0 |
| Ethanol | 24 |
| Water | 80 |
| Strontium titanate | 300 |
| *Use 1.00 in all calculations. |  |

## Example 21.12 Finding the dielectric constant

A parallel-plate capacitor is charged using a 100 V battery; then the battery is removed. If a dielectric slab is slid between the plates, filling the space inside, the capacitor voltage drops to 30 V . What is the dielectric constant of the dielectric?

## Example 21.12 Finding the dielectric constant (cont.)

PREPARE The capacitor voltage remains $\left(\Delta V_{\mathrm{C}}\right)_{1}=100 \mathrm{~V}$ when it is disconnected from the battery. Placing the dielectric between the plates reduces the voltage to $\left(\Delta V_{\mathrm{C}}\right)_{2}=30 \mathrm{~V}$. Because the plates are not connected when the dielectric is inserted, the charge on the plates remains constant.

## Example 21.12 Finding the dielectric constant (cont.)

SOLVE Because the plates are not connected, the charge on the capacitor is constant, so we have

$$
Q_{1}=C_{1}\left(\Delta V_{\mathrm{C}}\right)_{1}=Q_{2}=C_{2}\left(\Delta V_{\mathrm{C}}\right)_{2}
$$

Inserting the dielectric increases the capacitance by a factor of $\kappa$, so that $C_{2}=\kappa C_{1}$.

## Example 21.12 Finding the dielectric constant (cont.)

Thus $C_{1}\left(\Delta V_{\mathrm{C}}\right)_{1}=\kappa C_{1}\left(\Delta V_{\mathrm{C}}\right) 2$ or, canceling $C_{1},\left(\Delta V_{\mathrm{C}}\right)_{1}=$ $\kappa\left(\Delta V_{\mathrm{C}}\right)_{2}$. The dielectric constant is then

$$
\kappa=\frac{\left(\Delta V_{\mathrm{C}}\right)_{1}}{\left(\Delta V_{\mathrm{C}}\right)_{2}}=\frac{100 \mathrm{~V}}{30 \mathrm{~V}}=3.3
$$

ASSESS The dielectric constant is greater than 1, as must be the case.

## Example Problem

A parallel-plate capacitor with a capacitance of 200 pF is charged to 100 V . Then the battery is removed. A sheet of teflon $(\kappa=2.0)$ is then slid between the plates.
A. By what factor does the charge on the plates change?
B. What is the final potential difference between the plates?

## Section 21.8 Energy and Capacitors

## Energy and Capacitors

- A charged capacitor stores energy as electric potential energy.
- The potential energy $U_{\mathrm{C}}$ stored in a charged capacitor is

$$
U_{\mathrm{C}}=Q \Delta V_{\text {average }}=\frac{1}{2} Q \Delta V_{\mathrm{C}}
$$

- Since, $Q=C \Delta V_{\mathrm{C}}$, the electric potential can be written

$$
U_{\mathrm{C}}=\frac{1}{2} \frac{Q^{2}}{C}=\frac{1}{2} C\left(\Delta V_{\mathrm{C}}\right)^{2}
$$

Electric potential energy of a capacitor with charge $Q$ and potential difference $\Delta V_{\mathrm{C}}$

## Example Problem

The capacitor bank used to power a large electromagnet is charged to $23,500 \mathrm{~V}$ and stores 8.4 MJ of energy. What is the total capacitance of the capacitor bank?

## Example 21.13 Energy in a camera flash

How much energy is stored in a $220 \mu \mathrm{~F}$ camera-flash capacitor that has been charged to 330 V ? What is the average power delivered to the flash lamp if this capacitor is discharged in 1.0 ms ?

## Example 21.13 Energy in a camera flash (cont.)

SOLVE The energy stored in the capacitor is

$$
U_{\mathrm{C}}=\frac{1}{2} C\left(\Delta V_{\mathrm{C}}\right)^{2}=\frac{1}{2}\left(220 \times 10^{-6} \mathrm{~F}\right)(330 \mathrm{~V})^{2}=12 \mathrm{~J}
$$

If this energy is released in 1.0 ms , the average power is

$$
P=\frac{\Delta E}{\Delta t}=\frac{12 \mathrm{~J}}{1.0 \times 10^{-3} \mathrm{~s}}=12,000 \mathrm{~W}
$$

## Example 21.13 Energy in a camera flash (cont.)

ASSESS The stored energy is equivalent to raising a 1 kg mass by 1.2 m . This is a rather large amount of energy; imagine the damage a 1 kg object could do after falling 1.2 m . When this energy is released very quickly, as is possible in an electronic circuit, the power is very high.

## Energy and Capacitors

- A capacitor can charge very slowly and then can release the energy very quickly.
- A medical application of this ability to rapidly deliver energy is the defibrillator.

- Fibrillation is the state in which the heart muscles twitch and cannot pump blood.
- A defibrillator is a large capacitor that can store up to 360 J of energy and release it in 2 milliseconds. The large shock can sometimes stop fibrillation.


## The Energy in the Electric Field

- The energy stored in the capacitor is

$$
U_{\mathrm{C}}=\frac{1}{2} C\left(\Delta V_{\mathrm{C}}\right)^{2}=\frac{1}{2} \frac{\kappa \epsilon_{0} A}{d}(E d)^{2}=\frac{1}{2} \kappa \epsilon_{0}(A d) E^{2}
$$

- The energy is stored in the capacitor's electric field.


## The Energy in the Electric Field

Capacitor plate with area $A$


The capacitor's energy is stored in the electric field in volume $A d$ between the plates.

## The Energy in the Electric Field

- Because the quantity $A d$, the volume inside the capacitor, is the volume in which the energy is stored, we can define the energy density $u_{\mathrm{E}}$ of the electric field:

$$
u_{\mathrm{E}}=\frac{\text { energy stored }}{\text { volume in which it is stored }}=\frac{U_{\mathrm{C}}}{A d}=\frac{1}{2} \kappa \epsilon_{0} E^{2}
$$

- The energy density has units $\mathrm{J} / \mathrm{m}^{3}$.


## QuickCheck 21.24

A capacitor charged to 1.5 V stores 2.0 mJ of energy. If the capacitor is charged to 3.0 V , it will store
A. 1.0 mJ
B. 2.0 mJ
C. 4.0 mJ
D. 6.0 mJ
E. 8.0 mJ

## QuickCheck 21.24

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A. 1.0 mJ
B. 2.0 mJ
C. 4.0 mJ
D. 6.0 mJ
E. 8.0 mJ
$U_{C} \propto(\Delta V)^{2}$

## Example 21.14 Finding the energy density for a defibrillator

A defibrillator unit contains a $150 \mu \mathrm{~F}$ capacitor that is charged to 2000 V . The capacitor plates are separated by a $0.010-\mathrm{mm}$-thick dielectric with $\kappa=300$.
a. What is the total area of the capacitor plates?
b. What is the energy density stored in the electric field when the capacitor is charged?

PREPARE Assume the capacitor can be modeled as a parallel-plate capacitor with a dielectric.

## Example 21.14 Finding the energy density for a defibrillator (cont.)

SOLVE a. The surface area of the electrodes is

$$
A=\frac{d C}{\kappa \epsilon_{0}}=\frac{\left(1.0 \times 10^{-5} \mathrm{~m}\right)\left(150 \times 10^{-6} \mathrm{~F}\right)}{(300)\left(8.85 \times 10^{-12} \mathrm{~F} / \mathrm{m}\right)}=0.56 \mathrm{~m}^{2}
$$

b. The electric field strength is

$$
E=\frac{\Delta V_{\mathrm{C}}}{d}=\frac{2000 \mathrm{~V}}{1.0 \times 10^{-5} \mathrm{~m}}=2.0 \times 10^{8} \mathrm{~V} / \mathrm{m}
$$

## Example 21.14 Finding the energy density for a defibrillator (cont.)

Consequently, the energy density in the electric field is

$$
\begin{aligned}
u_{\mathrm{E}} & =\frac{1}{2} \kappa \epsilon_{0} E^{2} \\
& =\frac{1}{2}(300)\left(8.85 \times 10^{-12} \mathrm{~F} / \mathrm{m}\right)\left(2.0 \times 10^{8} \mathrm{~V} / \mathrm{m}\right)^{2} \\
& =5.3 \times 10^{7} \mathrm{~J} / \mathrm{m}^{3}
\end{aligned}
$$

## Example 21.14 Finding the energy density for a defibrillator (cont.)

ASSESS For comparison, the energy density of gasoline is about $3 \times 10^{9} \mathrm{~J} / \mathrm{m}^{3}$, about 60 times higher than this capacitor. Capacitors store less energy than some other devices, but they can deliver this energy very rapidly.

## Summary: General Principles

## Electric Potential and Potential Energy

The electric potential $V$ is created by charges and exists at every point surrounding those charges.

When a charge $q$ is brought near these charges, it acquires an electric potential energy


$$
U_{\mathrm{elec}}=q \mathrm{~V}
$$

at a point where the other charges have created an electric potential $V$.
Energy is conserved for a charged particle in an electric potential:

$$
K_{\mathrm{f}}+q V_{\mathrm{f}}=K_{\mathrm{i}}+q V_{\mathrm{i}}
$$

or

$$
\Delta K=-q \Delta V
$$

## Summary: General Principles

## Sources of Potential

Potential differences $\Delta V$ are created by a separation of charge. Two important sources of potential difference are

- A battery, which uses chemical means to separate charge and produce a potential difference.
- The opposite charges on the plates of a capacitor, which create a potential difference between the plates.
The electric potential of a point charge $q$ is $V=K \frac{q}{r}$
Connecting potential and field


Text: p. 693

## Summary: Important Concepts

## For a conductor in electrostatic equilibrium

- Any excess charge is on the surface.
- The electric field inside is zero.
- The exterior electric field is perpendicular to the surface.
- The field strength is largest at sharp corners.

- The entire conductor is at the same potential and so the surface is an equipotential.

Text: p. 693

## Summary: Important Concepts

## Graphical representations of the potential



Potential graph


Equipotential surfaces


Equipotential map

Text: p. 693

## Summary: Applications

## Capacitors and dielectrics

The charge $\pm Q$ on two conductors and the potential difference $\Delta V_{\mathrm{C}}$ between them are proportional:

$$
Q=C \Delta V_{\mathrm{C}}
$$


where $C$ is the capacitance of the two conductors.
A parallel-plate capacitor with plates of area $A$ and separation $d$ has a capacitance

$$
C=\epsilon_{0} A / d
$$

When a dielectric is inserted between the plates of a capacitor, its capacitance is increased by a factor $\kappa$, the dielectric constant of the material.
The energy stored in a capacitor is $U_{\mathrm{C}}=\frac{1}{2} C\left(\Delta V_{\mathrm{C}}\right)^{2}$.
This energy is stored in the electric field, which has energy density

$$
u_{\mathrm{E}}=\frac{1}{2} \kappa \epsilon_{0} E^{2}
$$

## Summary: Applications

## Parallel-plate capacitor

For a capacitor charged to $\Delta V_{\mathrm{C}}$ the potential at distance $x$ from the negative plate is

$$
V=\frac{x}{d} \Delta V_{\mathrm{C}}
$$

The electric field inside is

$$
E=\Delta V_{\mathrm{C}} / d
$$



## Units

- Electric potential: $1 \mathrm{~V}=1 \mathrm{~J} / \mathrm{C}$
- Electric field: $1 \mathrm{~V} / \mathrm{m}=1 \mathrm{~N} / \mathrm{C}$
- Energy: 1 electron volt $=1 \mathrm{eV}=1.60 \times 10^{-19} \mathrm{~J}$ is the kinetic energy gained by an electron upon accelerating through a potential difference of 1 V .


## Summary

## GENERAL PRINCIPLES

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Text: p. 693

## Summary

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