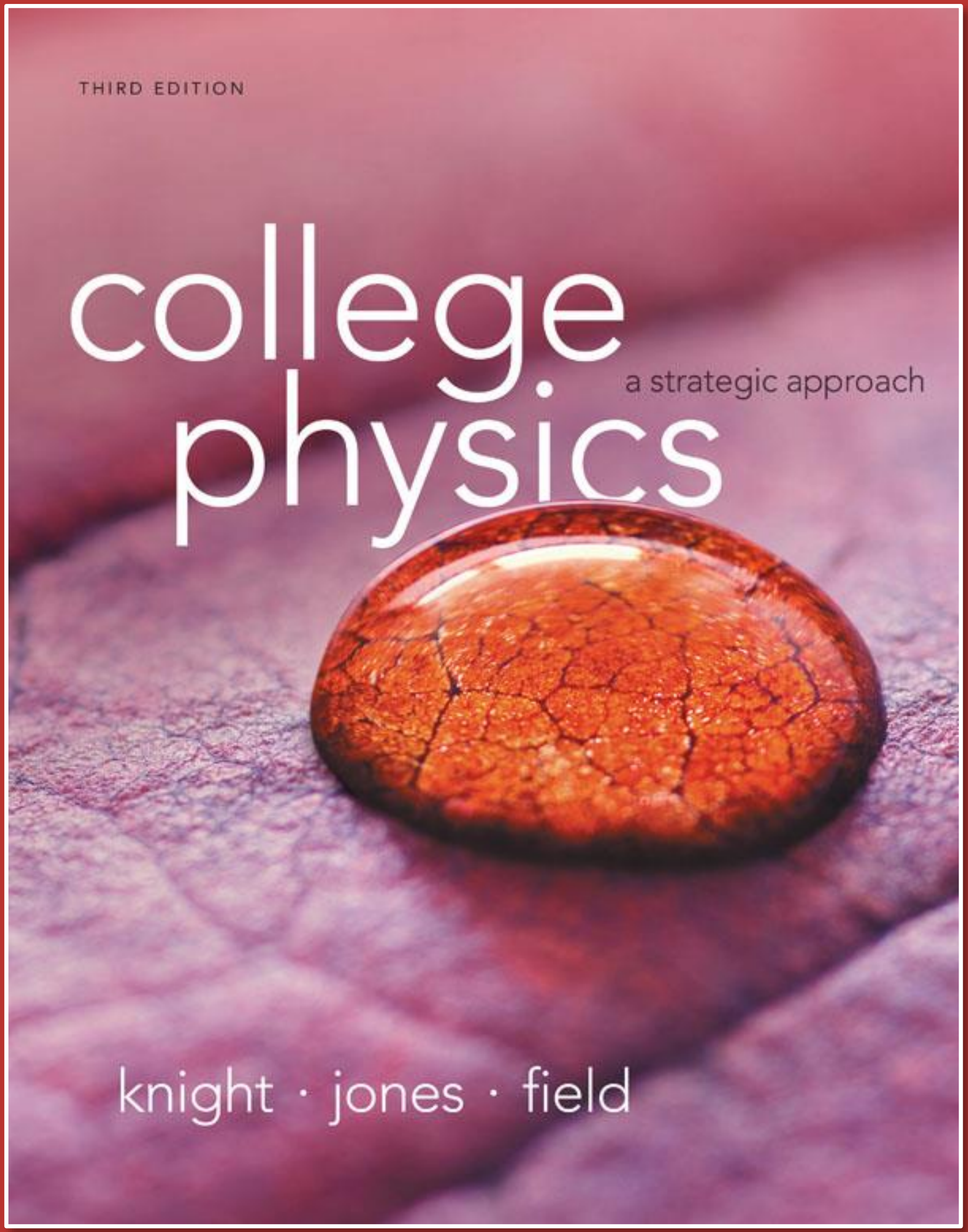


Lecture Presentation

Chapter 14

Oscillations

Suggested Videos for Chapter 14

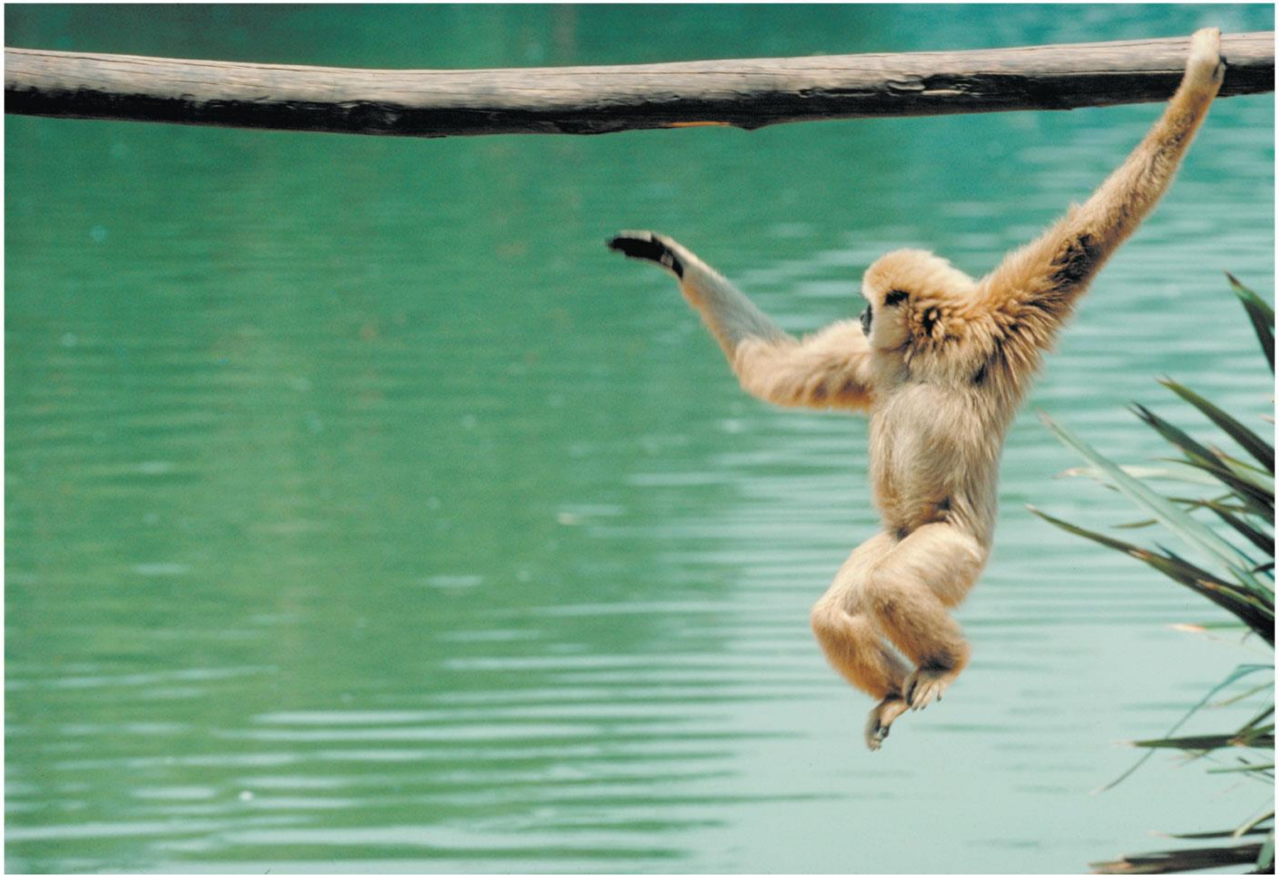
- **Prelecture Videos**
  - *Describing Simple Harmonic Motion*
  - *Details of SHM*
  - *Damping and Resonance*
- **Class Videos**
  - *Oscillations*
  - *Basic Oscillation Problems*
- **Video Tutor Solutions**
  - *Oscillations*

Suggested Simulations for Chapter 14

- **ActivPhysics**
  - *9.1–9.12*

- **PhETs**
  - *Masses & Springs*
  - *Motion in 2D*
  - *Pendulum Lab*

Chapter 14 Oscillations



Chapter Goal: To understand systems that oscillate with simple harmonic motion.

Chapter 14 Preview
Looking Ahead: Motion that Repeats

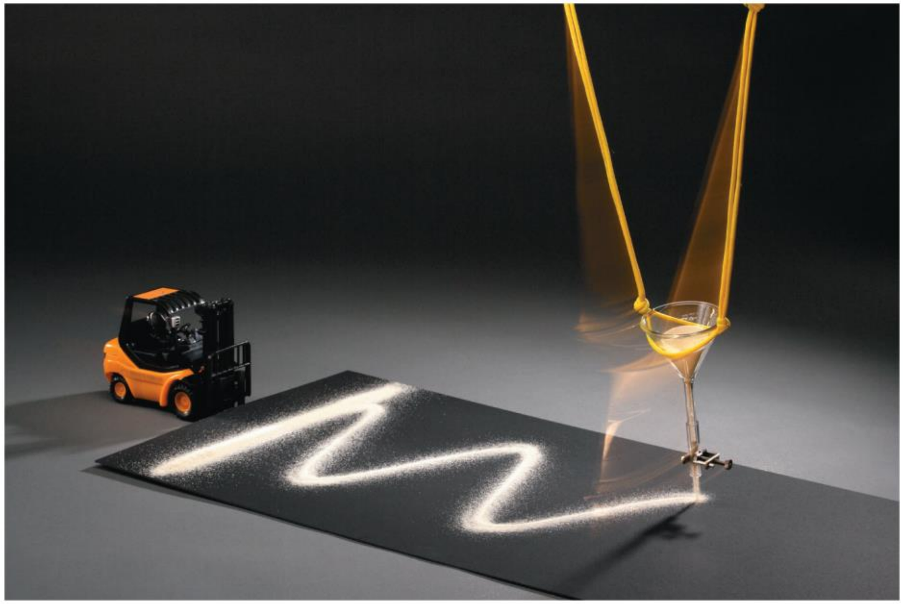
- When the woman moves down, the springy ropes pull up. This restoring force produces an **oscillation**: one bounce after another.

- You'll see many examples of systems with restoring forces that lead to oscillatory motion.

Chapter 14 Preview

Looking Ahead: Simple Harmonic Motion

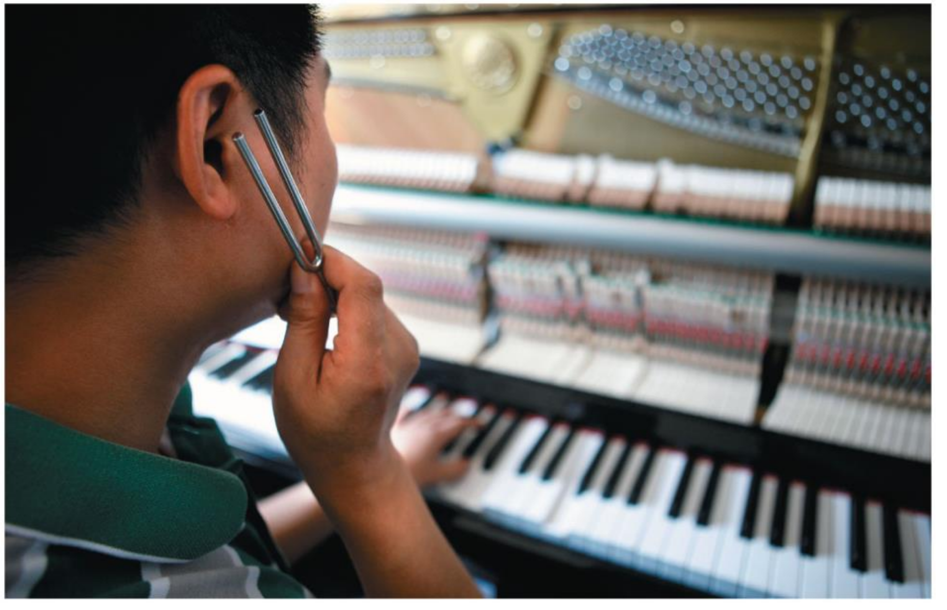
- The sand records the motion of the oscillating pendulum. The **sinusoidal** shape tells us that this is **simple harmonic motion**.



- All oscillations show a similar form. You'll learn to describe and analyze oscillating systems.

Chapter 14 Preview
Looking Ahead: Resonance

- When you make a system oscillate at its **natural frequency,** you can get a large amplitude. We call this **resonance**.



- You'll learn how resonance of a membrane in the inner ear lets you determine the **pitch** of a musical note.

Chapter 14 Preview
Looking Ahead

Motion that Repeats

When the woman moves down, the springy ropes pull up. This restoring force produces an **oscillation**: one bounce after another.

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Simple Harmonic Motion

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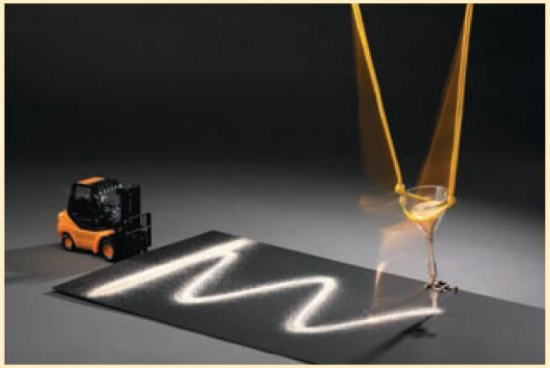


All oscillations show a similar form. You'll learn to describe and analyze oscillating systems.

Resonance

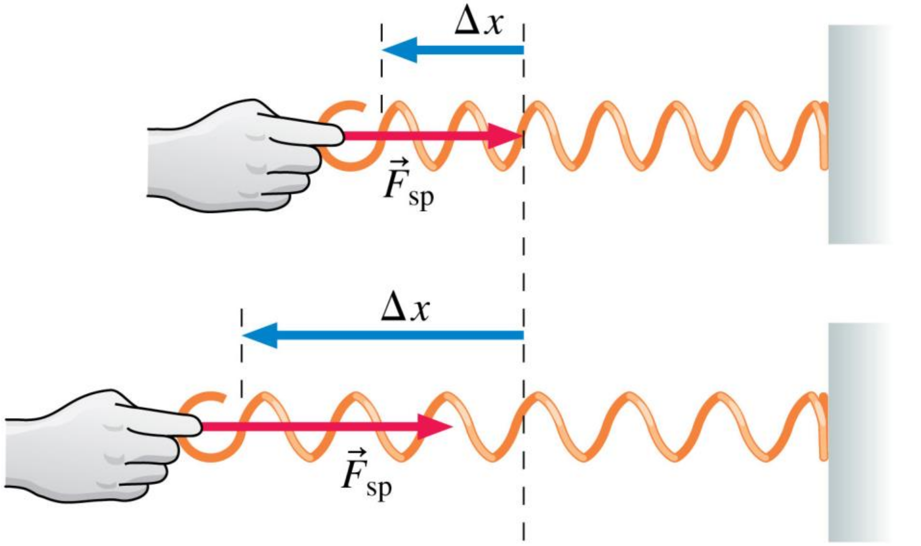
When you make a system oscillate at its **natural frequency,** you can get a large amplitude. We call this **resonance.**

You'll learn how resonance of a membrane in the inner ear lets you determine the **pitch** of a musical note.

Text: p. 438

Chapter 14 Preview

Looking Back: Springs and Restoring Forces



- In Chapter 8, you learned that a stretched spring exerts a restoring force proportional to the stretch:

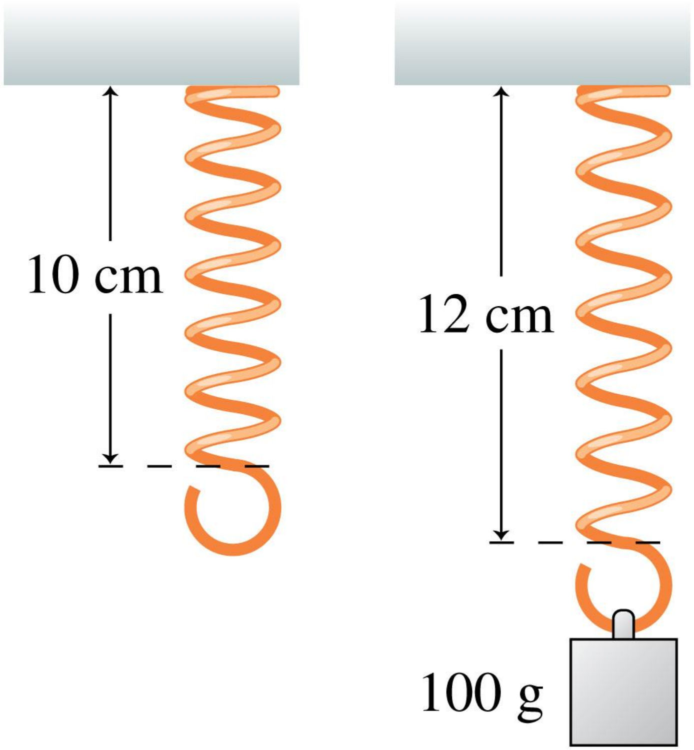
$$F_{sp} = -k\Delta x$$

- In this chapter, you'll see how this linear restoring force leads to an oscillation, with a frequency determined by the spring constant *k*.

Chapter 14 Preview
Stop to Think

A hanging spring has length 10 cm. A 100 g mass is hung from the spring, stretching it to 12 cm. What will be the length of the spring if this mass is replaced by a 200 g mass?

A. 14 cm
B. 16 cm
C. 20 cm
D. 24 cm



Reading Question 14.1

The type of function that describes simple harmonic motion is

A. Linear.
B. Exponential.
C. Quadratic.
D. Sinusoidal.
E. Inverse.

Reading Question 14.1

The type of function that describes simple harmonic motion is

A. Linear.
B. Exponential.
C. Quadratic.
✔ D. Sinusoidal.
E. Inverse.

Reading Question 14.2

When you displace an object from its equilibrium position and the force pushing it back toward equilibrium is __________, the resulting motion is simple harmonic motion.

A. Sinusoidal
B. Exponential
C. Quadratic
D. Linear

Reading Question 14.2

When you displace an object from its equilibrium position and the force pushing it back toward equilibrium is __________, the resulting motion is simple harmonic motion.

A. Sinusoidal
B. Exponential
C. Quadratic
✔ D. Linear

Reading Question 14.3

A mass is bobbing up and down on a spring. If you increase the amplitude of the motion, how does this affect the time for one oscillation?

A. The time increases.
B. The time decreases.
C. The time does not change.

Reading Question 14.3

A mass is bobbing up and down on a spring. If you increase the amplitude of the motion, how does this affect the time for one oscillation?

A. The time increases.
B. The time decreases.
✔ C. The time does not change.

Reading Question 14.4

A mass tied to the end of a 1.0-m-long string is swinging back and forth. During each swing, it moves 4 cm from its lowest point to the right, then 4 cm to the left. One complete swing takes about 2 s. If the amplitude of motion is doubled, so the mass swings 8 cm to one side and then the other, the period of the motion will be

A. 2 s
B. 4 s
C. 6 s
D. 8 s

Reading Question 14.4

A mass tied to the end of a 1.0-m-long string is swinging back and forth. During each swing, it moves 4 cm from its lowest point to the right, then 4 cm to the left. One complete swing takes about 2 s. If the amplitude of motion is doubled, so the mass swings 8 cm to one side and then the other, the period of the motion will be

✔ A. 2 s
B. 4 s
C. 6 s
D. 8 s

Reading Question 14.5

If you drive an oscillator, it will have the largest amplitude if you drive it at its ________ frequency.

A. Special
B. Positive
C. Resonant
D. Damped
E. Pendulum

Reading Question 14.5

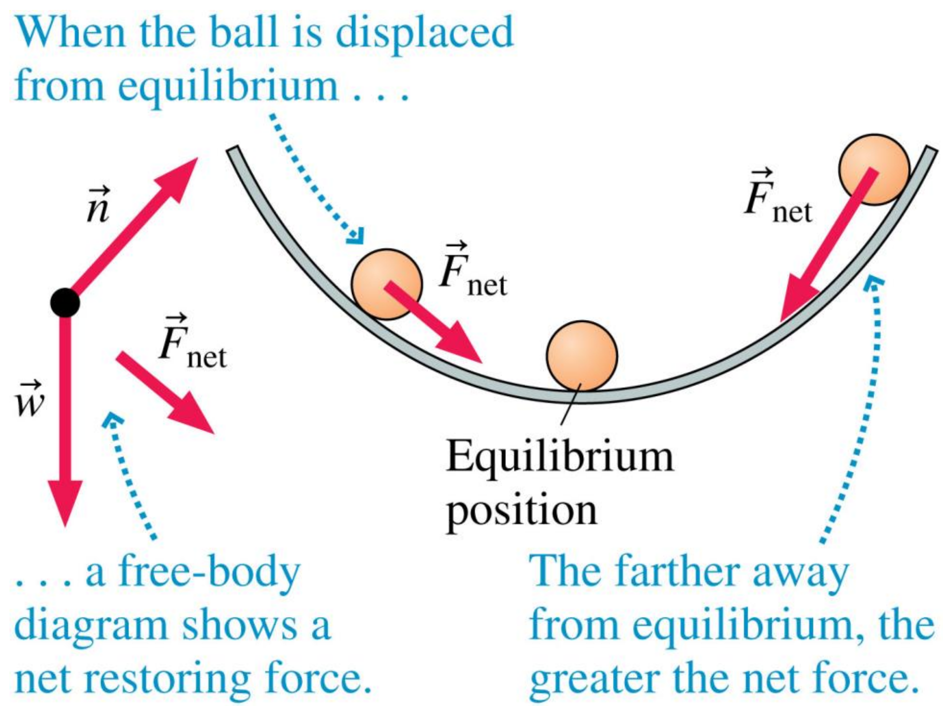
If you drive an oscillator, it will have the largest amplitude if you drive it at its ________ frequency.

A. Special
B. Positive
C. Resonant
D. Damped
E. Pendulum

Section 14.1 Equilibrium and Oscillation

Equilibrium and Oscillation

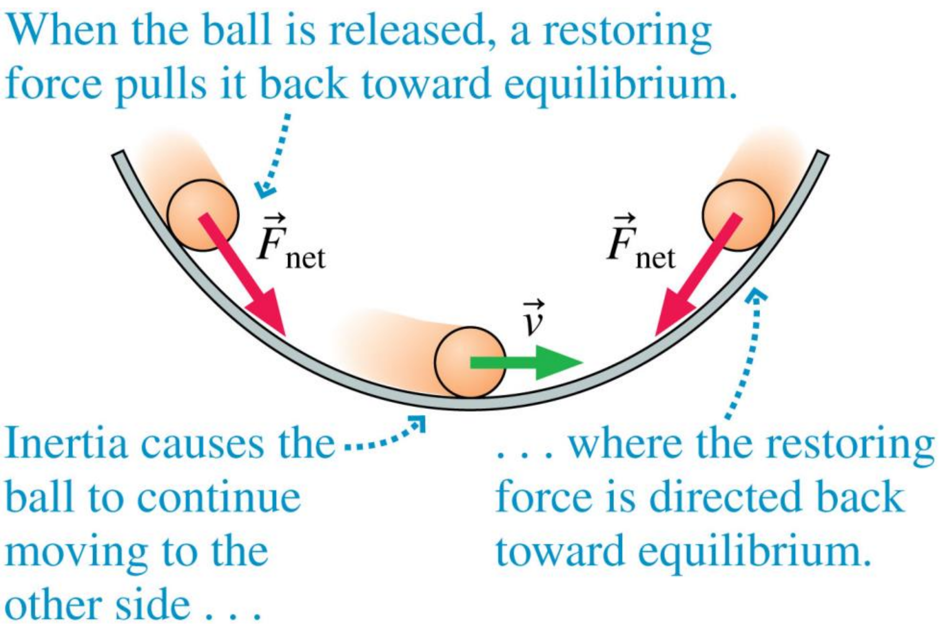
- A marble that is free to roll inside a spherical bowl has an **equilibrium position** at the bottom of the bowl where it will rest with no net force on it.



- If pushed away from equilibrium, the marble's weight leads to a net force toward the equilibrium position. This force is the **restoring force.**

Equilibrium and Oscillation

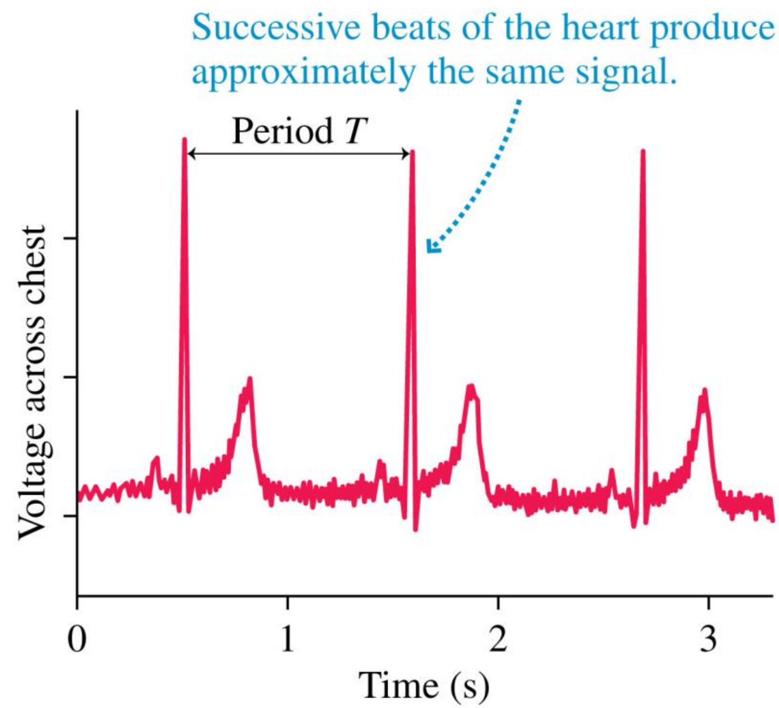
- When the marble is released from the side, it does not stop at the bottom of the bowl; it rolls up and down each side of the bowl, moving through the equilibrium position.



- This repetitive motion is called **oscillation**.
- Any oscillation is characterized by a *period* and *frequency*.

Frequency and Period

- For an oscillation, the time to complete one full cycle is called the **period** (T) of the oscillation.
- The number of cycles per second is called the **frequency** (f) of the oscillation.



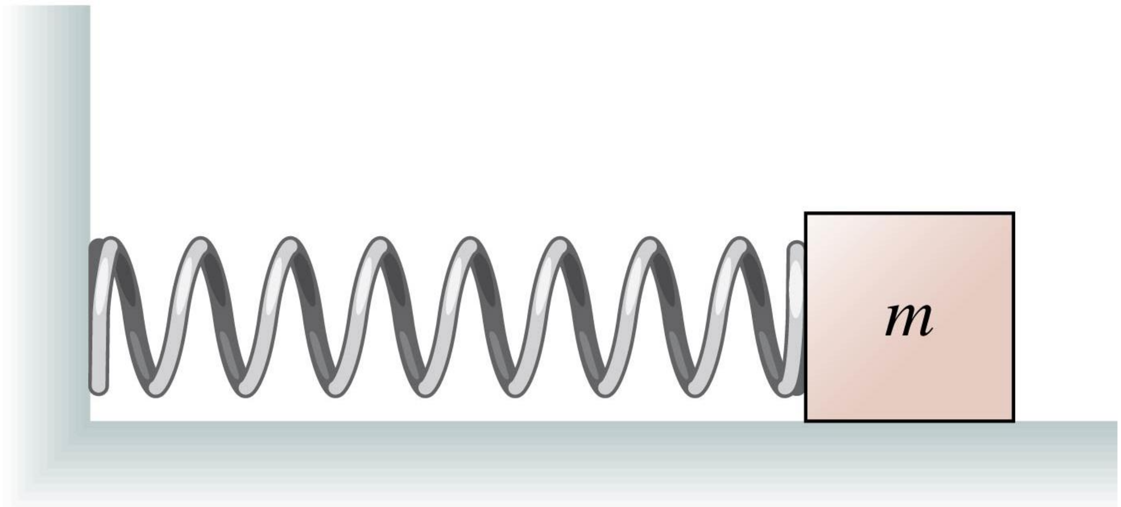
$$f = \frac{1}{T} \quad \text{or} \quad T = \frac{1}{f}$$

- The units of frequency are **hertz** (Hz), or 1 s^{-1}.

QuickCheck 14.1

A mass oscillates on a horizontal spring with period $T = 2.0$ s. What is the frequency?

A. 0.50 Hz
B. 1.0 Hz
C. 2.0 Hz
D. 3.0 Hz
E. 4.0 Hz



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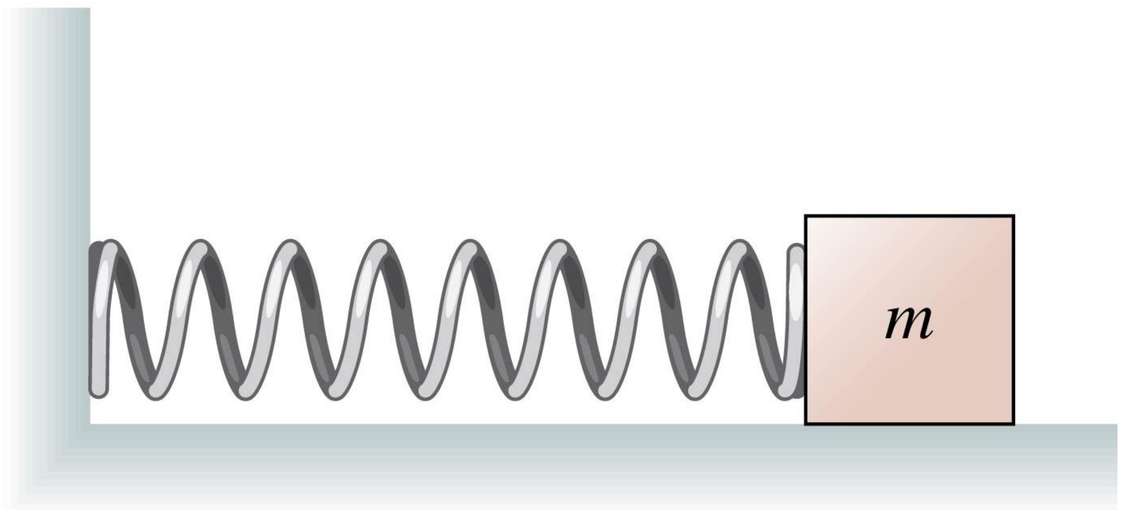
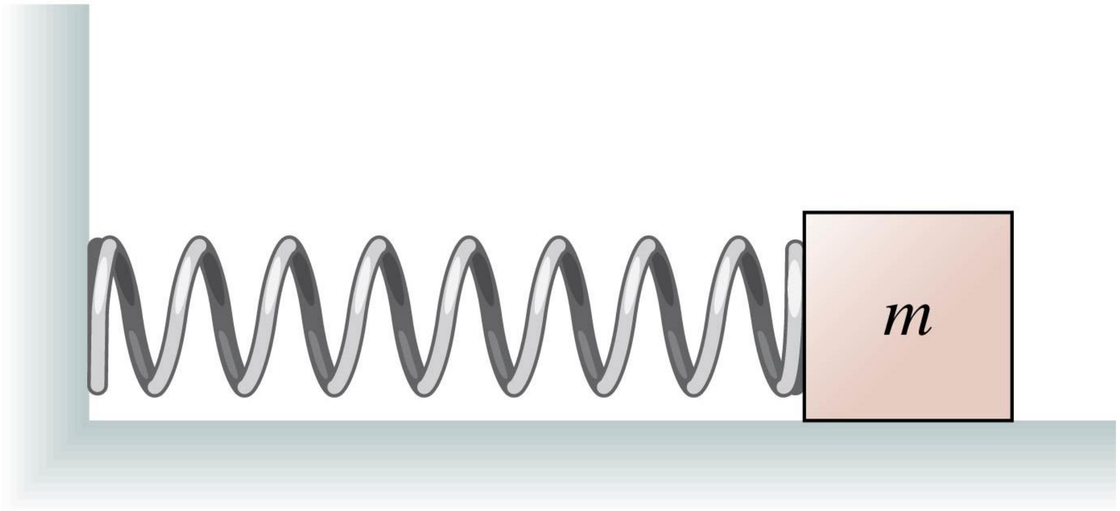


QuickCheck 14.2

A mass oscillates on a horizontal spring with period $T = 2.0$ s. If the mass is pulled to the right and then released, how long will it take for the mass to reach the leftmost point of its motion?

A. 1.0 s
B. 1.4 s
C. 2.0 s
D. 2.8 s
E. 4.0 s



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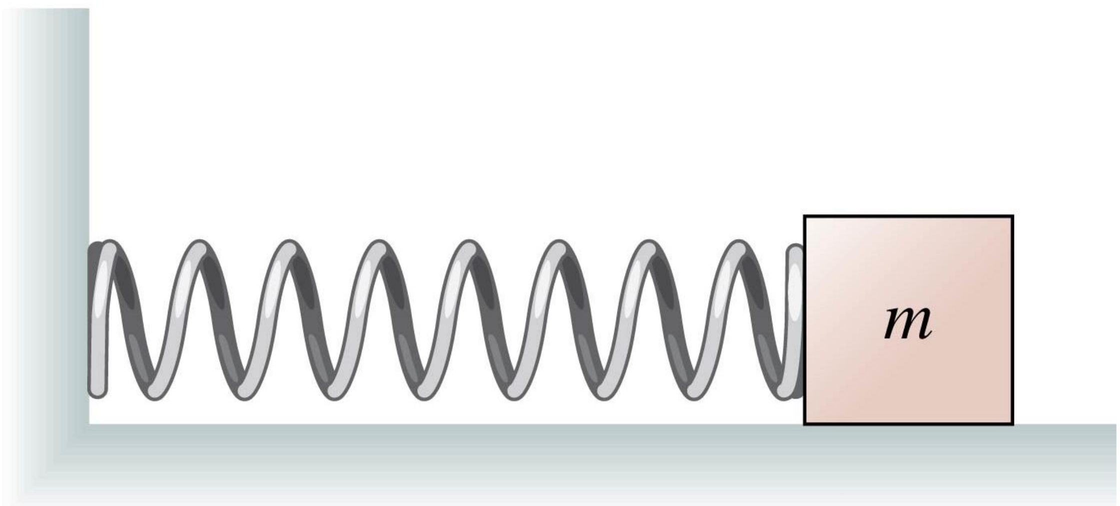


QuickCheck 14.3

A typical earthquake produces vertical oscillations of the earth. Suppose a particular quake oscillates the ground at a frequency of 0.15 Hz. As the earth moves up and down, what time elapses between the highest point of the motion and the lowest point?

A. 1 s
B. 3.3 s
C. 6.7 s
D. 13 s

QuickCheck 14.3

A typical earthquake produces vertical oscillations of the earth. Suppose a particular quake oscillates the ground at a frequency of 0.15 Hz. As the earth moves up and down, what time elapses between the highest point of the motion and the lowest point?

A. 1 s
B. 3.3 s
C. 6.7 s
D. 13 s

Frequency and Period

TABLE 14.1 Common units of frequency

Frequency	Period
10^3 Hz = 1 kilohertz = 1 kHz	1 ms
10^6 Hz = 1 megahertz = 1 MHz	1 μs
10^9 Hz = 1 gigahertz = 1 GHz	1 ns

Example 14.1 Frequency and period of a radio station

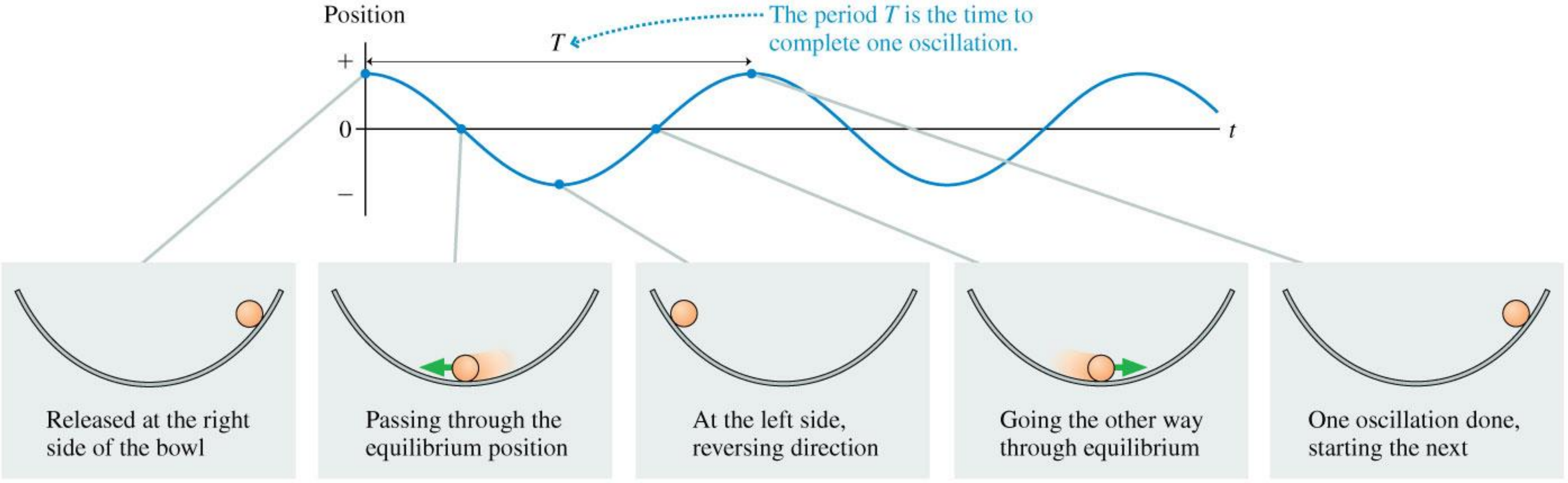
An FM radio station broadcasts an oscillating radio wave at a frequency of 100 MHz. What is the period of the oscillation?

SOLVE The frequency *f* of oscillations in the radio transmitter is 100 MHz = 1.0 × 10 8 Hz. The period is the inverse of the frequency; hence,

$$T = \frac{1}{f} = \frac{1}{1.0 \times 10^8 \text{ Hz}} = 1.0 \times 10^{-8} \text{ s} = 10 \text{ ns}$$

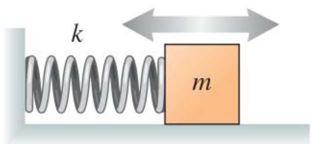
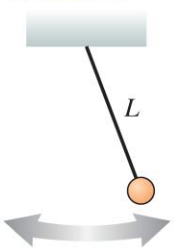
Oscillatory Motion

- The graph of an oscillatory motion has the form of a *cosine function*.
- A graph or a function that has the form of a sine or cosine function is called **sinusoidal**.
- A sinusoidal oscillation is called **simple harmonic motion** (SHM).



Oscillatory Motion

Examples of simple harmonic motion

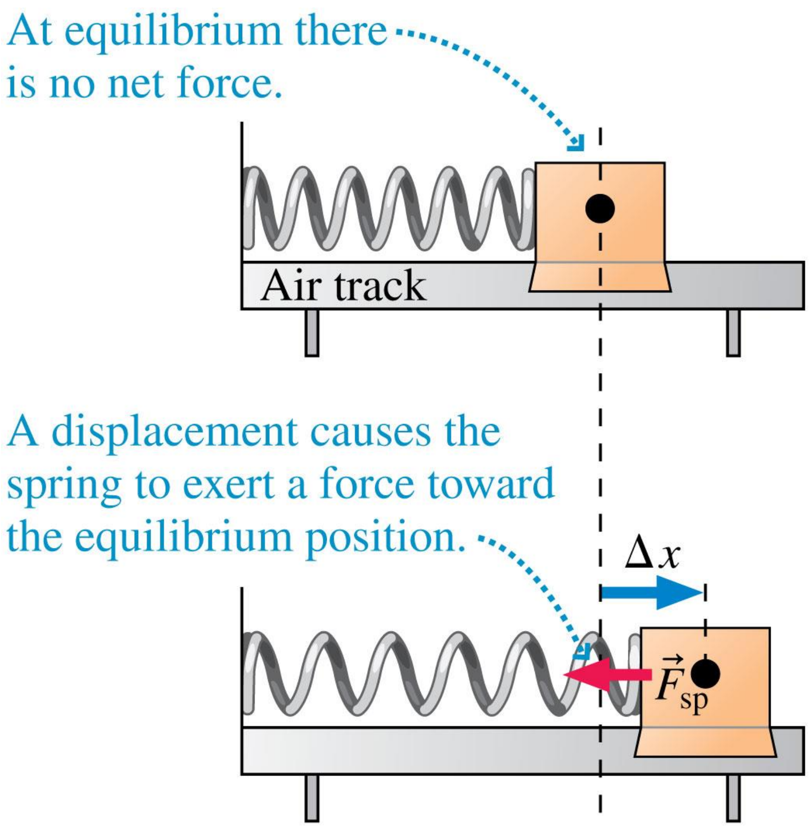
Oscillating system	Related real-world example BIO
Mass on a spring  The mass oscillates back and forth due to the restoring force of the spring. The period depends on the mass and the stiffness of the spring.	Vibrations in the ear  Sound waves entering the ear cause the oscillation of a membrane in the cochlea. The vibration can be modeled as a mass on a spring. The period of oscillation of a segment of the membrane depends on mass (the thickness of the membrane) and stiffness (the rigidity of the membrane).
Pendulum  The mass oscillates back and forth due to the restoring gravitational force. The period depends on the length of the pendulum and the free-fall acceleration g.	Motion of legs while walking  The motion of a walking animal's legs can be modeled as pendulum motion. The rate at which the legs swing depends on the length of the legs and the free-fall acceleration g.

Text: p. 440

Section 14.2 Linear Restoring Forces and SHM

Linear Restoring Forces and SHM

- If we displace a glider attached to a spring from its equilibrium position, the spring exerts a restoring force back toward equilibrium.

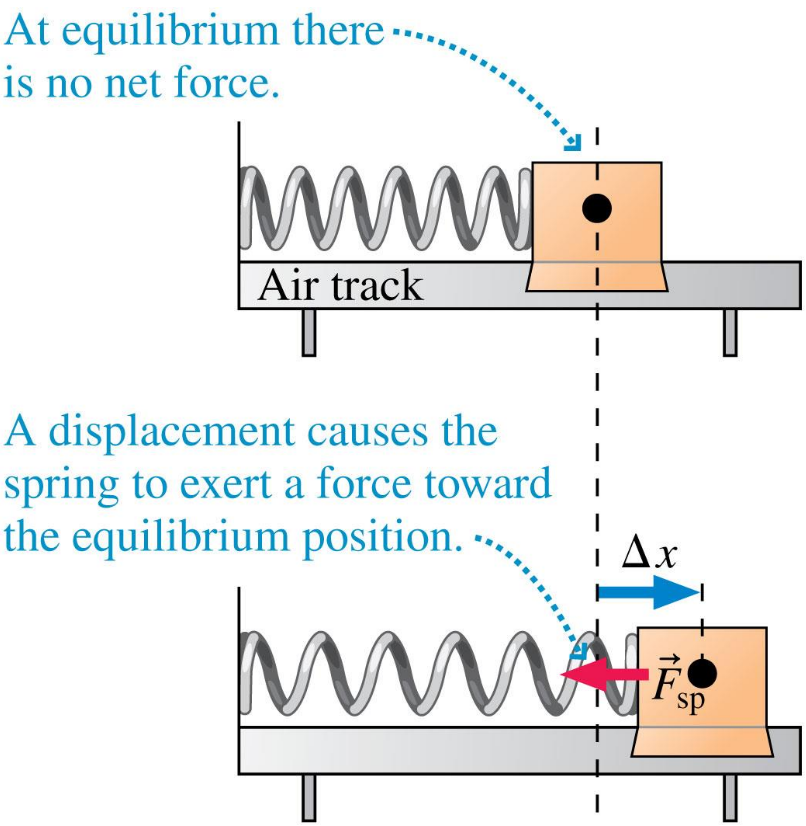


$$(F_{\text{net}})_x = -kx$$

The negative sign tells us that this is a restoring force because the force is in the direction opposite the displacement. If we pull the glider to the right (x is positive), the force is to the left (negative)—back toward equilibrium.

Linear Restoring Forces and SHM

- This is a **linear restoring force; the net force is toward the equilibrium position and is proportional to the distance from equilibrium.**

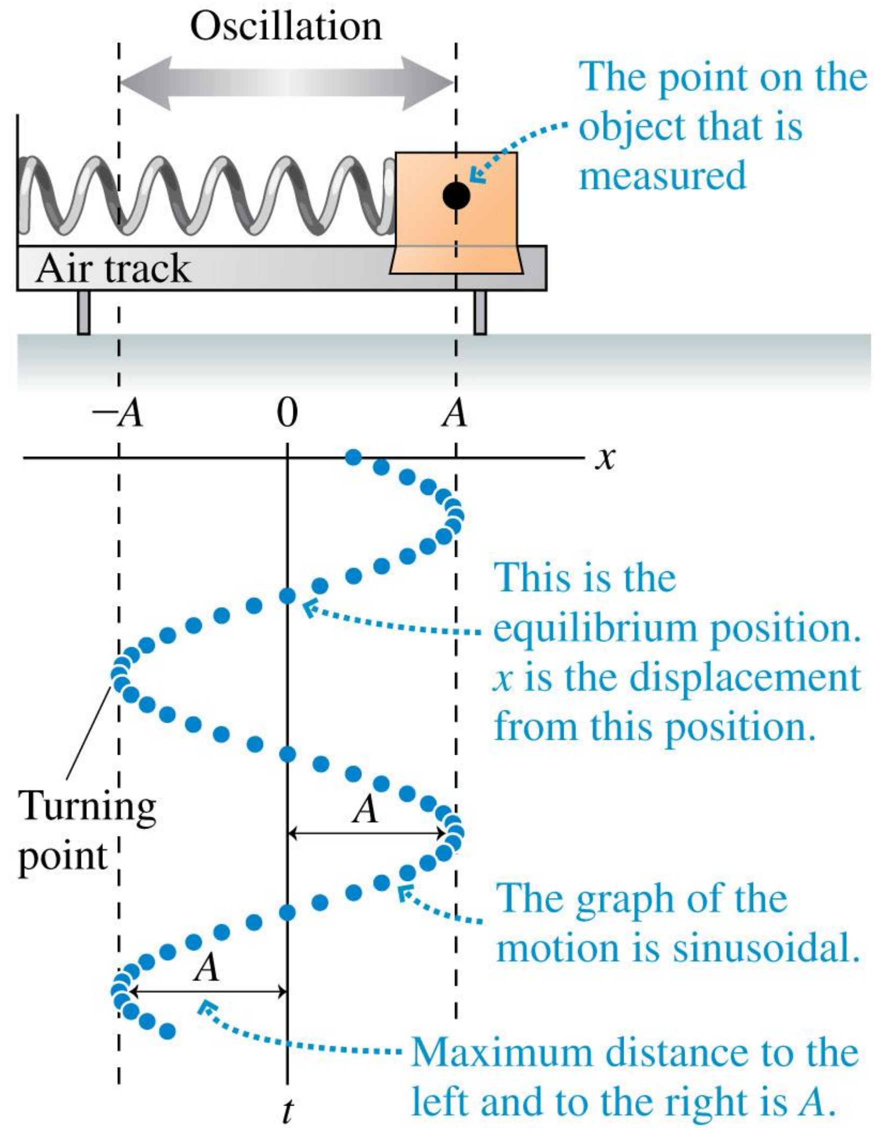


$$(F_{net})_x = -kx$$

The negative sign tells us that this is a restoring force because the force is in the direction opposite the displacement. If we pull the glider to the right (x is positive), the force is to the left (negative)—back toward equilibrium.

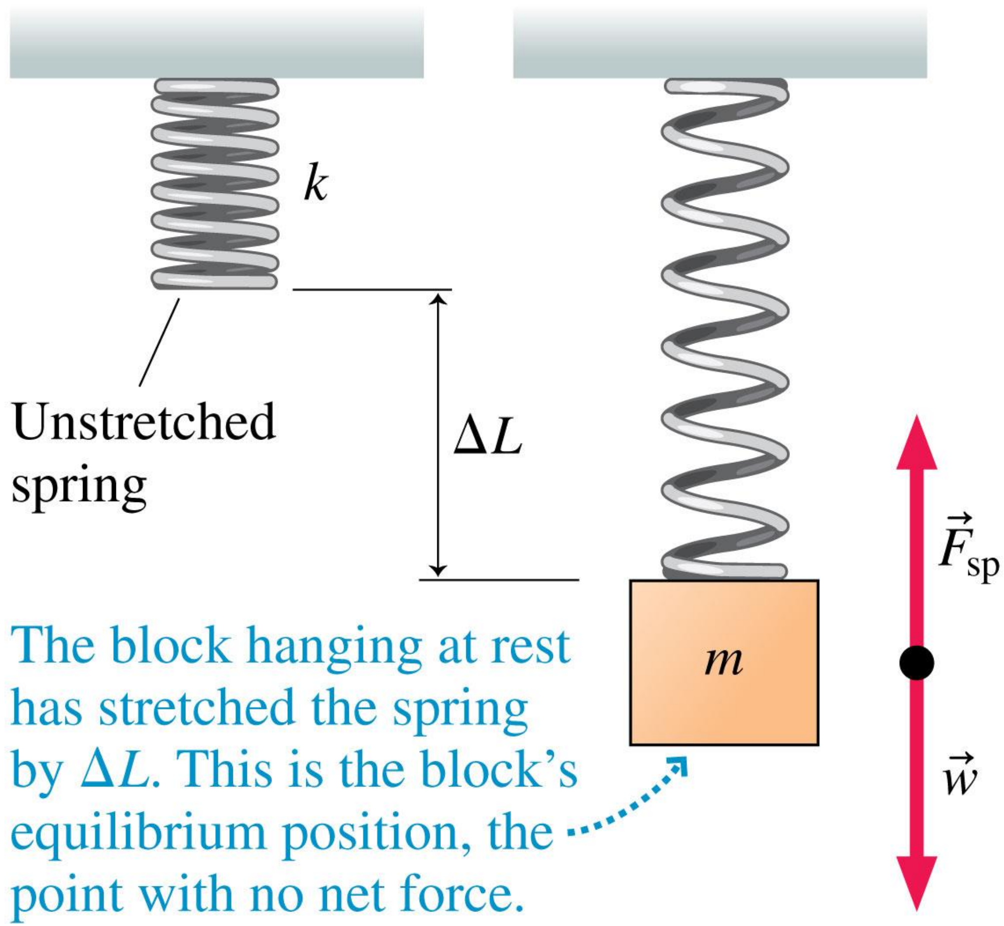
Motion of a Mass on a Spring

- The **amplitude** *A* is the object's maximum displacement from equilibrium.
- **Oscillation about an equilibrium position with a linear restoring force is always simple harmonic motion.**



Vertical Mass on a Spring

- For a hanging weight, the equilibrium position of the block is where it hangs motionless. The spring is stretched by ΔL.



Vertical Mass on a Spring

- The value of ΔL is determined by solving the static-equilibrium problem.
- Hooke's Law says

$$(F_{\text{sp}})_y = k\,\Delta L$$

- Newton's first law for the block in equilibrium is

$$(F_{\text{net}})_y = (F_{\text{sp}})_y + w_y = k\,\Delta L - mg = 0$$

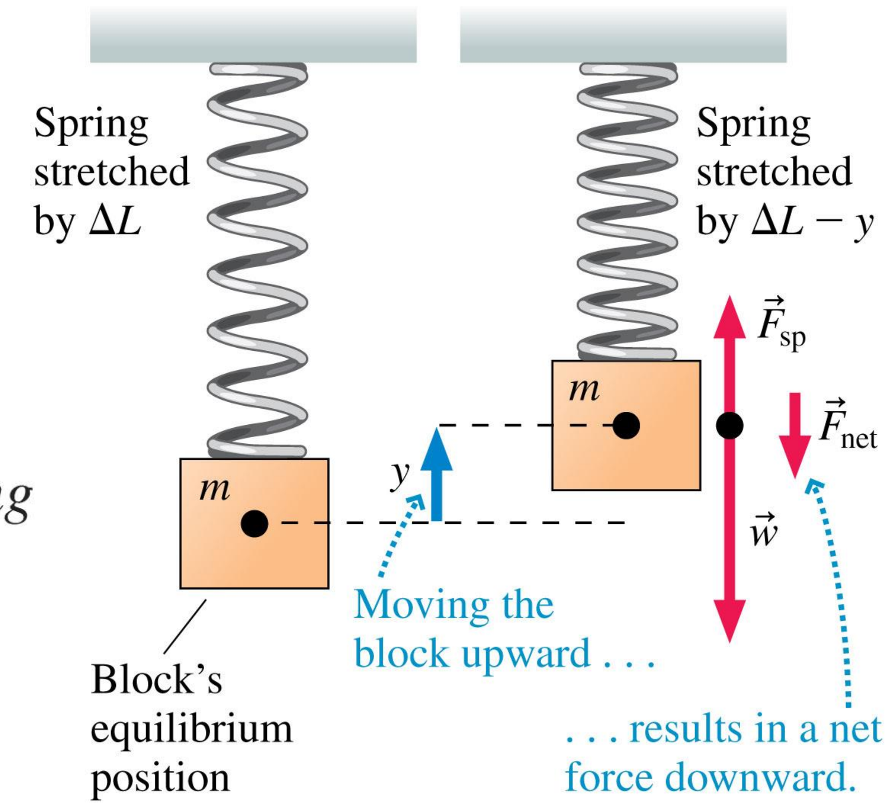
- Therefore the length of the spring at the equilibrium position is

$$\Delta L = \frac{mg}{k}$$

Vertical Mass on a Spring

- When the block is above the equilibrium position, the spring is still *stretched* by an amount $\Delta L - y$.
- The net force on the block is

$$(F_{\text{net}})_y = (F_{\text{sp}})_y + w_y = k(\Delta L - y) - mg$$
$$= (k\,\Delta L - mg) - ky$$

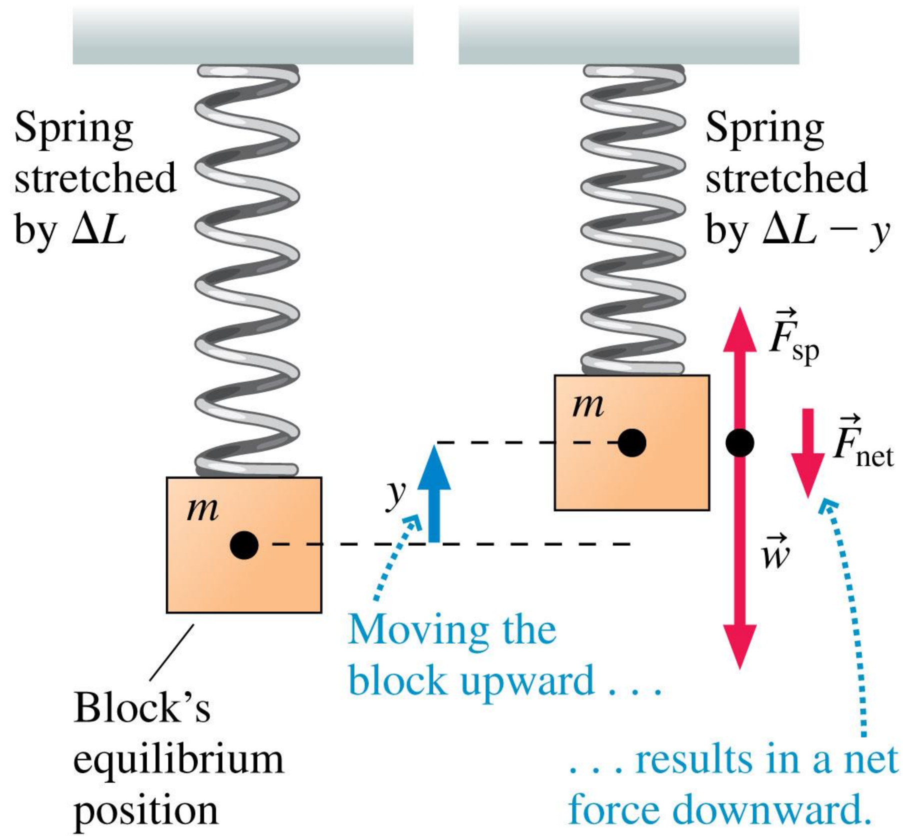


- But $k\,\Delta L - mg = 0$, from Equation 14.4, so the net force on the block is

$$(F_{\text{net}})_y = -ky$$

Vertical Mass on a Spring

- The role of gravity is to determine where the equilibrium position is, but it doesn't affect the restoring force for displacement from the equilibrium position.
- Because it has a linear restoring force, **a mass on a vertical spring oscillates with simple harmonic motion.**



Example Problem

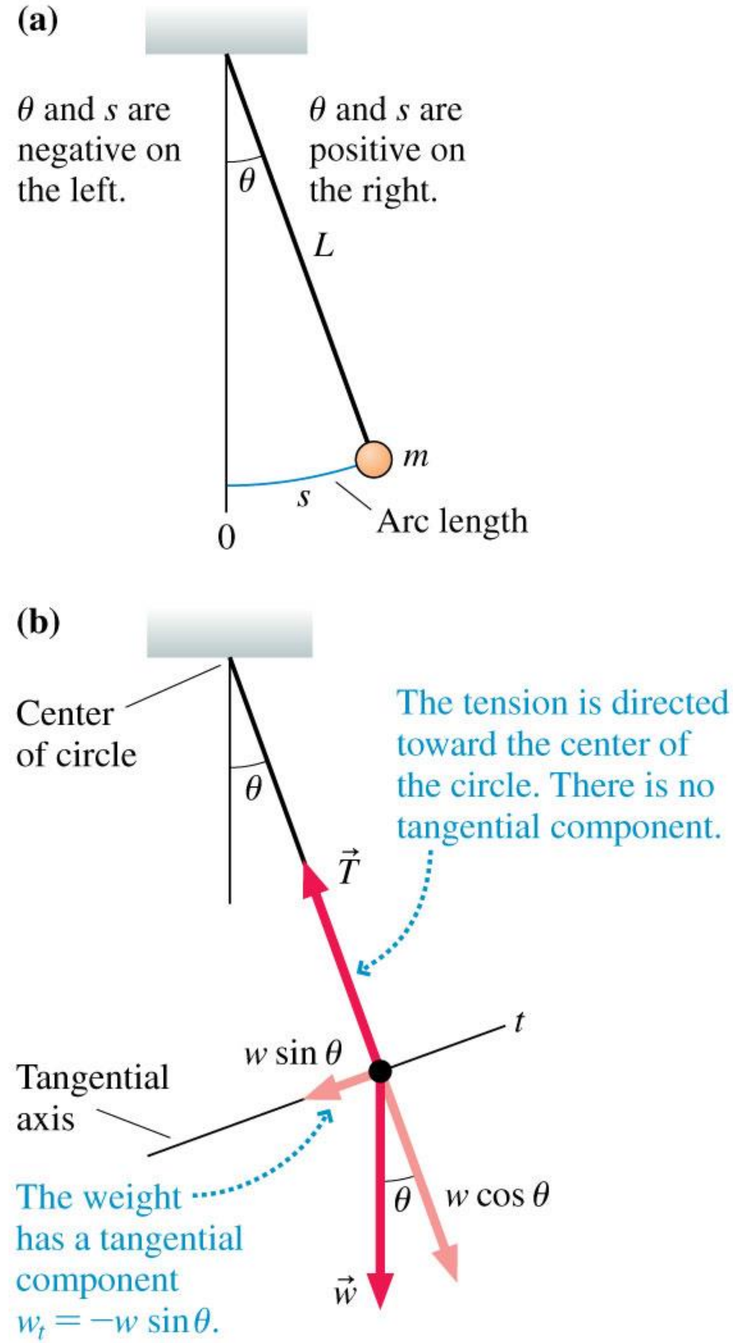
A car rides on four wheels that are connected to the body of the car by springs that allow the car to move up and down as the wheels go over bumps and dips in the road. Each spring supports approximately 1/4 the mass of the vehicle. A lightweight car has a mass of 2400 lbs. When a 160 lb person sits on the left front fender, this corner of the car dips by about ½″.

A. What is the spring constant of this spring?
B. When four people of this mass are in the car, what is the oscillation frequency of the vehicle on the springs?

The Pendulum

- A **pendulum** is a mass suspended from a pivot point by a light string or rod.
- The mass moves along a circular arc. The net force is the tangential component of the weight:

$$(F_{\text{net}})_t = \sum F_t = w_t = -mg\sin\theta$$



The Pendulum

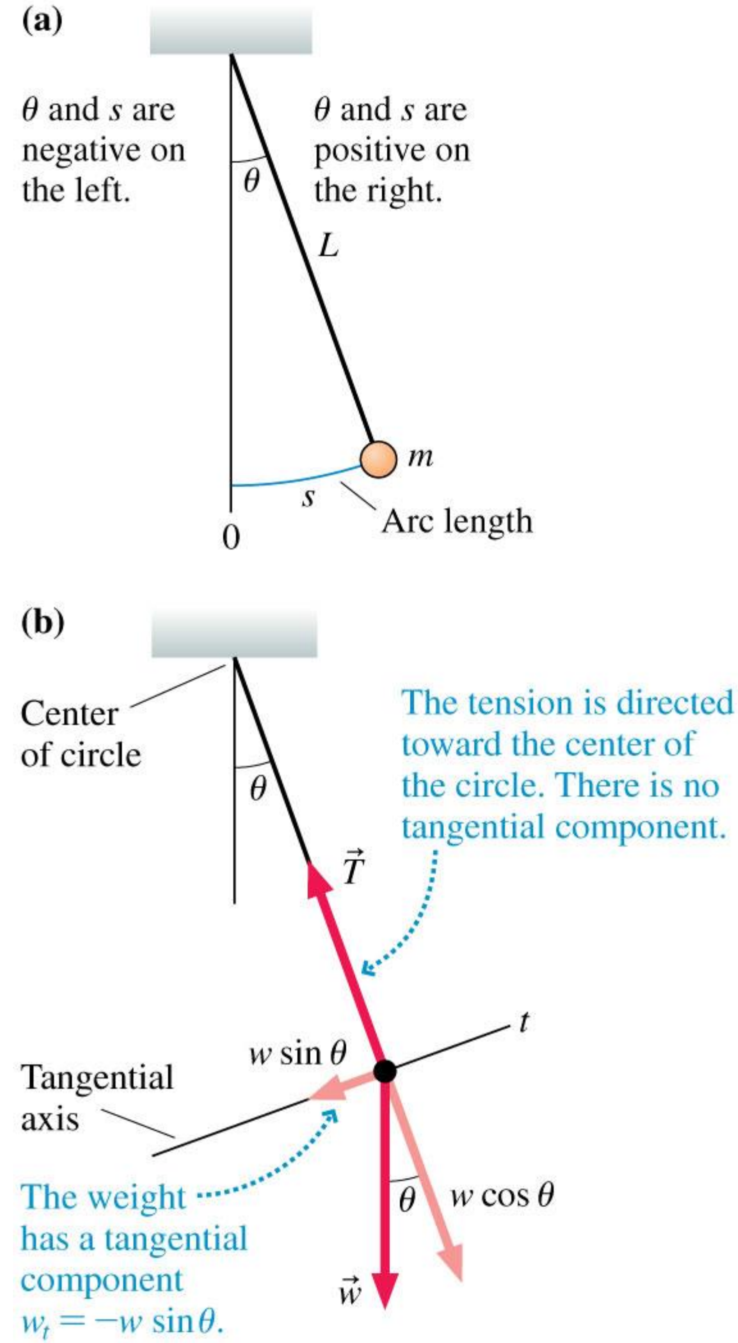
- The equation is simplified for small angles because

$$\sin\theta \approx \theta$$

- This is called the **small-angle approximation.** Therefore the restoring force is

$$(F_{\text{net}})_t = -mg\sin\theta \approx -mg\theta = -mg\frac{s}{L} = -\left(\frac{mg}{L}\right)s$$

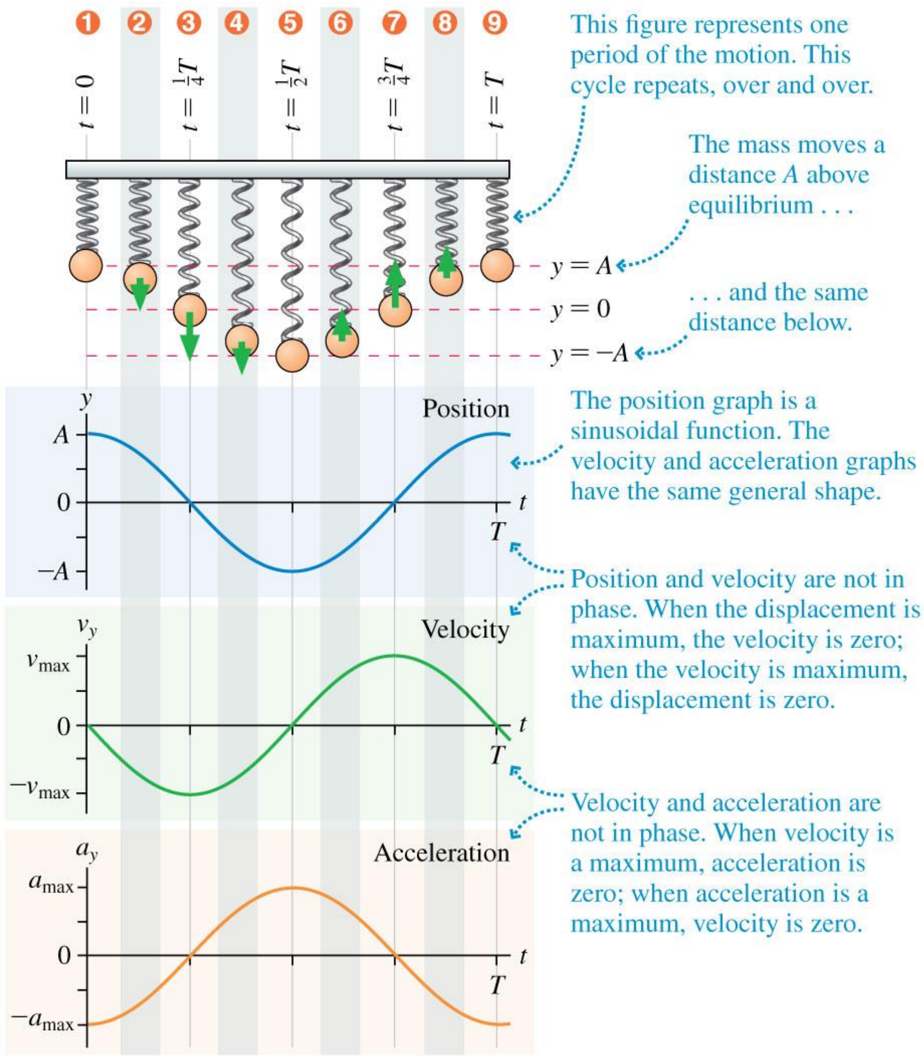
- **The force on a pendulum is a linear restoring force for small angles, so the pendulum will undergo simple harmonic motion.**



Section 14.3 Describing Simple Harmonic Motion

Describing Simple Harmonic Motion

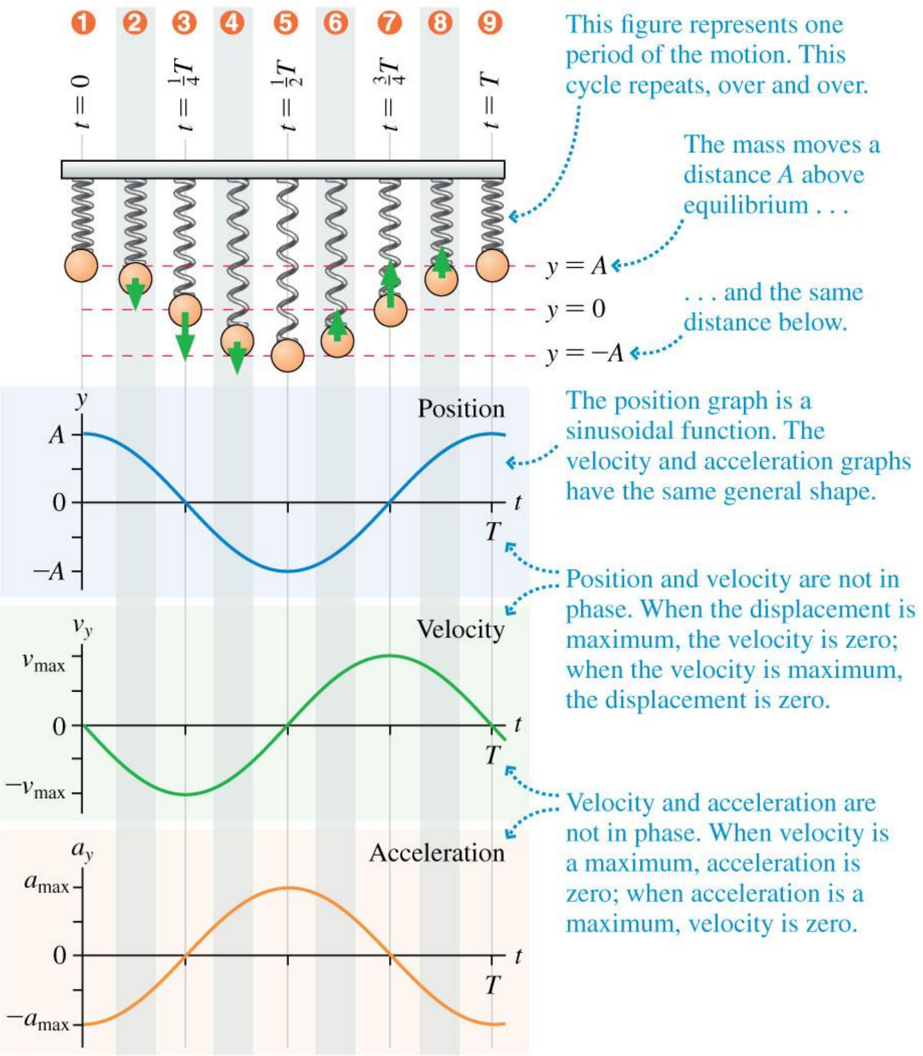
1. The mass starts at its maximum positive displacement, $y = A$. The velocity is zero, but the acceleration is negative because there is a net downward force.
2. The mass is now moving downward, so the velocity is negative. As the mass nears equilibrium, the restoring force—and thus the magnitude of the acceleration—decreases.
3. At this time the mass is moving downward with its maximum speed. It's at the equilibrium position, so the net force—and thus the acceleration—is zero.



Text: p. 443

Describing Simple Harmonic Motion

4. The velocity is still negative but its magnitude is decreasing, so the acceleration is positive.
5. The mass has reached the lowest point of its motion, a **turning point.** The spring is at its maximum extension, so there is a net upward force and the acceleration is positive.
6. The mass has begun moving upward; the velocity and acceleration are positive.



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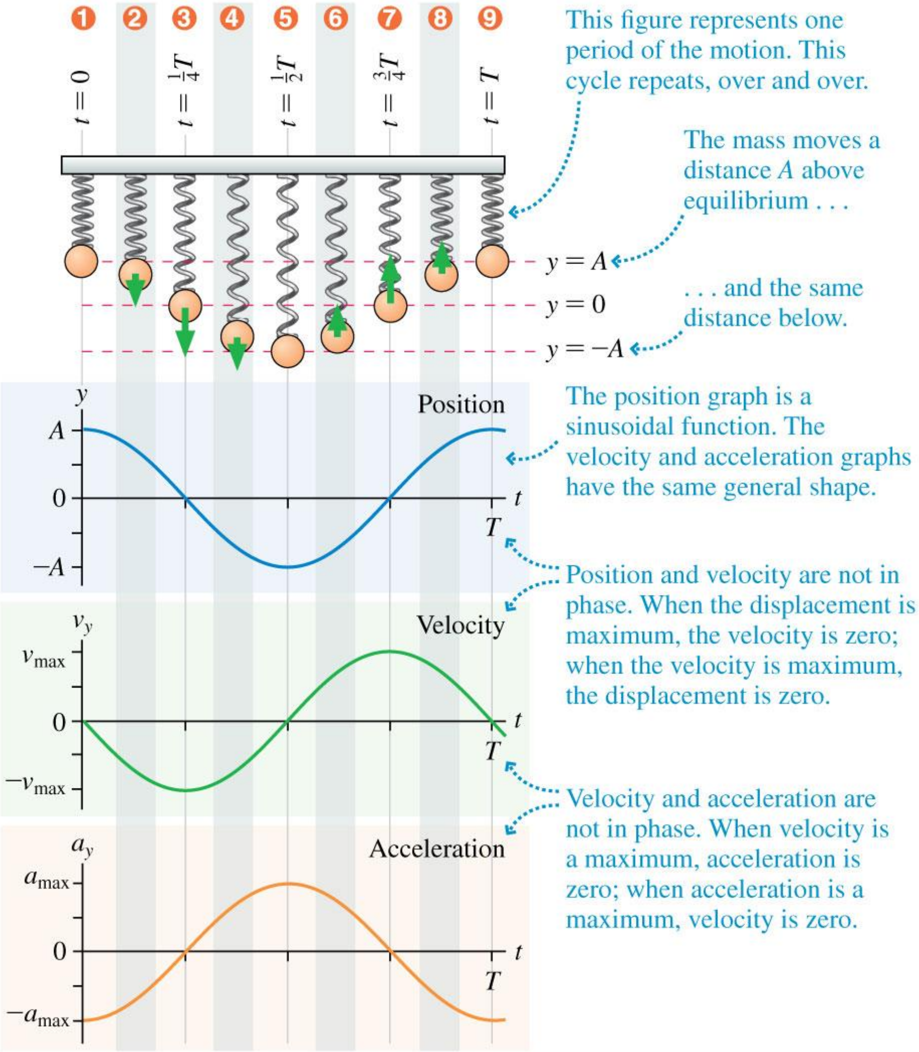
Describing Simple Harmonic Motion

7. The mass is passing through the equilibrium position again, in the opposite direction, so it has a positive velocity. There is no net force, so the acceleration is zero.
8. The mass continues moving upward. The velocity is positive but its magnitude is decreasing, so the acceleration is negative.
9. The mass is now back at its starting position. This is another turning point. The mass is at rest but will soon begin moving downward, and the cycle will repeat.



Text: p. 443

Describing Simple Harmonic Motion

- The position-versus-time graph for oscillatory motion is a cosine curve:

$$x(t) = A\cos\left(\frac{2\pi t}{T}\right)$$

- $x(t)$ indicates that the position is a *function* of time.
- The cosine function can be written in terms of frequency:

$$x(t) = A\cos(2\pi f t)$$

Describing Simple Harmonic Motion

- The velocity graph is an upside-down sine function with the same period T:

$$v_x(t) = -v_{\max}\sin\left(\frac{2\pi t}{T}\right) = -v_{\max}\sin(2\pi f t)$$

- The restoring force causes an acceleration:

$$a_x = \frac{(F_{\text{net}})_x}{m} = -\frac{k}{m}x$$

- The acceleration-versus-time graph is inverted from the position-versus-time graph and can also be written

$$a_x(t) = -a_{\max}\cos\left(\frac{2\pi t}{T}\right) = -a_{\max}\cos(2\pi f t)$$

Describing Simple Harmonic Motion

Sinusoidal relationships

A quantity that oscillates in time and can be written

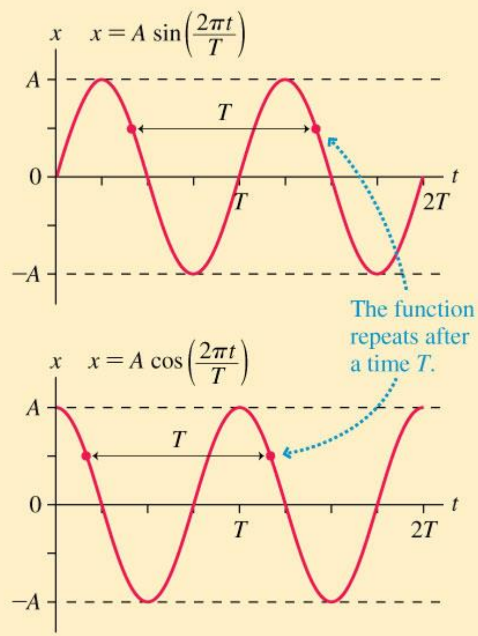
$$x = A\sin\left(\frac{2\pi t}{T}\right)$$

or

$$x = A\cos\left(\frac{2\pi t}{T}\right)$$

is called a **sinusoidal function** with **period** T. The argument of the functions, $2\pi t/T$, is in radians.

The graphs of both functions have the same shape, but they have different initial values at $t = 0$ s.



LIMITS If x is a sinusoidal function, then x is:

- *Bounded*—it can take only values between A and $-A$.
- *Periodic*—it repeats the same sequence of values over and over again. Whatever value x has at time t, it has the same value at $t + T$.

SPECIAL VALUES The function x has special values at certain times:

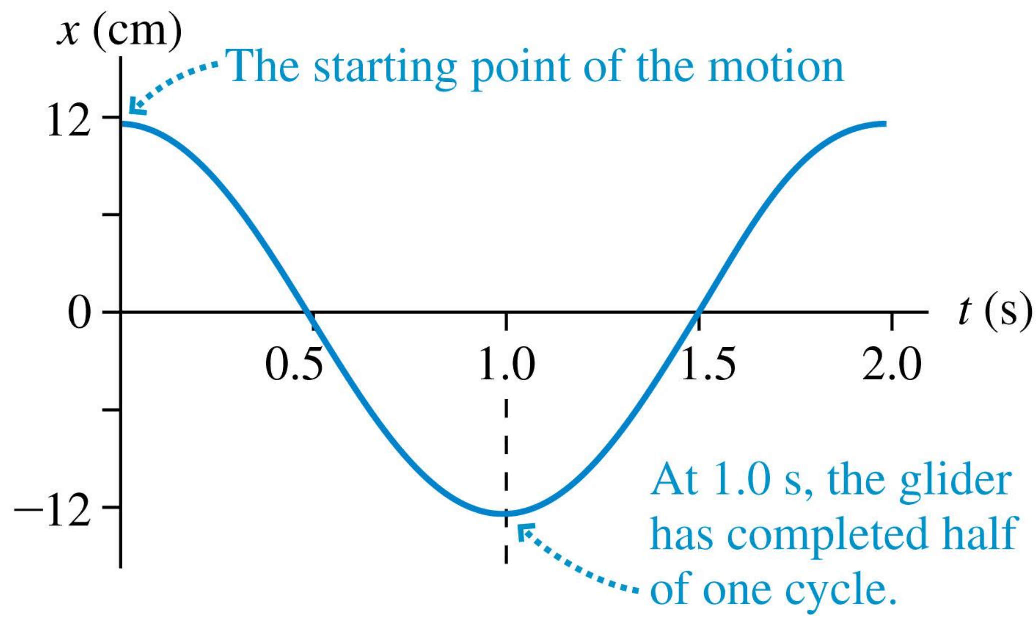
	$t = 0$	$t = \frac{1}{4}T$	$t = \frac{1}{2}T$	$t = \frac{3}{4}T$	$t = T$
$x = A\sin(2\pi t/T)$	0	A	0	$-A$	0
$x = A\cos(2\pi t/T)$	A	0	$-A$	0	A

Exercise 6

Text: p. 445

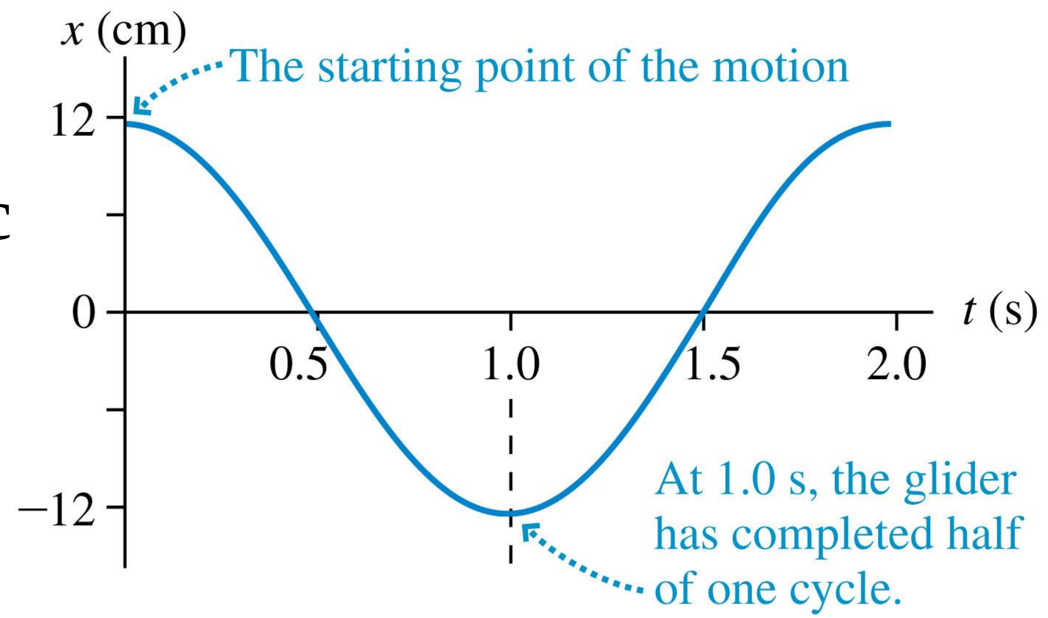
Example 14.2 Motion of a glider on a spring

An air-track glider oscillates horizontally on a spring at a frequency of 0.50 Hz. Suppose the glider is pulled to the right of its equilibrium position by 12 cm and then released. Where will the glider be 1.0 s after its release? What is its velocity at this point?

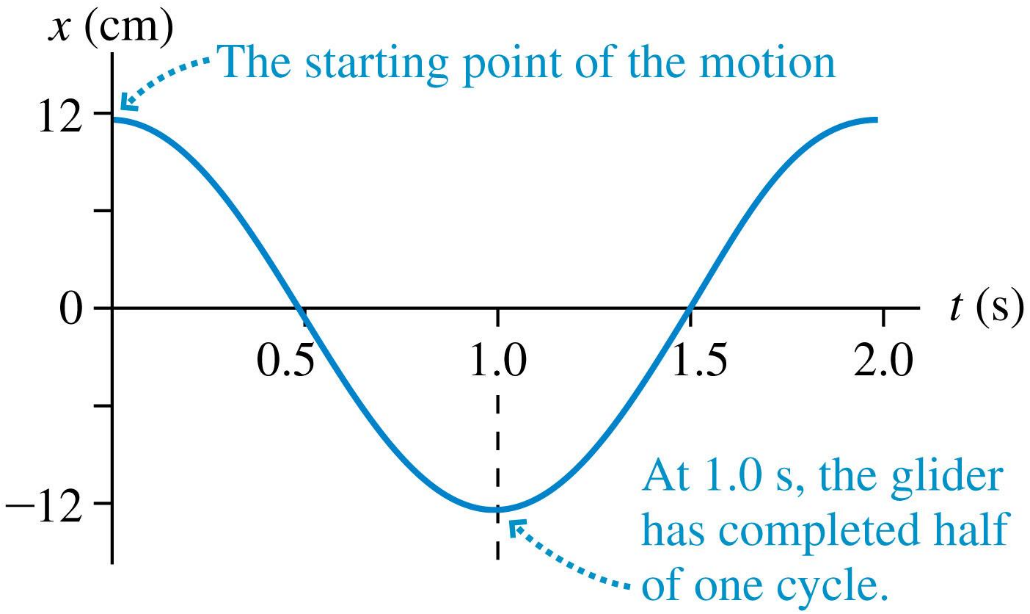


Example 14.2 Motion of a glider on a spring (cont.)

PREPARE The glider undergoes simple harmonic motion with amplitude 12 cm. The frequency is 0.50 Hz, so the period is $T = 1/f = 2.0$ s. The glider is released at maximum extension from the equilibrium position, meaning that we can take this point to be $t = 0$.



Example 14.2 Motion of a glider on a spring (cont.)



SOLVE 1.0 s is exactly half the period. As the graph of the motion in **FIGURE 14.10** shows, half a cycle brings the glider to its left turning point, 12 cm to the left of the equilibrium position. The velocity at this point is zero.

ASSESS Drawing a graph was an important step that helped us make sense of the motion.

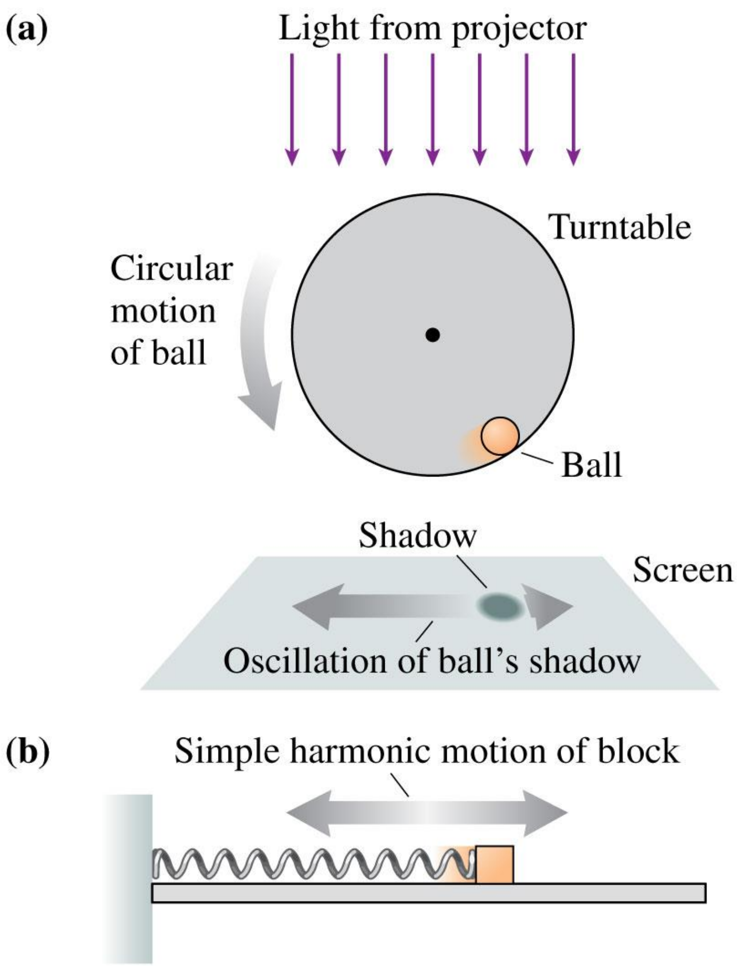
Example Problem

A 500 g block is attached to a spring on a frictionless horizontal surface. The block is pulled to stretch the spring by 10 cm, then gently released. A short time later, as the block passes through the equilibrium position, its speed is 1.0 m/s.

A. What is the block's period of oscillation?
B. What is the block's speed at the point where the spring is compressed by 5.0 cm?

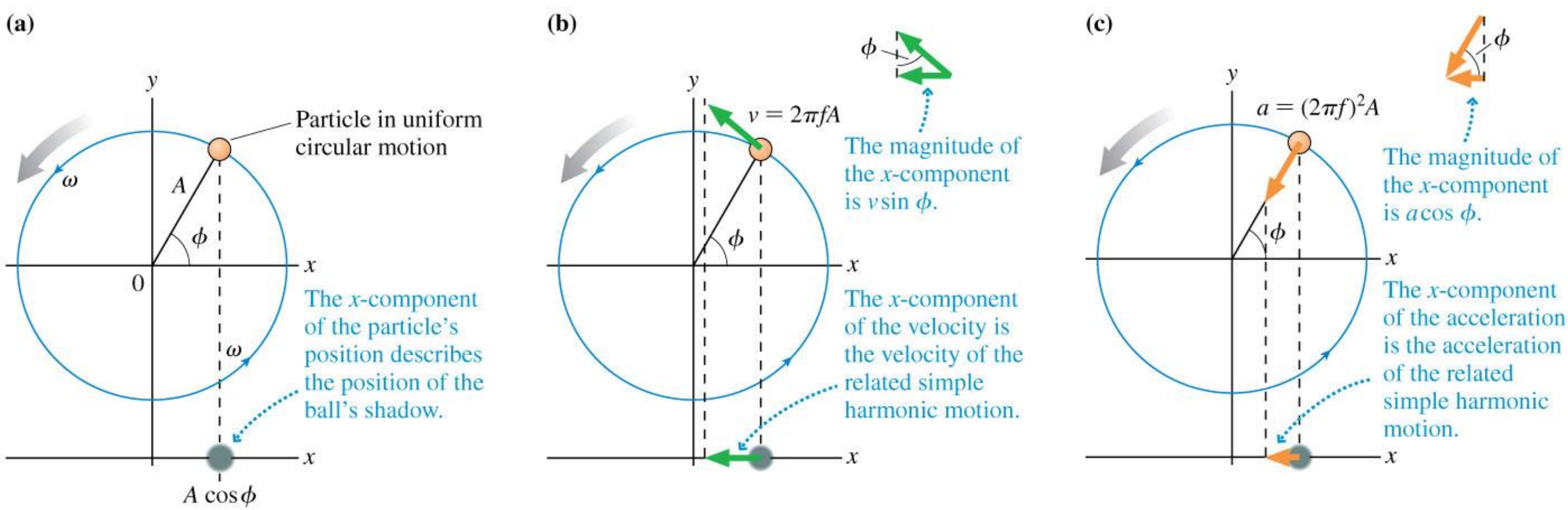
Connection to Uniform Circular Motion

- Circular motion and simple harmonic motion are motions that repeat.
- **Uniform circular motion projected onto one dimension is simple harmonic motion.**



Connection to Uniform Circular Motion

- The x-component of the circular motion when the particle is at angle ϕ is $x = A\cos\phi$.
- The angle at a later time is $\phi = \omega t$.
- ω is the particle's *angular velocity*: $\omega = 2\pi f$.

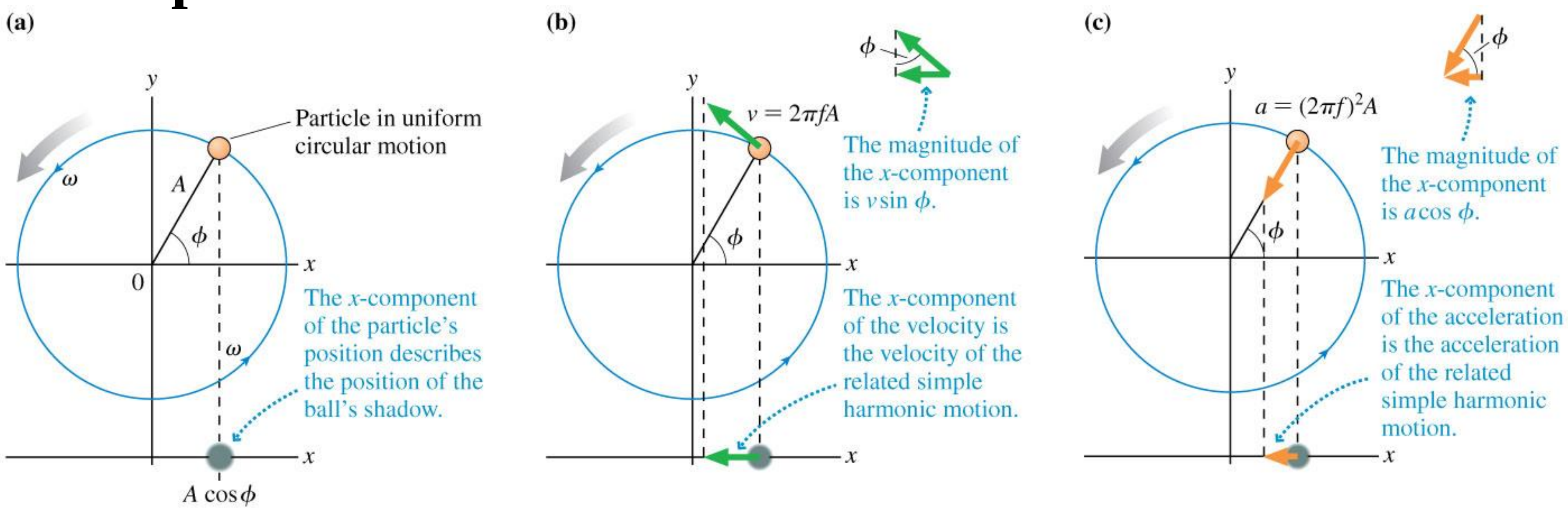


Connection to Uniform Circular Motion

- Therefore the particle's *x*-component is expressed

$$x(t) = A\cos(2\pi f t)$$

- This is the same equation for the position of a mass on a spring.
- **The *x*-component of a particle in uniform circular motion is simple harmonic motion.**



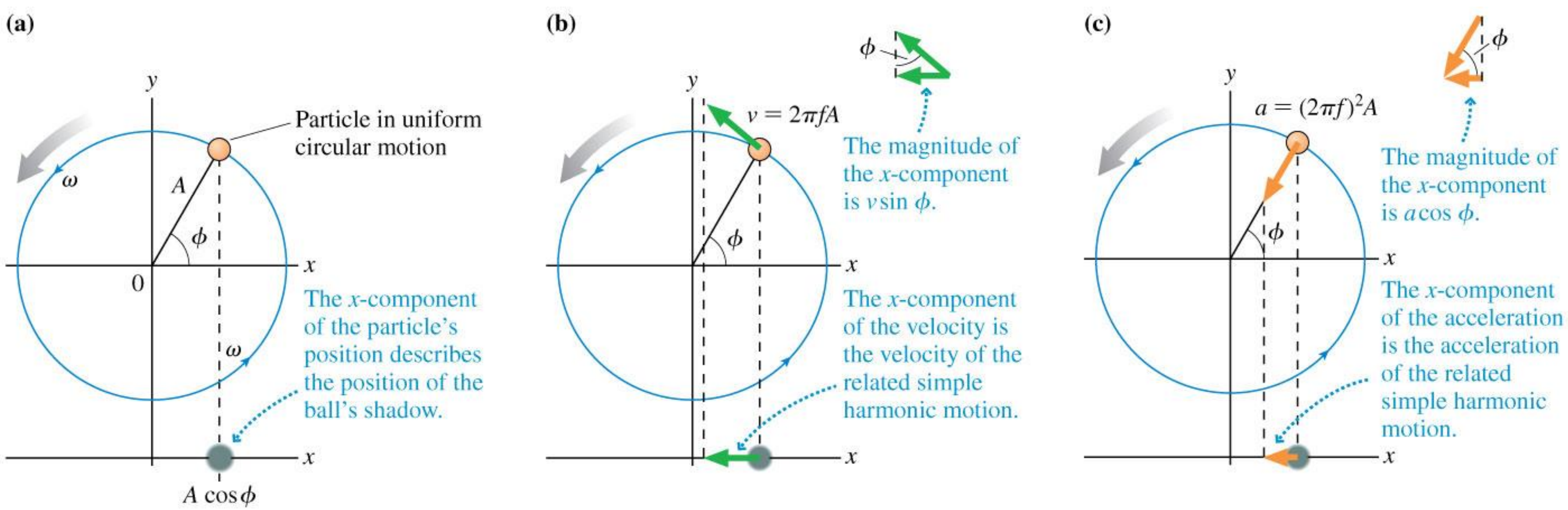
Connection to Uniform Circular Motion

- The x-component of the velocity vector is

$$v_x = -v \sin \phi = -(2\pi f)A \sin(2\pi f t)$$

- This corresponds to simple harmonic motion if we define the maximum speed to be

$$v_{max} = 2\pi f A$$



Connection to Uniform Circular Motion

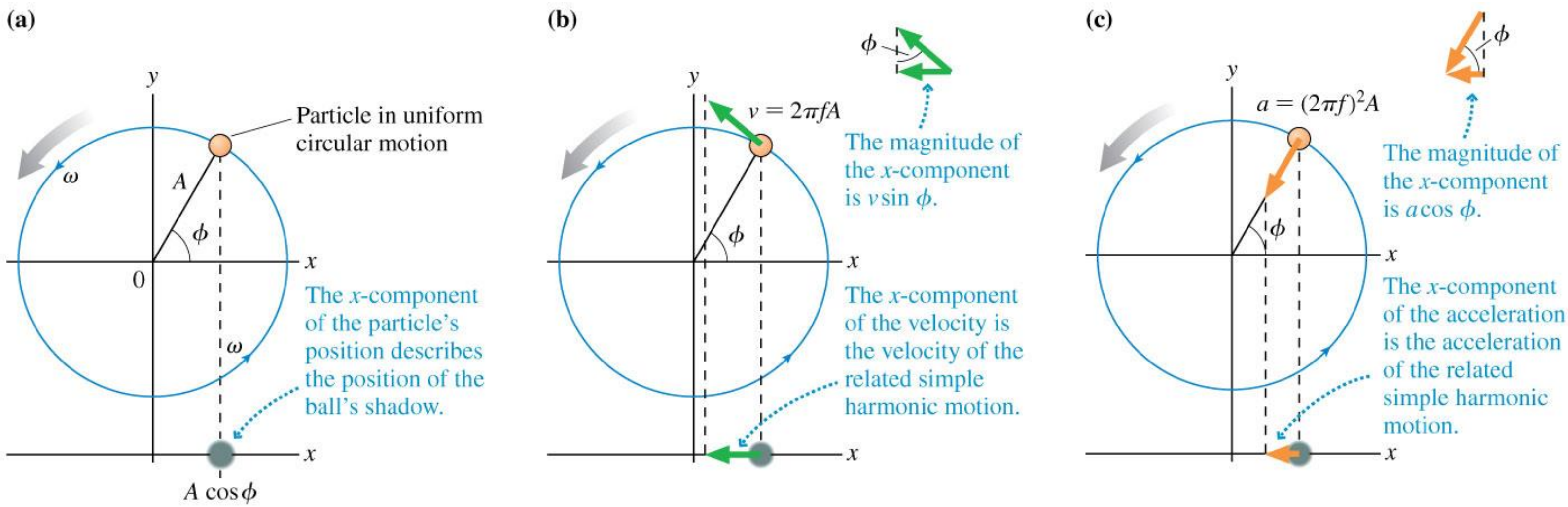
- The *x*-component of the acceleration vector is

$$a_x = -a\cos\phi = -(2\pi f)^2 A\cos(2\pi f t)$$

- The maximum acceleration is thus

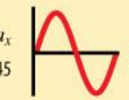
$$a_{\text{max}} = (2\pi f)^2 A$$

- For simple harmonic motion, **if you know the amplitude and frequency, the motion is completely specified.**



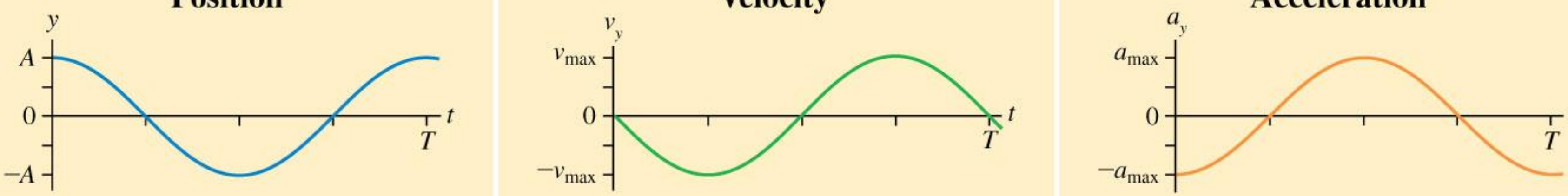
Connection to Uniform Circular Motion

SYNTHESIS 14.1 **Describing simple harmonic motion**



The position, velocity, and acceleration of objects undergoing simple harmonic motion are related sinusoidal functions.

Position **Velocity** **Acceleration**



At time t, the displacement, velocity and acceleration are given by:

$$x(t) = A\cos(2\pi ft) \qquad v_x(t) = -v_{max}\sin(2\pi ft) \qquad a_x(t) = -a_{max}\cos(2\pi ft)$$

The maximum values of the displacement, velocity, and acceleration are determined by the amplitude A and the frequency f:

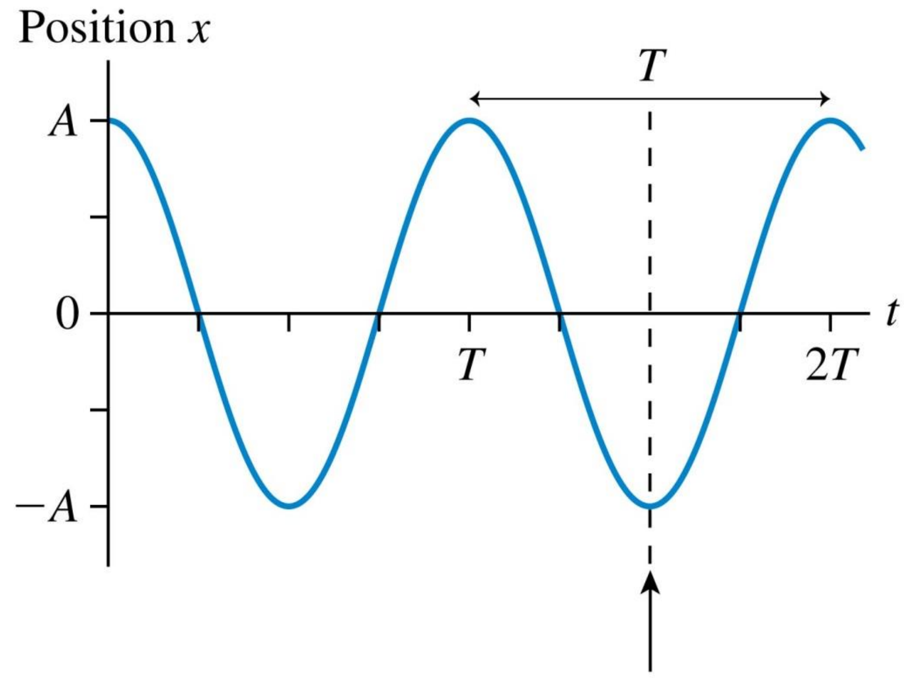
$$x_{max} = A \qquad v_{max} = 2\pi fA \qquad a_{max} = (2\pi f)^2 A$$

Text: p. 447

QuickCheck 14.9

A mass oscillates on a horizontal spring. It's velocity is v_x and the spring exerts force F_x. At the time indicated by the arrow,

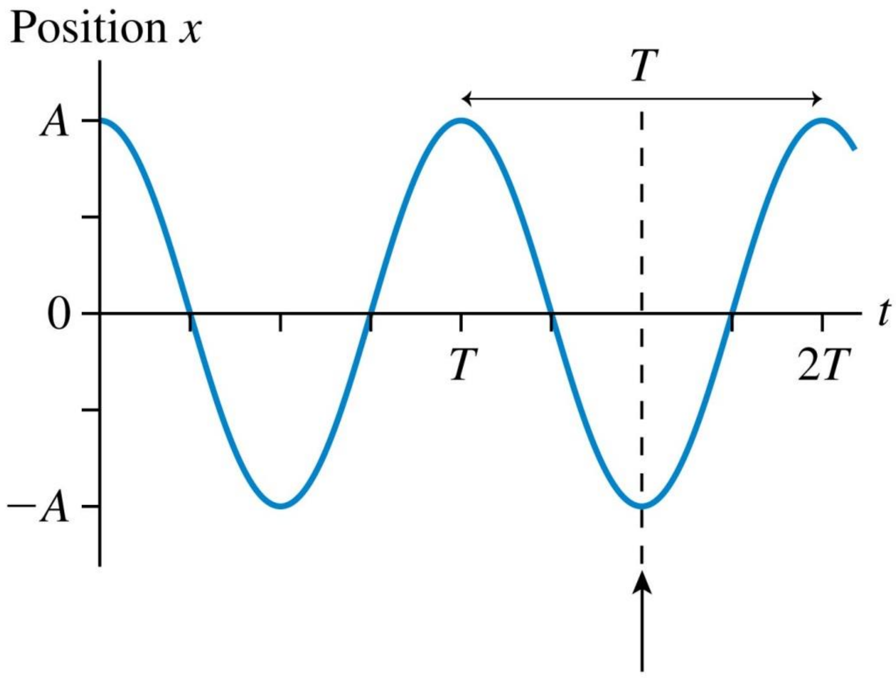
A. v_x is + and F_x is +
B. v_x is + and F_x is –
C. v_x is – and F_x is 0
D. v_x is 0 and F_x is +
E. v_x is 0 and F_x is –



QuickCheck 14.9

A mass oscillates on a horizontal spring. It's velocity is v_x and the spring exerts force F_x. At the time indicated by the arrow,

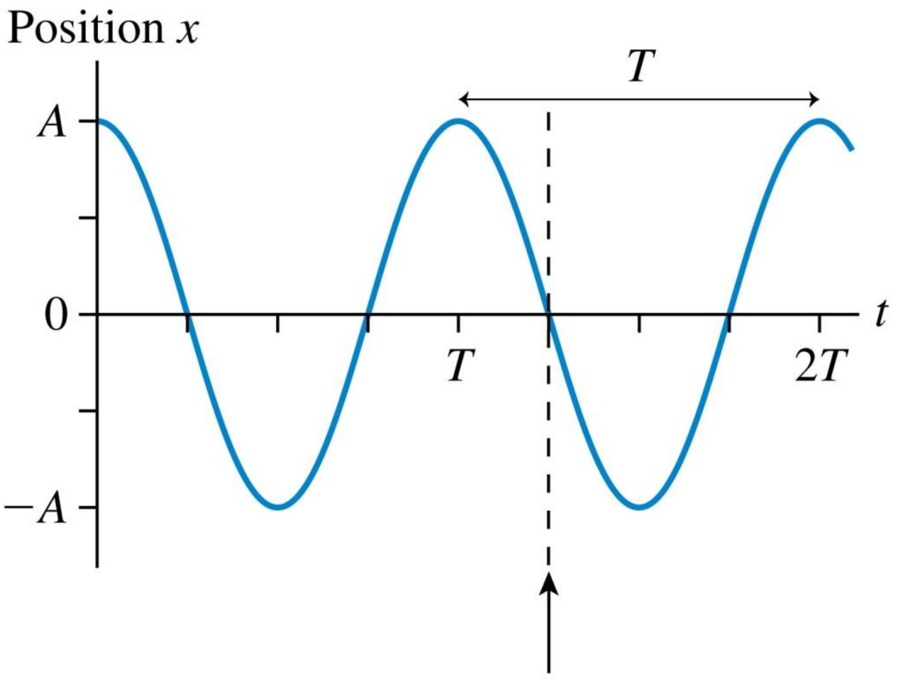
A. v_x is + and F_x is +
B. v_x is + and F_x is –
C. v_x is – and F_x is 0
✔ D. v_x is 0 and F_x is +
E. v_x is 0 and F_x is –



QuickCheck 14.10

A mass oscillates on a horizontal spring. It's velocity is v_x and the spring exerts force F_x. At the time indicated by the arrow,

A. v_x is + and F_x is +
B. v_x is + and F_x is –
C. v_x is – and F_x is 0
D. v_x is 0 and F_x is +
E. v_x is 0 and F_x is –



QuickCheck 14.10

A mass oscillates on a horizontal spring. It's velocity is v_x and the spring exerts force F_x. At the time indicated by the arrow,

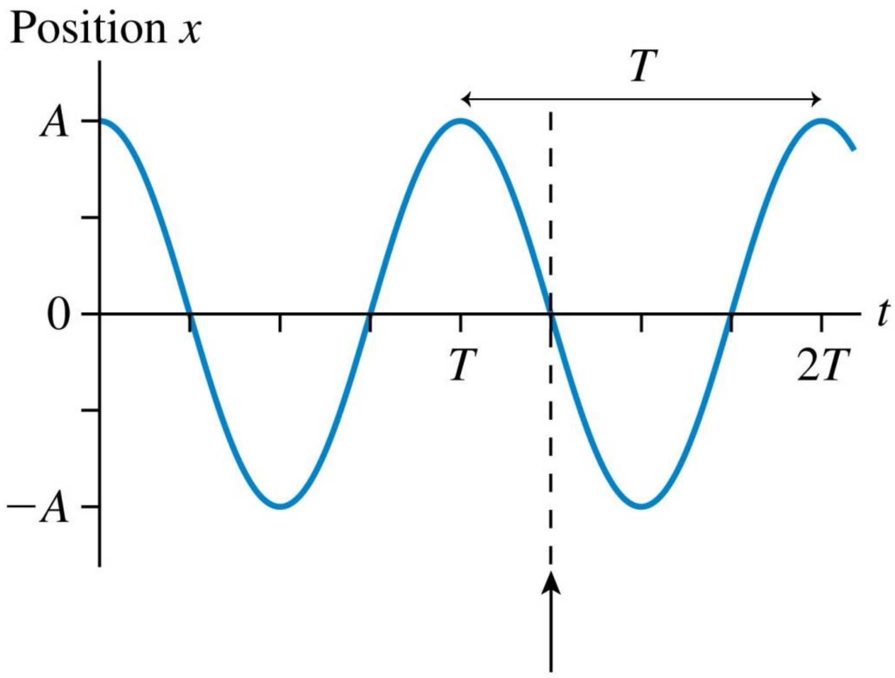
A. v_x is + and F_x is +

B. v_x is + and F_x is –

C. v_x is – and F_x is 0

D. v_x is 0 and F_x is +

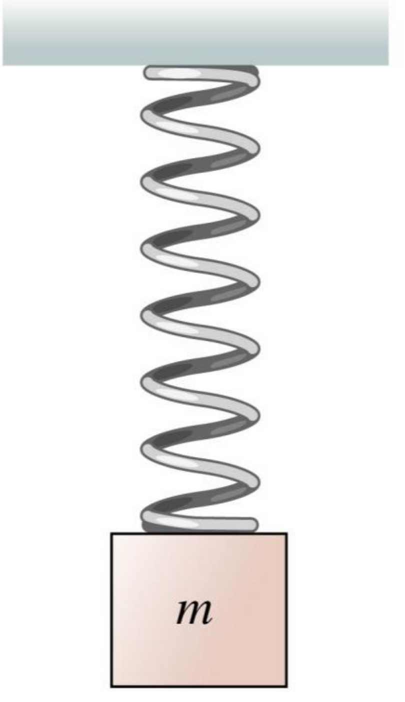
E. v_x is 0 and F_x is –



QuickCheck 14.11

A block oscillates on a vertical spring. When the block is at the lowest point of the oscillation, it's acceleration a_y is

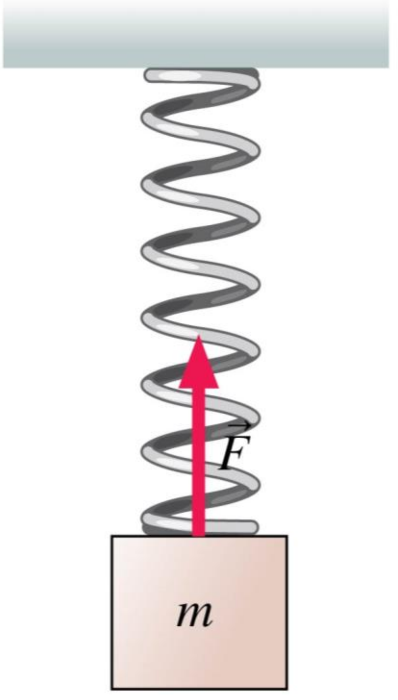
A. Negative.
B. Zero.
C. Positive.



QuickCheck 14.11

A block oscillates on a vertical spring. When the block is at the lowest point of the oscillation, it's acceleration a_y is

A. Negative.
B. Zero.
✔ C. Positive.



Try It Yourself: SHM in Your Microwave

The next time you are warming a cup of water in a microwave oven, try this: As the turntable rotates, moving the cup in a circle, stand in front of the oven with your eyes level with the cup and watch it, paying attention to the side-to-side motion. You'll see something like the turntable demonstration. The cup's apparent motion is the horizontal component of the turntable's circular motion—simple harmonic motion!

Example 14.3 Measuring the sway of a tall building

The John Hancock Center in Chicago is 100 stories high. Strong winds can cause the building to sway, as is the case with all tall buildings. On particularly windy days, the top of the building oscillates with an amplitude of 40 cm (≈16 in) and a period of 7.7 s. What are the maximum speed and acceleration of the top of the building?

Example 14.3 Measuring the sway of a tall building

PREPARE We will assume that the oscillation of the building is simple harmonic motion with amplitude A = 0.40 m. The frequency can be computed from the period:

$$f = \frac{1}{T} = \frac{1}{7.7\ \text{s}} = 0.13\ \text{Hz}$$

Example 14.3 Measuring the sway of a tall building (cont.)

SOLVE We can use the equations for maximum velocity and acceleration in Synthesis 14.1 to compute:

$$v_{\text{max}} = 2\pi f A = 2\pi(0.13\ \text{Hz})(0.40\ \text{m}) = 0.33\ \text{m/s}$$

$$a_{\text{max}} = (2\pi f\,)^2 A = [2\pi(0.13\ \text{Hz})]^2(0.40\ \text{m}) = 0.27\ \text{m/s}^2$$

In terms of the free-fall acceleration, the maximum acceleration is $a_{\text{max}} = 0.027g$.

Example 14.3 Measuring the sway of a tall building (cont.)

ASSESS The acceleration is quite small, as you would expect; if it were large, building occupants would certainly complain! Even if they don't notice the motion directly, office workers on high floors of tall buildings may experience a bit of nausea when the oscillations are large because the acceleration affects the equilibrium organ in the inner ear.

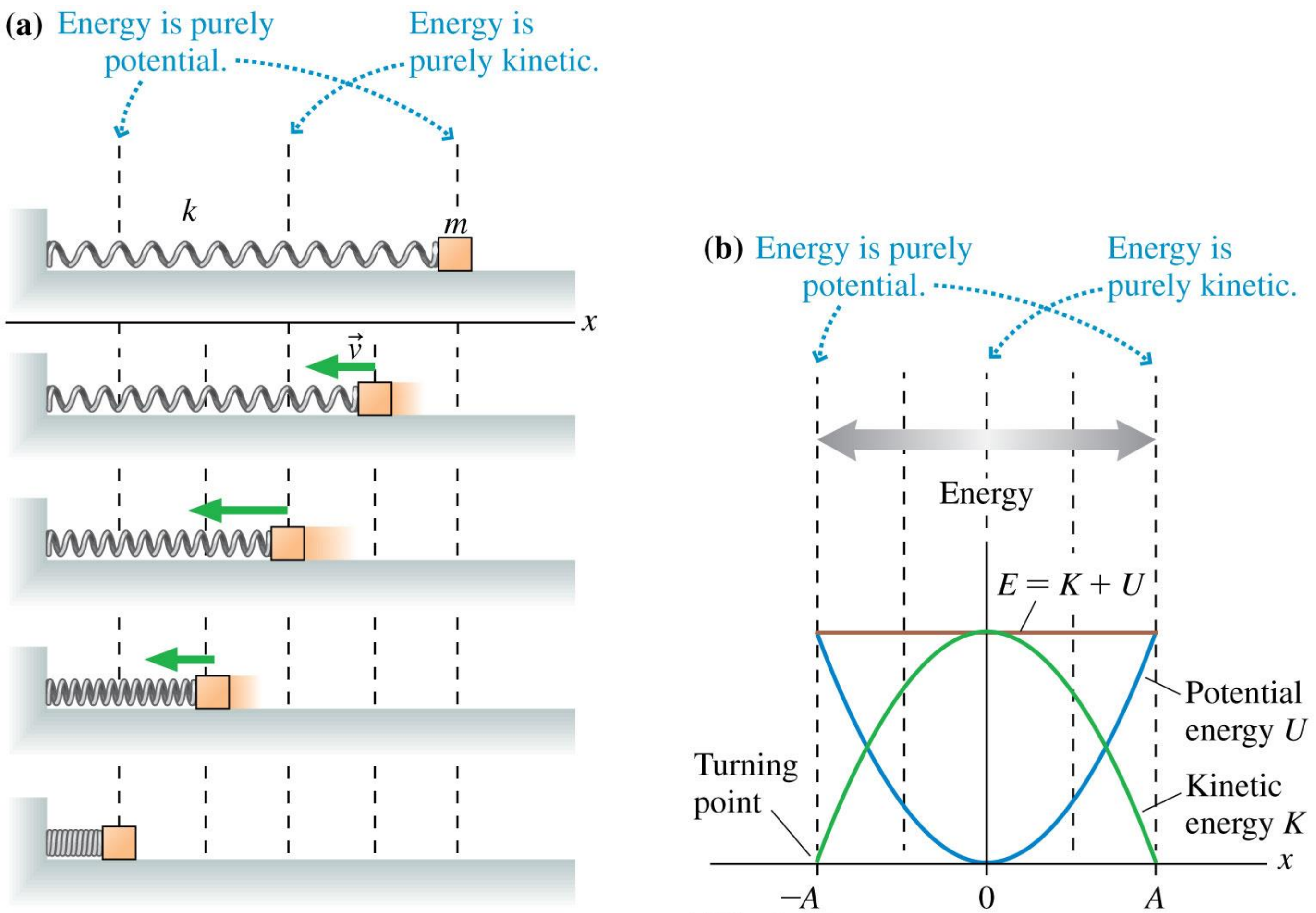
Example Problem

A 5.0 kg mass is suspended from a spring. Pulling the mass down by an additional 10 cm takes a force of 20 N. If the mass is then released, it will rise up and then come back down. How long will it take for the mass to return to its starting point 10 cm below its equilibrium position?

Section 14.4 Energy in Simple Harmonic Motion

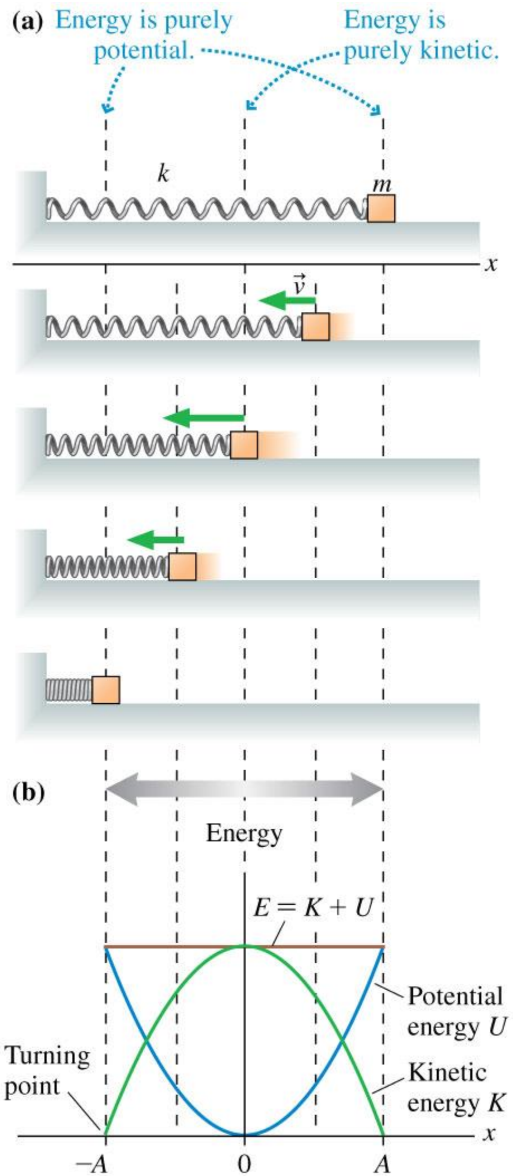
Energy in Simple Harmonic Motion

- The interplay between kinetic and potential energy is very important for understanding simple harmonic motion.



Energy in Simple Harmonic Motion

- For a mass on a spring, when the object is at rest the potential energy is a maximum and the kinetic energy is 0.
- At the equilibrium position, the kinetic energy is a maximum and the potential energy is 0.



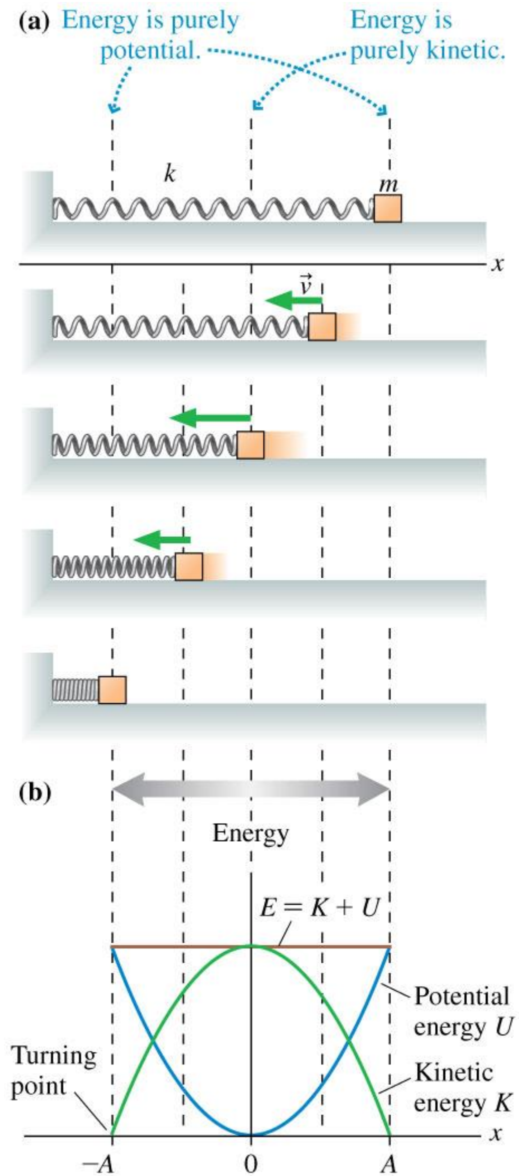
Energy in Simple Harmonic Motion

- The potential energy for the mass on a spring is

$$U = \frac{1}{2}kx^2$$

- The conservation of energy can be written:

$$E = K + U = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \text{constant}$$



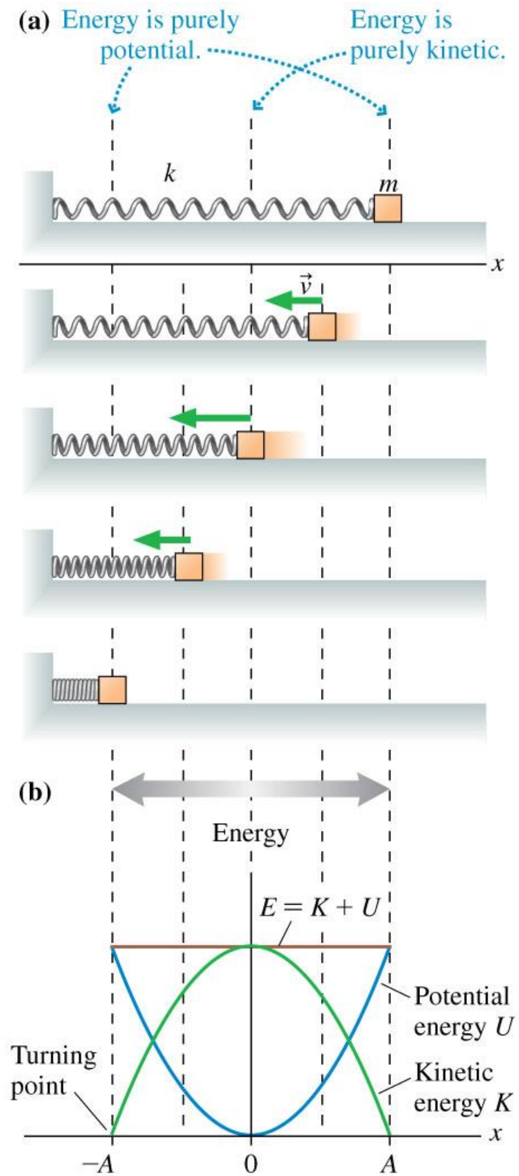
Energy in Simple Harmonic Motion

- At maximum displacement, the energy is purely potential:

$$E(\text{at } x = \pm A) = U_{\text{max}} = \frac{1}{2}kA^2$$

- At $x = 0$, the equilibrium position, the energy is purely kinetic:

$$E(\text{at } x = 0) = K_{\text{max}} = \frac{1}{2}m(v_{\text{max}})^2$$



Finding the Frequency for Simple Harmonic Motion

- Because of conservation of energy, the maximum potential energy must be equal to the maximum kinetic energy:

$$\frac{1}{2}m(v_{\text{max}})^2 = \frac{1}{2}kA^2$$

- Solving for the maximum velocity we find

$$v_{\text{max}} = \sqrt{\frac{k}{m}}A$$

- Earlier we found that

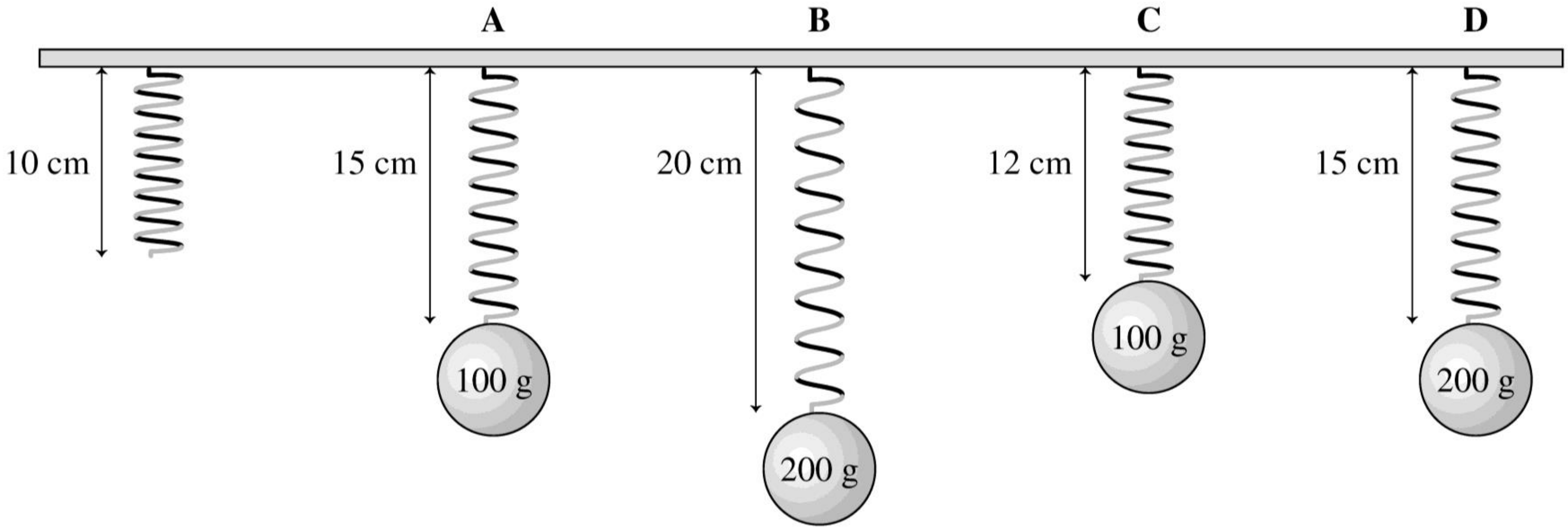
$$v_{\text{max}} = 2\pi f A$$

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \quad \text{and} \quad T = 2\pi\sqrt{\frac{m}{k}}$$

Frequency and period of SHM for mass *m* on a spring with spring constant *k*

QuickCheck 14.6

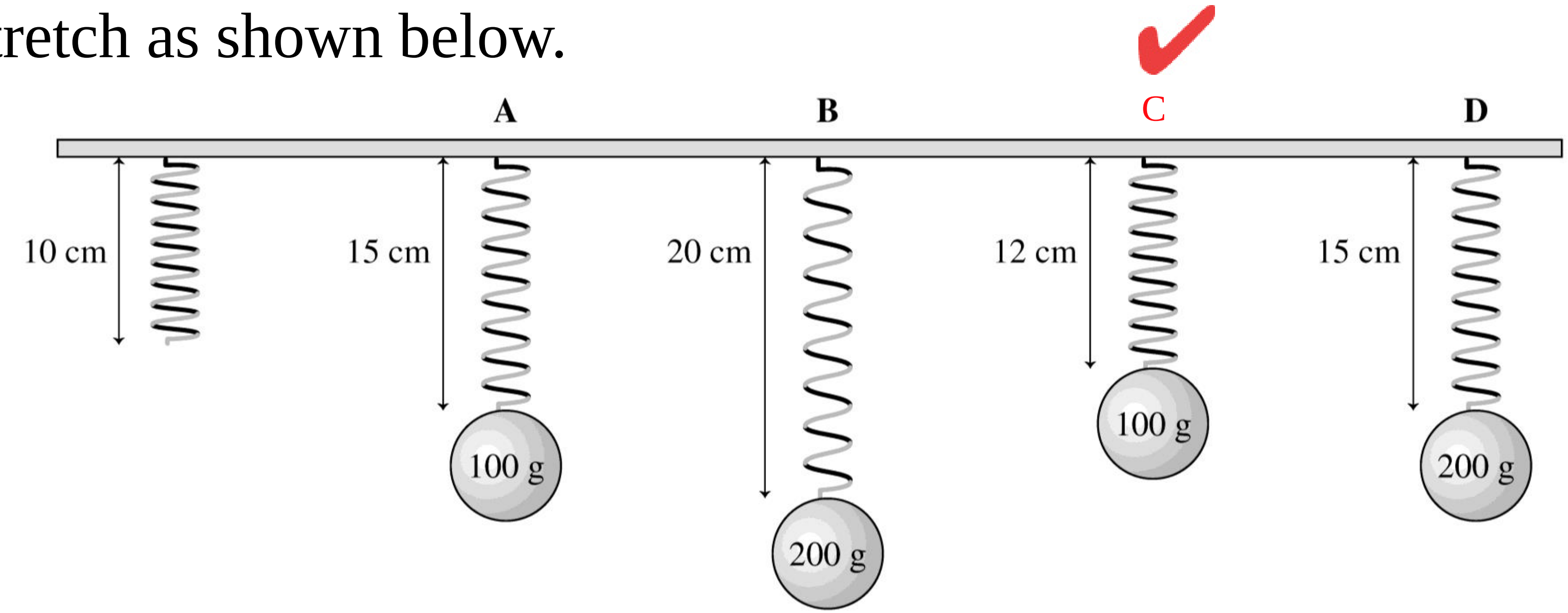
A set of springs all have initial length 10 cm. Each spring now has a mass suspended from its end, and the different springs stretch as shown below.



Now, each mass is pulled down by an additional 1 cm and released, so that it oscillates up and down. Which of the oscillating systems has the highest frequency?

QuickCheck 14.6

A set of springs all have initial length 10 cm. Each spring now has a mass suspended from its end, and the different springs stretch as shown below.

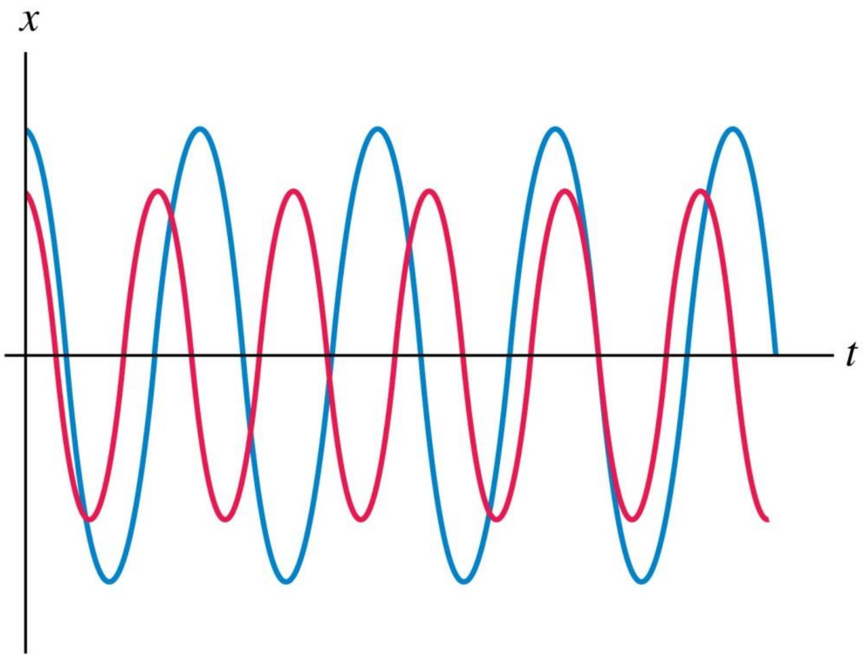


Now, each mass is pulled down by an additional 1 cm and released, so that it oscillates up and down. Which of the oscillating systems has the highest frequency?

QuickCheck 14.7

Two identical blocks oscillate on different horizontal springs. Which spring has the larger spring constant?

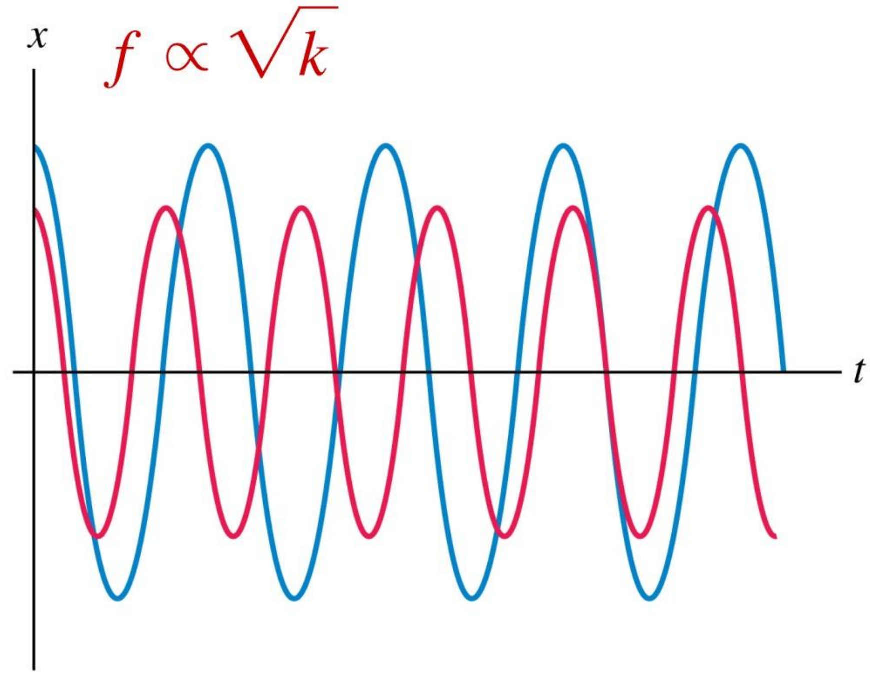
A. The red spring
B. The blue spring
C. There's not enough information to tell.



QuickCheck 14.7

Two identical blocks oscillate on different horizontal springs. Which spring has the larger spring constant?

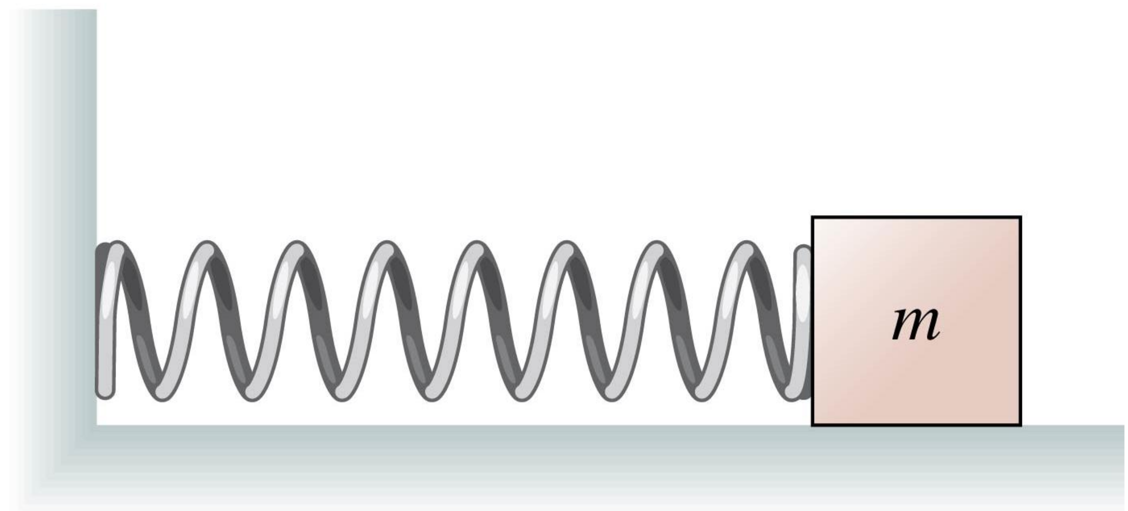
✔ A. The red spring
B. The blue spring
C. There's not enough information to tell.



QuickCheck 14.8

A block of mass m oscillates on a horizontal spring with period $T = 2.0$ s. If a second identical block is glued to the top of the first block, the new period will be

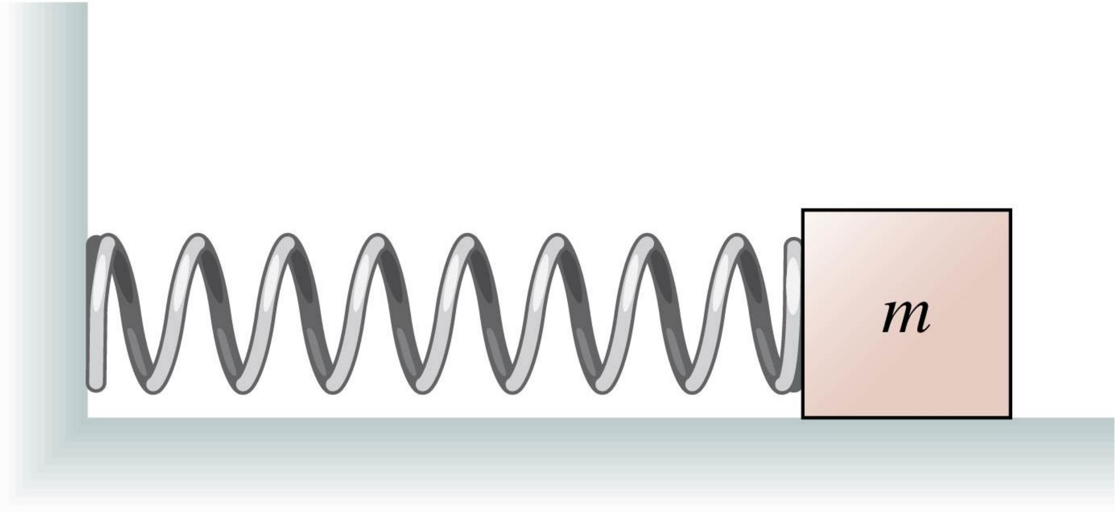
A. 1.0 s
B. 1.4 s
C. 2.0 s
D. 2.8 s
E. 4.0 s



QuickCheck 14.8

A block of mass *m* oscillates on a horizontal spring with period $T = 2.0$ s. If a second identical block is glued to the top of the first block, the new period will be

A. 1.0 s
B. 1.4 s
C. 2.0 s
D. 2.8 s $T \propto \sqrt{m}$
E. 4.0 s



Finding the Frequency for Simple Harmonic Motion

- **The frequency and period of simple harmonic motion are determined by the physical properties of the oscillator.**
- **The frequency and period of simple harmonic motion do not depend on the amplitude *A*.**

High frequency
Stiff spring,
low mass

Low frequency
Soft spring,
high mass

Increasing the stiffness of the spring will increase the restoring force. This *increases* the frequency.

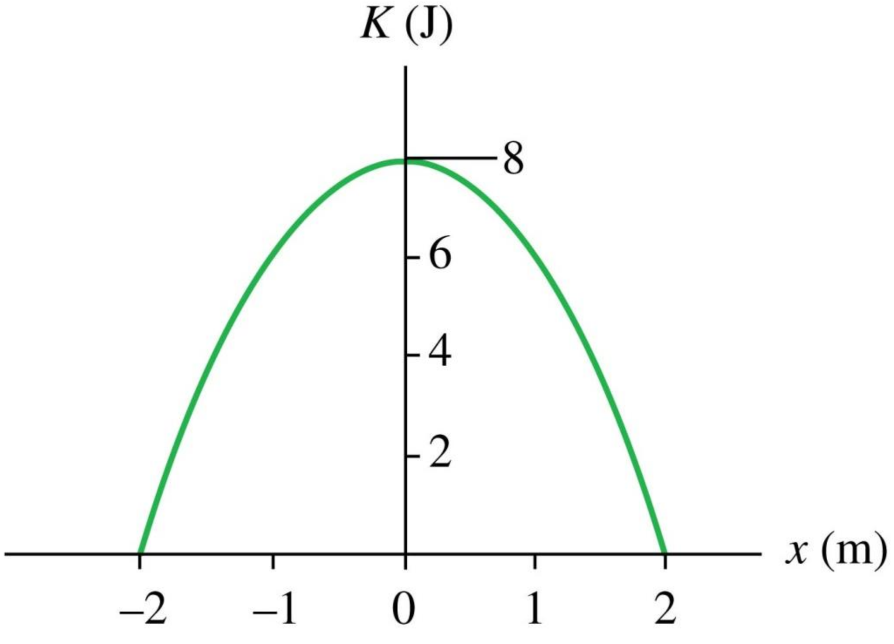
$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

Increasing the mass will increase the inertia of the system. This *decreases* the frequency.

QuickCheck 14.4

A block oscillates on a very long horizontal spring. The graph shows the block's kinetic energy as a function of position. What is the spring constant?

A. 1 N/m
B. 2 N/m
C. 4 N/m
D. 8 N/m



QuickCheck 14.4

A block oscillates on a very long horizontal spring. The graph shows the block's kinetic energy as a function of position. What is the spring constant?

A. 1 N/m

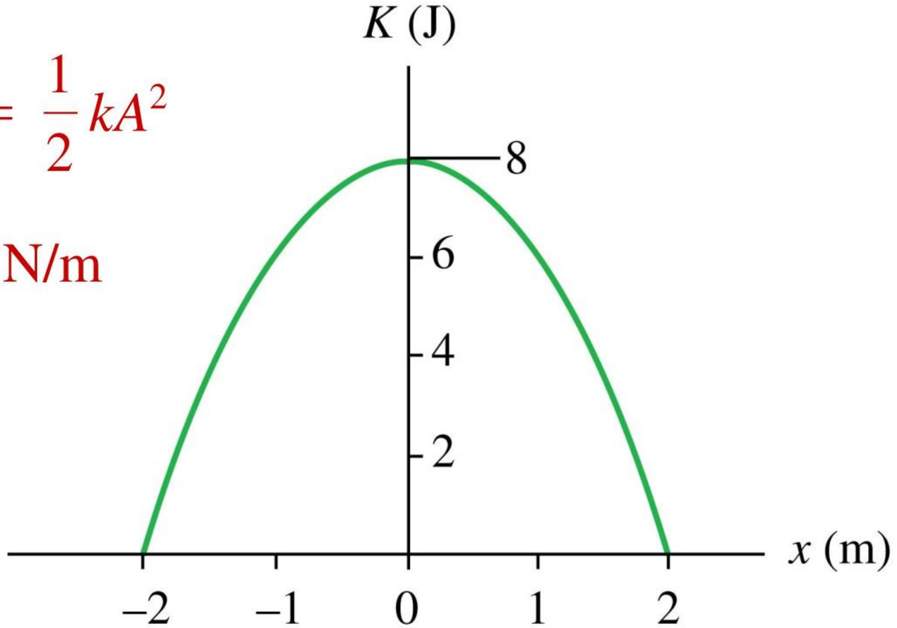
B. 2 N/m

C. 4 N/m

D. 8 N/m

$$E = K_{max} = 8\text{ J} = \frac{1}{2}kA^2$$

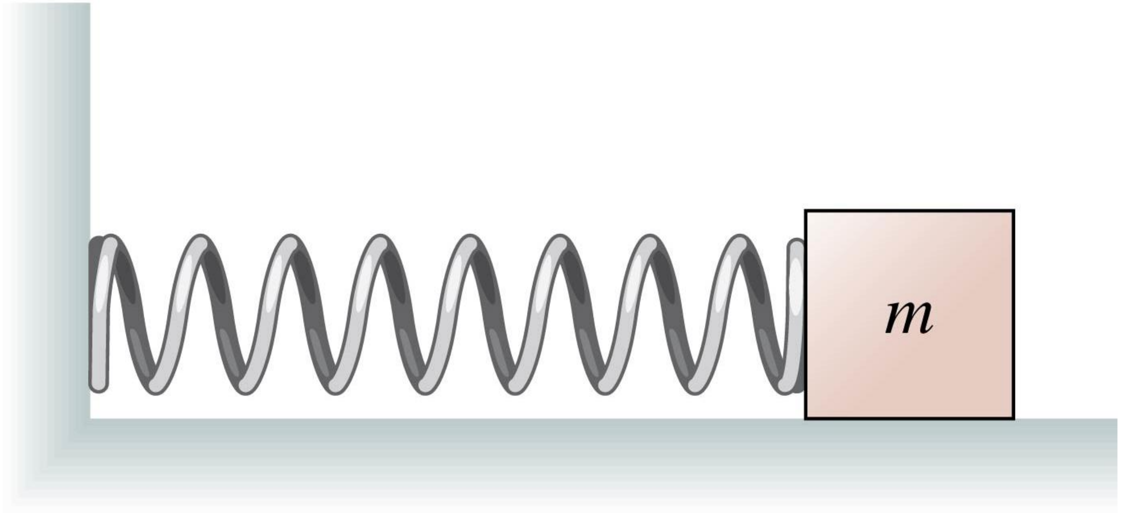
$$k = \frac{16\text{ J}}{(2\text{ m})^2} = 4\text{ N/m}$$



QuickCheck 14.5

A mass oscillates on a horizontal spring with period $T = 2.0$ s. If the amplitude of the oscillation is doubled, the new period will be

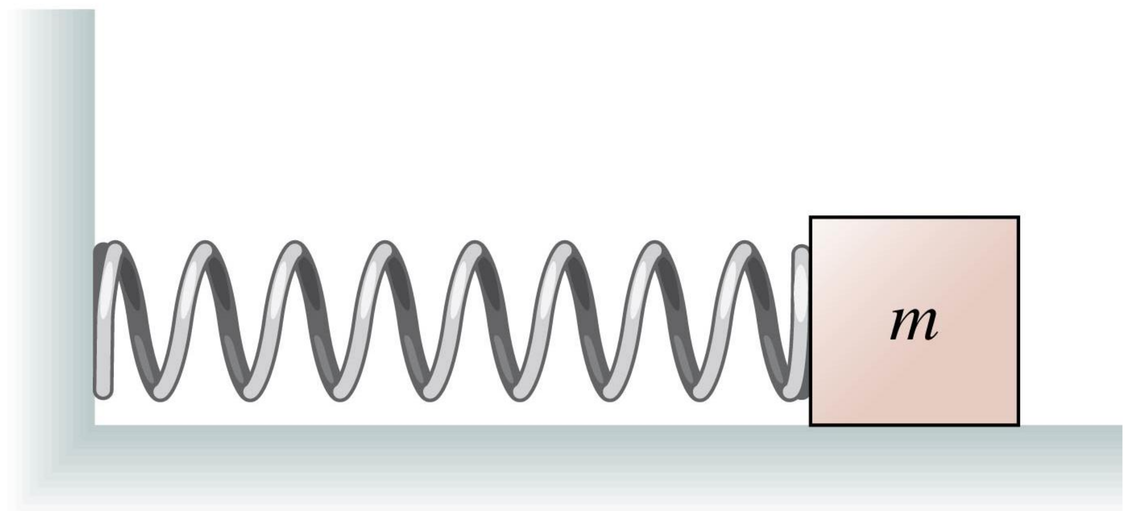
A. 1.0 s
B. 1.4 s
C. 2.0 s
D. 2.8 s
E. 4.0 s



QuickCheck 14.5

A mass oscillates on a horizontal spring with period $T = 2.0$ s. If the amplitude of the oscillation is doubled, the new period will be

A. 1.0 s
B. 1.4 s
C. 2.0 s
D. 2.8 s
E. 4.0 s



Finding the Frequency for Simple Harmonic Motion

TACTICS BOX 14.1 **Identifying and analyzing simple harmonic motion**

1. If the net force acting on a particle is a linear restoring force, the motion is simple harmonic motion around the equilibrium position.
2. The position, velocity, and acceleration as a function of time are given in Synthesis 14.1. The equations are given in terms of x, but they can be written in terms of y or some other variable if the situation calls for it.
3. The amplitude A is the maximum value of the displacement from equilibrium. The maximum speed and the maximum magnitude of the acceleration are given in Synthesis 14.1.
4. The frequency f (and hence the period $T = 1/f$) depends on the physical properties of the particular oscillator, but f does *not* depend on A.

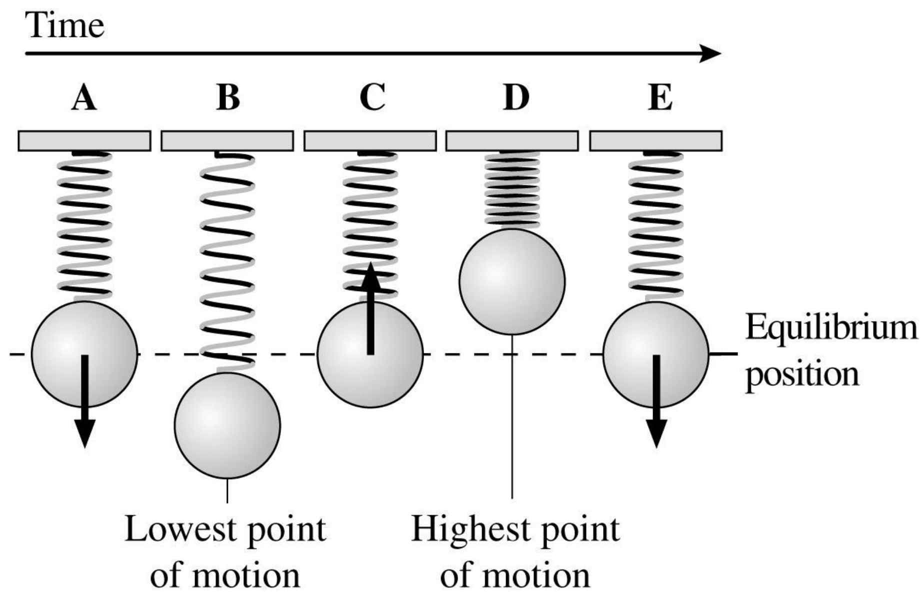
   For a mass on a spring, the frequency is given by $f = \dfrac{1}{2\pi}\sqrt{\dfrac{k}{m}}$.
5. The sum of potential energy plus kinetic energy is constant. As the oscillation proceeds, energy is transformed from kinetic into potential energy and then back again.

Exercise 11

Text: p. 451

QuickCheck 14.12

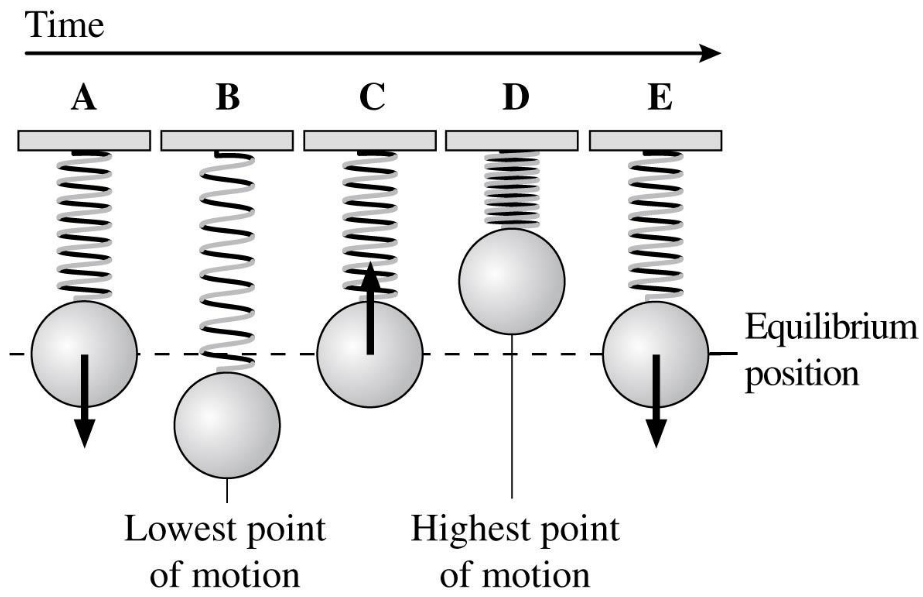
A mass oscillates up and down on a spring; the motion is illustrated at right.



1. At which time or times shown is the acceleration zero?
2. At which time or times shown is the kinetic energy a maximum?
3. At which time or times shown is the potential energy a maximum?

QuickCheck 14.12

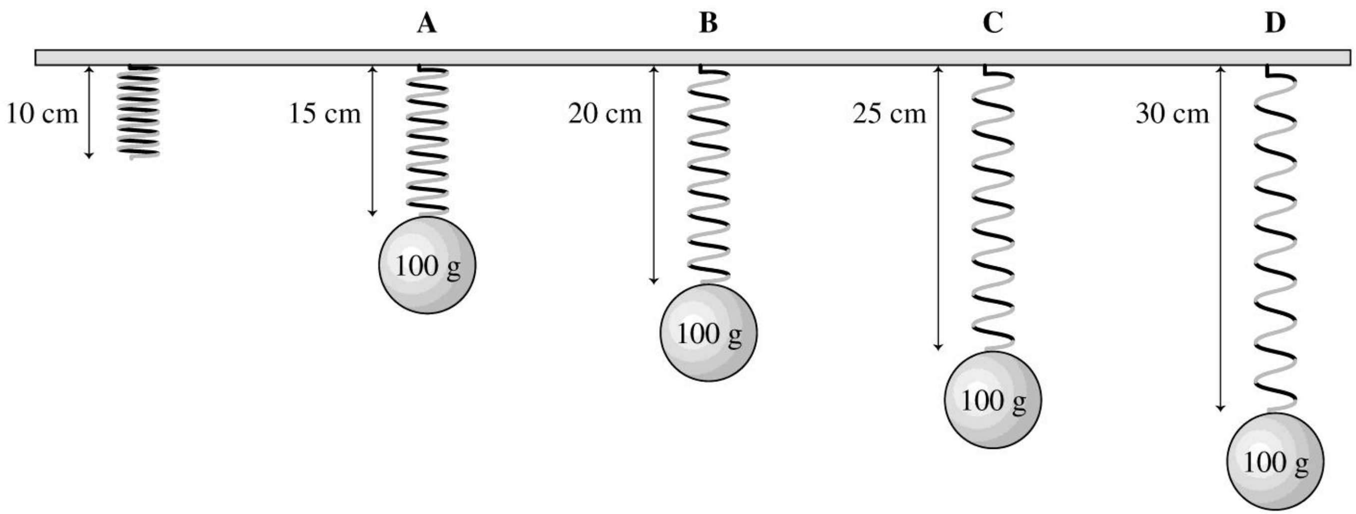
A mass oscillates up and down on a spring; the motion is illustrated at right.



1. At which time or times shown is the acceleration zero? A, C, E
2. At which time or times shown is the kinetic energy a maximum? A, C, E
3. At which time or times shown is the potential energy a maximum? B, D

QuickCheck 14.13

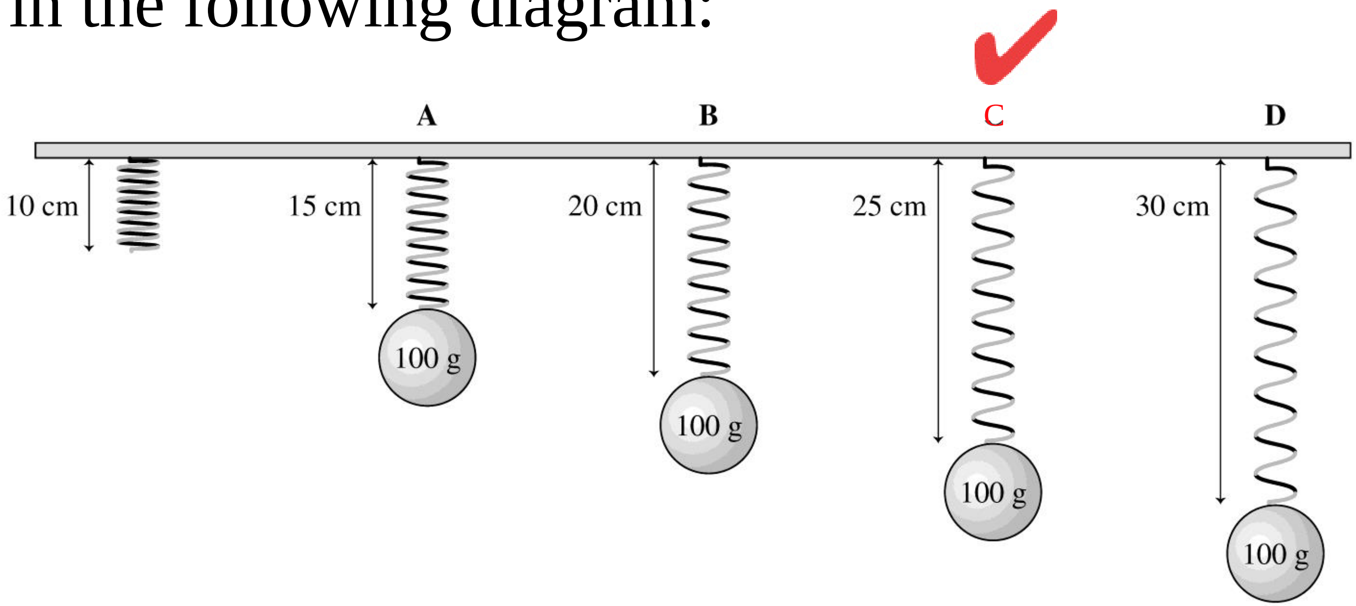
Four different masses are hung from four springs with an unstretched length of 10 cm, causing the springs to stretch as noted in the following diagram:



Now, each mass is pulled down by an additional 1 cm and released, so that it oscillates up and down. Which of the oscillating systems has the highest frequency?

QuickCheck 14.13

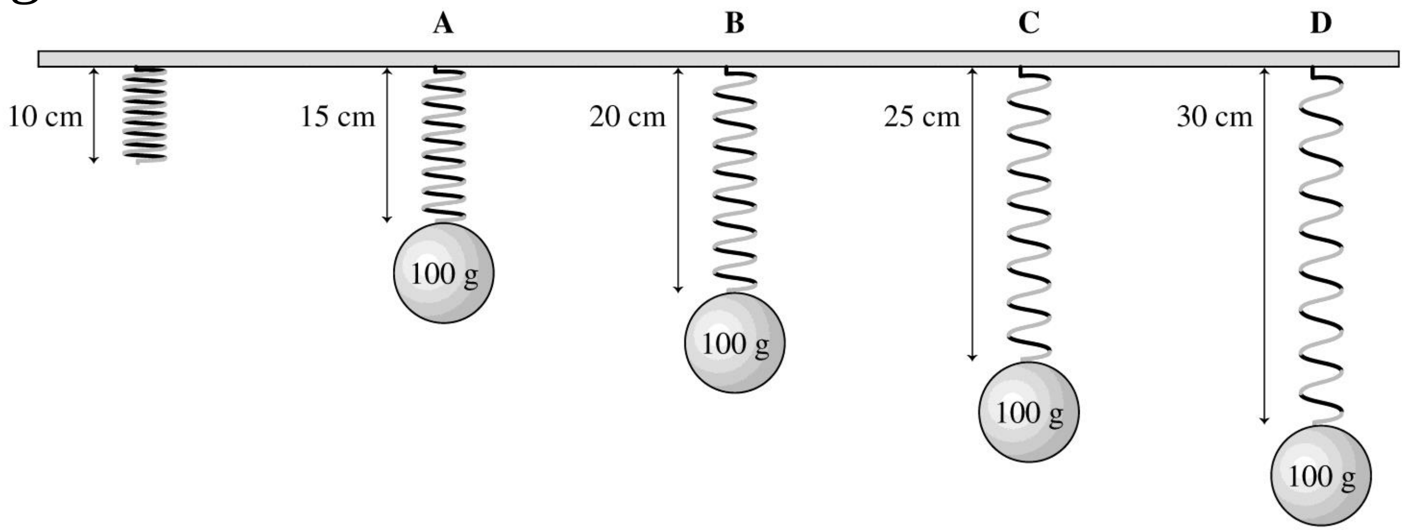
Four different masses are hung from four springs with an unstretched length of 10 cm, causing the springs to stretch as noted in the following diagram:



Now, each mass is pulled down by an additional 1 cm and released, so that it oscillates up and down. Which of the oscillating systems has the highest frequency?

QuickCheck 14.14

Four 100-g masses are hung from four springs, each with an unstretched length of 10 cm. The four springs stretch as noted in the following diagram:

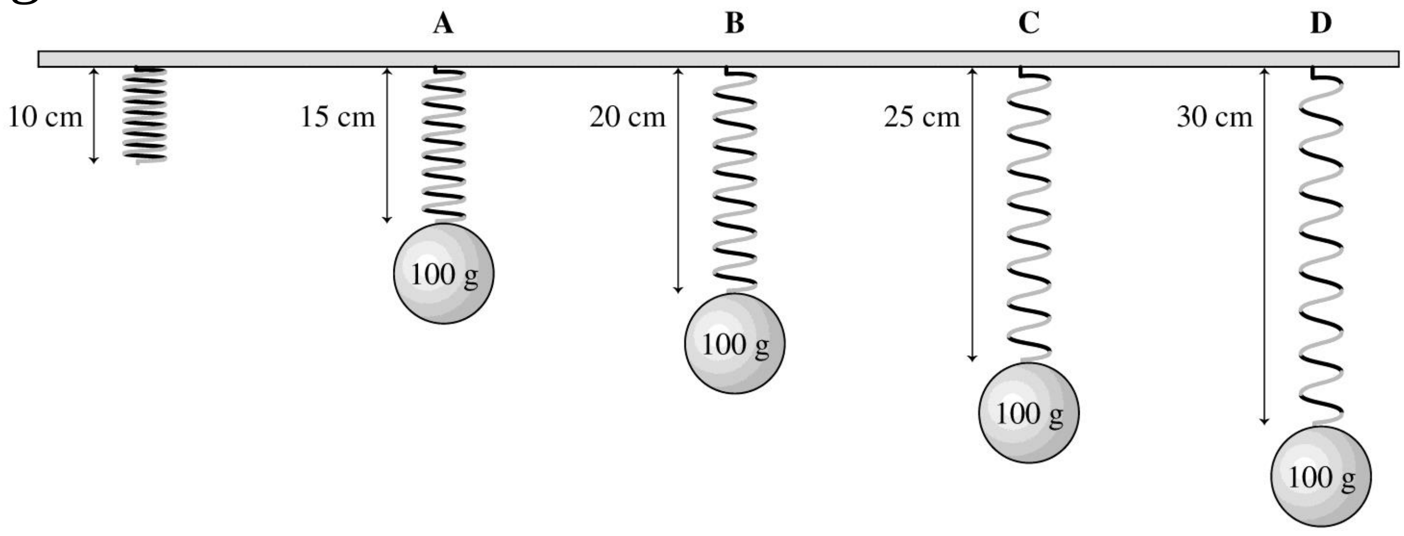


Now, each of the masses is lifted a small distance, released, and allowed to oscillate. Which mass oscillates with the highest frequency?

A. Mass A
B. Mass B
C. Mass C
D. Mass D
E. All masses oscillate with the same frequency.

QuickCheck 14.14

Four 100-g masses are hung from four springs, each with an unstretched length of 10 cm. The four springs stretch as noted in the following diagram:

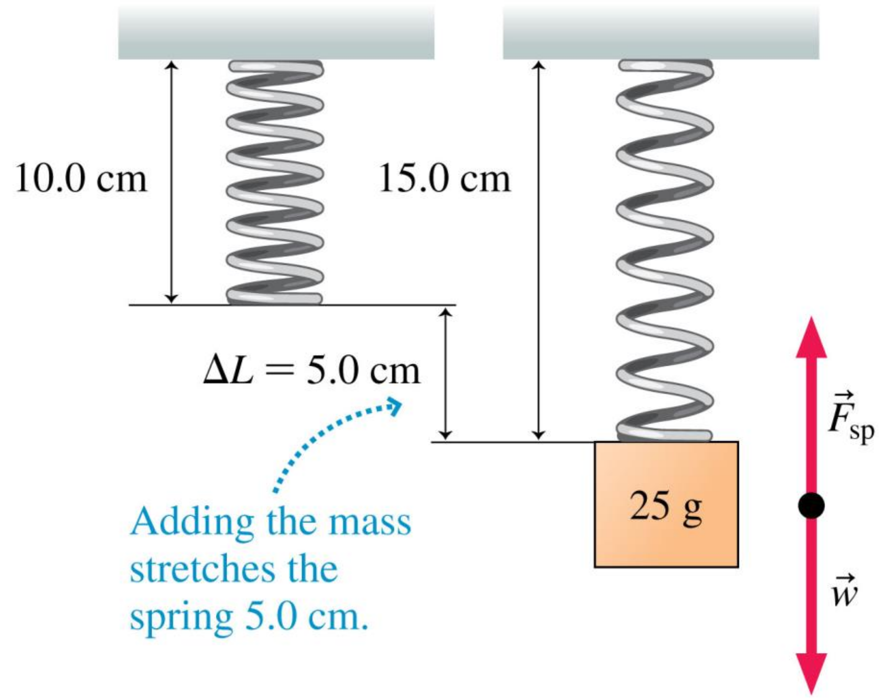


Now, each of the masses is lifted a small distance, released, and allowed to oscillate. Which mass oscillates with the highest frequency?

A. Mass A
B. Mass B
C. Mass C
D. Mass D
E. All masses oscillate with the same frequency.

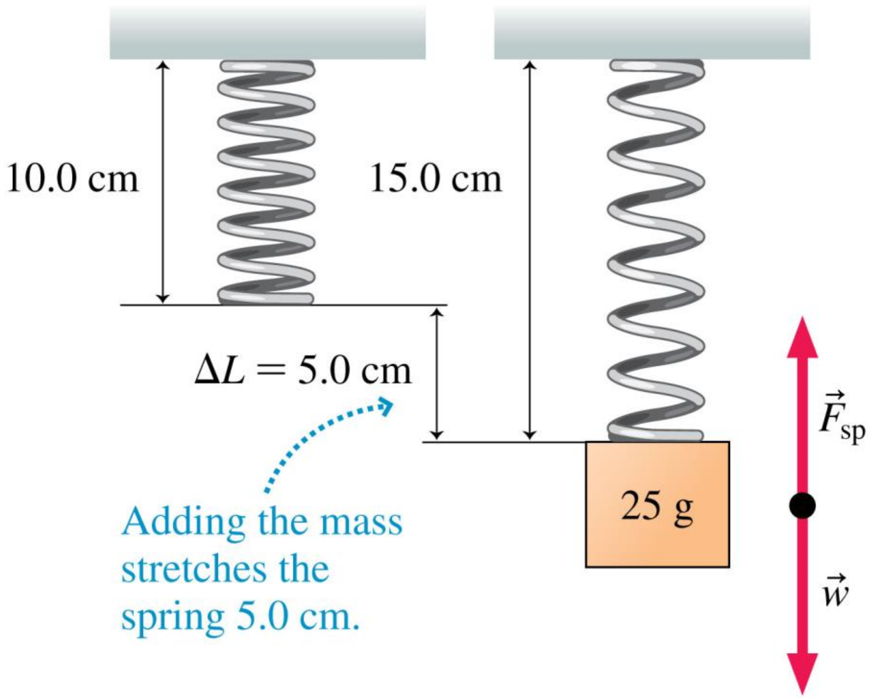
Example 14.7 Finding the frequency of an oscillator

A spring has an unstretched length of 10.0 cm. A 25 g mass is hung from the spring, stretching it to a length of 15.0 cm. If the mass is pulled down and released so that it oscillates, what will be the frequency of the oscillation?



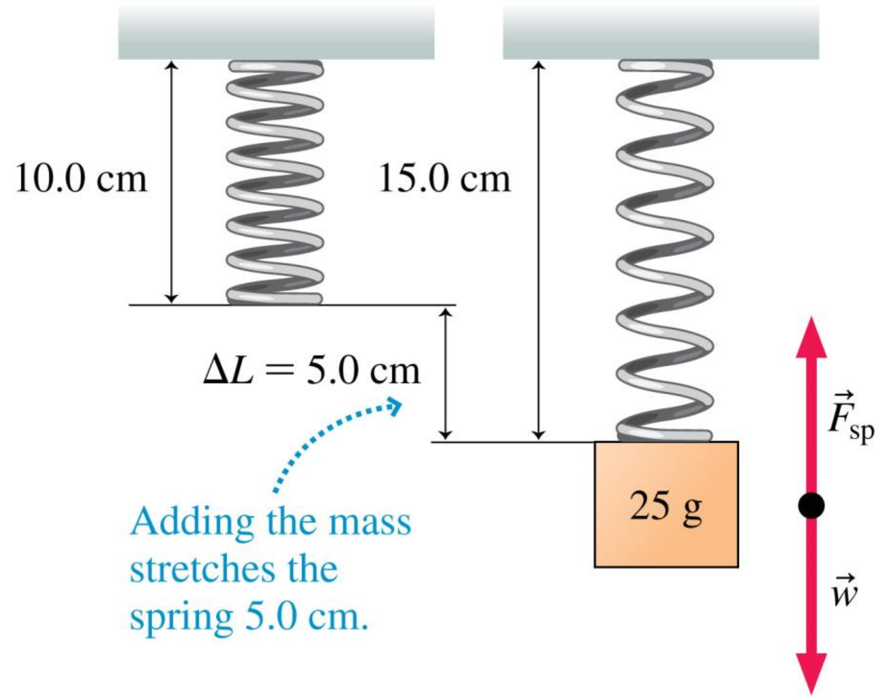
Example 14.7 Finding the frequency of an oscillator (cont.)

PREPARE The spring provides a linear restoring force, so the motion will be simple harmonic, as noted in Tactics Box 14.1. The oscillation frequency depends on the spring constant, which we can determine from the stretch of the spring. **FIGURE 14.17** gives a visual overview of the situation.



Example 14.7 Finding the frequency of an oscillator (cont.)

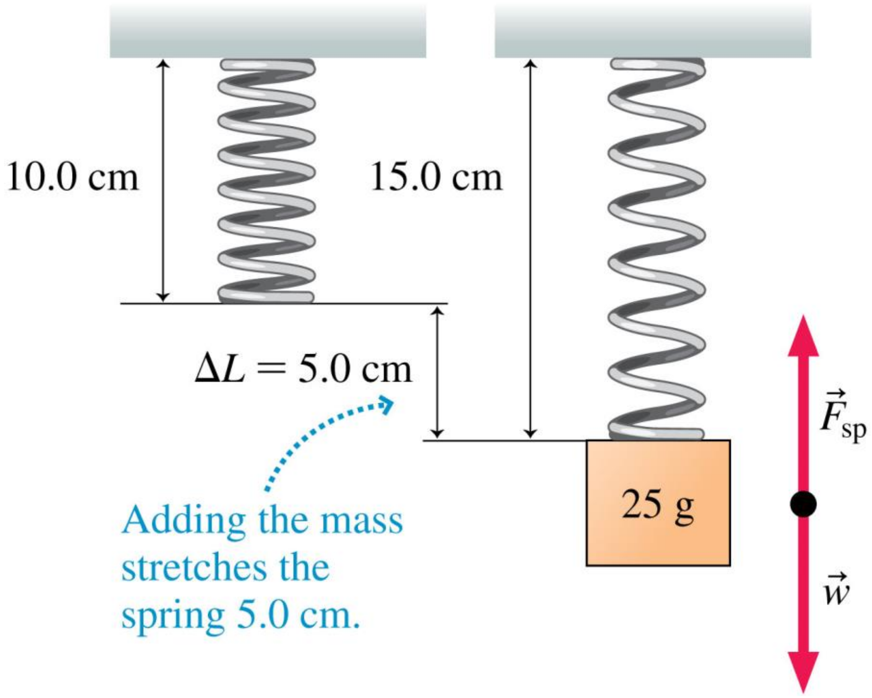
SOLVE When the mass hangs at rest, after stretching the spring to 15 cm, the net force on it must be zero. Thus the magnitude of the upward spring force equals the downward weight, giving $k\,\Delta L = mg$. The spring constant is thus



$$k = \frac{mg}{\Delta L} = \frac{(0.025\ \text{kg})(9.8\ \text{m/s}^2)}{0.050\ \text{m}} = 4.9\ \text{N/m}$$

Example 14.7 Finding the frequency of an oscillator (cont.)

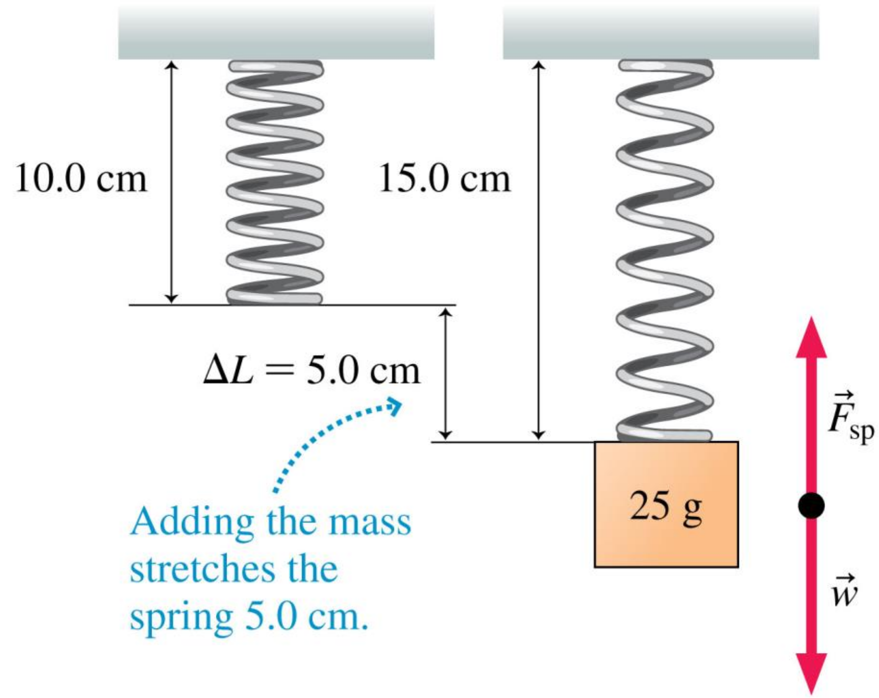
Now that we know the spring constant, we can compute the oscillation frequency:



$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{4.9 \text{ N/m}}{0.025 \text{ kg}}} = 2.2 \text{ Hz}$$

Example 14.7 Finding the frequency of an oscillator (cont.)

ASSESS 2.2 Hz is 2.2 oscillations per second. This seems like a reasonable frequency for a mass on a spring. A frequency in the kHz range (thousands of oscillations per second) would have been suspect!



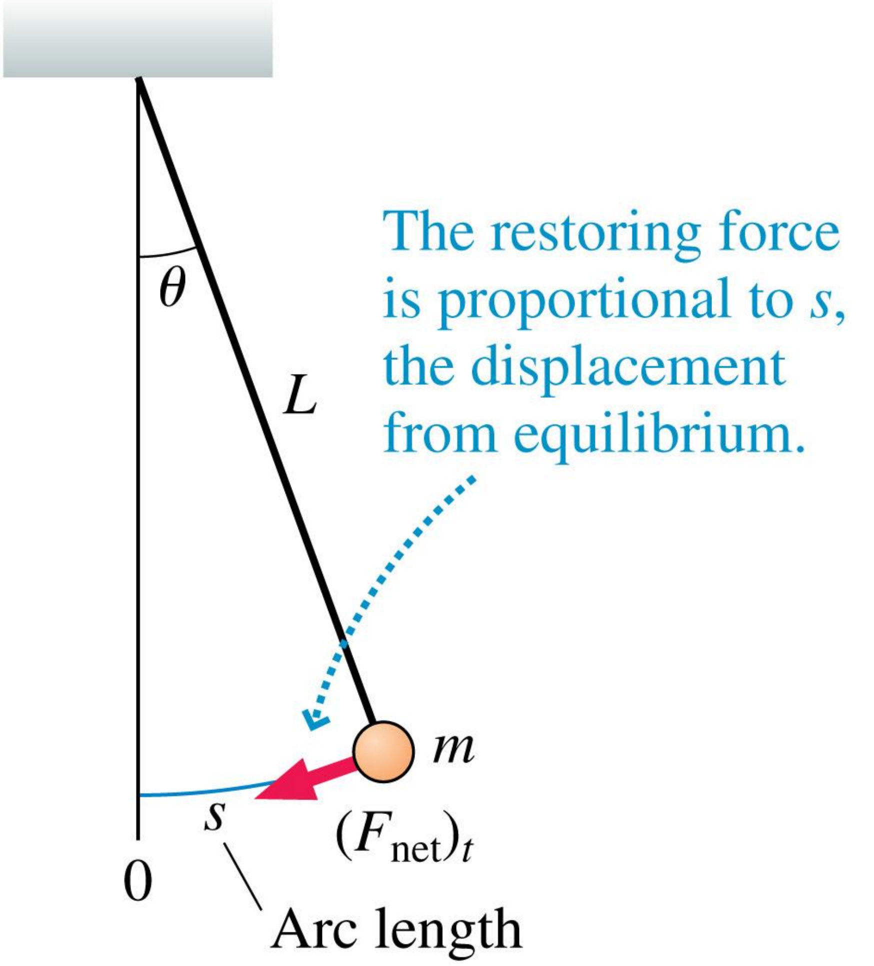
Section 14.5 Pendulum Motion

Pendulum Motion

- The tangential restoring force for a pendulum of length L displaced by arc length s is

$$(F_{\text{net}})_t = -\frac{mg}{L}s$$

- This is the same linear restoring force as the spring but with the constants mg/L instead of k.



Pendulum Motion

- The oscillation of a pendulum is simple harmonic motion; the equations of motion can be written for the arc length or the angle:

$$s(t) = A\cos(2\pi f t)$$

or

$$\theta(t) = \theta_{\max}\cos(2\pi f t)$$

High frequency
Short pendulum, strong gravity

Low frequency
Long pendulum, weak gravity

$$f = \frac{1}{2\pi}\sqrt{\frac{g}{L}}$$

Stronger gravity will increase the restoring force. This *increases* the frequency.

Increasing the length of the pendulum *decreases* the frequency.

Pendulum Motion

- The frequency can be obtained from the equation for the frequency of the mass on a spring by substituting mg/L in place of k:

$$f = \frac{1}{2\pi}\sqrt{\frac{g}{L}} \quad \text{and} \quad T = 2\pi\sqrt{\frac{L}{g}}$$

Frequency and period of a pendulum of length L with free-fall acceleration g

High frequency
Short pendulum, strong gravity

Low frequency
Long pendulum, weak gravity

$$f = \frac{1}{2\pi}\sqrt{\frac{g}{L}}$$

Stronger gravity will increase the restoring force. This *increases* the frequency.

Increasing the length of the pendulum *decreases* the frequency.

Pendulum Motion

- As for a mass on a spring, the frequency does not depend on the amplitude. Note also that **the frequency, and hence the period, is independent of the mass.** It depends only on the length of the pendulum.

High frequency
Short pendulum,
strong gravity

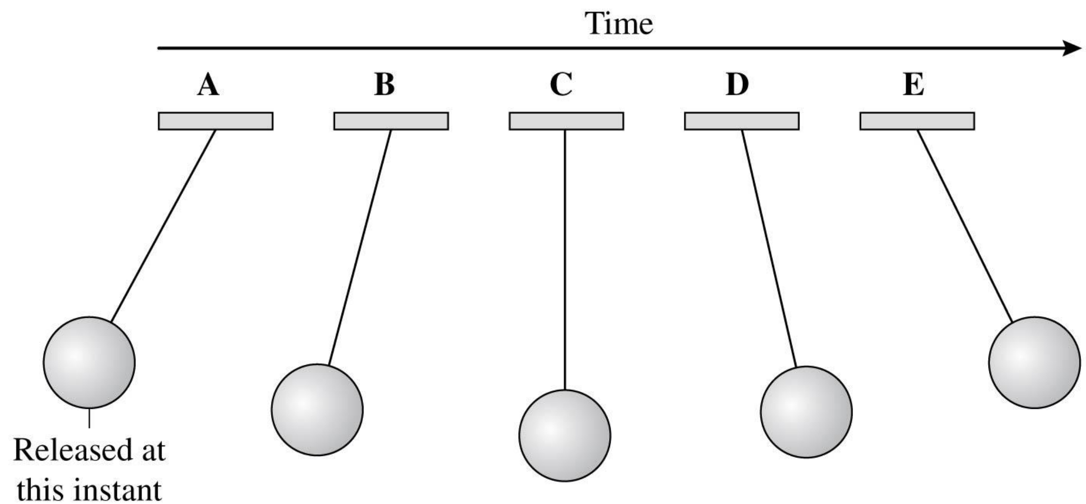
Low frequency
Long pendulum,
weak gravity

Stronger gravity will increase the restoring force. This *increases* the frequency.

$$f = \frac{1}{2\pi}\sqrt{\frac{g}{L}}$$

Increasing the length of the pendulum *decreases* the frequency.

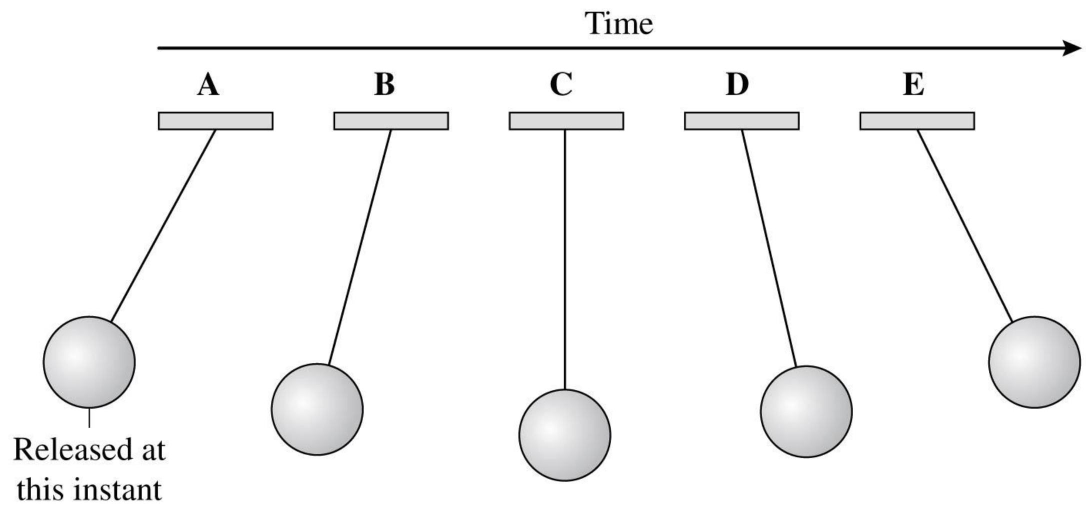
QuickCheck 14.15



A pendulum is pulled to the side and released. The mass swings to the right as shown. The diagram shows positions for half of a complete oscillation.

1. At which point or points is the speed the highest?
2. At which point or points is the acceleration the greatest?
3. At which point or points is the restoring force the greatest?

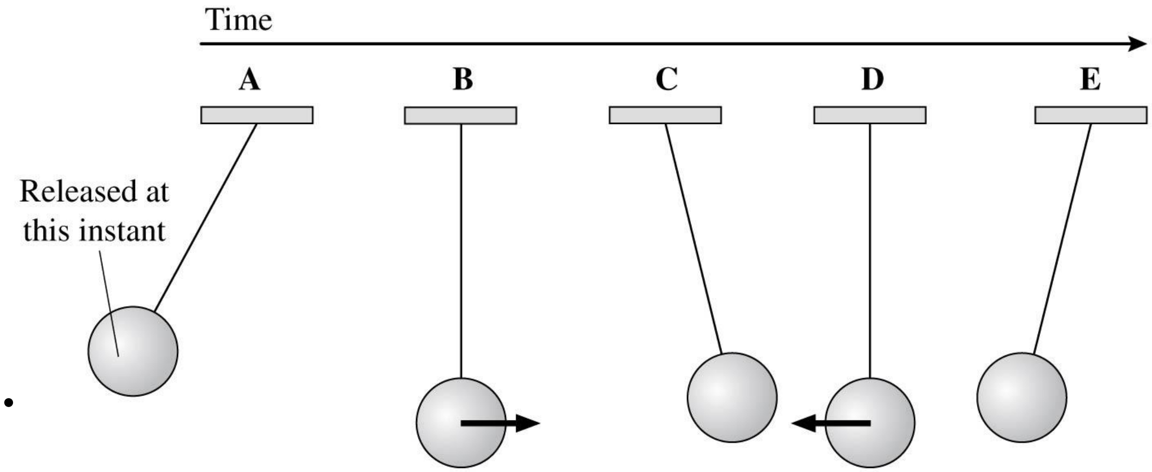
QuickCheck 14.15



A pendulum is pulled to the side and released. The mass swings to the right as shown. The diagram shows positions for half of a complete oscillation.

1. At which point or points is the speed the highest? C
2. At which point or points is the acceleration the greatest? A, E
3. At which point or points is the restoring force the greatest? A, E

QuickCheck 14.16

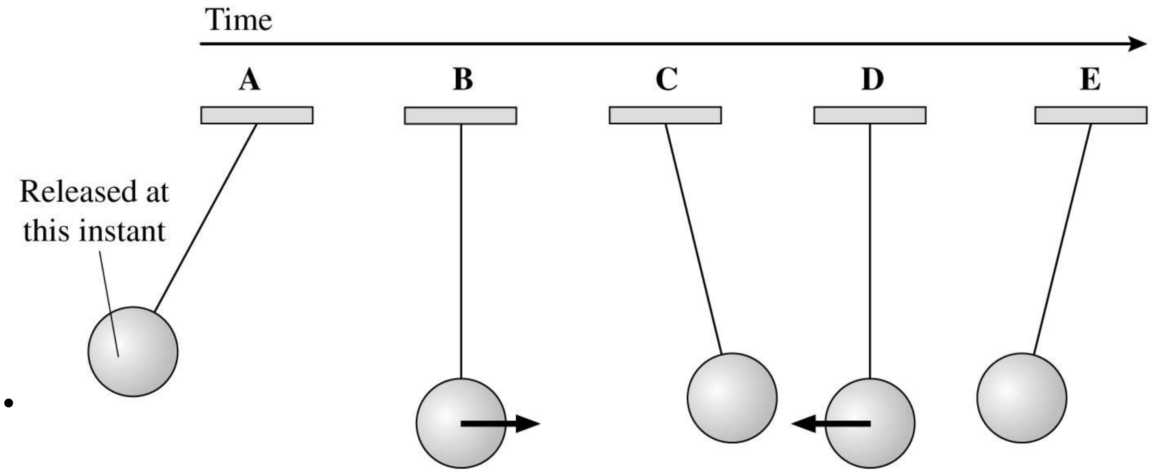


A mass on the end of a string is pulled to the side and released.

1. At which time or times shown is the acceleration zero?
2. At which time or times shown is the kinetic energy a maximum?
3. At which time or times shown is the potential energy a maximum?

QuickCheck 14.16

A mass on the end of a string is pulled to the side and released.

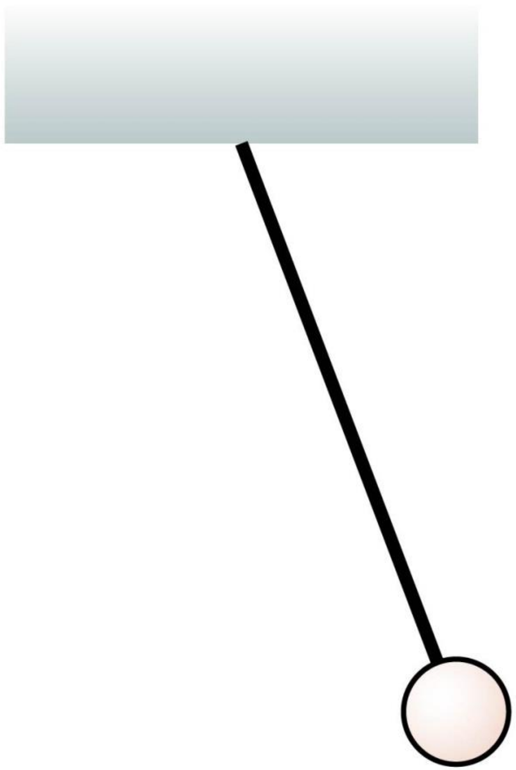


1. At which time or times shown is the acceleration zero? B, D
2. At which time or times shown is the kinetic energy a maximum? B, D
3. At which time or times shown is the potential energy a maximum? A, C, E

QuickCheck 14.17

A ball on a massless, rigid rod oscillates as a simple pendulum with a period of 2.0 s. If the ball is replaced with another ball having twice the mass, the period will be

A. 1.0 s
B. 1.4 s
C. 2.0 s
D. 2.8 s
E. 4.0 s



QuickCheck 14.17

A ball on a massless, rigid rod oscillates as a simple pendulum with a period of 2.0 s. If the ball is replaced with another ball having twice the mass, the period will be

A. 1.0 s
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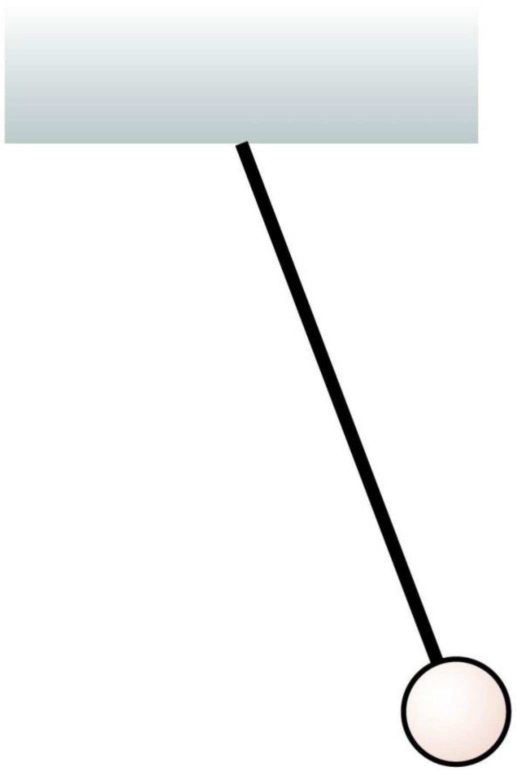
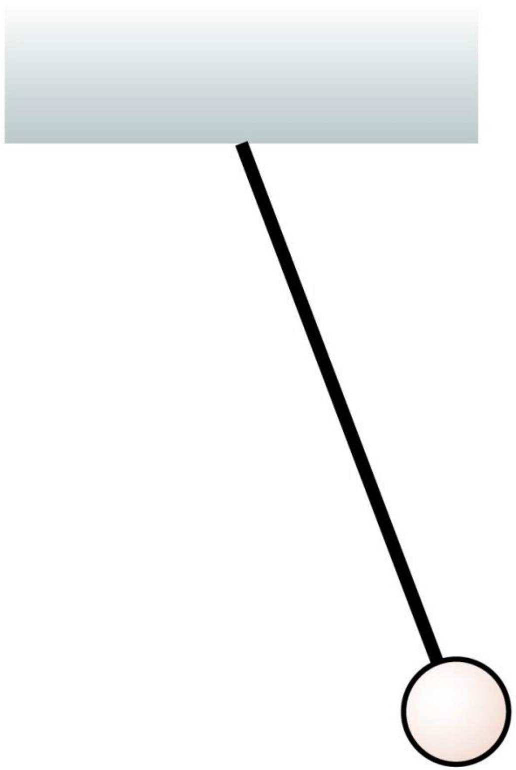


QuickCheck 14.18

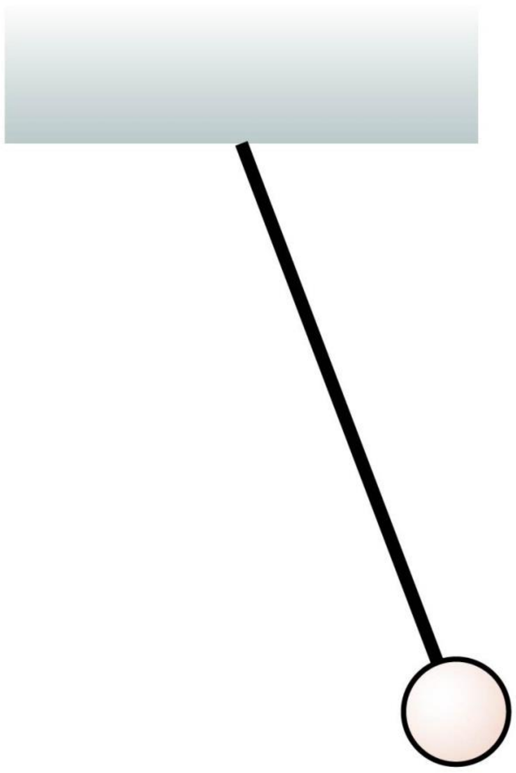
On Planet X, a ball on a massless, rigid rod oscillates as a simple pendulum with a period of 2.0 s. If the pendulum is taken to the moon of Planet X, where the free-fall acceleration *g* is half as big, the period will be



A. 1.0 s
B. 1.4 s
C. 2.0 s
D. 2.8 s
E. 4.0 s

QuickCheck 14.18

On Planet X, a ball on a massless, rigid rod oscillates as a simple pendulum with a period of 2.0 s. If the pendulum is taken to the moon of Planet X, where the free-fall acceleration g is half as big, the period will be



A. 1.0 s

B. 1.4 s

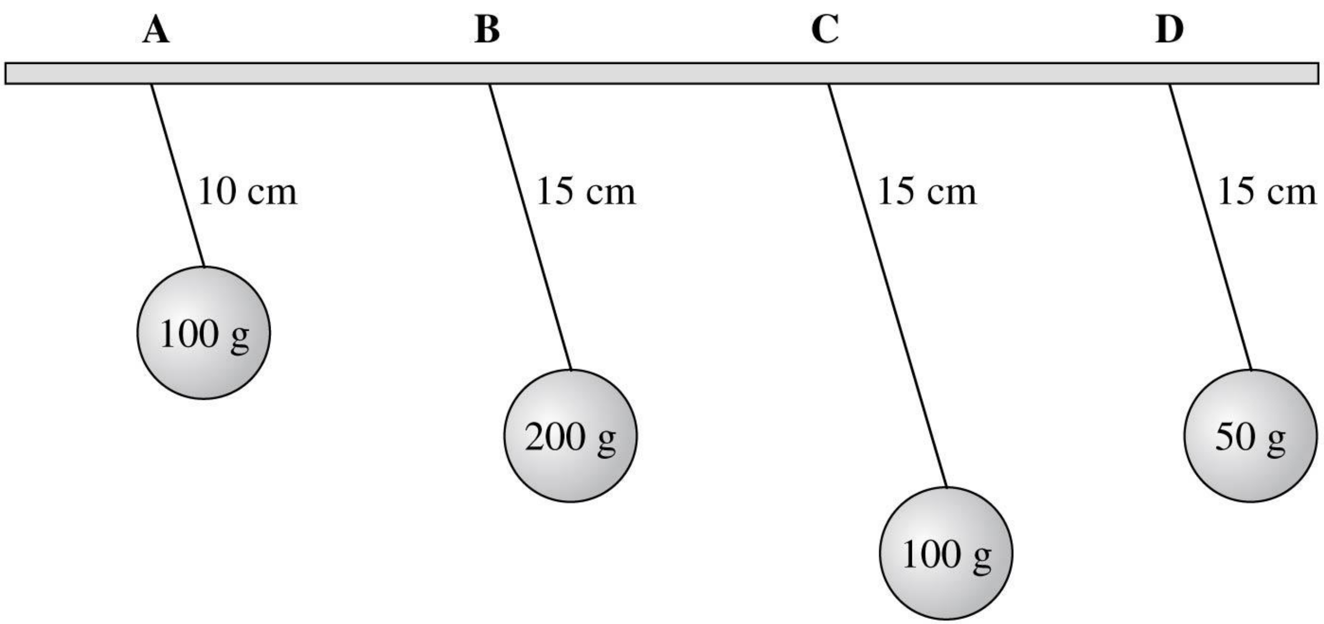
C. 2.0 s

✔ D. 2.8 s $T \propto \dfrac{1}{\sqrt{g}}$

E. 4.0 s

QuickCheck 14.19

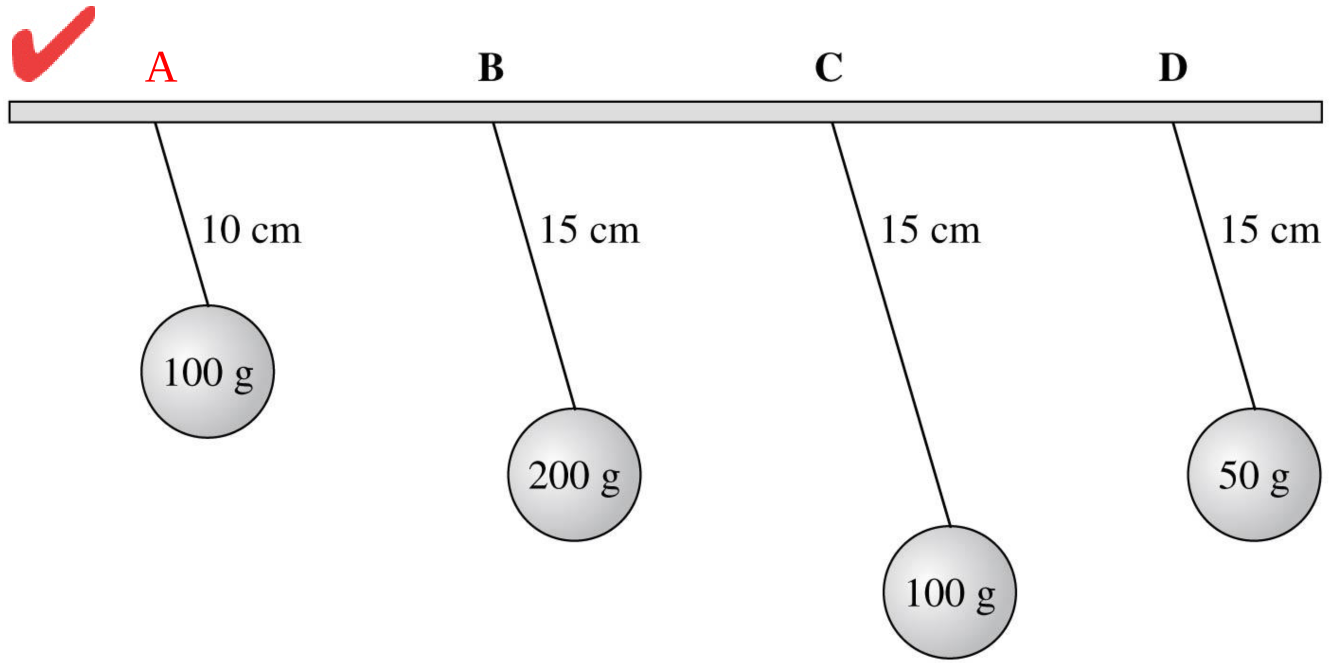
A series of pendulums with different length strings and different masses is shown below. Each pendulum is pulled to the side by the same (small) angle, the pendulums are released, and they begin to swing from side to side.



Which of the pendulums oscillates with the highest frequency?

QuickCheck 14.19

A series of pendulums with different length strings and different masses is shown below. Each pendulum is pulled to the side by the same (small) angle, the pendulums are released, and they begin to swing from side to side.



Which of the pendulums oscillates with the highest frequency?

Example 14.10 Designing a pendulum for a clock

A grandfather clock is designed so that one swing of the pendulum in either direction takes 1.00 s. What is the length of the pendulum?

PREPARE One period of the pendulum is two swings, so the period is $T = 2.00$ s.

Example 14.10 Designing a pendulum for a clock (cont.)

SOLVE The period is independent of the mass and depends only on the length. From Equation 14.27,

$$T = \frac{1}{f} = 2\pi\sqrt{\frac{L}{g}}$$

Solving for L, we find

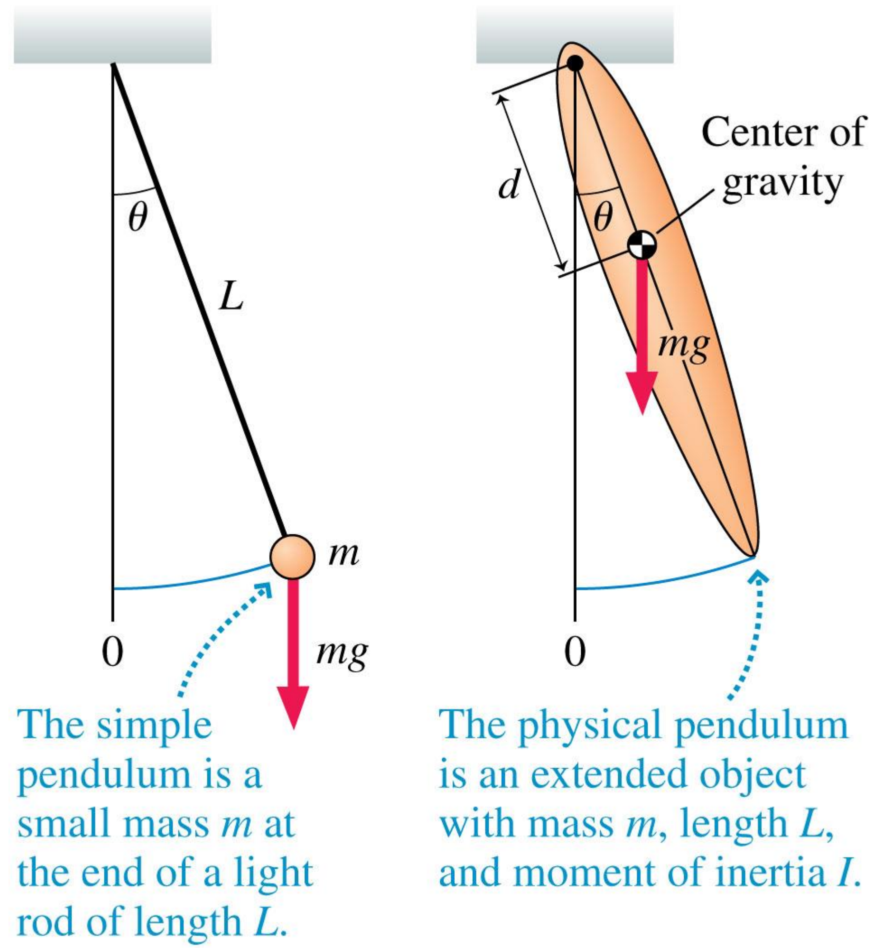
$$L = g\left(\frac{T}{2\pi}\right)^2 = (9.80\ \text{m/s}^2)\left(\frac{2.00\ \text{s}}{2\pi}\right)^2 = 0.993\ \text{m}$$

Example 14.10 Designing a pendulum for a clock (cont.)

ASSESS A pendulum clock with a "tick" or "tock" each second requires a long pendulum of about 1 m—which is why these clocks were original known as "tall case clocks."

Physical Pendulums and Locomotives

- A **physical pendulum** is a pendulum whose mass is distributed along its length.
- The position of the center of gravity of the physical pendulum is at a distance *d* from the pivot.



Physical Pendulums and Locomotives

- The moment of inertia I is a measure of an object's resistance to rotation. Increasing the moment of inertia while keeping other variables equal should cause the frequency to decrease. In an expression for the frequency of the physical pendulum, we would expect I to appear in the denominator.
- When the pendulum is pushed to the side, a gravitational torque pulls it back. The greater the distance d of the center of gravity from the pivot point, the greater the torque. Increasing this distance while keeping the other variables constant should cause the frequency to increase. In an expression for the frequency of the physical pendulum, we would expect d to appear in the numerator.

$$f = \frac{1}{2\pi}\sqrt{\frac{mgd}{I}}$$

Frequency of a physical pendulum of mass m, moment of inertia I, with center of gravity distance d from the pivot

Example 14.11 Finding the frequency of a swinging leg

A student in a biomechanics lab measures the length of his leg, from hip to heel, to be 0.90 m. What is the frequency of the pendulum motion of the student's leg? What is the period?

PREPARE We can model a human leg reasonably well as a rod of uniform cross section, pivoted at one end (the hip). Recall from Chapter 7 that the moment of inertia of a rod pivoted about its end is $1/3mL^2$. The center of gravity of a uniform leg is at the midpoint, so $d = L/2$.

Example 14.11 Finding the frequency of a swinging leg (cont.)

SOLVE The frequency of a physical pendulum is given by Equation 14.28. Before we put in numbers, we will use symbolic relationships and simplify:

$$f = \frac{1}{2\pi}\sqrt{\frac{mgd}{I}} = \frac{1}{2\pi}\sqrt{\frac{mg(L/2)}{\frac{1}{3}mL^2}} = \frac{1}{2\pi}\sqrt{\frac{3}{2}\frac{g}{L}}$$

Example 14.11 Finding the frequency of a swinging leg (cont.)

The expression for the frequency is similar to that for the simple pendulum, but with an additional numerical factor of 3/2 inside the square root. The numerical value of the frequency is

$$f = \frac{1}{2\pi}\sqrt{\left(\frac{3}{2}\right)\left(\frac{9.8\ \text{m/s}^2}{0.90\ \text{m}}\right)} = 0.64\ \text{Hz}$$

The period is

$$T = \frac{1}{f} = 1.6\ \text{s}$$

Example 14.11 Finding the frequency of a swinging leg (cont.)

ASSESS Notice that we didn't need to know the mass of the leg to find the period. The period of a physical pendulum does not depend on the mass, just as it doesn't for the simple pendulum. The period depends only on the *distribution* of mass. When you walk, swinging your free leg forward to take another stride corresponds to half a period of this pendulum motion. For a period of 1.6 s, this is 0.80 s. For a normal walking pace, one stride in just under one second sounds about right.

Try It Yourself: How Do You Hold Your Arms?

You maintain your balance when walking or running by moving your arms back and forth opposite the motion of your legs. You hold your arms so that the natural period of their motion matches that of your legs. At a normal walking pace, your arms are extended and naturally swing at the same period as your legs. When you run, your gait is more rapid. To decrease the period of the pendulum motion of your arms to match, you bend them at the elbows, shortening their effective length and increasing the natural frequency of oscillation. To test this for yourself, try running fast with your arms fully extended. It's quite awkward!

Section 14.6 Damped Oscillations

Damped Oscillation

- An oscillation that runs down and stops is called a **damped oscillation.**
- For a pendulum, the main energy loss is air resistance, or the *drag force*.
- As an oscillation decays, the *rate* of decay decreases; the difference between successive peaks is less.

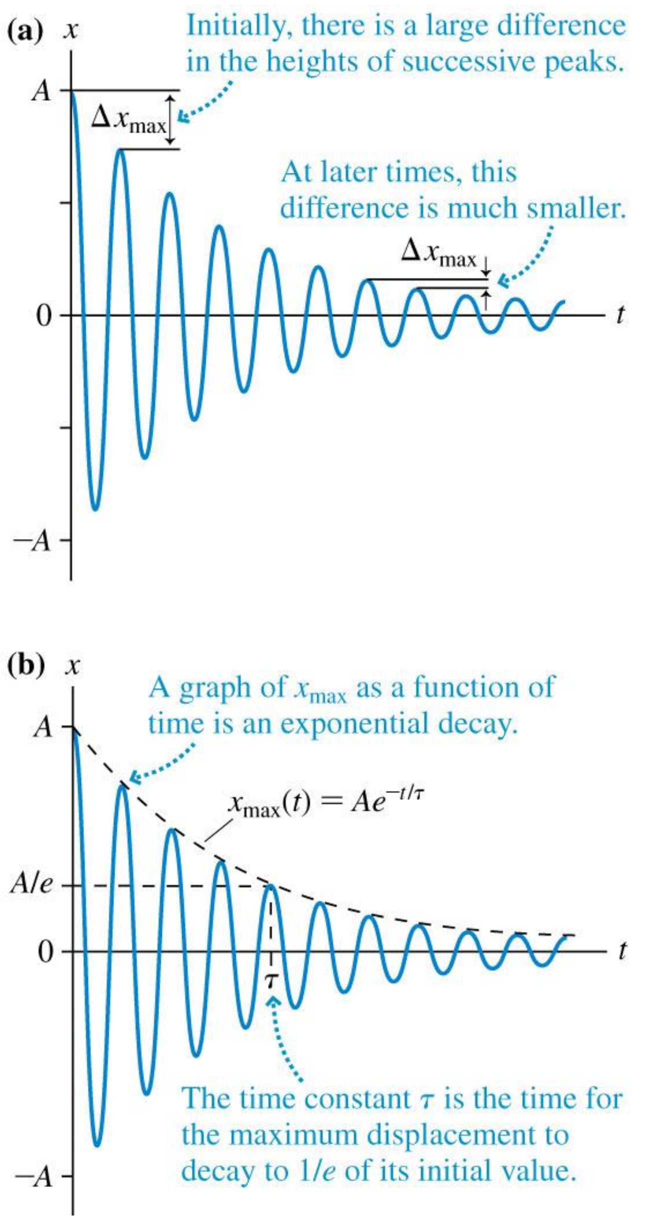
Damped Oscillation

- Damped oscillation causes $x_{\max}$ to decrease with time as

$$x_{\max}(t) = Ae^{-t/\tau}$$

where $e \approx 2.718$ is the base of the natural logarithm and A is the initial amplitude.

- The steady decrease in $x_{\max}$ is the **exponential decay.**
- The constant τ is the **time constant.**



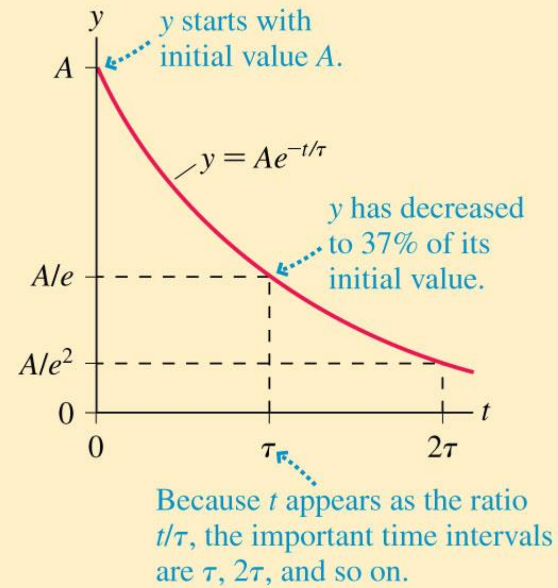
Damped Oscillation

Exponential decay

Exponential decay occurs when a quantity y is proportional to the number e taken to the power $-t/\tau$. The quantity τ is known as the **time constant**. We write this mathematically as

$$y = Ae^{-t/\tau}$$

y is proportional to $e^{-t/\tau}$



SCALING Whenever t increases by one time constant, y decreases by a factor of $1/e$. For instance:

- At time $t = 0$, $y = A$.
- Increasing time to $t = \tau$ reduces y to A/e.
- A further increase to $t = 2\tau$ reduces y by another factor of $1/e$ to A/e^2.

Generally, we can say:

At $t = n\tau$, y has the value A/e^n.

LIMITS As t becomes large, y becomes very small and approaches zero.

Exercises 14–17

Text: p. 456

Example Problem

A 500 g mass on a string oscillates as a pendulum. The pendulum's energy decays to 50% of its initial value in 30 s. What is the value of the damping constant?

Different Amounts of Damping

- Mathematically, the oscillation never ceases, however the amplitude will be so small that it is undetectable.
- For practical purposes, the time constant τ is the *lifetime* of the oscillation—the measure of how long it takes to decay.
- If $\tau << T$, the oscillation persists over many periods and the amplitude decrease is small.
- If $\tau >> T$, the oscillation will damp quickly.

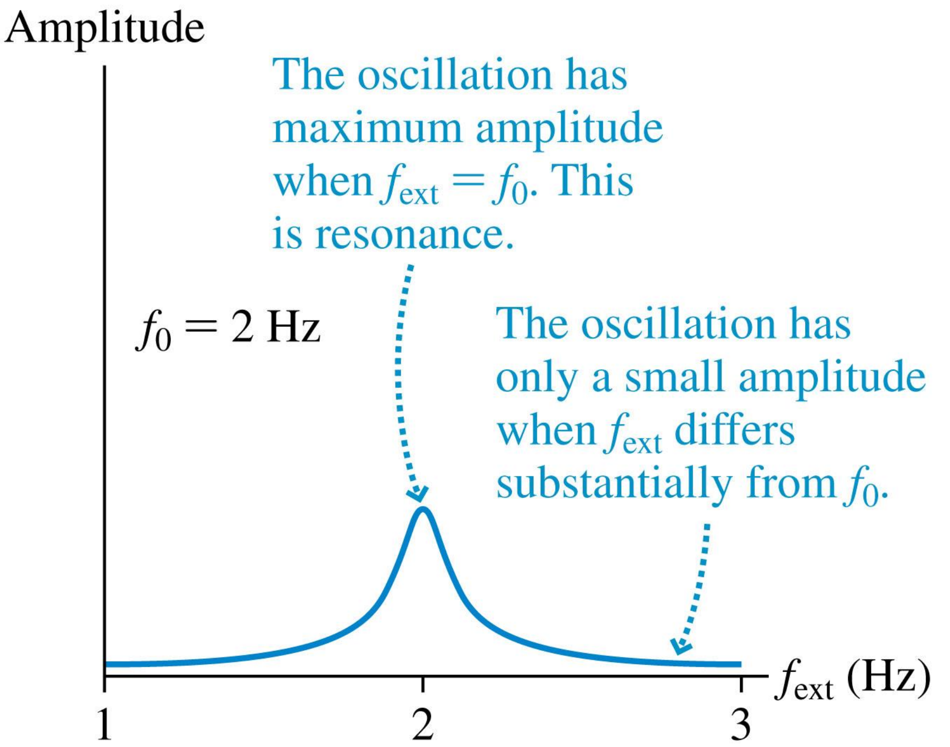
Section 14.7 Driven Oscillations and Resonance

Driven Oscillations and Resonance

- **Driven oscillation** is the motion of an oscillator that is subjected to a periodic external force.
- The **natural frequency** f_0 of an oscillator is the frequency of the system if it is displaced from equilibrium and released.
- The **driving frequency** f_{ext} is a periodic external force of frequency. It is independent of the natural frequency.

Driven Oscillations and Resonance

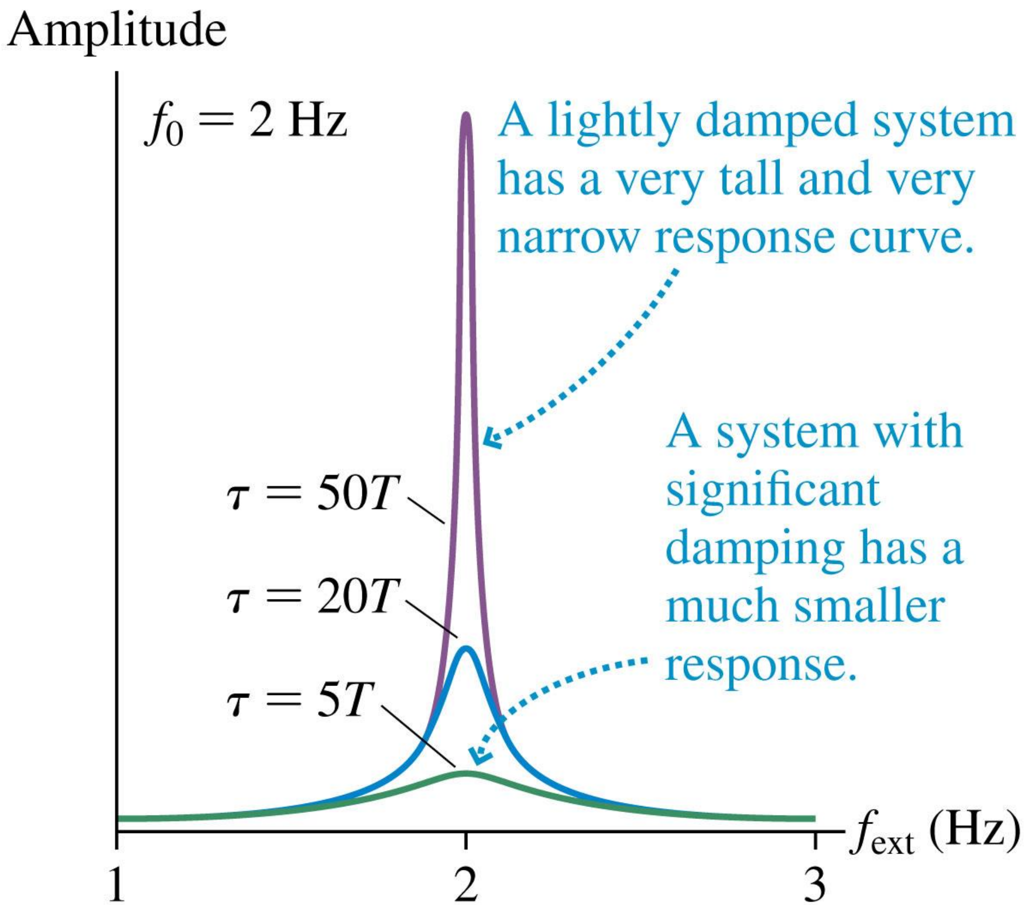
- An oscillator's **response curve** is the graph of amplitude versus driving frequency.
- A **resonance** is the large-amplitude response to a driving force whose frequency matches the natural frequency of the system.



- The natural frequency is often called **the resonance frequency.**

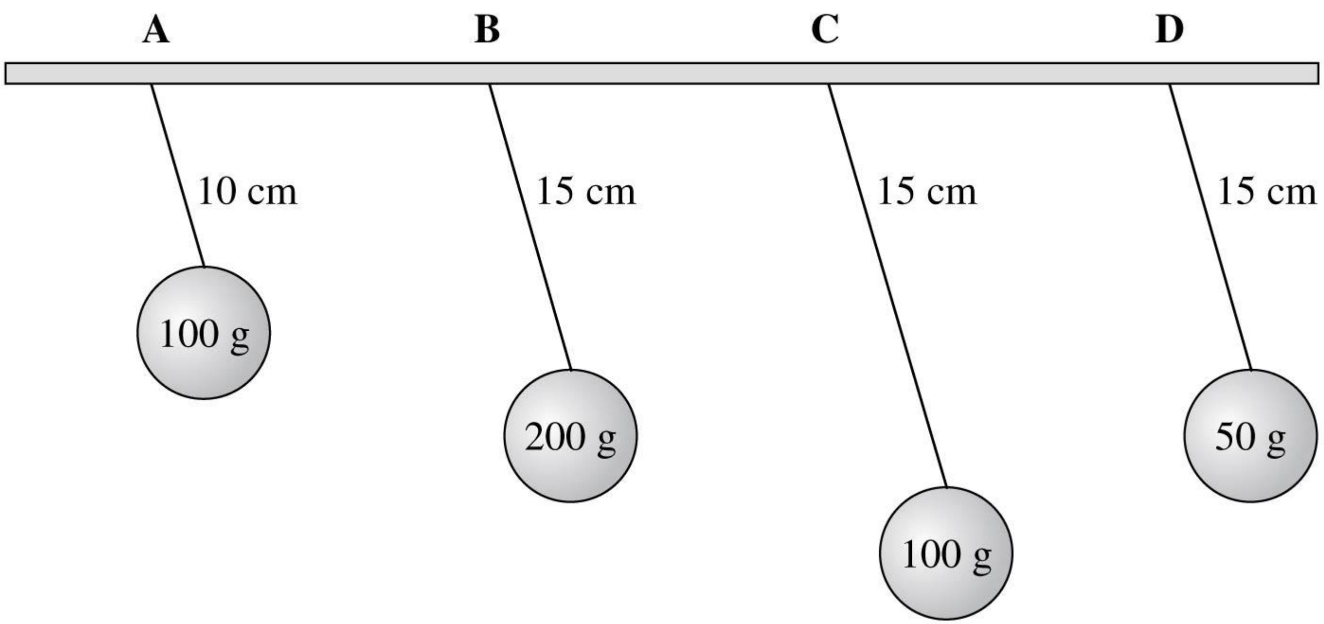
Driven Oscillations and Resonance

- The amplitude can become exceedingly large when the frequencies match, especially when there is very little damping.



QuickCheck 14.20

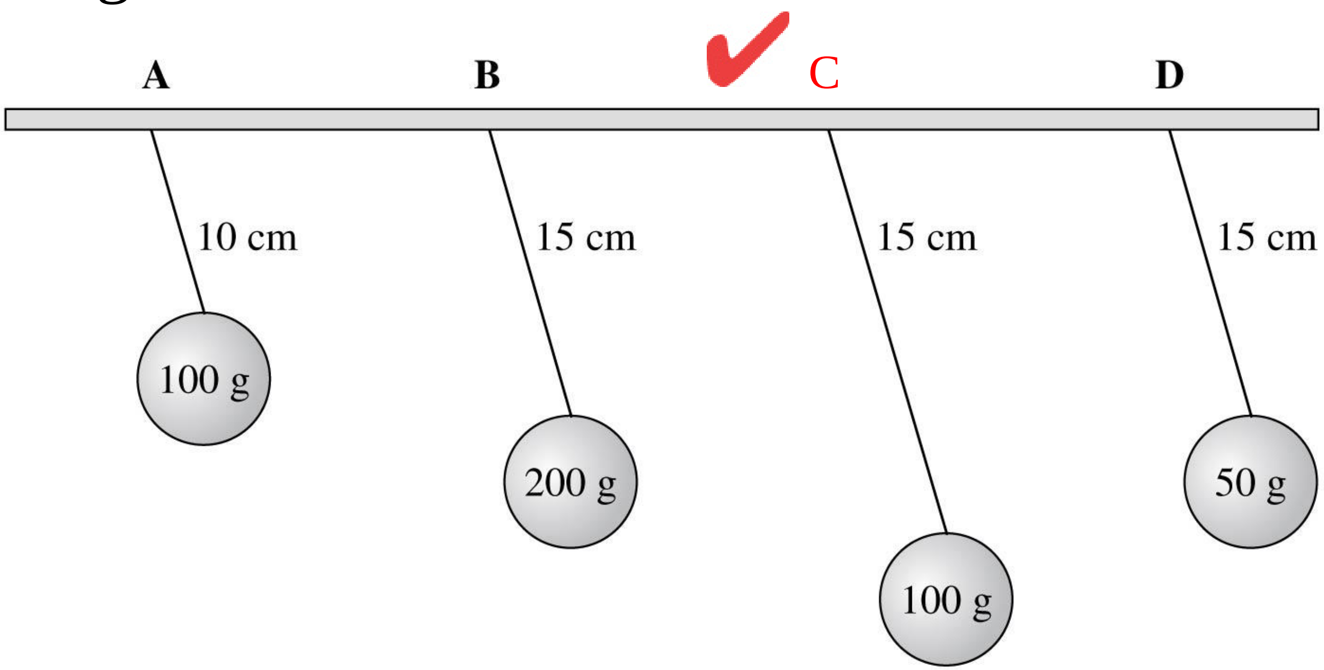
A series of pendulums with different length strings and different masses is shown below. Each pendulum is pulled to the side by the same (small) angle, the pendulums are released, and they begin to swing from side to side.



Which of the pendulums oscillates with the lowest frequency?

QuickCheck 14.20

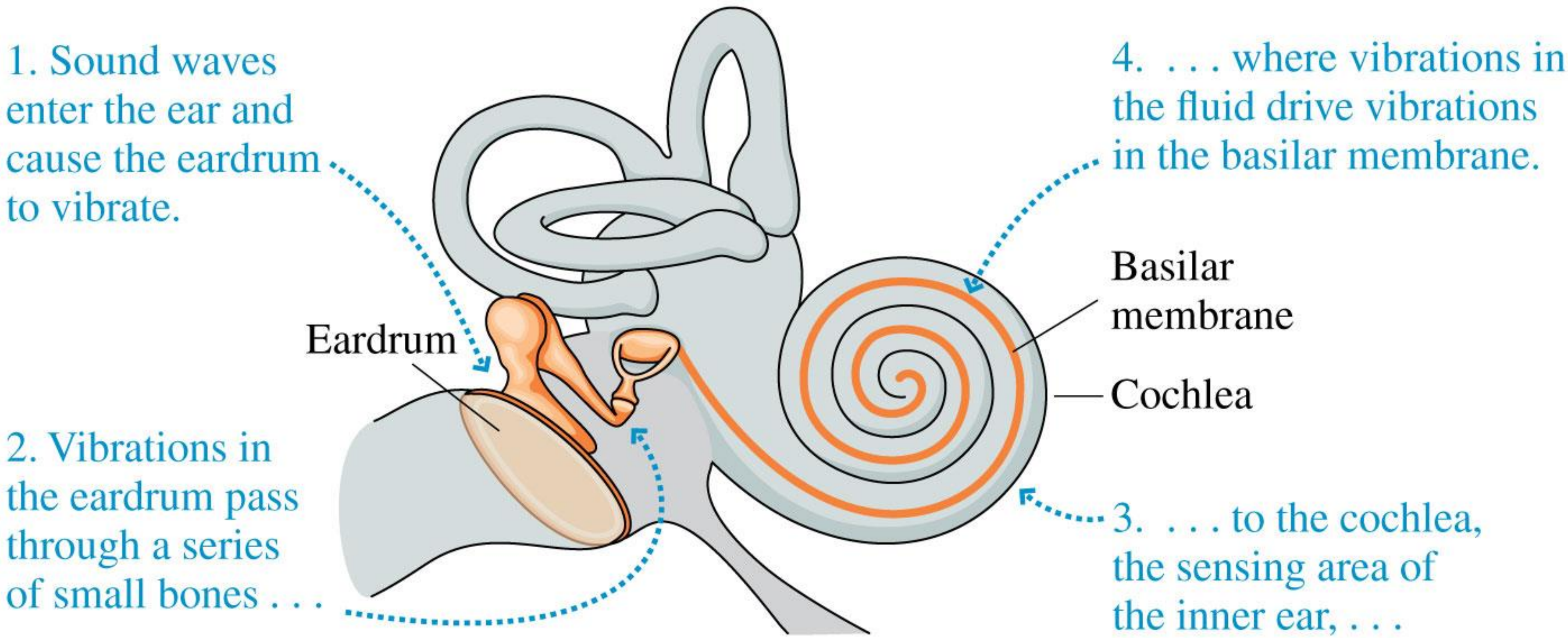
A series of pendulums with different length strings and different masses is shown below. Each pendulum is pulled to the side by the same (small) angle, the pendulums are released, and they begin to swing from side to side.



Which of the pendulums oscillates with the lowest frequency?

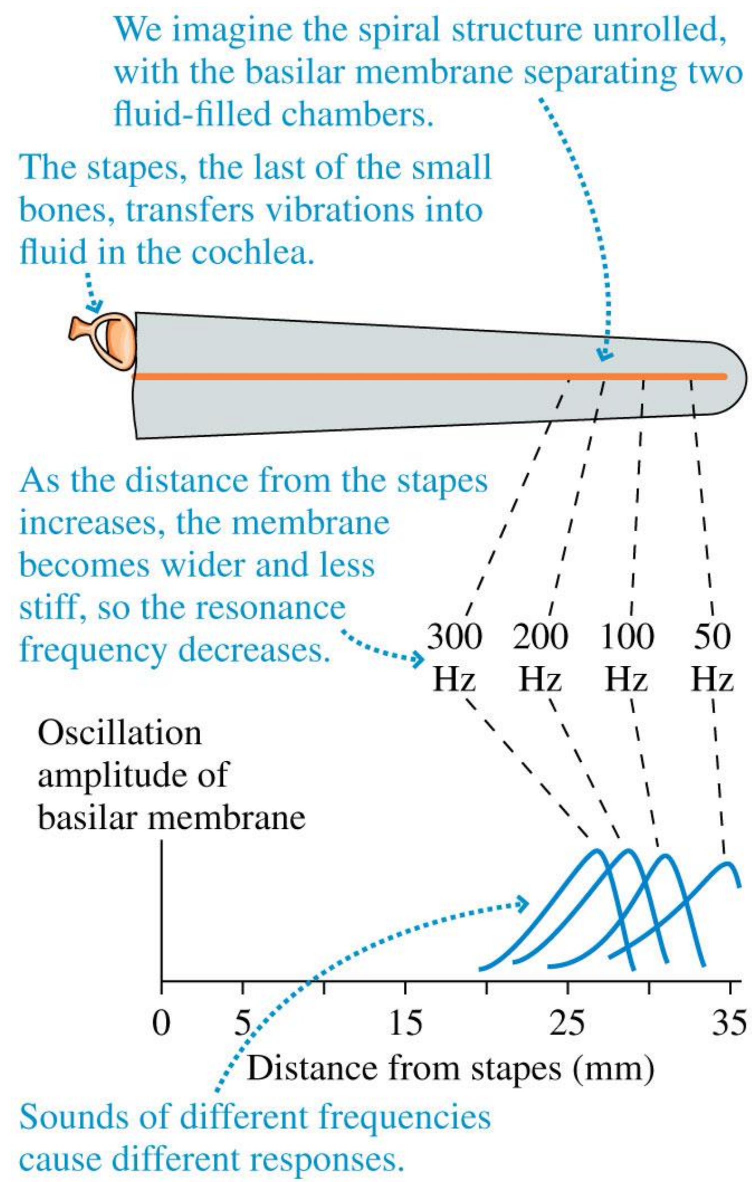
Resonance and Hearing

- Resonance in a system means that certain frequencies produce a large response and others do not. Resonances enable frequency discrimination in the ear.



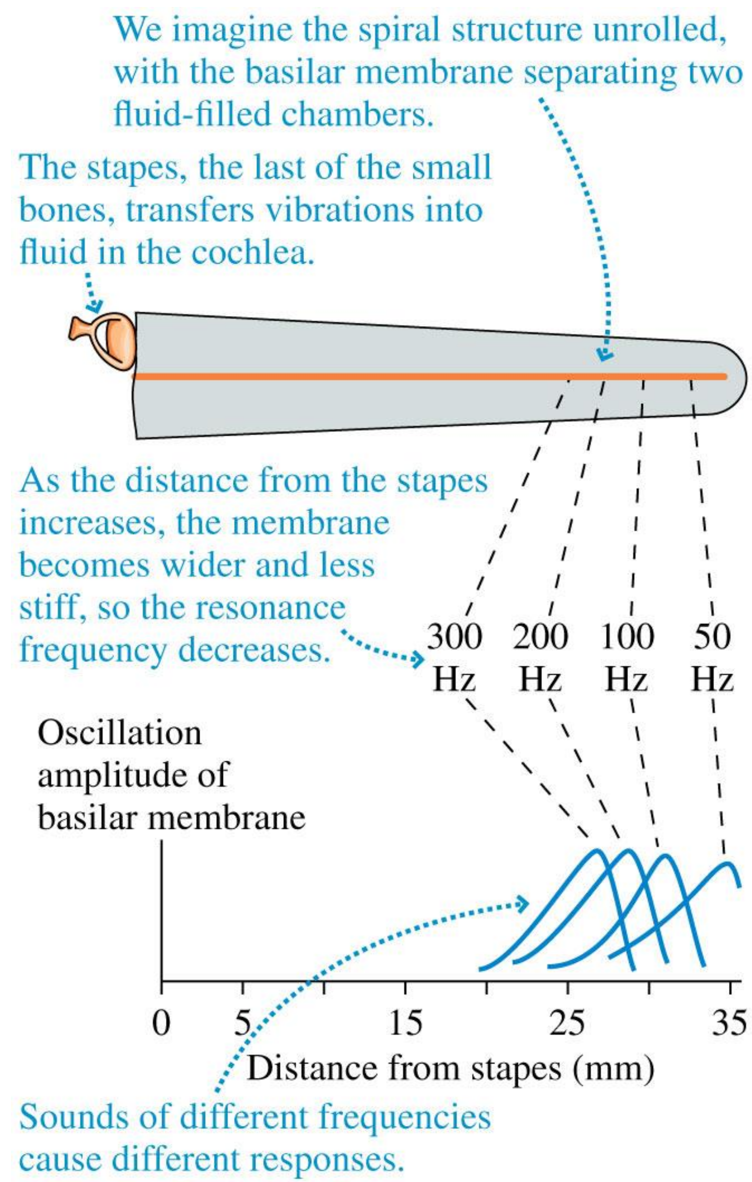
Resonance and Hearing

- In a simplified model of the cochlea, sound waves produce large-amplitude vibrations of the basilar membrane at resonances. Lower-frequency sound causes a response farther from the stapes.
- Hair cells sense the vibration and send signals to the brain.



Resonance and Hearing

- The fact that different frequencies produce maximal response at different positions allows your brain to very accurately determine frequency because a small shift in frequency causes a detectable change in the position of the maximal response.



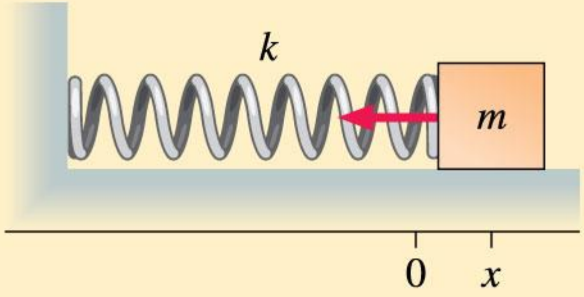
Summary: General Principles

Frequency and Period

Text: p. 462

SHM occurs when a **linear restoring force** acts to return a system to an equilibrium position. Frequency and the period depend on the force and on masses or lengths. **Frequency and period do not depend on amplitude.**

Mass on spring

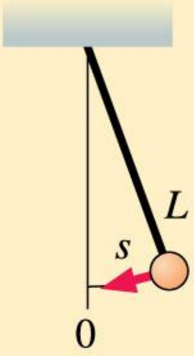


$$(F_{\text{net}})_x = -kx$$

The frequency and period of a mass on a spring depend on the mass and the spring constant: They are the same for horizontal and vertical systems.

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \qquad T = 2\pi\sqrt{\frac{m}{k}}$$

Pendulum



$$(F_{\text{net}})_t = -\left(\frac{mg}{L}\right)s$$

The frequency and period of a pendulum depend on the length and the free-fall acceleration. They do not depend on the mass.

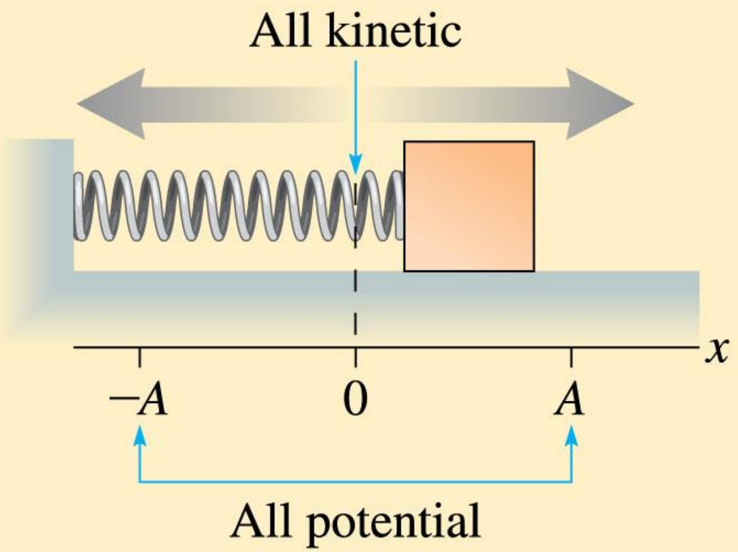
$$f = \frac{1}{2\pi}\sqrt{\frac{g}{L}} \qquad T = 2\pi\sqrt{\frac{L}{g}}$$

Summary: General Principles

Energy

If there is no friction or dissipation, kinetic and potential energies are alternately transformed into each other in SHM, with the sum of the two conserved.

$$E = \frac{1}{2}mv_x^2 + \frac{1}{2}kx^2$$
$$= \frac{1}{2}mv_{\max}^2$$
$$= \frac{1}{2}kA^2$$

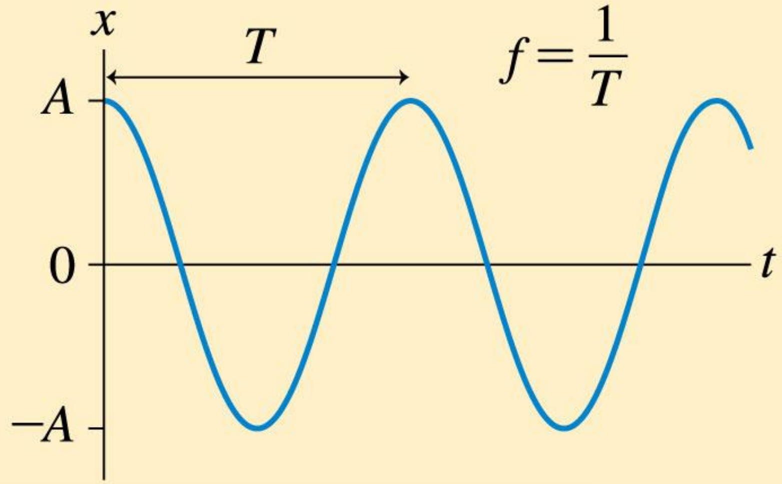


Text: p. 462

Summary: Important Concepts

Oscillation

An **oscillation** is a repetitive motion about an equilibrium position. The **amplitude** A is the maximum displacement from equilibrium. The period T is the time for one cycle. We may also characterize an oscillation by its frequency f.



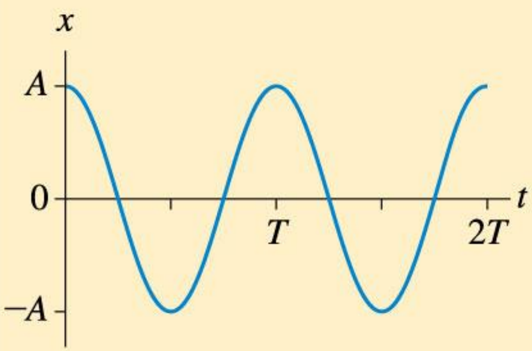
Text: p. 462

Summary: Important Concepts

Simple Harmonic Motion (SHM)

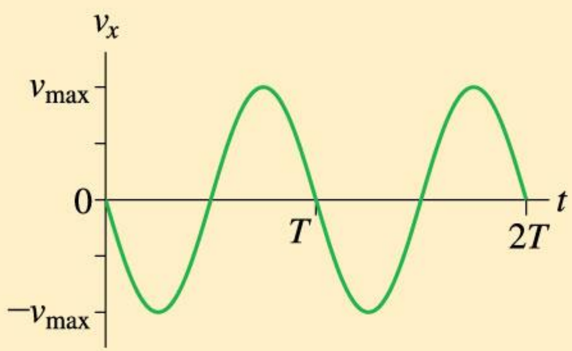
SHM is an oscillation that is described by a sinusoidal function. All systems that undergo SHM can be described by the same functional forms.

Position-versus-time is a cosine function.



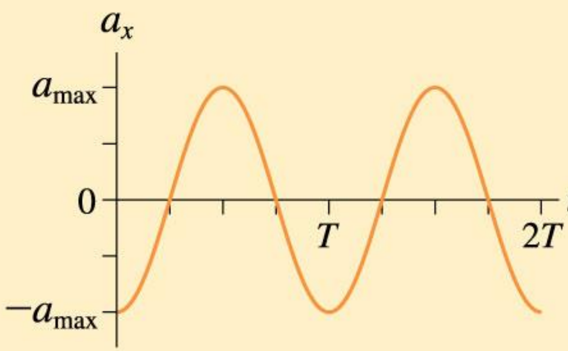
$$x(t) = A\cos(2\pi ft)$$

Velocity-versus-time is an inverted sine function.



$$v_x(t) = -[(2\pi f)A]\sin(2\pi ft)$$

Acceleration-versus-time is an inverted cosine function.



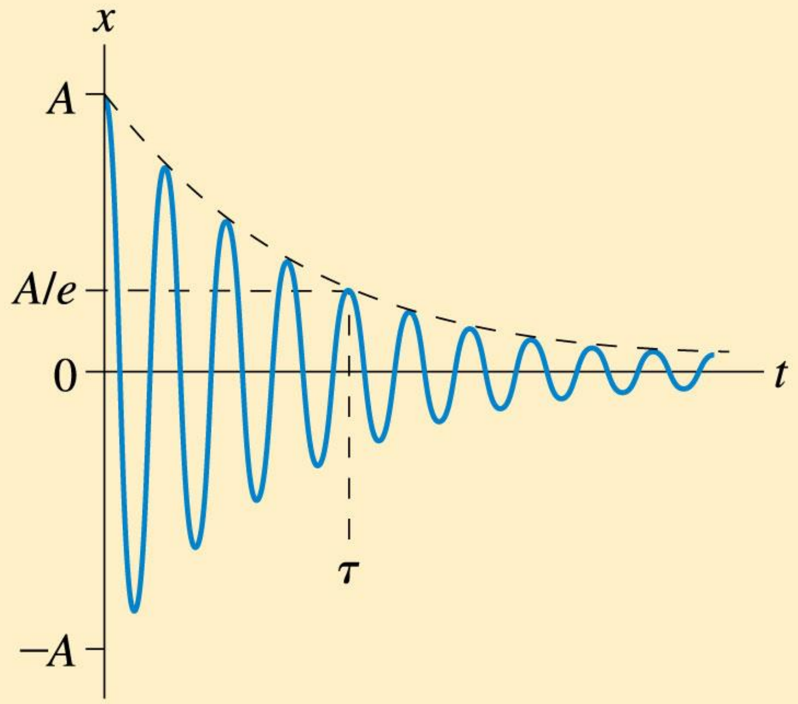
$$a_x(t) = -[(2\pi f)^2A]\cos(2\pi ft)$$

Text: p. 462

Summary: Applications

Damping

Simple harmonic motion with damping (due to drag) decreases in amplitude over time. The **time constant** τ determines how quickly the amplitude decays.



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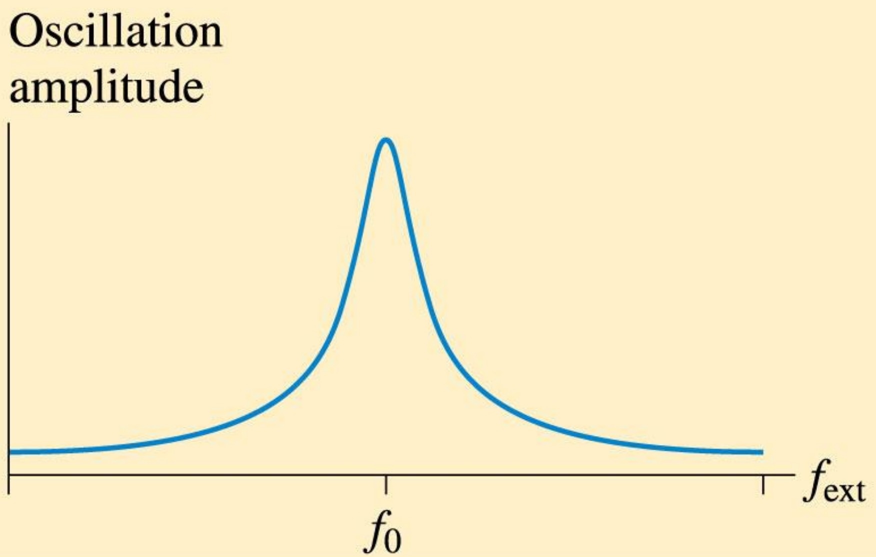
Summary: Applications

Resonance

A system that oscillates has a **natural frequency** of oscillation f_0. **Resonance** occurs if the system is driven with a frequency f_{ext} that matches this natural frequency. This may produce a large amplitude of oscillation.



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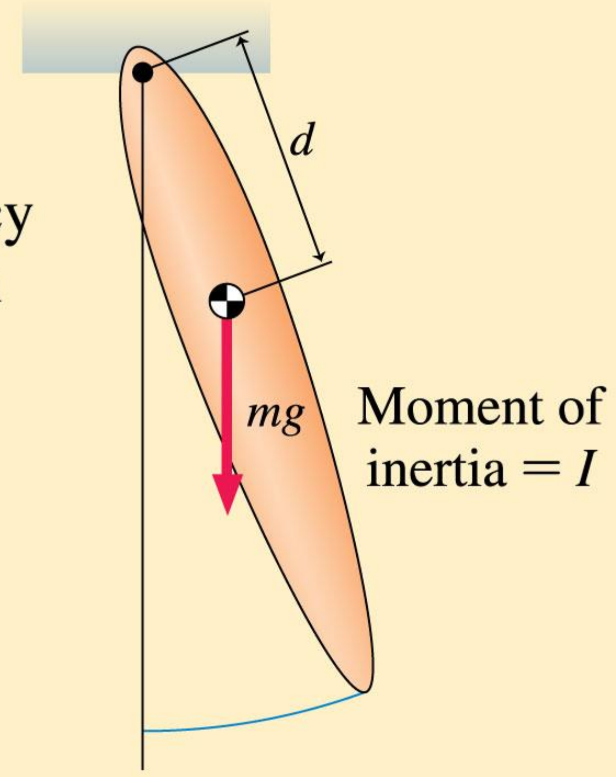
Summary: Applications

Physical pendulum

A **physical pendulum** is a pendulum with mass distributed along its length. The frequency depends on the position of the center of gravity and the moment of inertia.

The motion of legs during walking can be described using a physical pendulum model.



$$f = \frac{1}{2\pi}\sqrt{\frac{mgd}{I}}$$

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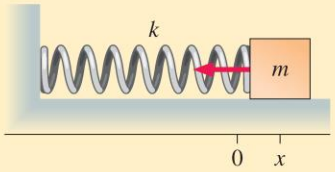
Summary

GENERAL PRINCIPLES

Frequency and Period

SHM occurs when a **linear restoring force** acts to return a system to an equilibrium position. Frequency and the period depend on the force and on masses or lengths. **Frequency and period do not depend on amplitude.**

Mass on spring

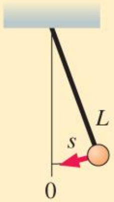


$$(F_{net})_x = -kx$$

The frequency and period of a mass on a spring depend on the mass and the spring constant: They are the same for horizontal and vertical systems.

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \qquad T = 2\pi\sqrt{\frac{m}{k}}$$

Pendulum



$$(F_{net})_t = -\left(\frac{mg}{L}\right)s$$

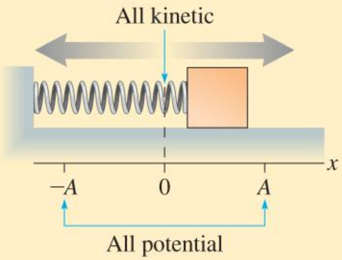
The frequency and period of a pendulum depend on the length and the free-fall acceleration. They do not depend on the mass.

$$f = \frac{1}{2\pi}\sqrt{\frac{g}{L}} \qquad T = 2\pi\sqrt{\frac{L}{g}}$$

Energy

If there is no friction or dissipation, kinetic and potential energies are alternately transformed into each other in SHM, with the sum of the two conserved.

$$E = \frac{1}{2}mv_x^2 + \frac{1}{2}kx^2 = \frac{1}{2}mv_{max}^2 = \frac{1}{2}kA^2$$



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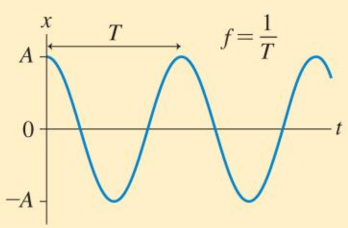
Summary

IMPORTANT CONCEPTS

Oscillation

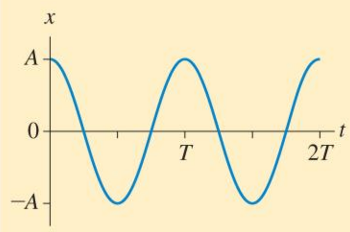
An **oscillation** is a repetitive motion about an equilibrium position. The **amplitude** A is the maximum displacement from equilibrium. The period T is the time for one cycle. We may also characterize an oscillation by its frequency f.



Simple Harmonic Motion (SHM)

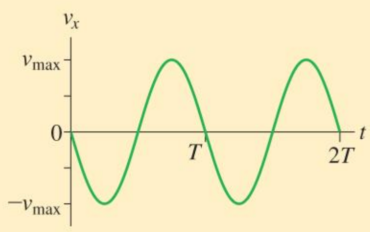
SHM is an oscillation that is described by a sinusoidal function. All systems that undergo SHM can be described by the same functional forms.

Position-versus-time is a cosine function.



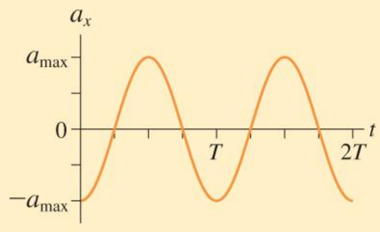
$x(t) = A\cos(2\pi ft)$

Velocity-versus-time is an inverted sine function.



$v_x(t) = -[(2\pi f)A]\sin(2\pi ft)$

Acceleration-versus-time is an inverted cosine function.



$a_x(t) = -[(2\pi f)^2 A]\cos(2\pi ft)$

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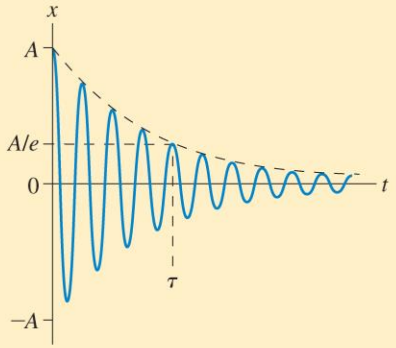
Summary

APPLICATIONS

Damping

Simple harmonic motion with damping (due to drag) decreases in amplitude over time. The **time constant** τ determines how quickly the amplitude decays.



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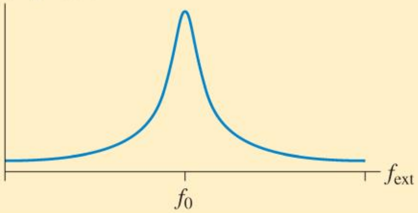

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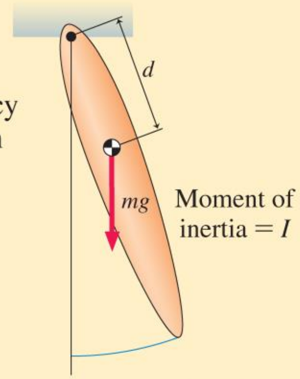

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