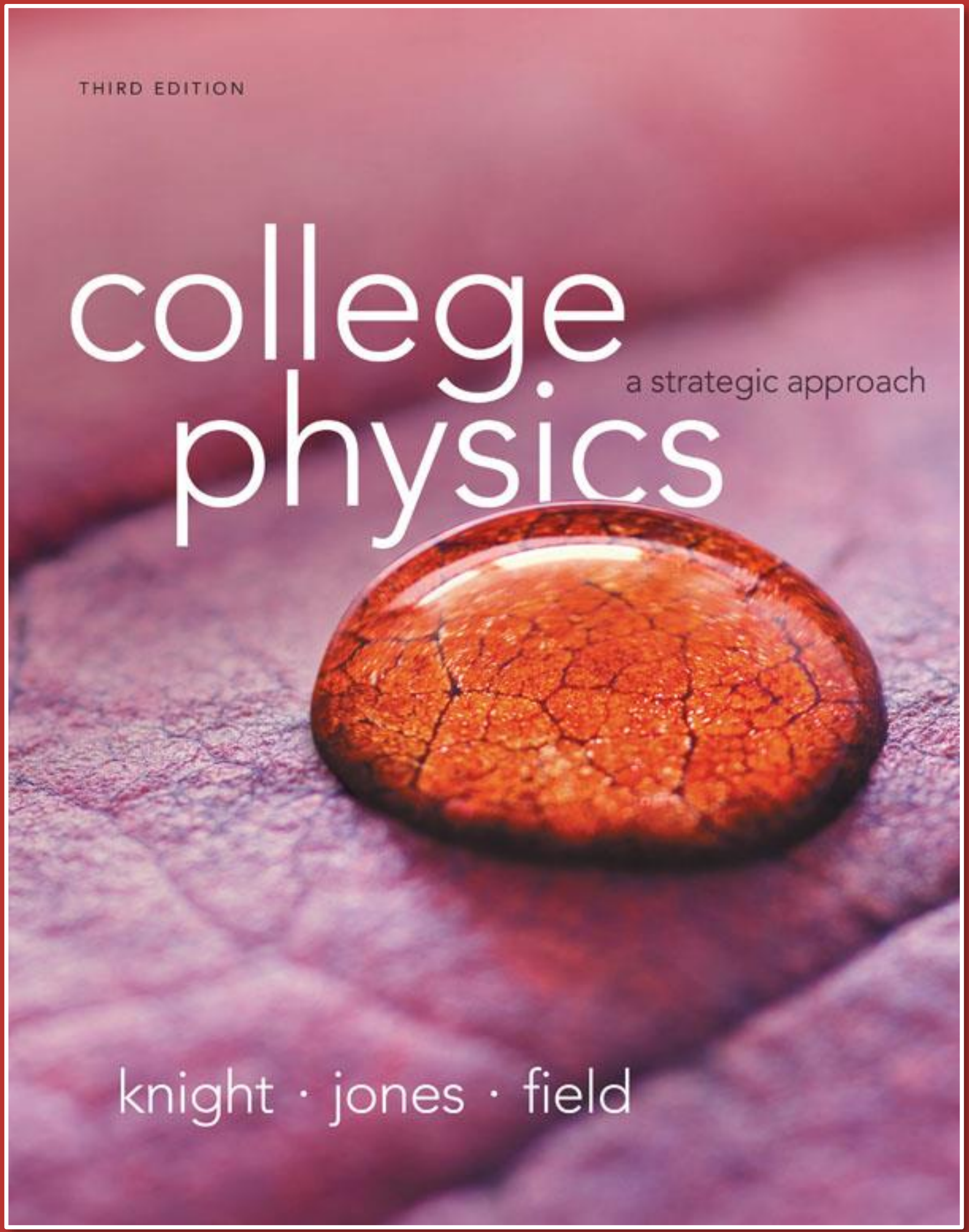


Lecture Presentation

Chapter 9

Momentum

Suggested Videos for Chapter 9

- **Prelecture Videos**
  - *Impulse and Momentum*
  - *Conservation of Momentum*
- **Video Tutor Solutions**
  - *Momentum*
- **Class Videos**
  - *Force and Momentum Change*
  - *Angular Momentum*
- **Video Tutor Demos**
  - *Water Rocket*
  - *Spinning Person Drops Weights*
  - *Off-Center Collision*

Suggested Simulations for Chapter 9

- **ActivPhysics**
  - *6.3, 6.4, 6.6, 6.7, 6.10*
  - *7.14*

- **PhETs**
  - *Lunar Lander*
  - *Torque*

Chapter 9 Momentum

Chapter Goal: To learn about impulse, momentum, and a new problem-solving strategy based on conservation laws.

Chapter 9 Preview
Looking Ahead: Impulse

- This golf club delivers an **impulse** to the ball as the club strikes it.

- You'll learn that a longer-lasting, stronger force delivers a greater impulse to an object.

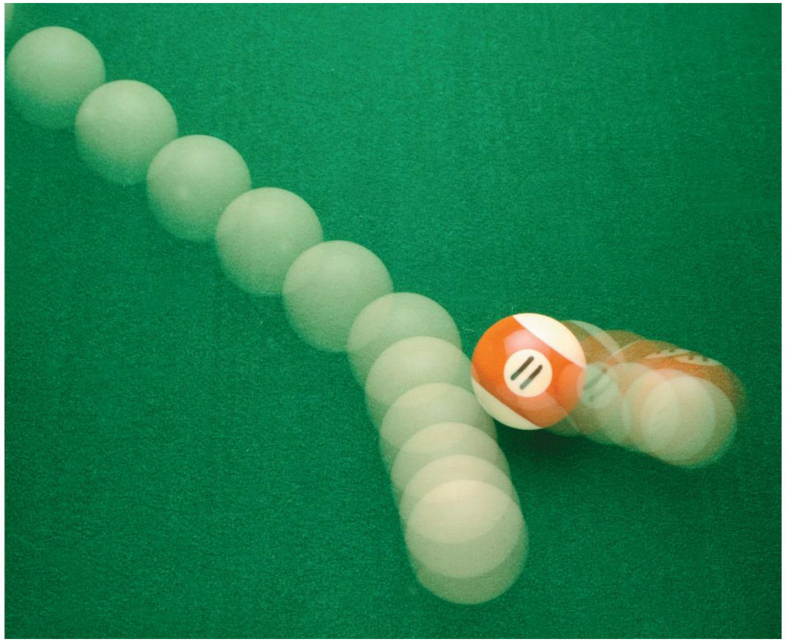
Chapter 9 Preview
Looking Ahead: Momentum and Impulse

- The impulse delivered by the player's head *changes* the ball's **momentum**.

- You'll learn how to calculate this momentum change using the **impulse-momentum theorem**.

Chapter 9 Preview
Looking Ahead: Conservation of Momentum

- The momentum of these pool balls before and after they collide is the *same*—it is **conserved**.



- You'll learn a powerful new *before-and-after* problem-solving strategy using this **law of conservation of momentum**.

Chapter 9 Preview
Looking Ahead

Impulse

This golf club delivers an **impulse** to the ball as the club strikes it.

You'll learn that a longer-lasting, stronger force delivers a greater impulse to an object.

Momentum and Impulse

The impulse delivered by the player's head *changes* the ball's **momentum.**

You'll learn how to calculate this momentum change using the **impulse-momentum theorem.**

Conservation of Momentum

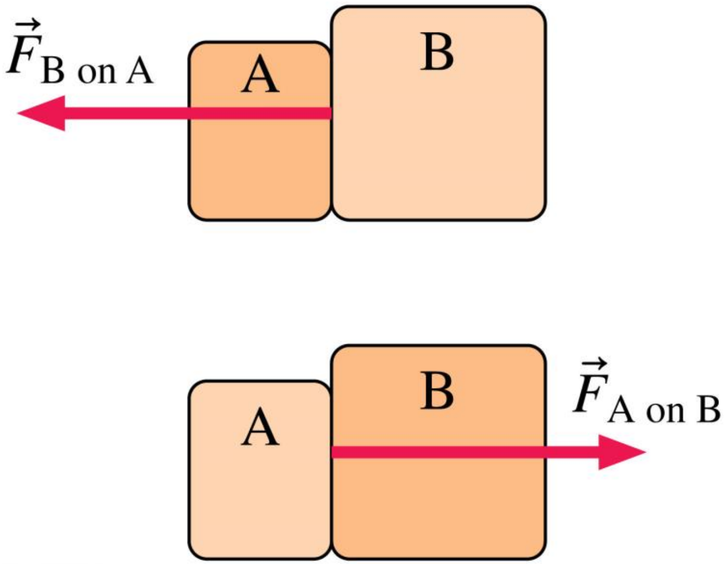
The momentum of these pool balls before and after they collide is the *same*—it is **conserved.**

You'll learn a powerful new *before-and-after* problem-solving strategy using this **law of conservation of momentum.**

Text: p. 254

Chapter 9 Preview
Looking Back: Newton's Third Law

- In Section 4.7, you learned about Newton's third law. In this chapter, you'll apply this law in order to understand the conservation of momentum.
- Newton's third law states that the force that object B exerts on A has *equal magnitude* but is *directed opposite to* the force that A exerts on B.



Chapter 9 Preview
Stop to Think

A hammer hits a nail. The force of the nail on the hammer is

A. Greater than the force of the hammer on the nail.
B. Less than the force of the hammer on the nail.
C. Equal to the force of the hammer on the nail.
D. Zero.

Reading Question 9.1

Impulse is

A. A force that is applied at a random time.
B. A force that is applied very suddenly.
C. The area under the force curve in a force-versus-time graph.
D. The time interval that a force lasts.

Reading Question 9.1

Impulse is

A. A force that is applied at a random time.

B. A force that is applied very suddenly.

✔ C. The area under the force curve in a force-versus-time graph.

D. The time interval that a force lasts.

Reading Question 9.2

In the *impulse approximation*,

A. A large force acts for a very short time.
B. The true impulse is approximated by a rectangular pulse.
C. No external forces act during the time the impulsive force acts.
D. The forces between colliding objects can be neglected.

Reading Question 9.2

In the *impulse approximation*,

✔ A. A large force acts for a very short time.

B. The true impulse is approximated by a rectangular pulse.

C. No external forces act during the time the impulsive force acts.

D. The forces between colliding objects can be neglected.

Reading Question 9.3

The total momentum of a system is conserved

A. Always.
B. If no external forces act on the system.
C. If no internal forces act on the system.
D. Never; it’s just an approximation.

Reading Question 9.3

The total momentum of a system is conserved

A. Always.
✔ B. If no external forces act on the system.
C. If no internal forces act on the system.
D. Never; it's just an approximation.

Reading Question 9.4

In an *inelastic collision*,

A. Impulse is conserved.
B. Momentum is conserved.
C. Force is conserved.
D. Energy is conserved.
E. Elasticity is conserved.

Reading Question 9.4

In an *inelastic collision*,

A. Impulse is conserved.
✔ B. Momentum is conserved.
C. Force is conserved.
D. Energy is conserved.
E. Elasticity is conserved.

Reading Question 9.5

An object's angular momentum is proportional to its

A. Mass.
B. Moment of inertia.
C. Kinetic energy.
D. Linear momentum.

Reading Question 9.5

An object's angular momentum is proportional to its

A. Mass.
✔ B. Moment of inertia.
C. Kinetic energy.
D. Linear momentum.

Section 9.1 Impulse

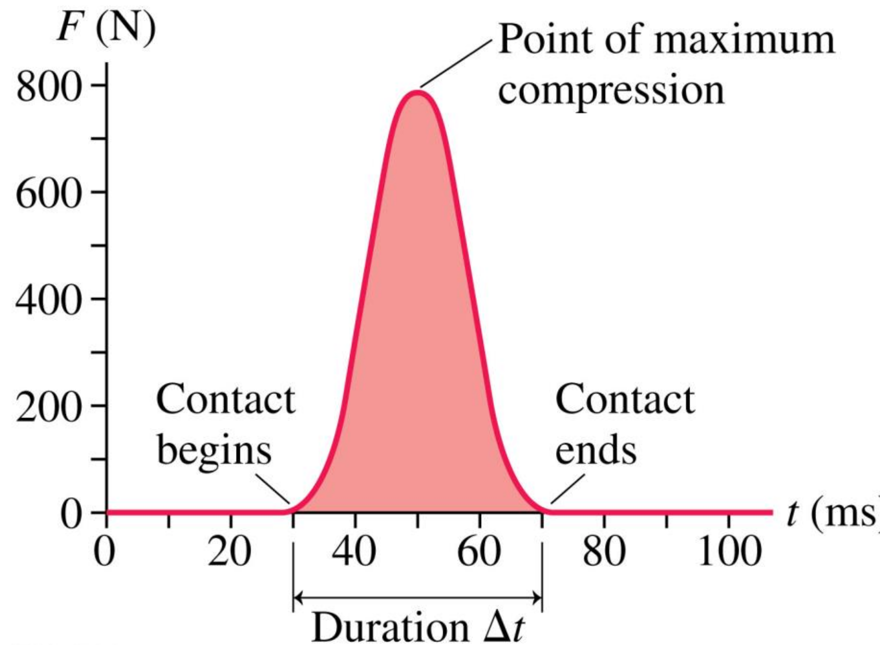
Impulse

- A **collision** is a short-duration interaction between two objects.

- During a collision, it takes time to compress the object, and it takes time for the object to re-expand.
- The duration of a collision depends on the materials.

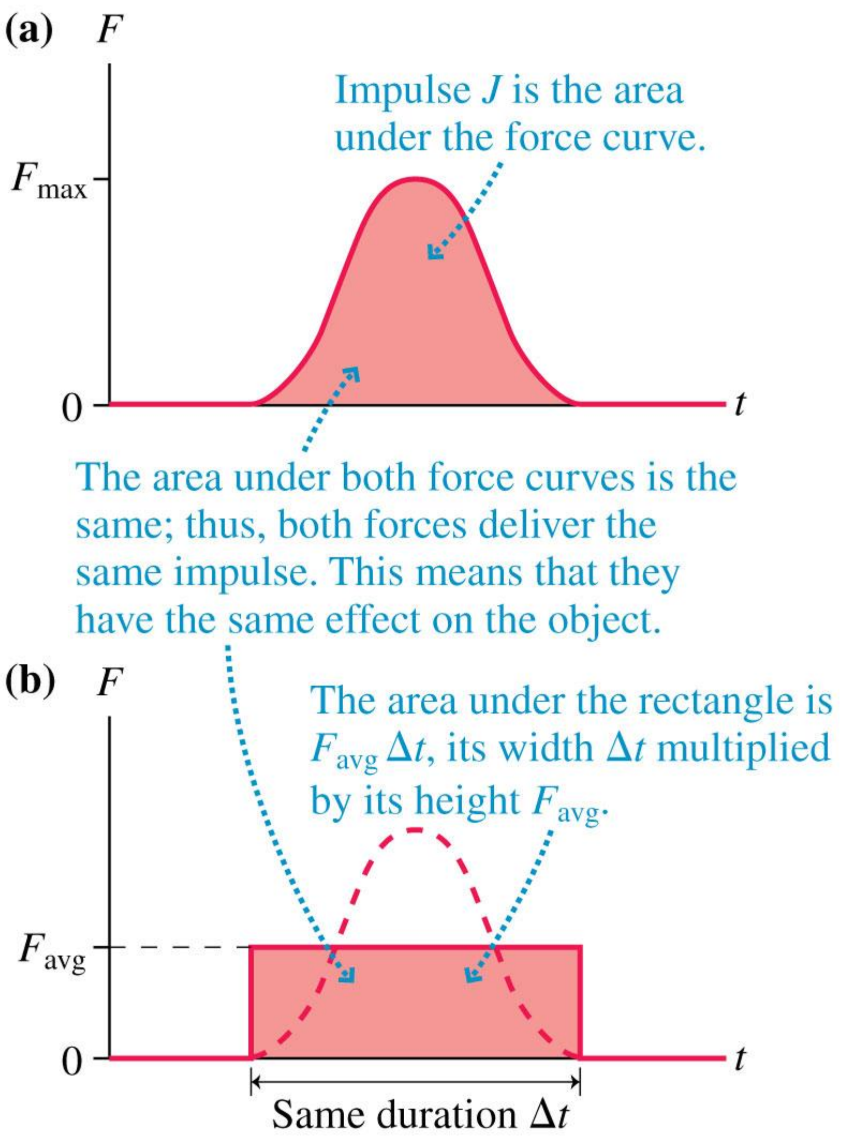
Impulse

- When kicking a soccer ball, the amount by which the ball is compressed is a measure of the magnitude of the force the foot exerts on the ball.
- The force is applied only while the ball is in contact with the foot.
- The **impulse force** is a large force exerted during a short interval of time.



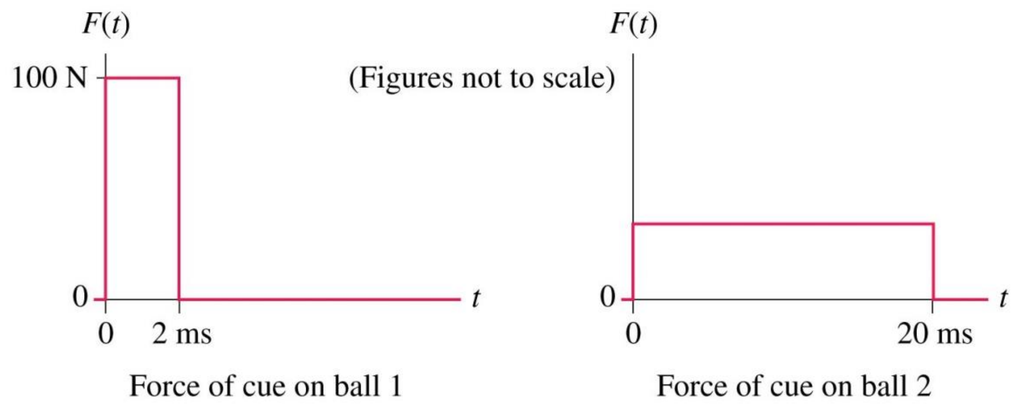
Impulse

- **The effect of an impulsive force is proportional to the area under the force-versus-time curve.**
- The area is called the **impulse** J of the force.



QuickCheck 9.6

Two 1.0 kg stationary cue balls are struck by cue sticks. The cues exert the forces shown. Which ball has the greater final speed?



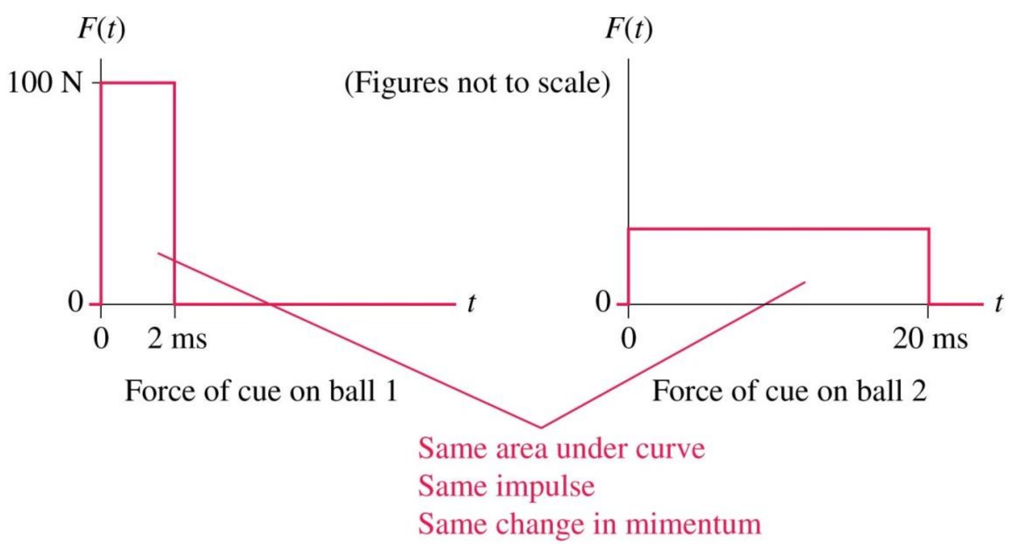
A. Ball 1

B. Ball 2

C. Both balls have the same final speed.

QuickCheck 9.6

Two 1.0 kg stationary cue balls are struck by cue sticks. The cues exert the forces shown. Which ball has the greater final speed?



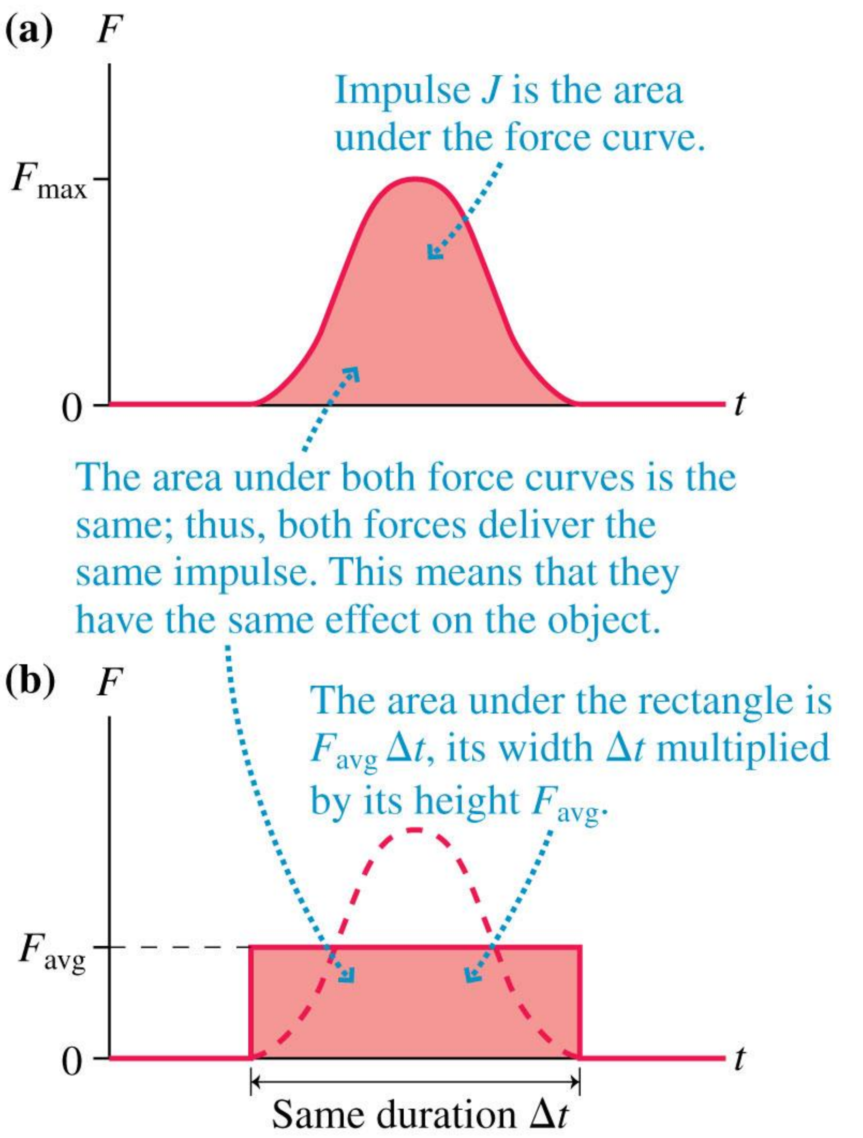
A. Ball 1

B. Ball 2

✔ C. Both balls have the same final speed.

Impulse

- It is useful to think of the collision in terms of an *average* force F_{avg}.
- F_{avg} is defined as the constant force that has the same duration Δt and the same area under the force curve as the real force.



Impulse

$$\text{impulse } J = \text{area under the force curve} = F_{avg}\,\Delta t$$

Impulse due to a force acting for a duration Δt

- Impulse has units of N · s, but N · s are equivalent to kg · m/s.
- The latter are the preferred units for impulse.
- The impulse is a *vector* quantity, pointing in the direction of the average force vector:

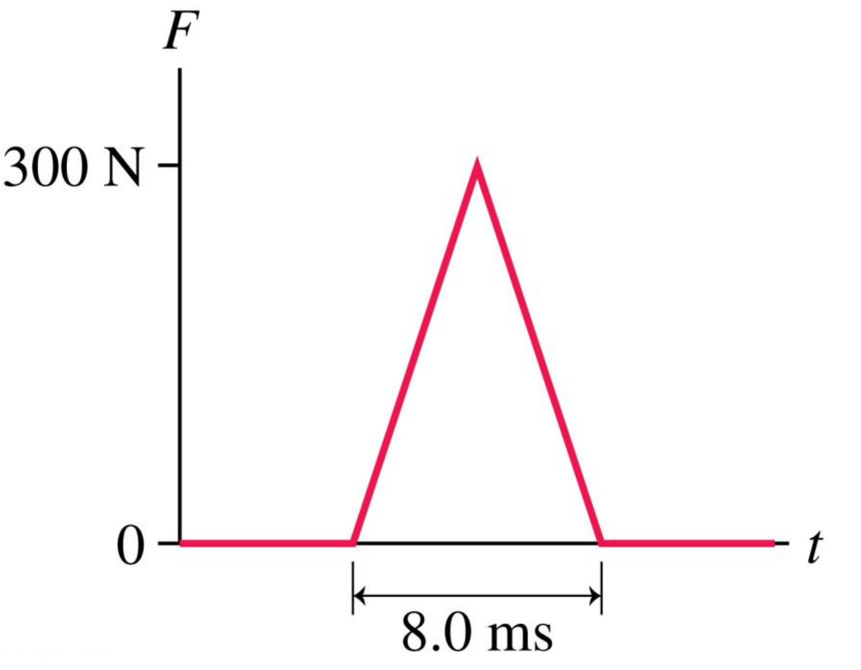
$$\vec{J} = \vec{F}_{avg}\,\Delta t$$

Example 9.1 Finding the impulse on a bouncing ball

A rubber ball experiences the force shown in FIGURE 9.4 as it bounces off the floor.

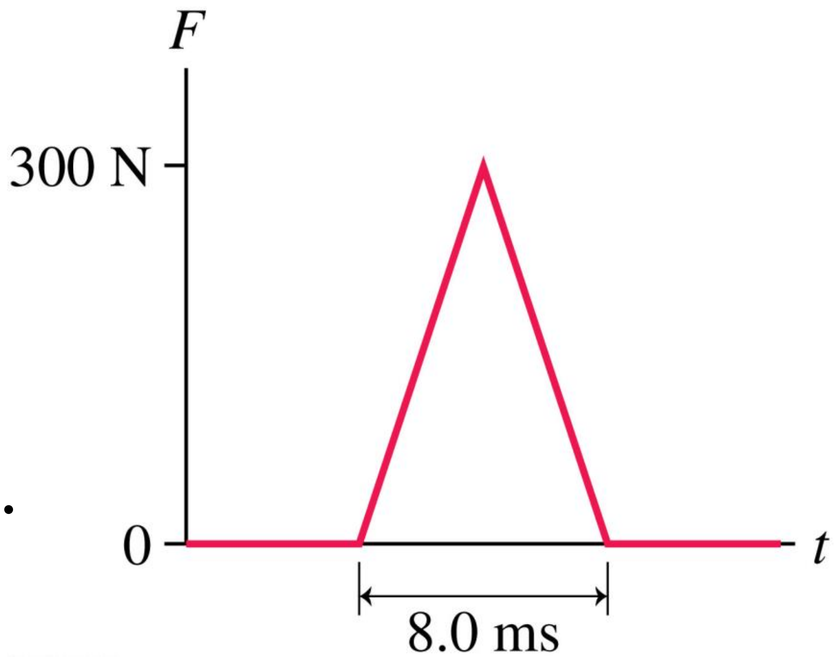
a. What is the impulse on the ball?

b. What is the average force on the ball?



Example 9.1 Finding the impulse on a bouncing ball (cont.)

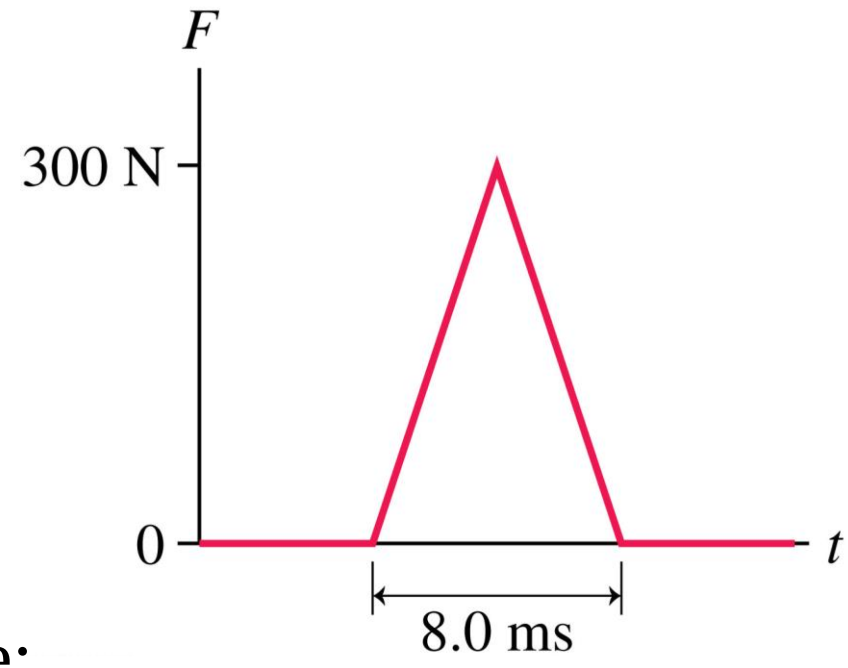
PREPARE The impulse is the area under the force curve. Here the shape of the graph is triangular, so we'll need to use the fact that the area of a triangle is $\frac{1}{2} \times \text{height} \times \text{base}$.



Example 9.1 Finding the impulse on a bouncing ball (cont.)

SOLVE a. The impulse is

$$J = \tfrac{1}{2}(300\ \text{N})(0.0080\ \text{s})$$
$$= 1.2\ \text{N}\cdot\text{s} = 1.2\ \text{kg}\cdot\text{m/s}$$

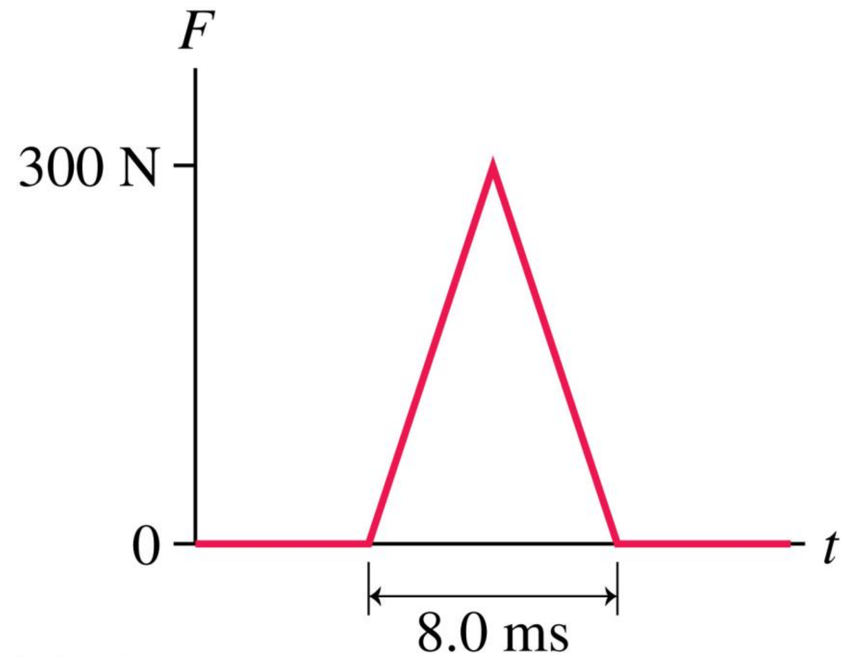


b. From Equation 9.1, $J = F_{avg}\,\Delta t$, we can find the average force that would give this same impulse:

$$F_{avg} = \frac{J}{\Delta t} = \frac{1.2\ \text{N}\cdot\text{s}}{0.0080\ \text{s}} = 150\ \text{N}$$

Example 9.1 Finding the impulse on a bouncing ball (cont.)

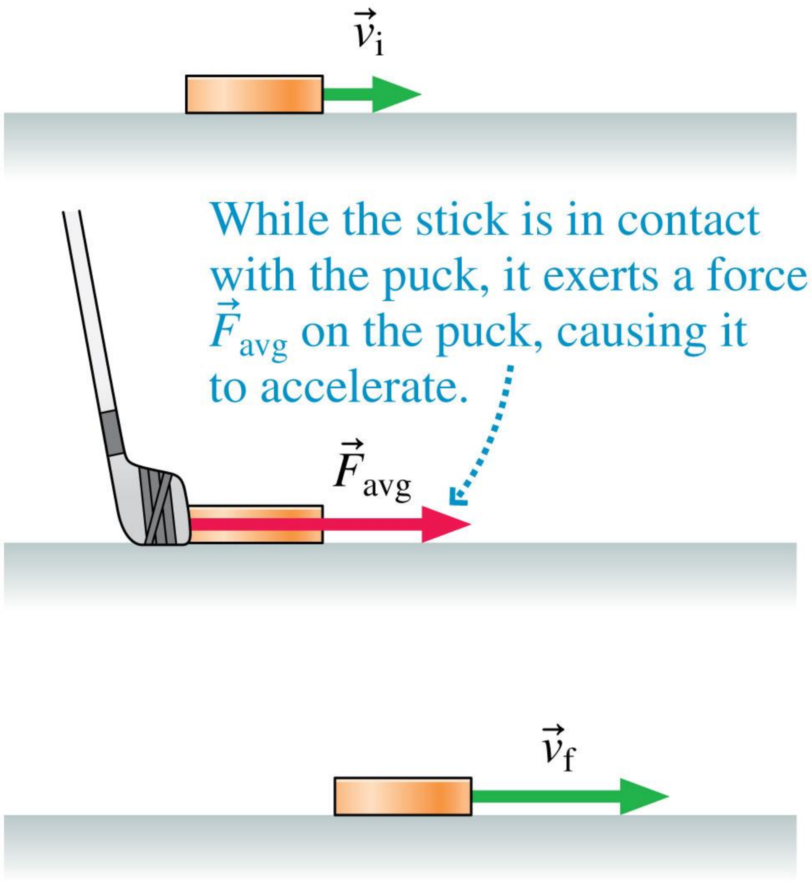
ASSESS In this particular example, the average value of the force is half the maximum value. This is not surprising for a triangular force because the area of a triangle is *half* the base times the height.



Section 9.2 Momentum and the Impulse-Momentum Theorem

Momentum and the Impulse-Momentum Theorem

- Intuitively we know that giving a kick to a heavy object will change its velocity much less than giving the same kick to a light object.
- We can calculate how the final velocity is related to the initial velocity.



Momentum and the Impulse-Momentum Theorem

- From Newton's second law, the average acceleration of an object during the time the force is being applied is

$$\vec{a}_{\text{avg}} = \frac{\vec{F}_{\text{avg}}}{m}$$

- The average acceleration is related to the change in the velocity by

$$\vec{a}_{\text{avg}} = \frac{\Delta\vec{v}}{\Delta t} = \frac{\vec{v}_{\text{f}} - \vec{v}_{\text{i}}}{\Delta t}$$

- We combine those two equations to find

$$\frac{\vec{F}_{\text{avg}}}{m} = \vec{a}_{\text{avg}} = \frac{\vec{v}_{\text{f}} - \vec{v}_{\text{i}}}{\Delta t}$$

Momentum and the Impulse-Momentum Theorem

- We can rearrange that equation in terms of impulse:

$$\vec{F}_{\text{avg}}\,\Delta t = m\vec{v}_{\text{f}} - m\vec{v}_{\text{i}}$$

- **Momentum** is the product of the object's mass and velocity. It has units of kg · m/s.

$$\vec{p} = m\vec{v}$$

Momentum of an object of mass m and velocity $\vec{v}$

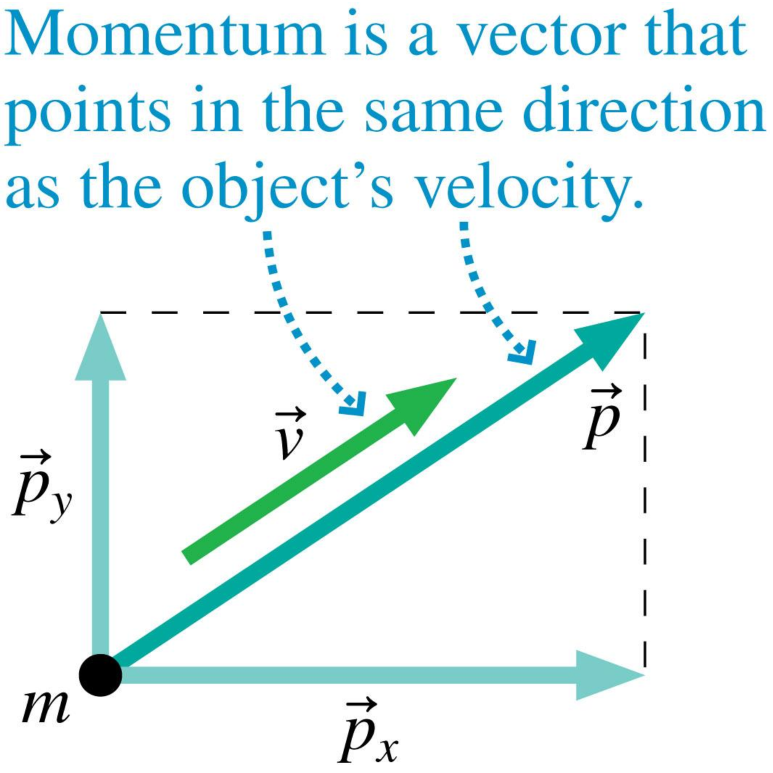
Momentum and the Impulse-Momentum Theorem

- Momentum is a *vector* quantity that points in the same direction as the velocity vector:

$$p_x = mv_x$$

$$p_y = mv_y$$

- The *magnitude* of an object's momentum is simply the product of the object's mass and speed.



Momentum and the Impulse-Momentum Theorem

TABLE 9.1 Some typical momenta (approximate)

Object	Mass (kg)	Speed (m/s)	Momentum (kg · m/s)
Falling raindrop	2×10^{-5}	5	10^{-4}
Bullet	0.004	500	2
Pitched baseball	0.15	40	6
Running person	70	3	200
Car on highway	1000	30	3×10^{4}

The Impulse-Momentum Theorem

- Impulse and momentum are related as:

$$\vec{J} = \vec{p}_{\mathrm{f}} - \vec{p}_{\mathrm{i}} = \Delta\vec{p}$$

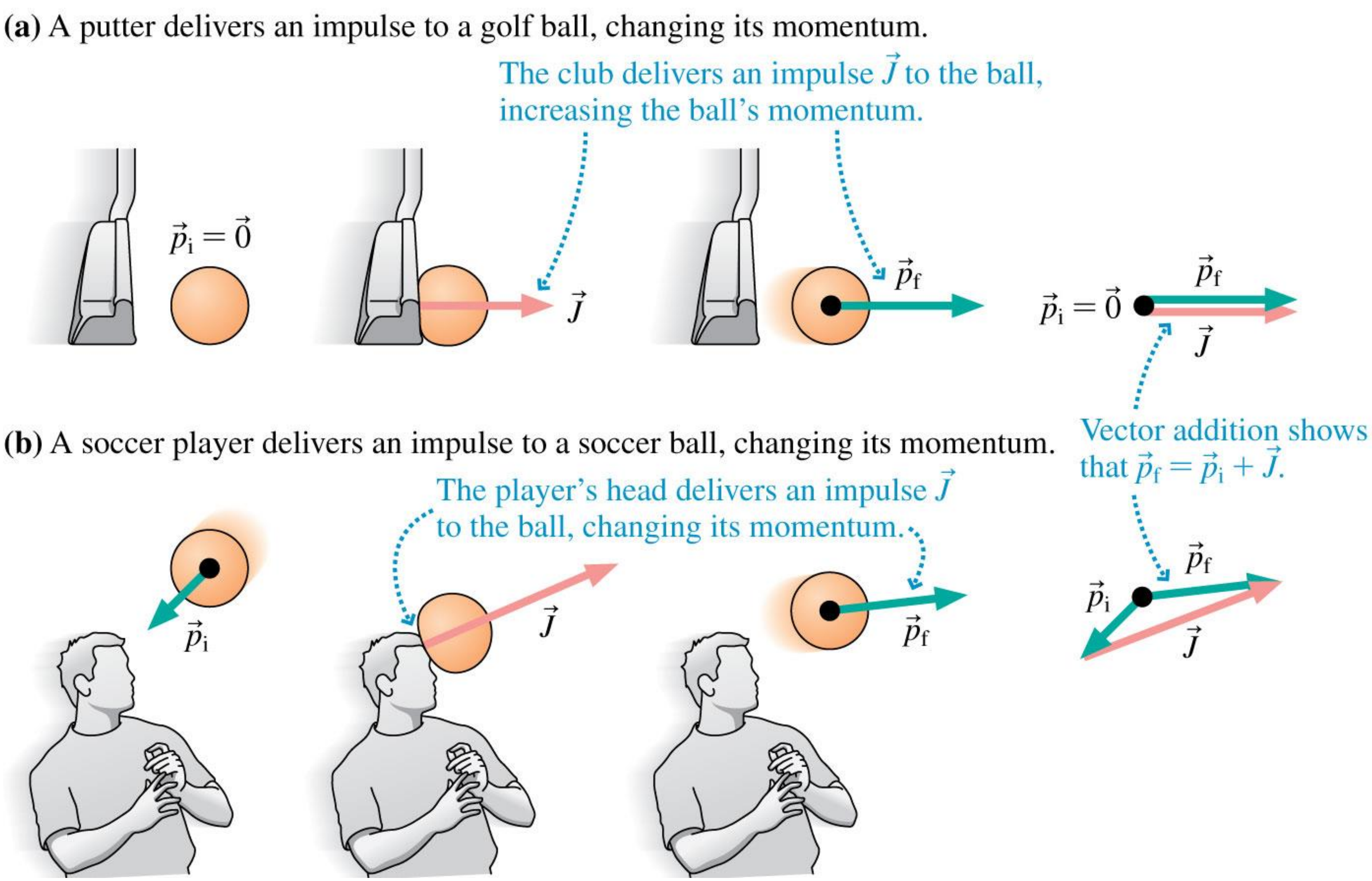
Impulse-momentum theorem

- **The impulse-momentum theorem states that an impulse delivered to an object causes the object's momentum to change.**
- Impulse can be written in terms of its *x*- and *y*-components:

$$J_x = \Delta p_x = (p_x)_{\mathrm{f}} - (p_x)_{\mathrm{i}} = m(v_x)_{\mathrm{f}} - m(v_x)_{\mathrm{i}}$$

$$J_y = \Delta p_y = (p_y)_{\mathrm{f}} - (p_y)_{\mathrm{i}} = m(v_y)_{\mathrm{f}} - m(v_y)_{\mathrm{i}}$$

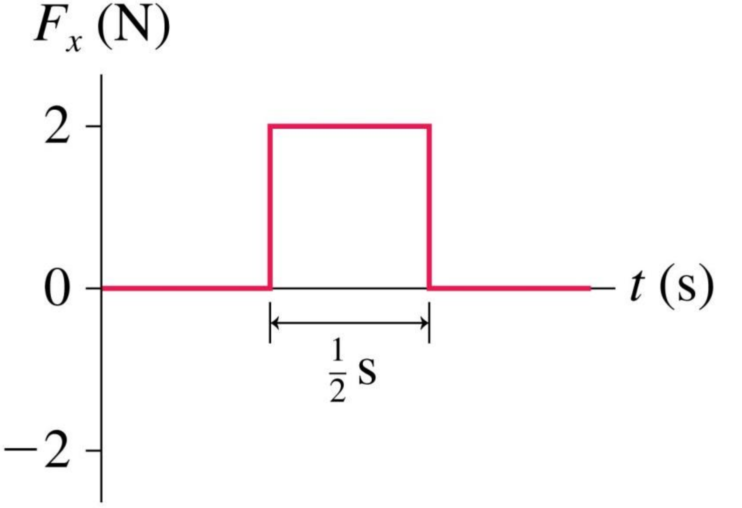
The Impulse-Momentum Theorem



QuickCheck 9.2

A 2.0 kg object moving to the right with speed 0.50 m/s experiences the force shown. What are the object's speed and direction after the force ends?

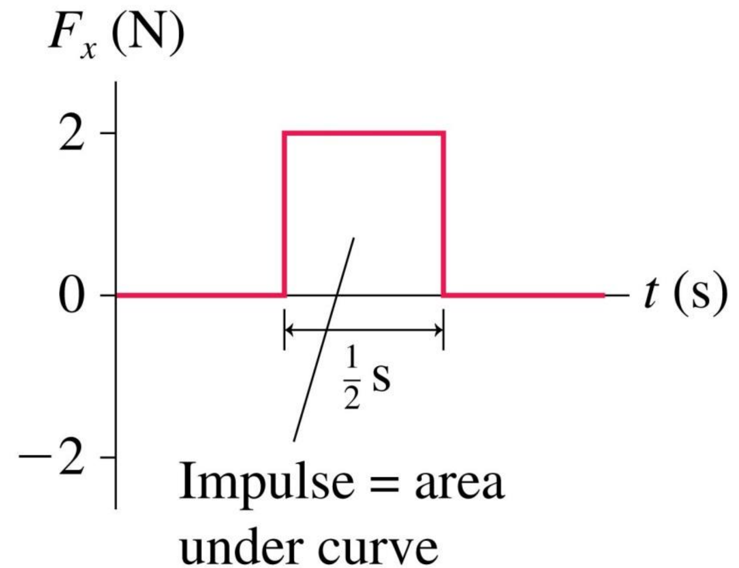
A. 0.50 m/s left
B. At rest
C. 0.50 m/s right
D. 1.0 m/s right
E. 2.0 m/s right



QuickCheck 9.2

A 2.0 kg object moving to the right with speed 0.50 m/s experiences the force shown. What are the object's speed and direction after the force ends?

A. 0.50 m/s left
B. At rest
C. 0.50 m/s right
✔ D. 1.0 m/s right
E. 2.0 m/s right

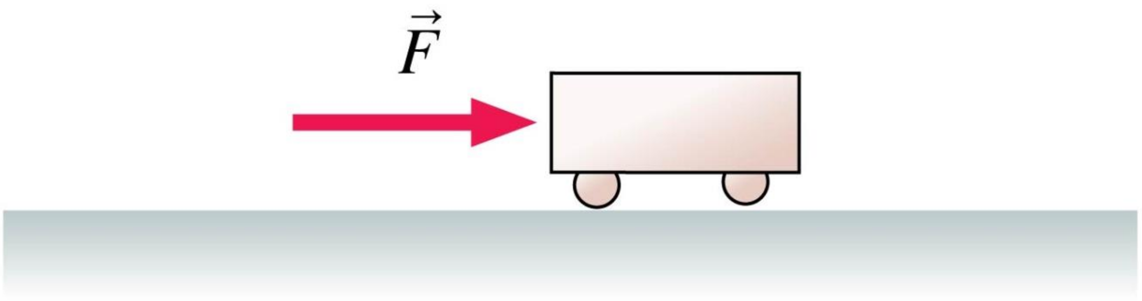


$$\Delta p_x = J_x \text{ or } p_{fx} = p_{ix} + J_x$$

QuickCheck 9.4

A force pushes the cart for 1 s, starting from rest. To achieve the same speed with a force half as big, the force would need to push for

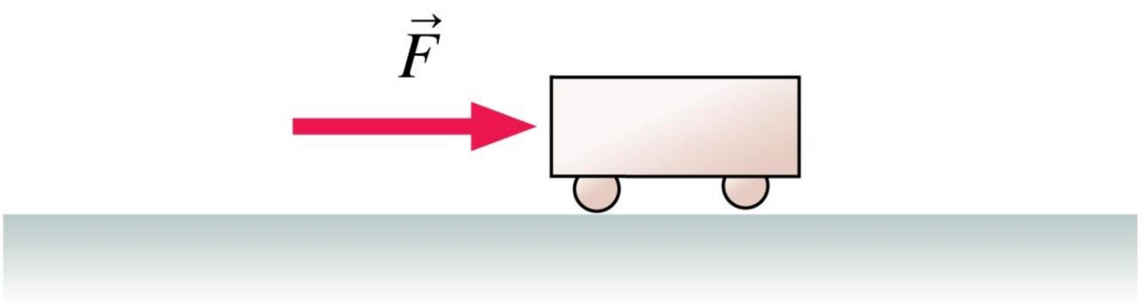
A. $\frac{1}{4}$ s
B. $\frac{1}{2}$ s
C. 1 s
D. 2 s
E. 4 s



QuickCheck 9.4

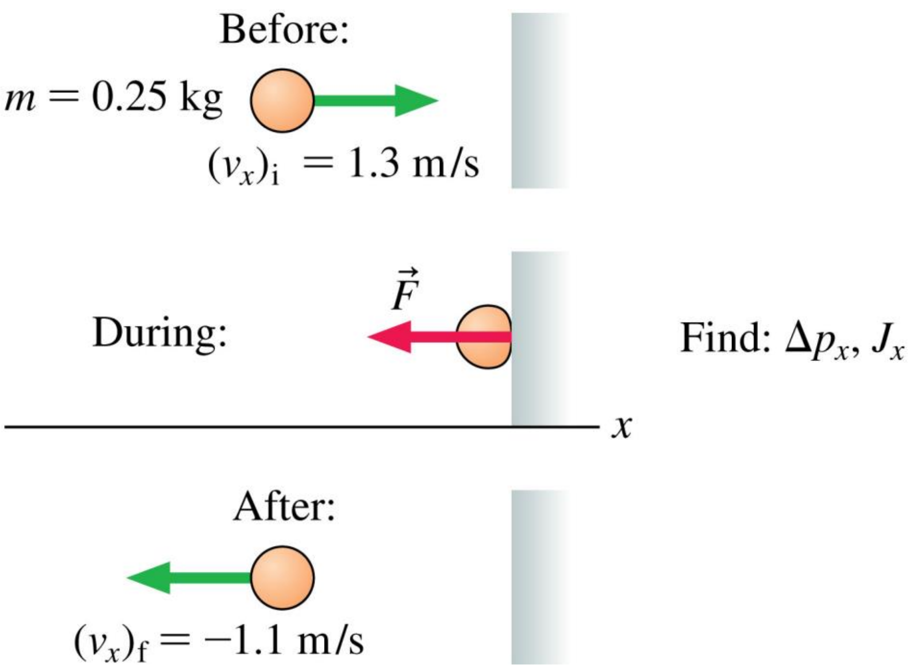
A force pushes the cart for 1 s, starting from rest. To achieve the same speed with a force half as big, the force would need to push for

A. $\frac{1}{4}$ s
B. $\frac{1}{2}$ s
C. 1 s
✔ D. 2 s
E. 4 s



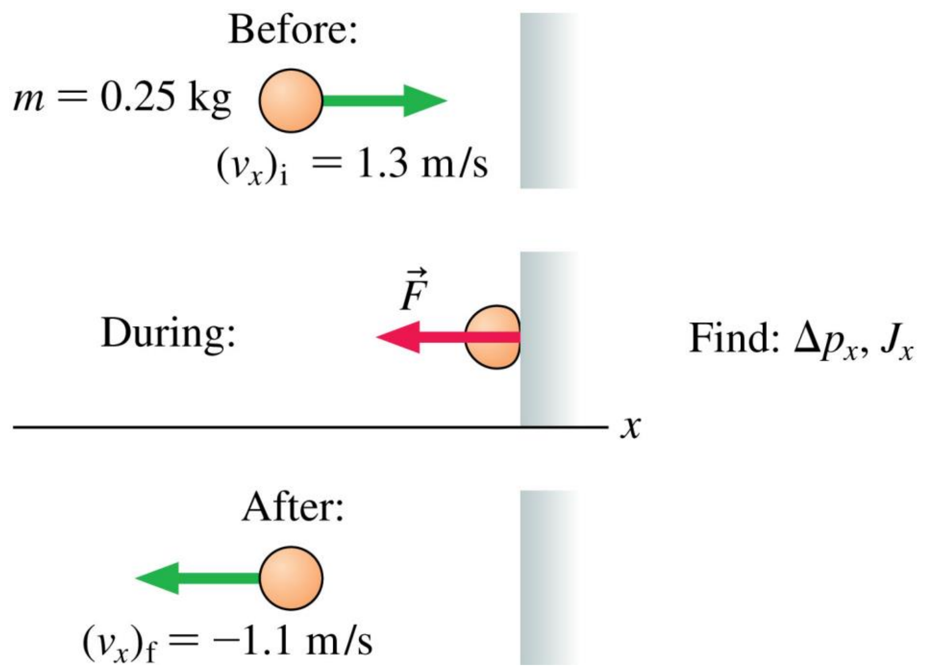
Example 9.2 Calculating the change in momentum

A ball of mass $m = 0.25$ kg rolling to the right at 1.3 m/s strikes a wall and rebounds to the left at 1.1 m/s. What is the change in the ball's momentum? What is the impulse delivered to it by the wall?



Example 9.2 Calculating the change in momentum (cont.)

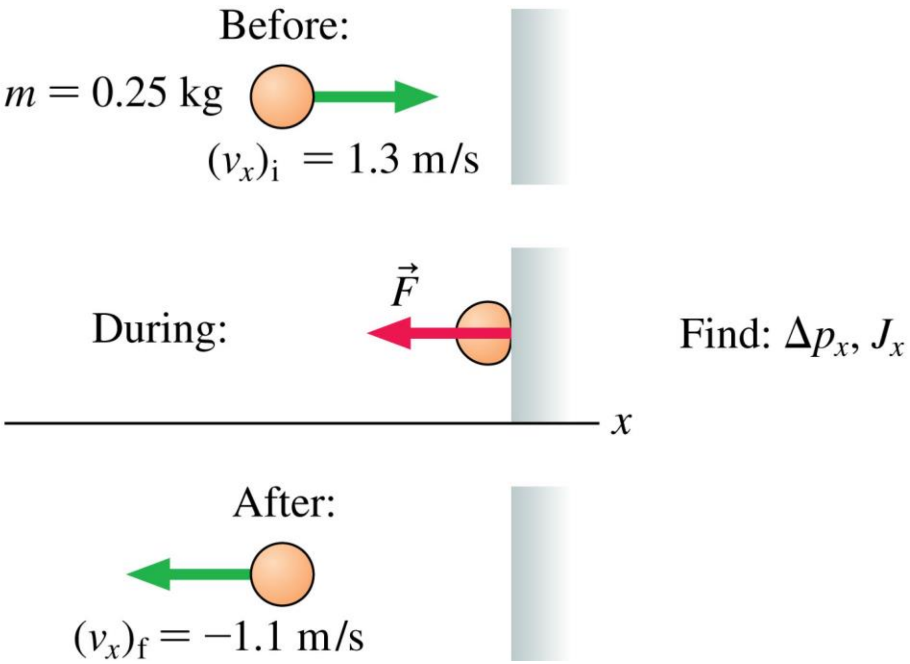
PREPARE A visual overview of the ball bouncing is shown in FIGURE 9.8. This is a new kind of visual overview, one in which we show the situation "before" and "after" the interaction. We'll have more to say about before-and-after pictures in the next section. The ball is moving along the *x*-axis, so we'll write the momentum in component form, as in Equation 9.7. The change in momentum is then the difference between the final and initial values of the momentum. By the impulse-momentum theorem, the impulse is equal to this change in momentum.



Example 9.2 Calculating the change in momentum (cont.)

SOLVE The *x*-component of the initial momentum is

$$\begin{aligned}(p_x)_\text{i} &= m(v_x)_\text{i} \\ &= (0.25\text{ kg})(1.3\text{ m/s}) \\ &= 0.325\text{ kg}\cdot\text{m/s}\end{aligned}$$

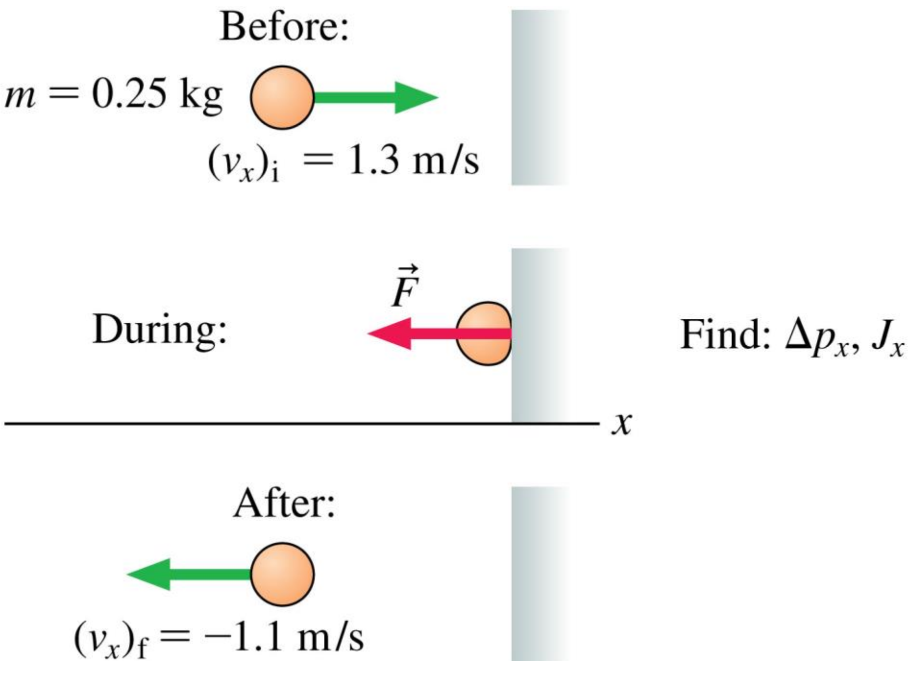


The *y*-component of the momentum is zero both before and after the bounce. After the ball rebounds, the *x*-component is

$$(p_x)_\text{f} = m(v_x)_\text{f} = (0.25\text{ kg})(-1.1\text{ m/s}) = -0.275\text{ kg}\cdot\text{m/s}$$

Example 9.2 Calculating the change in momentum (cont.)

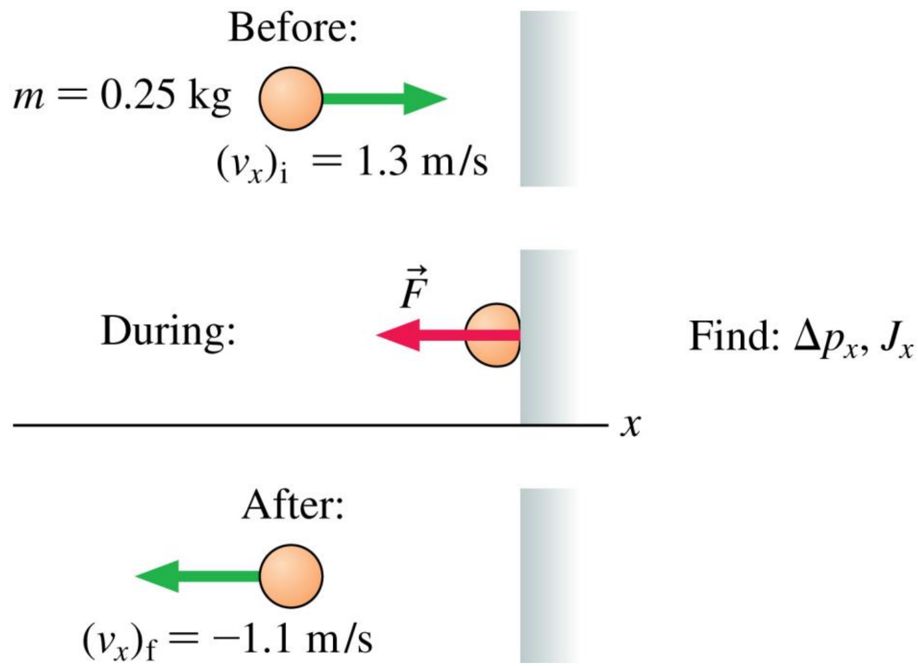
It is particularly important to notice that the *x*-component of the momentum, like that of the velocity, is negative. This indicates that the ball is moving to the left. The change in momentum is



$$\Delta p_x = (p_x)_f - (p_x)_i = (-0.275 \text{ kg} \cdot \text{m/s}) - (0.325 \text{ kg} \cdot \text{m/s})$$
$$= -0.60 \text{ kg} \cdot \text{m/s}$$

Example 9.2 Calculating the change in momentum (cont.)

By the impulse-momentum theorem, the impulse delivered to the ball by the wall is equal to this change, so



$$J_x = \Delta p_x = -0.60\ \text{kg} \cdot \text{m/s}$$

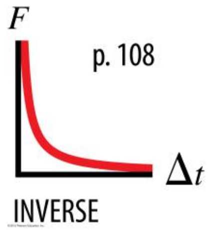
ASSESS The impulse is negative, indicating that the force causing the impulse is pointing to the left, which makes sense.

The Impulse-Momentum Theorem

- The impulse-momentum theorem tells us

$$\vec{J} = \vec{F}_{\text{avg}}\,\Delta t = \Delta\vec{p} = \vec{p}_{\text{f}} - \vec{p}_{\text{i}} = -\vec{p}_{\text{i}}$$

$$\vec{F}_{\text{avg}} = -\frac{\vec{p}_{\text{i}}}{\Delta t}$$



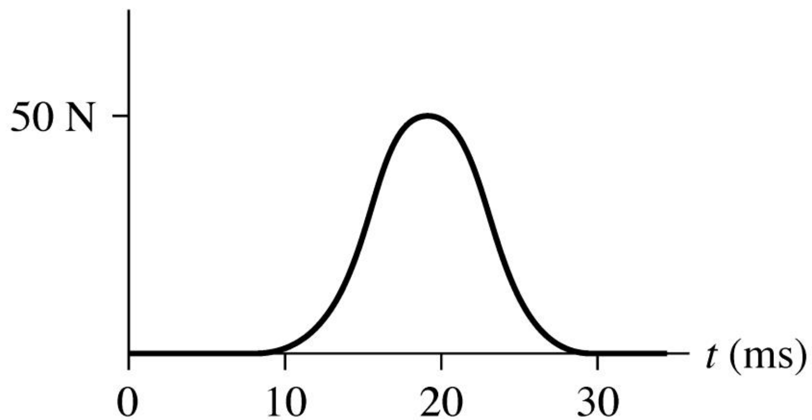
- The average force needed to stop an object is *inversely proportional* to the duration of the collision.
- **If the duration of the collision can be increased, the force of the impact will be decreased.**

The Impulse-Momentum Theorem

- The spines of a hedgehog obviously help protect it from predators. But they serve another function as well. If a hedgehog falls from a tree—a not uncommon occurrence—it simply rolls itself into a ball before it lands. Its thick spines then cushion the blow by increasing the time it takes for the animal to come to rest. Indeed, hedgehogs have been observed to fall out of trees on purpose to get to the ground!

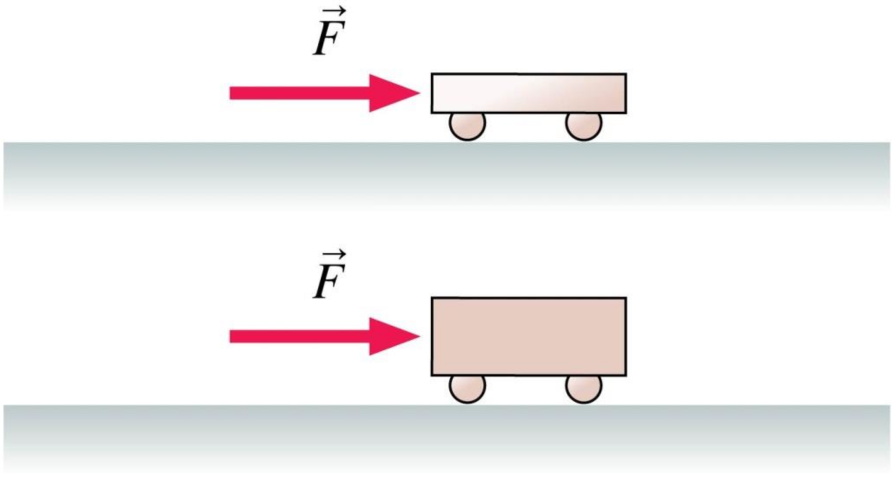
Example Problem

A 0.5 kg hockey puck slides to the right at 10 m/s. It is hit with a hockey stick that exerts the force shown. What is its approximate final speed?



QuickCheck 9.5

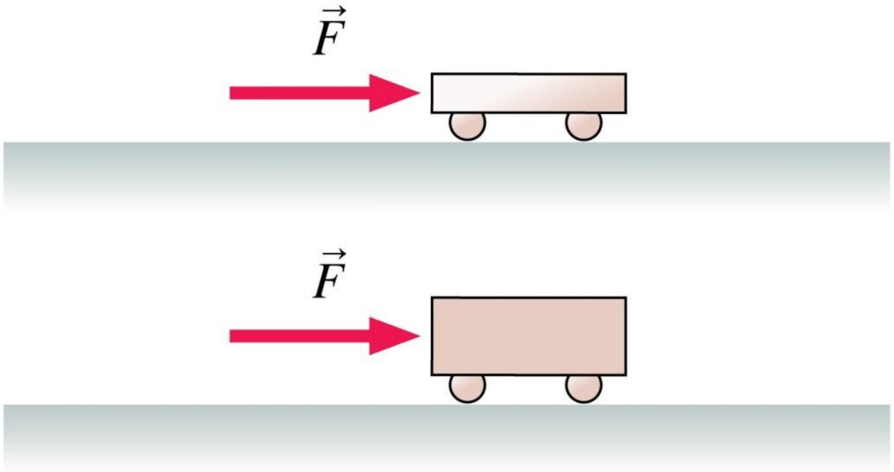
A light plastic cart and a heavy steel cart are both pushed with the same force for 1.0 s, starting from rest. After the force is removed, the momentum of the light plastic cart is ________ that of the heavy steel cart.



A. Greater than
B. Equal to
C. Less than
D. Can’t say. It depends on how big the force is.

QuickCheck 9.5

A light plastic cart and a heavy steel cart are both pushed with the same force for 1.0 s, starting from rest. After the force is removed, the momentum of the light plastic cart is _________ that of the heavy steel cart.



A. Greater than

✔ B. Equal to

C. Less than

D. Can't say. It depends on how big the force is.

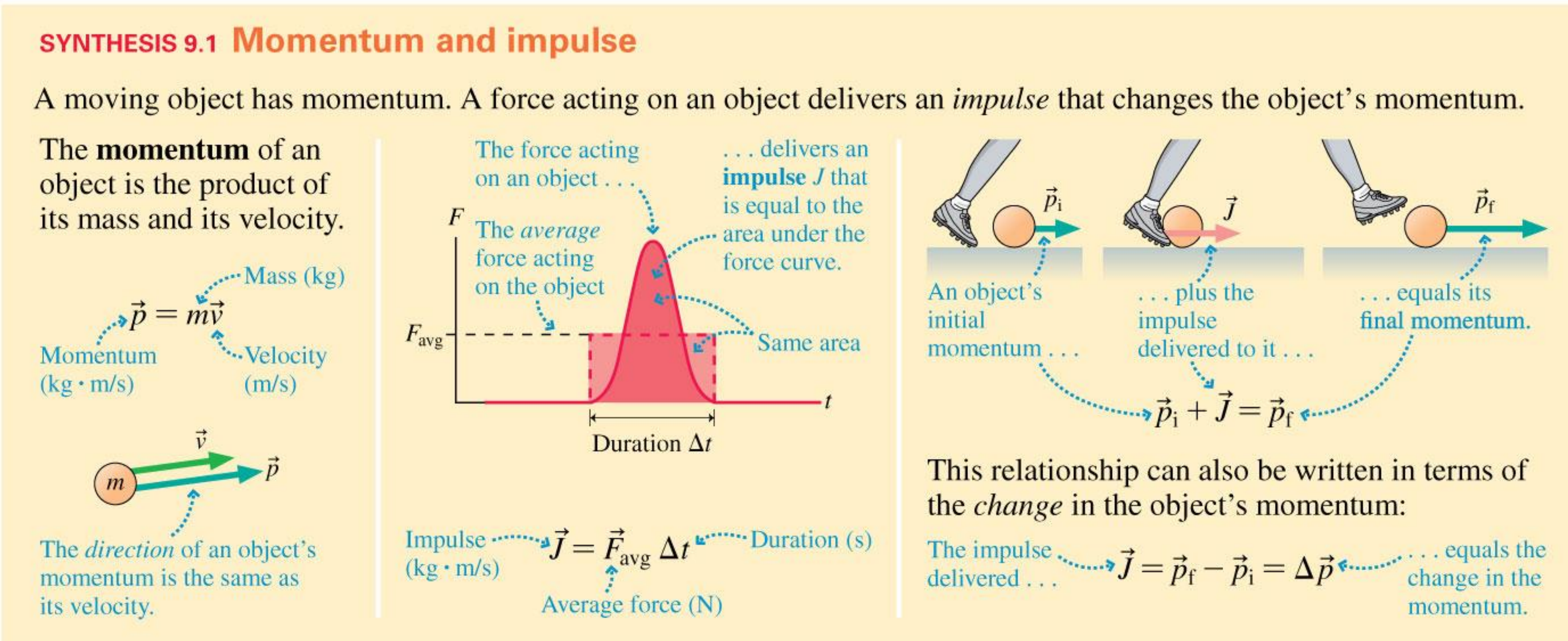
Same force, same time $\Rightarrow$ same impulse

Same impulse $\Rightarrow$ same change of momentum

Try It Yourself: Water Balloon Catch

If you've ever tried to catch a water balloon, you may have learned the hard way not to catch it with your arms rigidly extended. The brief collision time implies a large, balloon-bursting force. A better way to catch a water balloon is to pull your arms in toward your body as you catch it, lengthening the collision time and hence reducing the force on the balloon.

The Impulse-Momentum Theorem

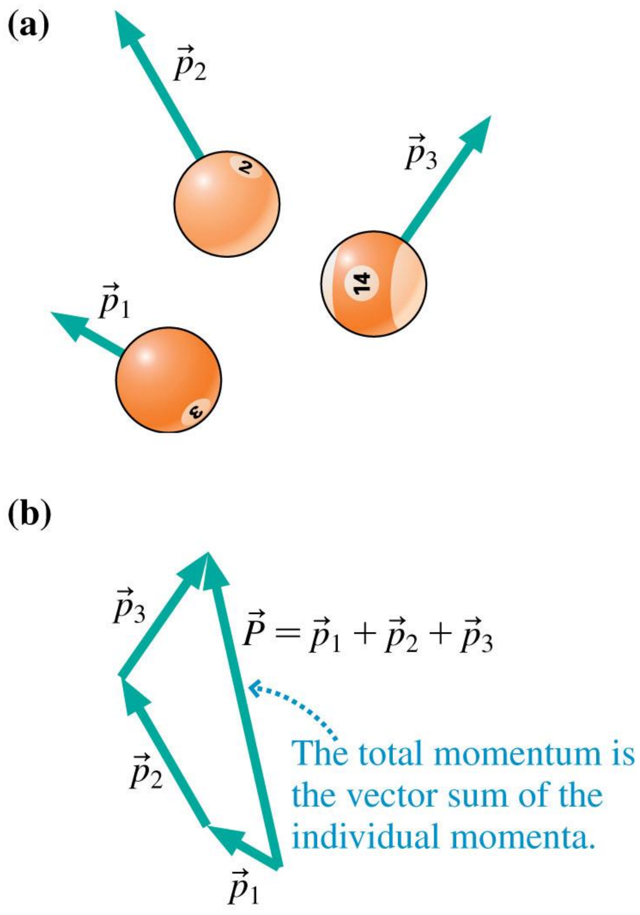


Text: p. 259

Total Momentum

- If there is a *system* of particles moving, then the system as a whole has an overall momentum.
- The **total momentum** of a system of particles is the vector sum of the momenta of the individual particles:

$$\vec{P} = \vec{p}_1 + \vec{p}_2 + \vec{p}_3 + \cdots$$



Section 9.3 Solving Impulse and Momentum Problems

Solving Impulse and Momentum Problems

TACTICS BOX 9.1 **Drawing a before-and-after visual overview**

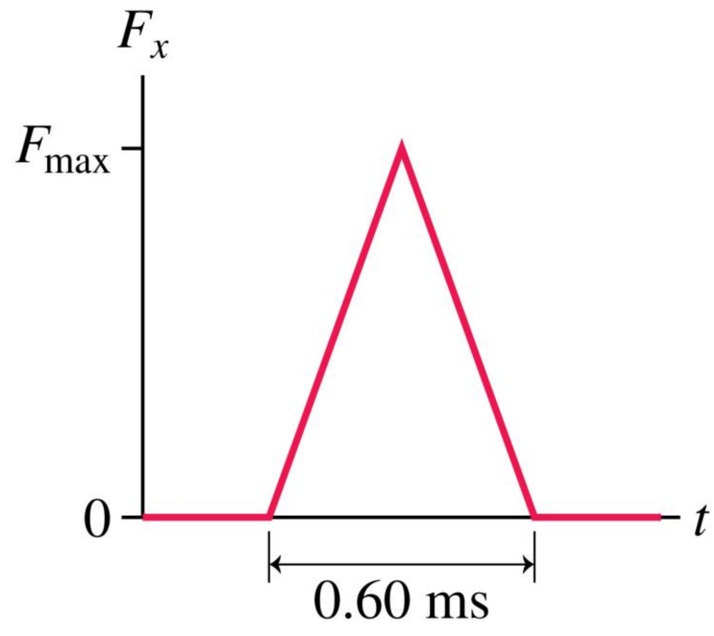
1. **Sketch the situation.** Use two drawings, labeled "Before" and "After," to show the objects *immediately before* they interact and again *immediately after* they interact.
2. **Establish a coordinate system.** Select your axes to match the motion.
3. **Define symbols.** Define symbols for the masses and for the velocities before and after the interaction. Position and time are not needed.
4. **List known information.** List the values of quantities known from the problem statement or that can be found quickly with simple geometry or unit conversions. Before-and-after pictures are usually simpler than the pictures you used for dynamics problems, so listing known information on the sketch is often adequate.
5. **Identify the desired unknowns.** What quantity or quantities will allow you to answer the question? These should have been defined as symbols in step 3.

Exercises 9–11

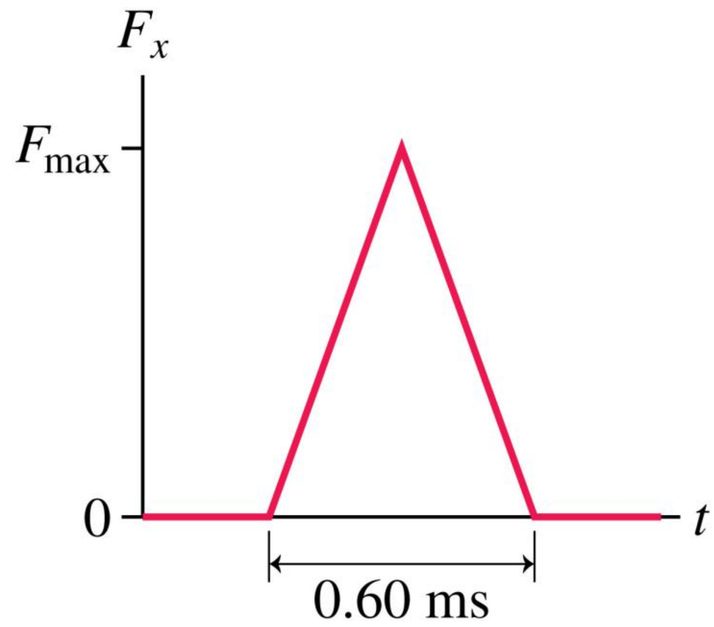
Text: p. 260

Example 9.3 Force in hitting a baseball

A 150 g baseball is thrown with a speed of 20 m/s. It is hit straight back toward the pitcher at a speed of 40 m/s. The impulsive force of the bat on the ball has the shape shown in FIGURE 9.10. What is the *maximum* force F_{max} that the bat exerts on the ball? What is the *average* force that the bat exerts on the ball?



Example 9.3 Force in hitting a baseball (cont.)



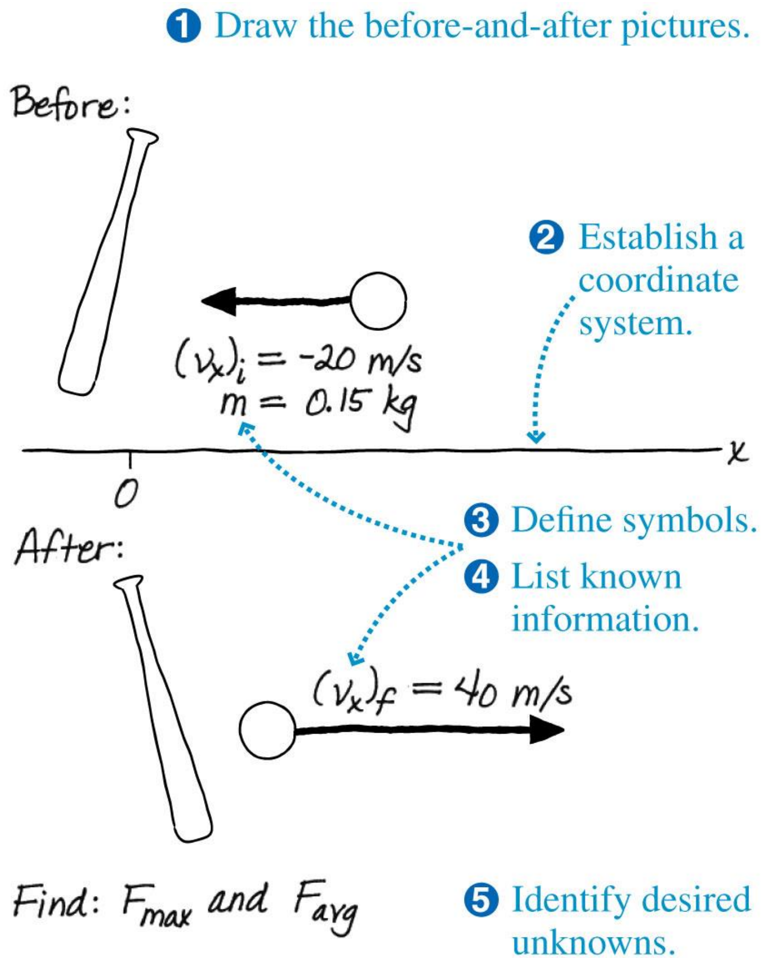
PREPARE We can model the interaction as a collision. FIGURE 9.11 is a before-and-after visual overview in which the steps from Tactics Box 9.1 are explicitly noted. Because F_x is positive (a force to the right), we know the ball was initially moving toward the left and is hit back toward the right. Thus we converted the statements about *speeds* into information about *velocities*, with $(v_x)_i$ negative.

Example 9.3 Force in hitting a baseball (cont.)

SOLVE In the last several chapters we've started the mathematical solution with Newton's second law. Now we want to use the impulse-momentum theorem:

$$\Delta p_x = J_x = \text{area under the force curve}$$

We know the velocities before and after the collision, so we can find the change in the ball's momentum:



$$\Delta p_x = m(v_x)_\mathrm{f} - m(v_x)_\mathrm{i} = (0.15\ \mathrm{kg})(40\ \mathrm{m/s} - (-20\ \mathrm{m/s}))$$
$$= 9.0\ \mathrm{kg \cdot m/s}$$

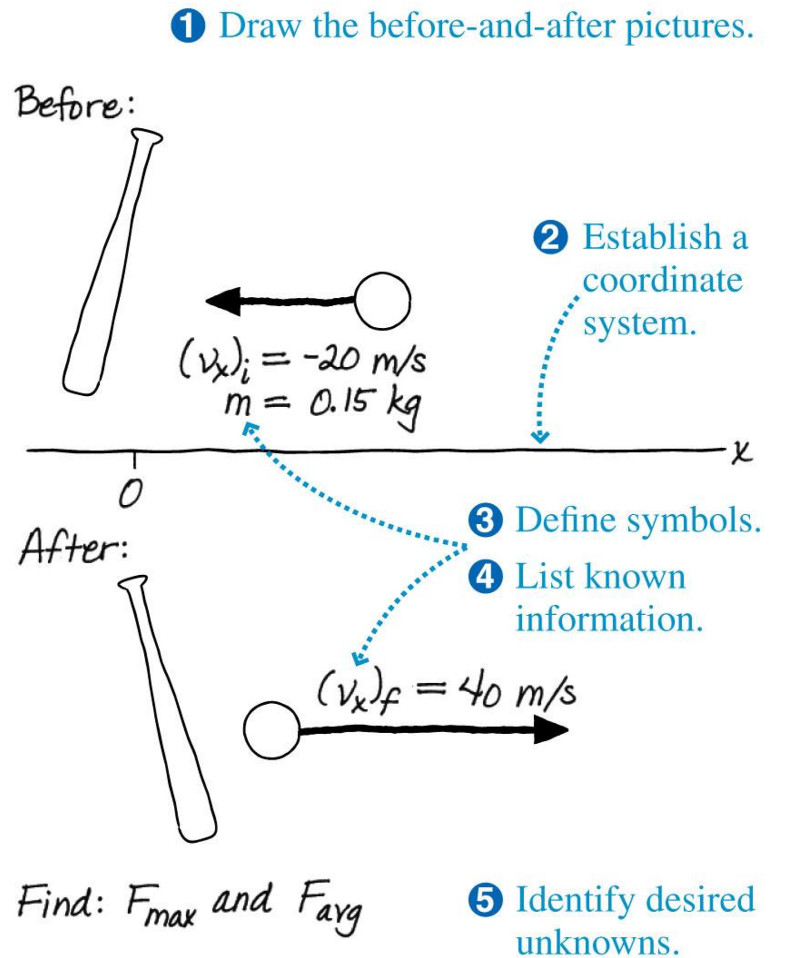
Example 9.3 Force in hitting a baseball (cont.)

The force curve is a triangle with height F_{max} and width 0.60 ms. As in Example 9.1, the area under the curve is

$$J_x = \text{area} = \tfrac{1}{2} \times F_{max} \times (6.0 \times 10^{-4}\ \text{s})$$
$$= (F_{max})(3.0 \times 10^{-4})$$

According to the impulse-momentum theorem, $\Delta px = Jx$, so we have

$$9.0\ \text{kg} \cdot \text{m/s} = (F_{max})(3.0 \times 10^{-4}\ \text{s})$$



Example 9.3 Force in hitting a baseball (cont.)

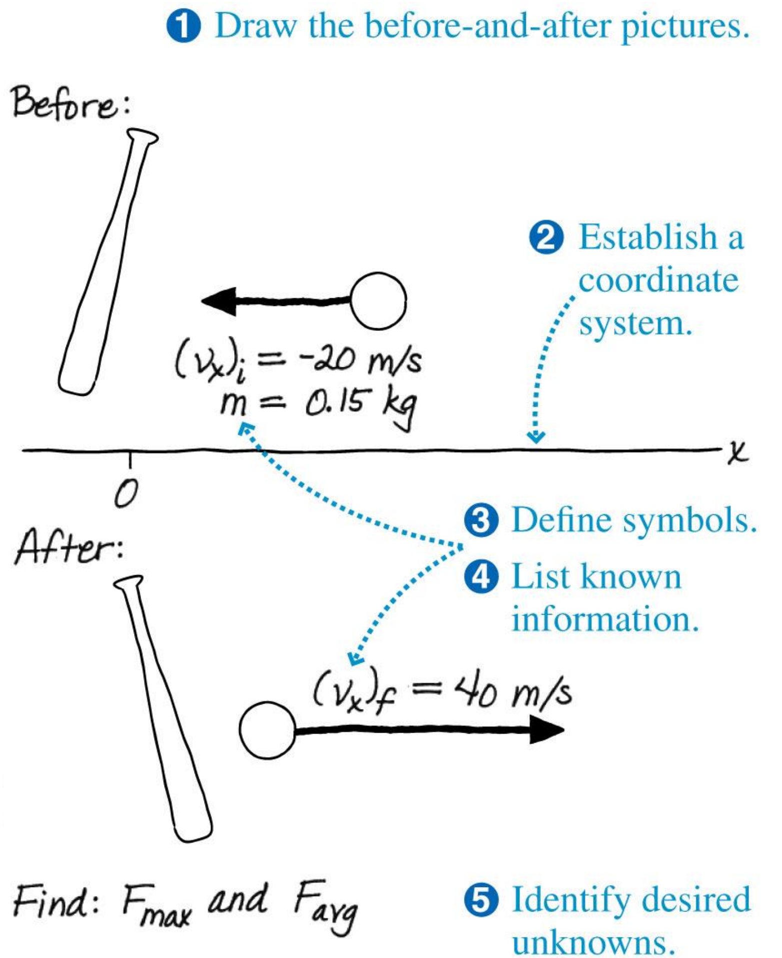
Thus the *maximum* force is

$$F_{\text{max}} = \frac{9.0\ \text{kg}\cdot\text{m/s}}{3.0 \times 10^{-4}\ \text{s}} = 30{,}000\ \text{N}$$

Using Equation 9.1, we find that the *average* force, which depends on the collision duration $\Delta t = 6.0 \times 10^{-4}$ s, has the smaller value:

$$F_{\text{avg}} = \frac{J_x}{\Delta t} = \frac{\Delta p_x}{\Delta t} = \frac{9.0\ \text{kg}\cdot\text{m/s}}{6.0 \times 10^{-4}\ \text{s}} = 15{,}000$$

ASSESS F_{max} is a large force, but quite typical of the impulsive forces during collisions.



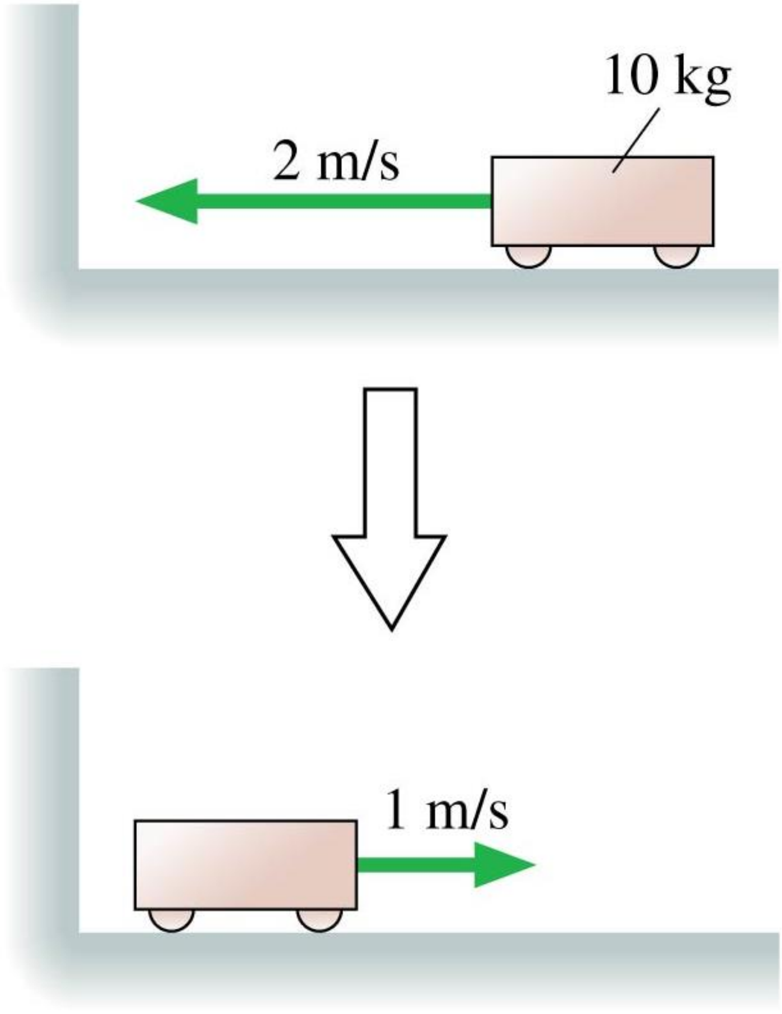
The Impulse Approximation

- The **impulse approximation** states that we can ignore the small forces that act *during* the brief time of the impulsive force.
- We consider only the momenta and velocities *immediately* before and *immediately* after the collisions.

QuickCheck 9.1

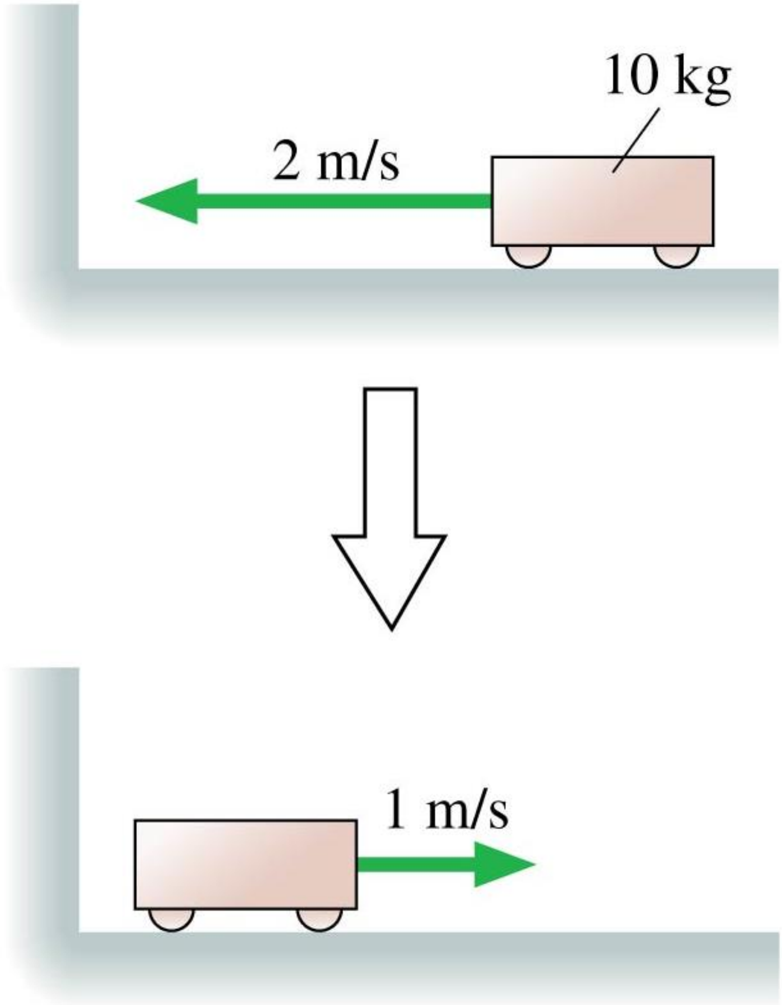
The cart's change of momentum Δp_x is

A. –20 kg m/s
B. –10 kg m/s
C. 0 kg m/s
D. 10 kg m/s
E. 30 kg m/s



QuickCheck 9.1

The cart's change of momentum Δp_x is



A. –20 kg m/s
B. –10 kg m/s
C. 0 kg m/s
D. 10 kg m/s
E. 30 kg m/s ✔

Δp_x = 10 kg m/s – (–20 kg m/s) = 30 kg m/s

Negative initial momentum because motion is to the left and $v_x < 0$.

Example Problem

A 500 kg rocket sled is coasting at 20 m/s. It then turns on its rocket engines for 5.0 s, with a thrust of 1000 N. What is its final speed?

Example Problem

A car traveling at 20 m/s crashes into a bridge abutment. Estimate the force on the driver if the driver is stopped by

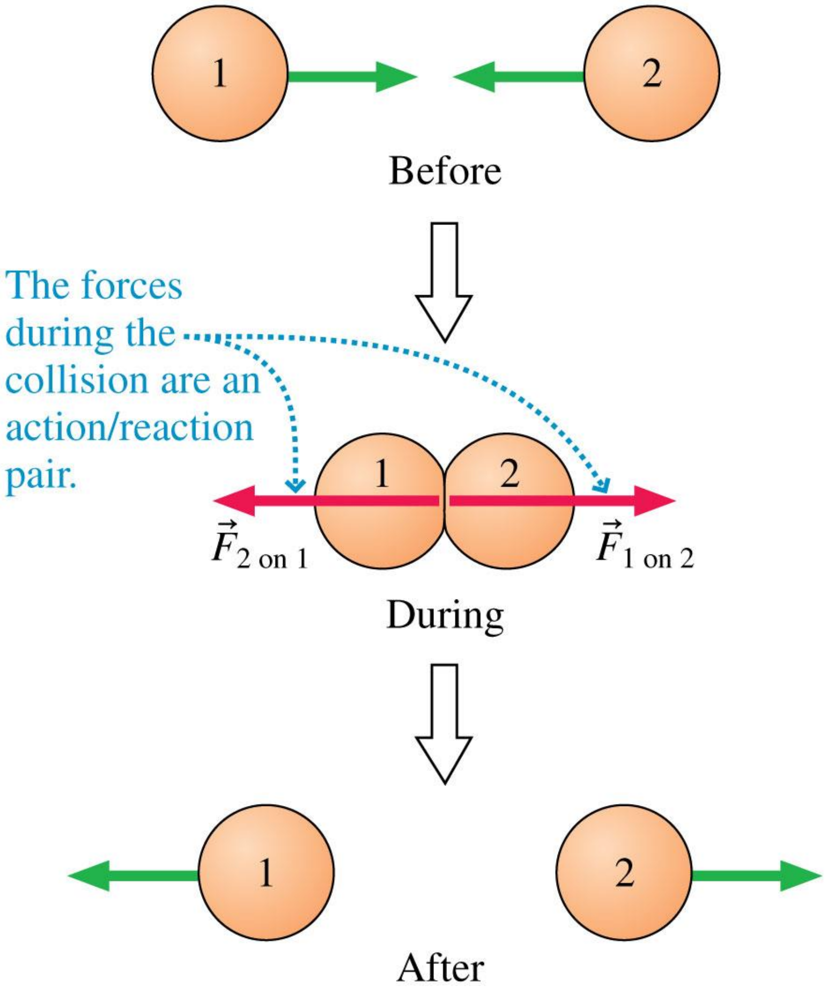
A. A 20-m-long row of water-filled barrels.

B. The crumple zone of her car (~1 m). Assume a constant acceleration.

Section 9.4 Conservation of Momentum

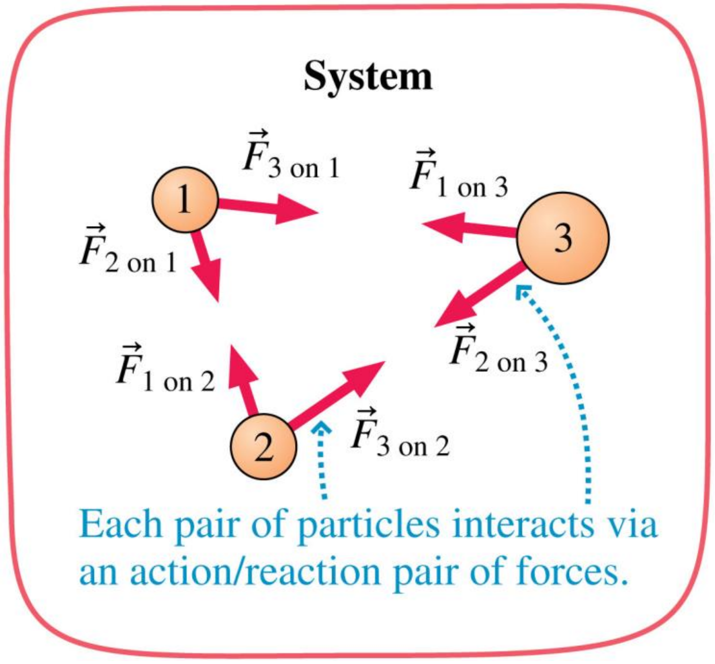
Conservation of Momentum

- The forces acting on two balls during a collision form an action/reaction pair. They have equal magnitude but opposite directions (Newton's third law).
- If the momentum of ball 1 increases, the momentum of ball 2 will decrease by the same amount.



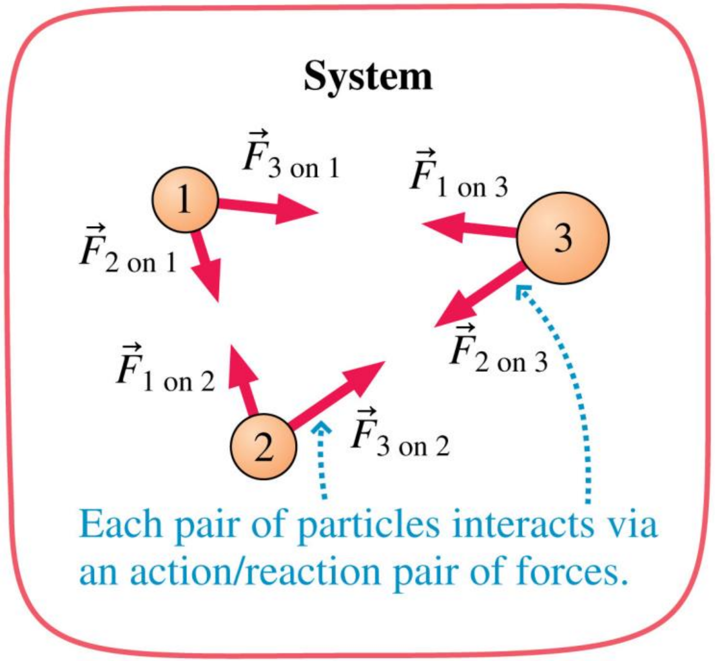
Law of Conservation of Momentum

- **There is no change in the *total* momentum of the system** no matter how complicated the forces are between the particles.
- The total momentum of the system is *conserved*.



Law of Conservation of Momentum

- **Internal forces** act only between particles within a system.
- **The total momentum of a system subjected to only internal forces is conserved.**



QuickCheck 9.7

You awake in the night to find that your living room is on fire. Your one chance to save yourself is to throw something that will hit the back of your bedroom door and close it, giving you a few seconds to escape out the window. You happen to have both a sticky ball of clay and a super-bouncy Superball next to your bed, both the same size and same mass. You've only time to throw one. Which will it be? Your life depends on making the right choice!

A. Throw the Superball.
B. Throw the ball of clay.
C. It doesn't matter. Throw either.

QuickCheck 9.7

You awake in the night to find that your living room is on fire. Your one chance to save yourself is to throw something that will hit the back of your bedroom door and close it, giving you a few seconds to escape out the window. You happen to have both a sticky ball of clay and a super-bouncy Superball next to your bed, both the same size and same mass. You've only time to throw one. Which will it be? Your life depends on making the right choice!

✔ A. Throw the Superball. Larger $\Delta p \Rightarrow$ more impulse to door

B. Throw the ball of clay.

C. It doesn't matter. Throw either.

QuickCheck 9.8

A mosquito and a truck have a head-on collision. Splat! Which has a larger change of momentum?

A. The mosquito
B. The truck
C. They have the same change of momentum.
D. Can't say without knowing their initial velocities.

QuickCheck 9.8

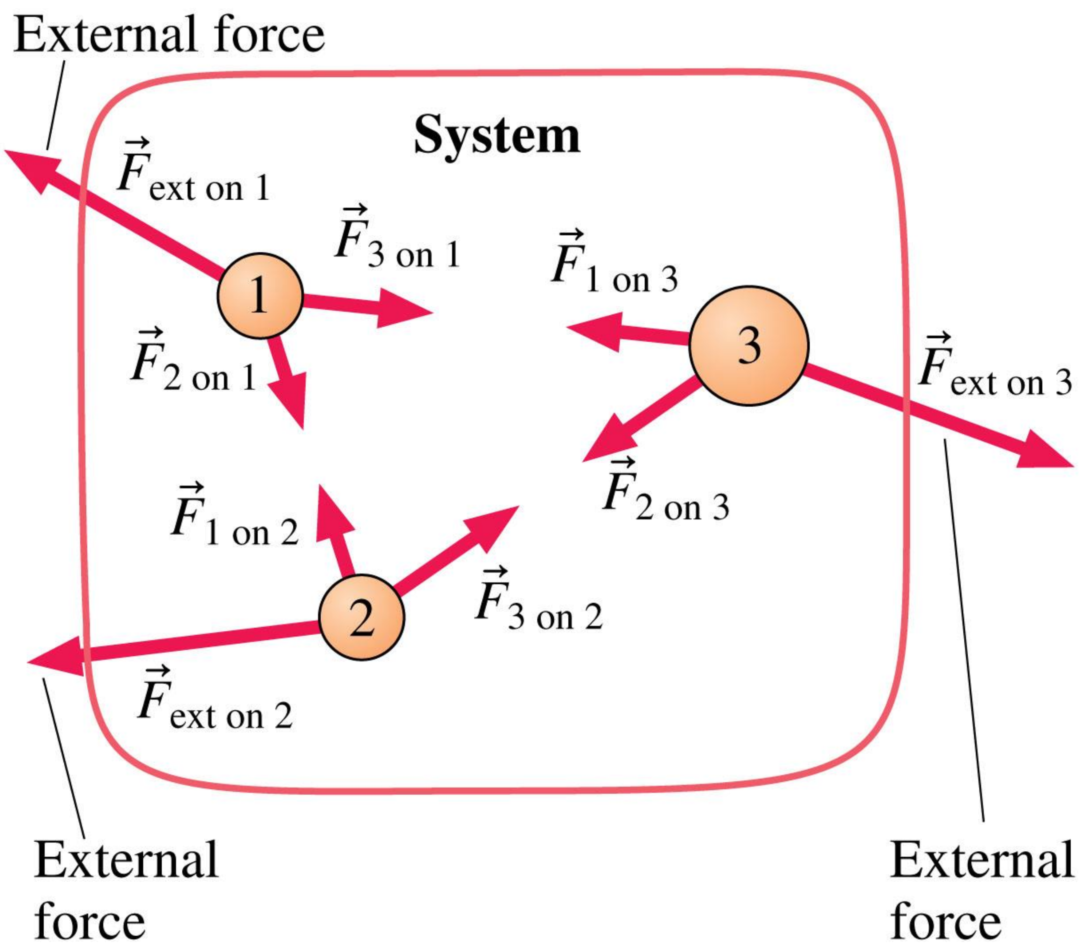
A mosquito and a truck have a head-on collision. Splat! Which has a larger change of momentum?

A. The mosquito
B. The truck
✔ C. They have the same change of momentum.
D. Can't say without knowing their initial velocities.

Momentum is conserved, so $\Delta p_{\text{mosquito}} + \Delta p_{\text{truck}} = 0$.
Equal magnitude (but opposite sign) changes in momentum.

Law of Conservation of Momentum

- **External forces** are forces from agents outside the system.
- External forces *can* change the momentum of the system.



Law of Conservation of Momentum

- The change in the total momentum is

$$\begin{aligned}\Delta \vec{P} &= \Delta \vec{p}_1 + \Delta \vec{p}_2 + \Delta \vec{p}_3 \\ &= (\vec{F}_{\text{ext on 1}}\,\Delta t) + (\vec{F}_{\text{ext on 2}}\,\Delta t) + (\vec{F}_{\text{ext on 3}}\,\Delta t) \\ &= (\vec{F}_{\text{ext on 1}} + \vec{F}_{\text{ext on 2}} + \vec{F}_{\text{ext on 3}})\,\Delta t \\ &= \vec{F}_{\text{net}}\,\Delta t\end{aligned}$$

- $\vec{F}_{\text{net}}$ is the net force due to *external forces*.
- **If $\vec{F}_{\text{net}} = \vec{0}$, the *total* momentum $\vec{P}$ of the system does not change.**
- An **isolated system** is a system with no net external force acting on it, leaving the momentum unchanged.

Law of Conservation of Momentum

Law of conservation of momentum The total momentum $\vec{P}$ of an isolated system is a constant. Interactions within the system do not change the system's total momentum.

- The law of conservation of momentum for an isolated system is written

$$\vec{P}_f = \vec{P}_i$$

Law of conservation of momentum for an isolated system

- **The total momentum after an interaction is equal to the total momentum before the interaction.**

Law of Conservation of Momentum

- Since momentum is a vector, we can rewrite the law of conservation of momentum for an isolated system:

Final momentum = Initial momentum

x-component:

$$(p_{1x})_\mathrm{f} + (p_{2x})_\mathrm{f} + (p_{3x})_\mathrm{f} + \cdots = (p_{1x})_\mathrm{i} + (p_{2x})_\mathrm{i} + (p_{3x})_\mathrm{i} + \cdots$$

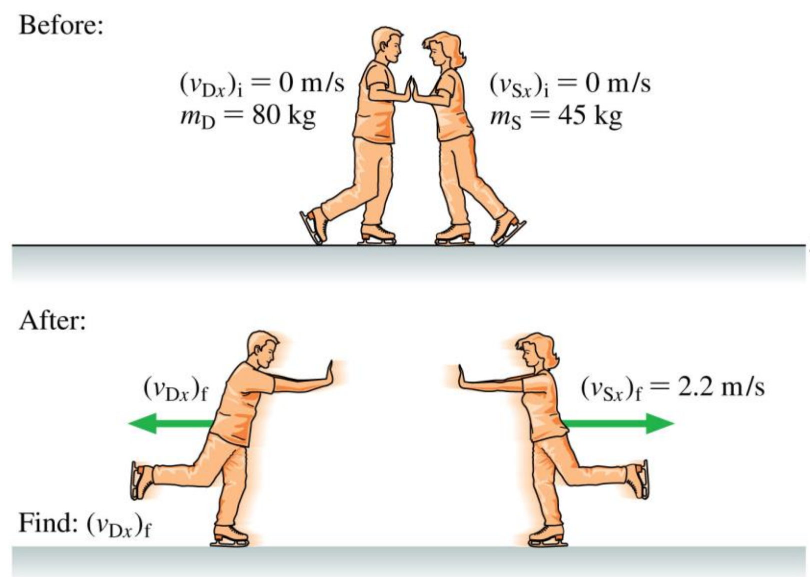
Particle 1 Particle 2 Particle 3

y-component:

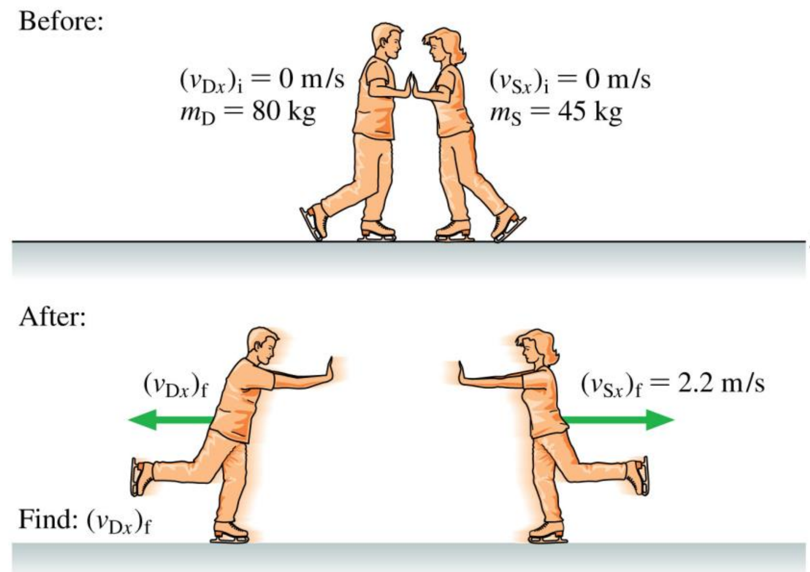
$$(p_{1y})_\mathrm{f} + (p_{2y})_\mathrm{f} + (p_{3y})_\mathrm{f} + \cdots = (p_{1y})_\mathrm{i} + (p_{2y})_\mathrm{i} + (p_{3y})_\mathrm{i} + \cdots$$

Example 9.5 Speed of ice skaters pushing off

Two ice skaters, Sandra and David, stand facing each other on frictionless ice. Sandra has a mass of 45 kg, David a mass of 80 kg. They then push off from each other. After the push, Sandra moves off at a speed of 2.2 m/s. What is David's speed?



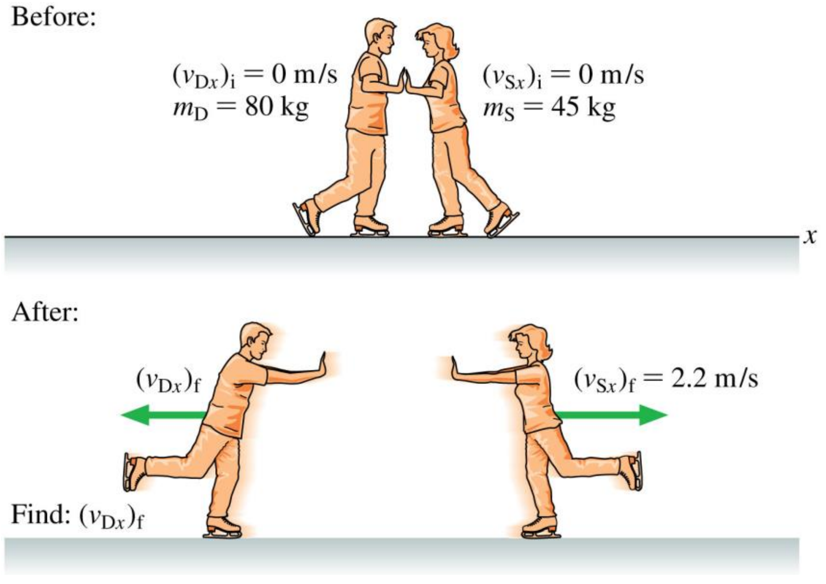
Example 9.5 Speed of ice skaters pushing off (cont.)



PREPARE The two skaters interact with each other, but they form an isolated system because, for each skater, the upward normal force of the ice balances their downward weight force to make $\vec{F}_{net} = \vec{0}$. Thus the total momentum of the system of the two skaters is conserved.

FIGURE 9.17 shows a before-and-after visual overview for the two skaters. The total momentum before they push off is $\vec{P}_i = \vec{0}$ because both skaters are at rest. Consequently, the total momentum will still be $\vec{0}$ after they push off.

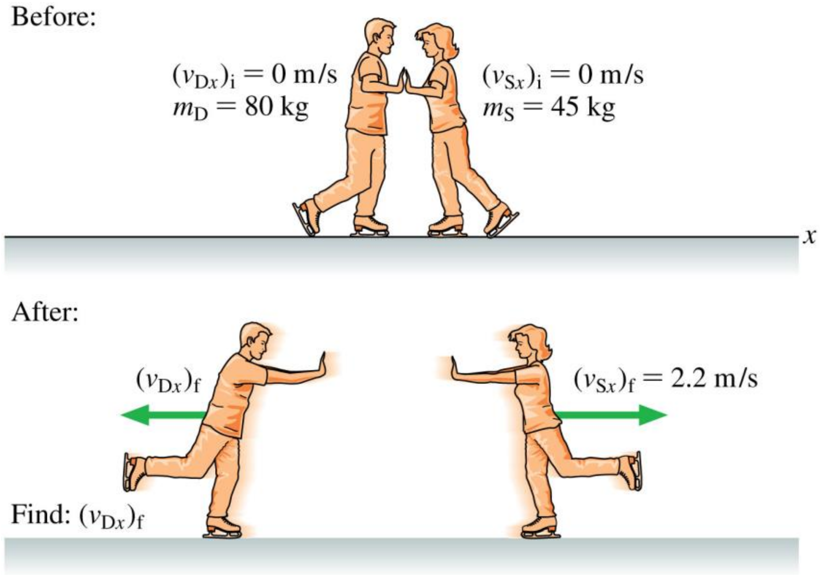
Example 9.5 Speed of ice skaters pushing off (cont.)



SOLVE Since the motion is only in the *x*-direction, we'll need to consider only *x*-components of momentum. We write Sandra's initial momentum as $(p_{Sx})_i = m_S(v_{Sx})_i$, where m_S is her mass and $(v_{Sx})_i$ her initial velocity. Similarly, we write David's initial momentum as $(p_{Dx})_i = m_D(v_{Dx})_i$. Both these momenta are zero because both skaters are initially at rest.

Example 9.5 Speed of ice skaters pushing off (cont.)

We can now apply the mathematical statement of momentum conservation, Equation 9.15. Writing the final momentum of Sandra as $m_S(v_{Sx})_f$ and that of David as $m_D(v_{Dx})_f$, we have



$$\underbrace{m_S(v_{Sx})_f + m_D(v_{Dx})_f}_{\text{The skaters' final momentum . . .}} = \underbrace{m_S(v_{Sx})_i + m_D(v_{Dx})_i}_{\text{. . . equals their initial momentum . . .}} = 0$$

. . . which was zero.

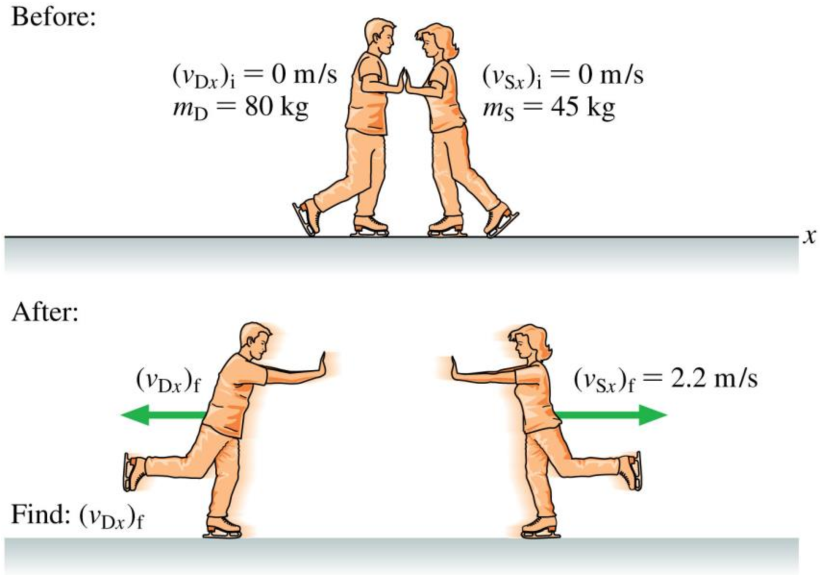
Example 9.5 Speed of ice skaters pushing off (cont.)

Solving for $(v_{\mathrm{D}x})_\mathrm{f}$, we find

$$(v_{\mathrm{D}x})_\mathrm{f} = -\frac{m_\mathrm{S}}{m_\mathrm{D}}(v_{\mathrm{S}x})_\mathrm{f}$$

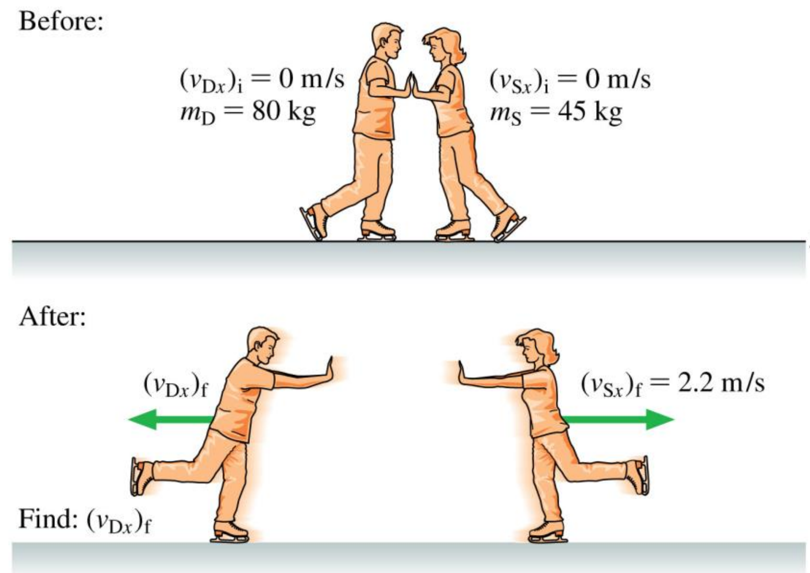
$$= -\frac{45\ \mathrm{kg}}{80\ \mathrm{kg}} \times 2.2\ \mathrm{m/s}$$

$$= -1.2\ \mathrm{m/s}$$



David moves backward with a *speed* of 1.2 m/s.

Example 9.5 Speed of ice skaters pushing off (cont.)



Notice that we didn't need to know any details about the force between David and Sandra in order to find David's final speed. Conservation of momentum *mandates* this result.

ASSESS It seems reasonable that Sandra, whose mass is less than David's, would have the greater final speed.

Law of Conservation of Momentum

PROBLEM-SOLVING STRATEGY 9.1 **Conservation of momentum problems**

We can use the law of conservation of momentum to relate the momenta and velocities of objects *after* an interaction to their values *before* the interaction.

PREPARE Clearly define the *system*.

- If possible, choose a system that is isolated ($\vec{F}_{net} = \vec{0}$) or within which the interactions are sufficiently short and intense that you can ignore external forces for the duration of the interaction (the impulse approximation). Momentum is then conserved.
- If it's not possible to choose an isolated system, try to divide the problem into parts such that momentum is conserved during one segment of the motion. Other segments of the motion can be analyzed using Newton's laws or, as you'll learn in Chapter 10, conservation of energy.

Text: p. 265

Law of Conservation of Momentum

PROBLEM-SOLVING STRATEGY 9.1 **Conservation of momentum problems**

Following Tactics Box 9.1, draw a before-and-after visual overview. Define symbols that will be used in the problem, list known values, and identify what you're trying to find.

SOLVE The mathematical representation is based on the law of conservation of momentum, Equations 9.15. Because we generally want to solve for the velocities of objects, we usually use Equations 9.15 in the equivalent form

$$m_1(v_{1x})_f + m_2(v_{2x})_f + \cdots = m_1(v_{1x})_i + m_2(v_{2x})_i + \cdots$$
$$m_1(v_{1y})_f + m_2(v_{2y})_f + \cdots = m_1(v_{1y})_i + m_2(v_{2y})_i + \cdots$$

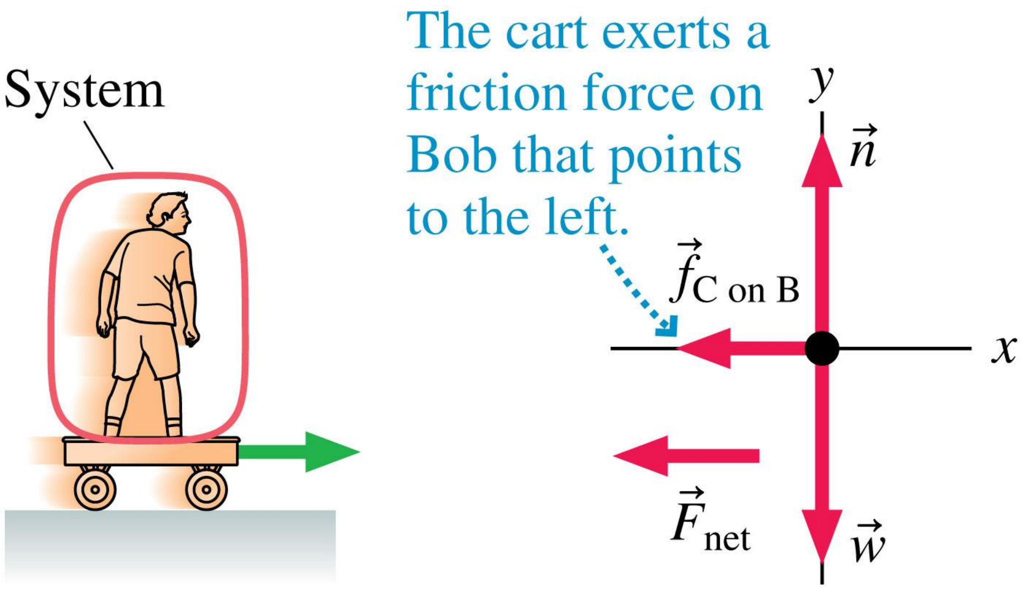
ASSESS Check that your result has the correct units, is reasonable, and answers the question.

Exercise 17

Text: p. 265

It Depends on the System

- **The goal is to choose a system where momentum will be conserved.**
- For a skateboarder, if we choose just the person, there is a nonzero net force on the system.



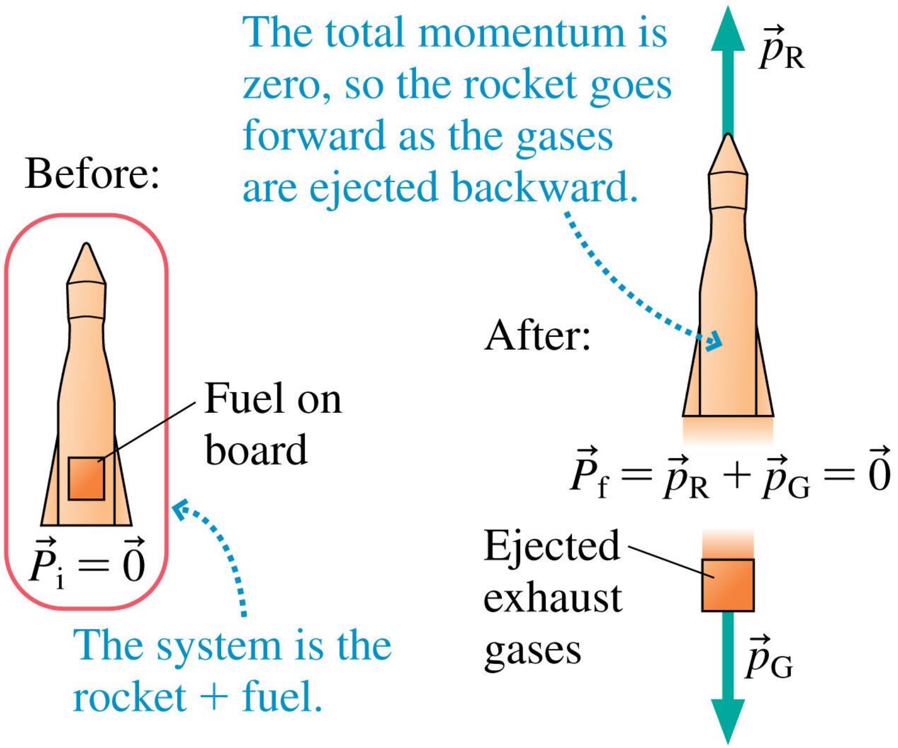
- If we choose the system to be the person *and* the cart, the net force is zero and the momentum is conserved.

Example Problem

Jack stands at rest on a skateboard. The mass of Jack and the skateboard together is 75 kg. Ryan throws a 3.0 kg ball horizontally to the right at 4.0 m/s to Jack, who catches it. What is the final speed of Jack and the skateboard?

Explosions

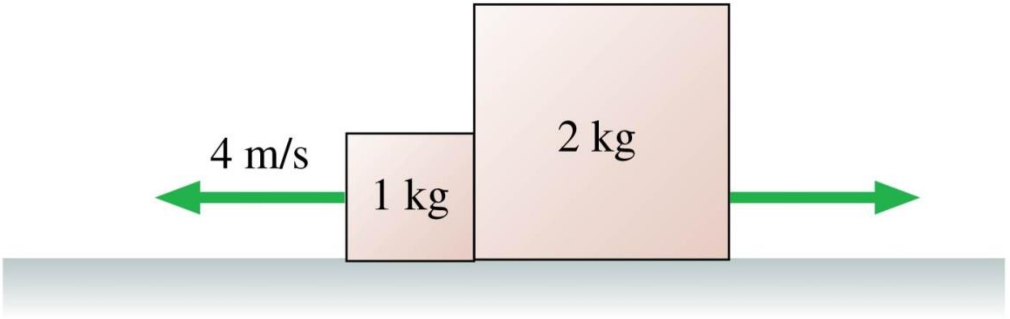
- An **explosion** is when the particles of the system move apart after a brief, intense interaction.
- An explosion is the opposite of a collision.
- The forces are *internal* forces and if the system is isolated, the total momentum is conserved.



QuickCheck 9.10

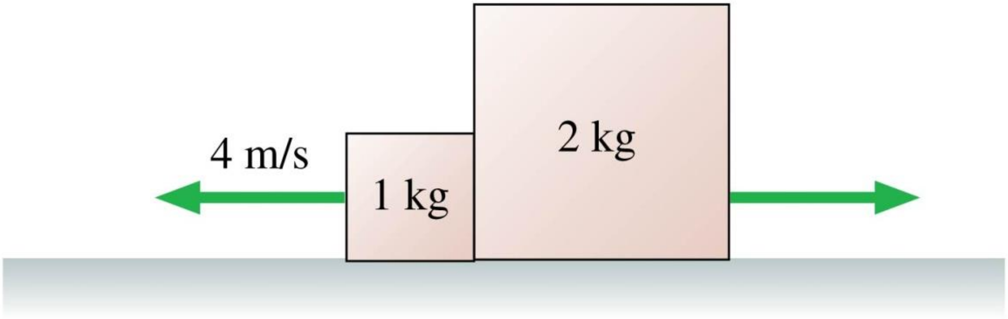
The two boxes are on a frictionless surface. They had been sitting together at rest, but an explosion between them has just pushed them apart. How fast is the 2-kg box going?

A. 1 m/s
B. 2 m/s
C. 4 m/s
D. 8 m/s
E. There's not enough information to tell.



QuickCheck 9.10

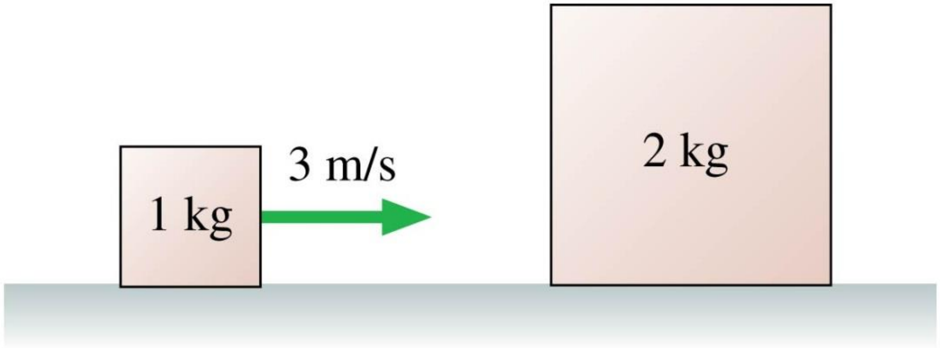
The two boxes are on a frictionless surface. They had been sitting together at rest, but an explosion between them has just pushed them apart. How fast is the 2-kg box going?



A. 1 m/s
✔ B. 2 m/s
C. 4 m/s
D. 8 m/s
E. There's not enough information to tell.

QuickCheck 9.11

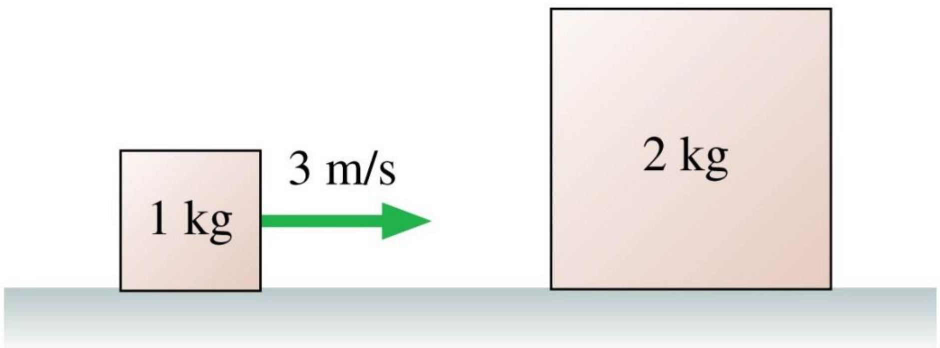
The 1-kg box is sliding along a frictionless surface. It collides with and sticks to the 2-kg box. Afterward, the speed of the two boxes is



A. 0 m/s
B. 1 m/s
C. 2 m/s
D. 3 m/s
E. There’s not enough information to tell.

QuickCheck 9.11

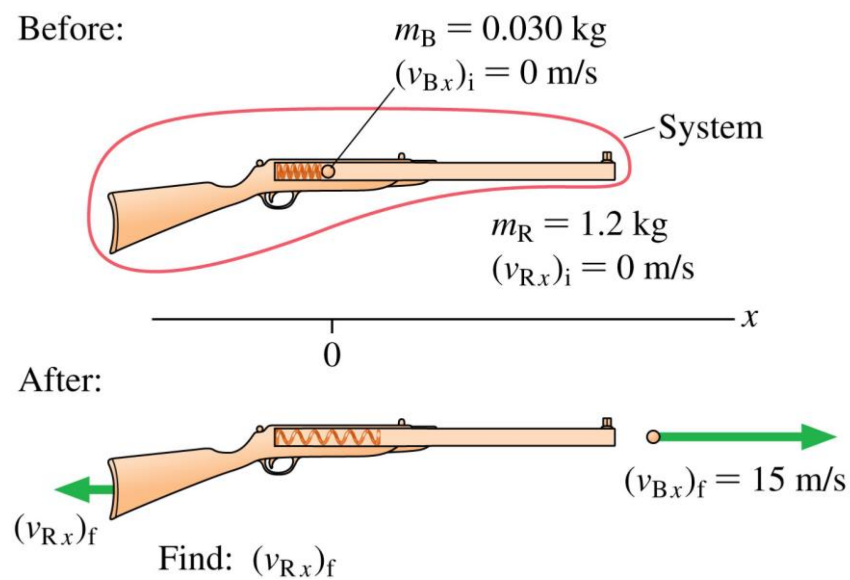
The 1-kg box is sliding along a frictionless surface. It collides with and sticks to the 2-kg box. Afterward, the speed of the two boxes is



A. 0 m/s
✔ B. 1 m/s
C. 2 m/s
D. 3 m/s
E. There's not enough information to tell.

Example 9.7 Recoil speed of a rifle

A 30 g ball is fired from a 1.2 kg spring-loaded toy rifle with a speed of 15 m/s. What is the recoil speed of the rifle?



PREPARE As the ball moves down the barrel, there are complicated forces exerted on the ball and on the rifle. However, if we take the system to be the ball + rifle, these are *internal* forces that do not change the total momentum.

Example 9.7 Recoil speed of a rifle (cont.)

The *external* forces of the rifle's and ball's weights are balanced by the external force exerted by the person holding the rifle, so $\vec{F}_{net} = \vec{0}$. This is an isolated system and the law of conservation of momentum applies.

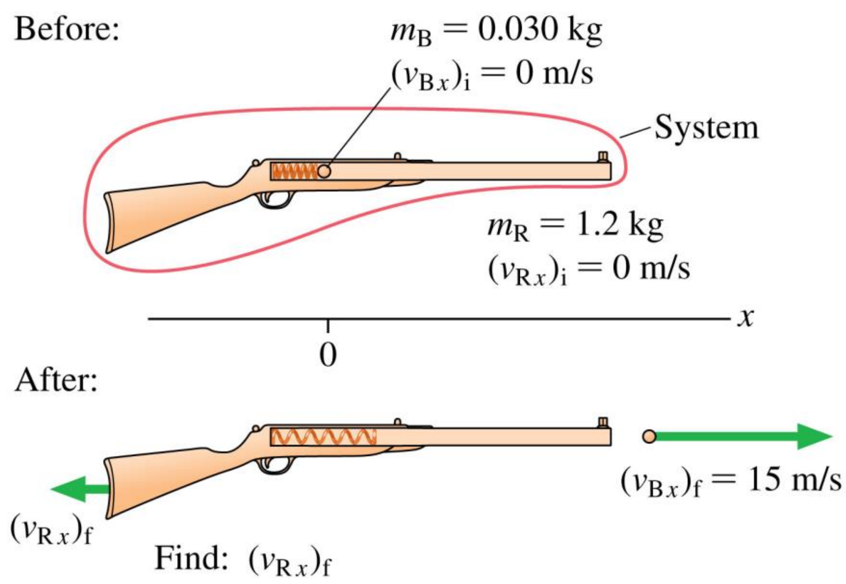
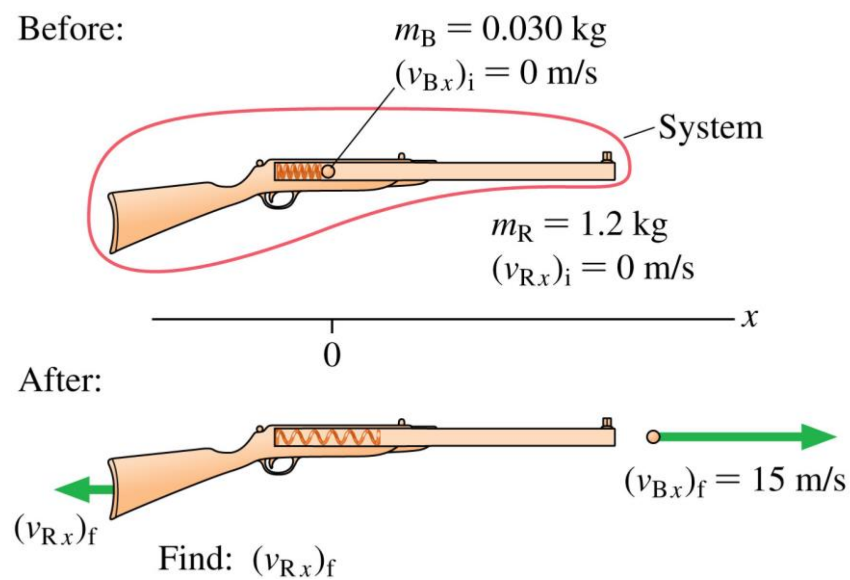


FIGURE 9.20 shows a visual overview before and after the ball is fired. We'll assume the ball is fired in the +*x*-direction.

Example 9.7 Recoil speed of a rifle (cont.)

SOLVE The x-component of the total momentum is $P_x = p_{Bx} + p_{Rx}$. Everything is at rest before the trigger is pulled, so the initial momentum is zero. After the trigger is pulled, the internal force of the spring pushes the ball down the barrel and pushes the rifle backward. Conservation of momentum gives



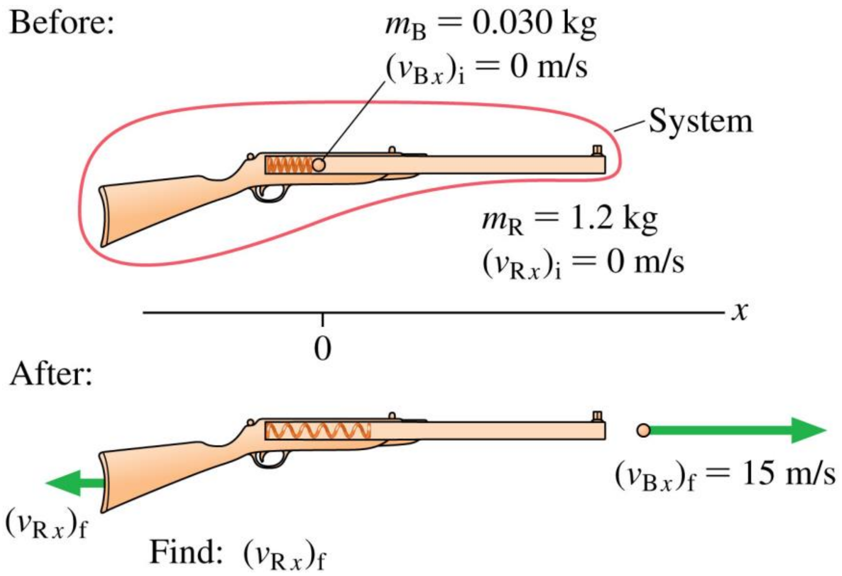
$$(P_x)_f = m_B(v_{Bx})_f + m_R(v_{Rx})_f = (P_x)_i = 0$$

Example 9.7 Recoil speed of a rifle (cont.)

Solving for the rifle's velocity, we find

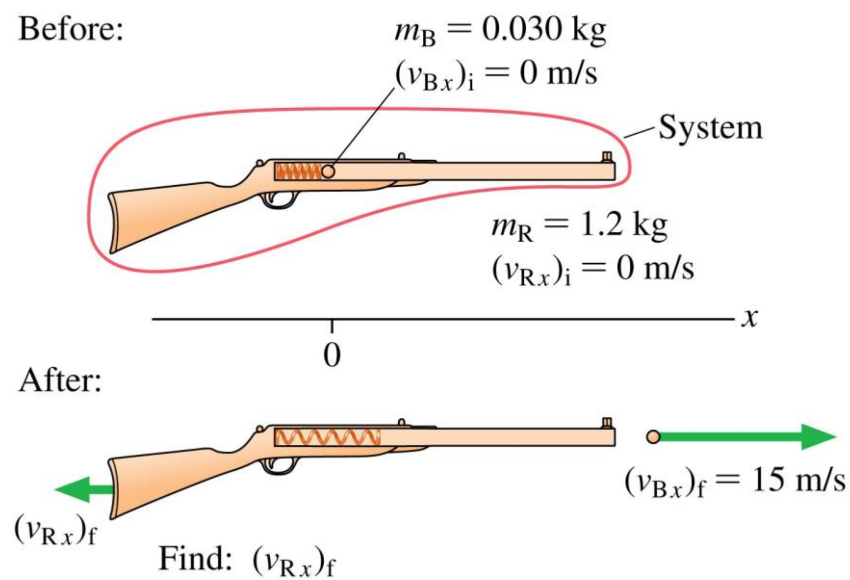
$$(v_{Rx})_f = -\frac{m_B}{m_R}(v_{Bx})_f = -\frac{0.030\ \text{kg}}{1.2\ \text{kg}} \times 15\ \text{m/s} = -0.38\ \text{m/s}$$

The minus sign indicates that the rifle's recoil is to the left. The recoil *speed* is 0.38 m/s.



Example 9.7 Recoil speed of a rifle (cont.)

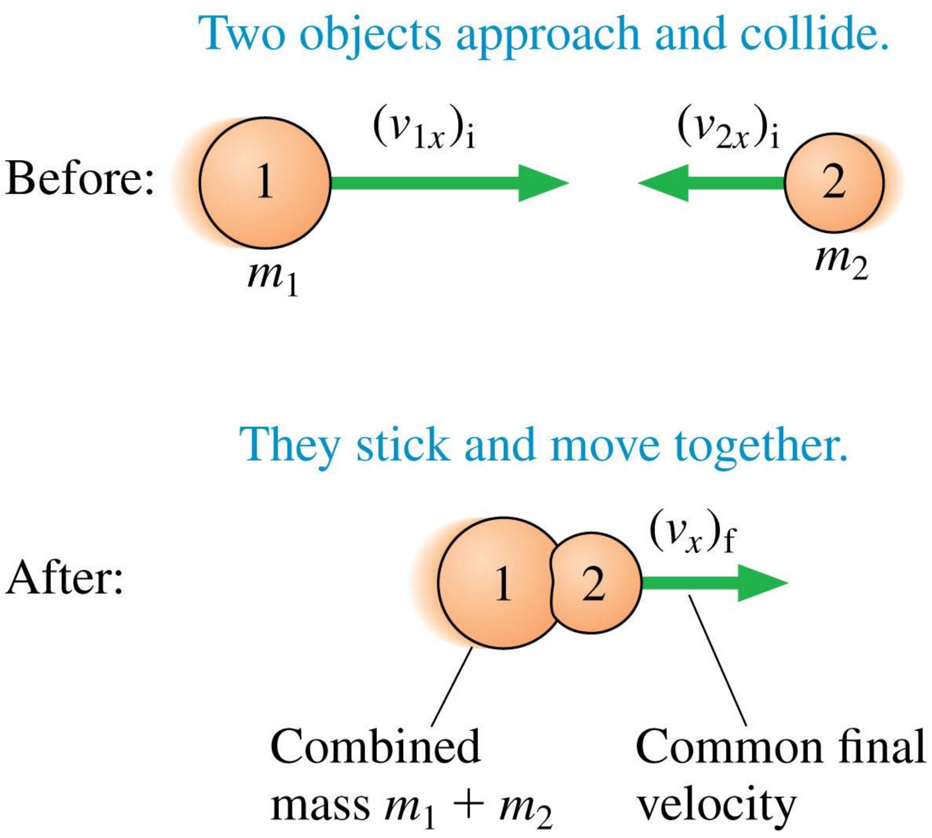
ASSESS Real rifles fire their bullets at much higher velocities, and their recoil is correspondingly higher. Shooters need to brace themselves against the "kick" of the rifle back against their shoulder.



Section 9.5 Inelastic Collisions

Inelastic Collisions

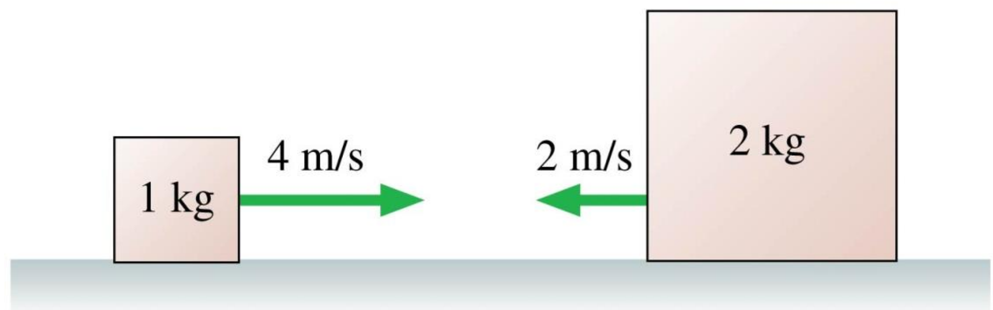
- A **perfectly inelastic collision** is a collision in which the two objects stick together and move with a common final velocity.
- Examples of perfectly inelastic collisions include clay hitting the floor and a bullet embedding itself in wood.



QuickCheck 9.9

The two boxes are sliding along a frictionless surface. They collide and stick together. Afterward, the velocity of the two boxes is

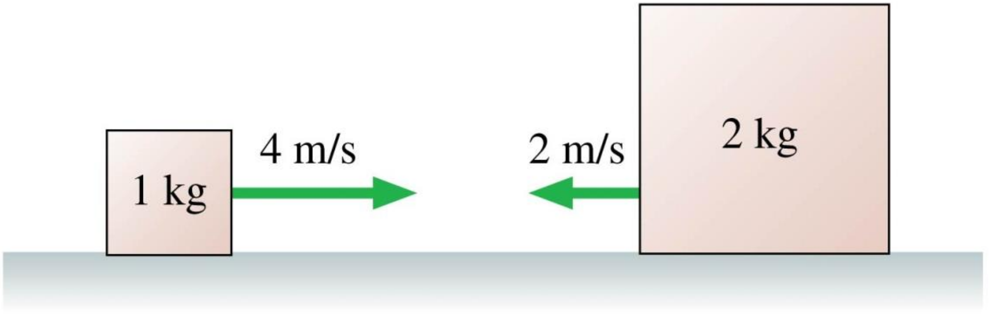
A. 2 m/s to the left
B. 1 m/s to the left
C. 0 m/s, at rest
D. 1 m/s to the right
E. 2 m/s to the right



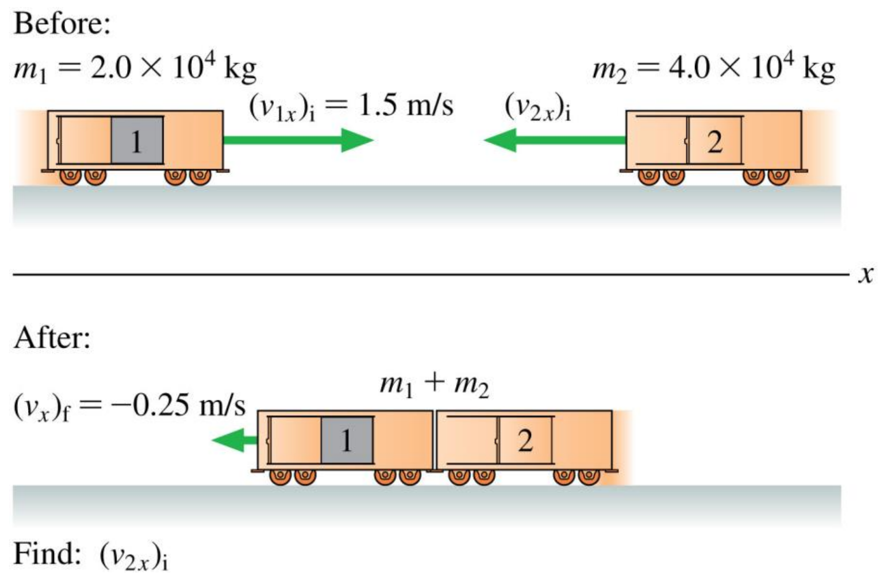
QuickCheck 9.9

The two boxes are sliding along a frictionless surface. They collide and stick together. Afterward, the velocity of the two boxes is

A. 2 m/s to the left
B. 1 m/s to the left
✔ C. 0 m/s, at rest
D. 1 m/s to the right
E. 2 m/s to the right

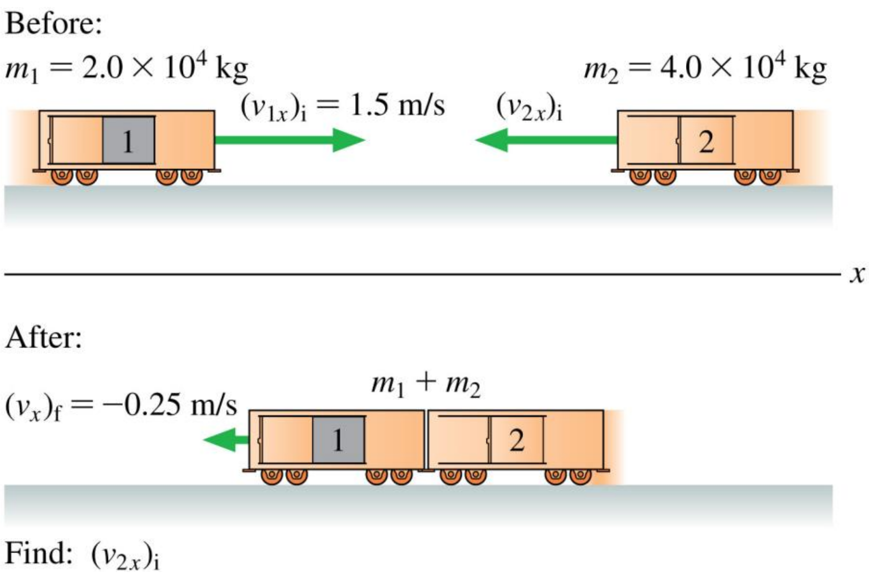


Example 9.8 A perfectly inelastic collision of railroad cars



In assembling a train from several railroad cars, two of the cars, with masses 2.0×10^4 kg and 4.0×10^4 kg, are rolled toward each other. When they meet, they couple and stick together. The lighter car has an initial speed of 1.5 m/s; the collision causes it to reverse direction at 0.25 m/s. What was the initial speed of the heavier car?

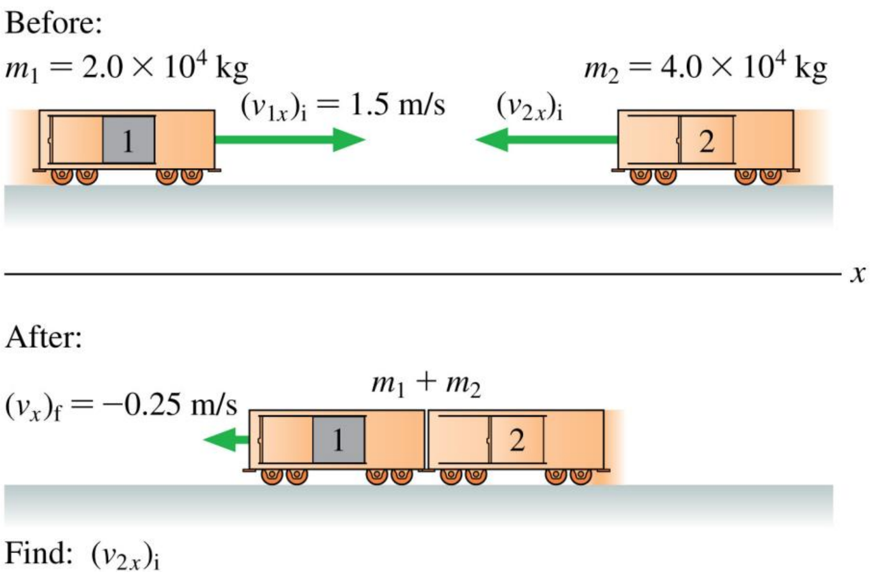
Example 9.8 A perfectly inelastic collision of railroad cars (cont.)



PREPARE We model the cars as particles and define the two cars as the system. This is an isolated system, so its total momentum is conserved in the collision. The cars stick together, so this is a perfectly inelastic collision.

FIGURE 9.23 shows a visual overview. We've chosen to let the 2.0×10^4 kg car (car 1) start out moving to the right, so $(v_{1x})_i$ is a positive 1.5 m/s. The cars move to the left after the collision, so their common final velocity is $(v_x)_f = -0.25$ m/s. You can see that velocity $(v_{2x})_i$ must be negative in order to "turn around" both cars.

Example 9.8 A perfectly inelastic collision of railroad cars (cont.)



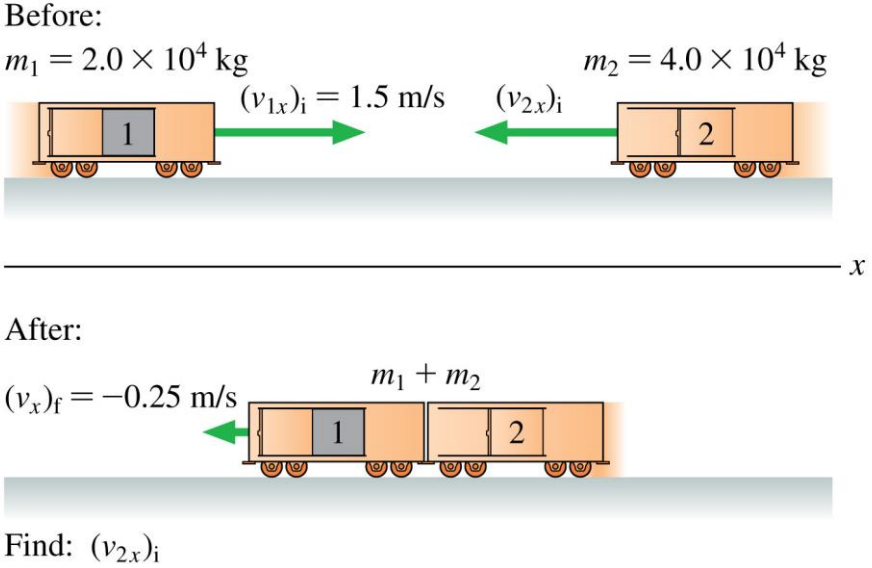
SOLVE The law of conservation of momentum, $(P_x)_f = (P_x)_i$, is

$$(m_1 + m_2)(v_x)_f = m_1(v_{1x})_i + m_2(v_{2x})_i$$

where we made use of the fact that the combined mass $m_1 + m_2$ moves together after the collision.

Example 9.8 A perfectly inelastic collision of railroad cars (cont.)

We can easily solve for the initial velocity of the 4.0×10^4 kg car:



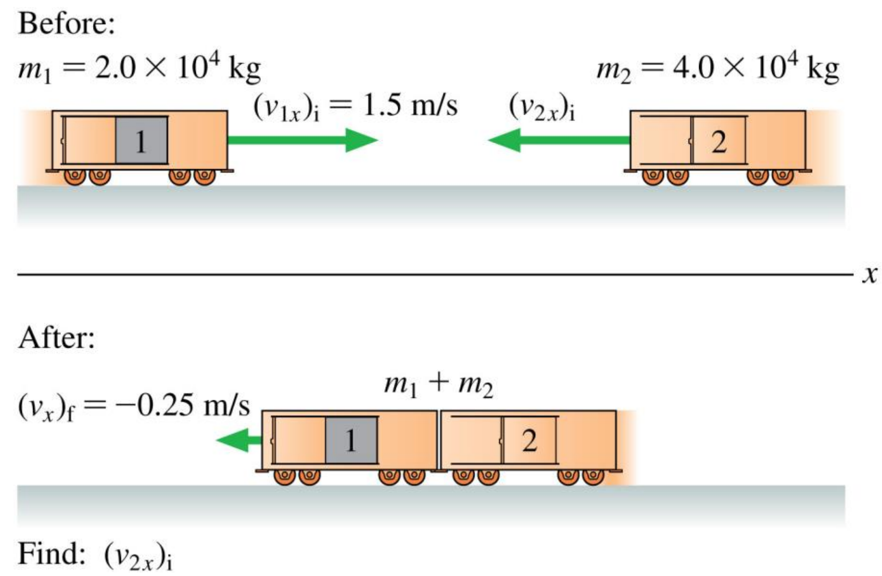
$$(v_{2x})_i = \frac{(m_1 + m_2)(v_x)_f - m_1(v_{1x})_i}{m_2}$$

$$= \frac{(6.0 \times 10^4 \text{ kg})(-0.25 \text{ m/s}) - (2.0 \times 10^4 \text{ kg})(1.5 \text{ m/s})}{4.0 \times 10^4 \text{ kg}}$$

$$= -1.1 \text{ m/s}$$

Example 9.8 A perfectly inelastic collision of railroad cars (cont.)

The negative sign, which we anticipated, indicates that the heavier car started out moving to the left. The initial speed of the car, which we were asked to find, is 1.1 m/s.



ASSESS The key step in solving inelastic collision problems is that both objects move after the collision with the same velocity. You should thus choose a single symbol (here, $(v_x)_f$) for this common velocity.

Example Problem

A 10 g bullet is fired into a 1.0 kg wood block, where it lodges. Subsequently the block slides 4.0 m across a floor (μ_k = 0.20 for wood on wood). What was the bullet's speed?

Section 9.6 Momentum and Collisions in Two Dimensions

Momentum and Collision in Two Dimensions

- When the motion of the collisions occur in two dimensions, we must solve for each component of the momentum:

$$(p_{1x})_f + (p_{2x})_f + (p_{3x})_f + \cdots = (p_{1x})_i + (p_{2x})_i + (p_{3x})_i + \cdots$$

$$(p_{1y})_f + (p_{2y})_f + (p_{3y})_f + \cdots = (p_{1y})_i + (p_{2y})_i + (p_{3y})_i + \cdots$$

- In these collisions, the individual momenta can change *but the total momentum does not.*

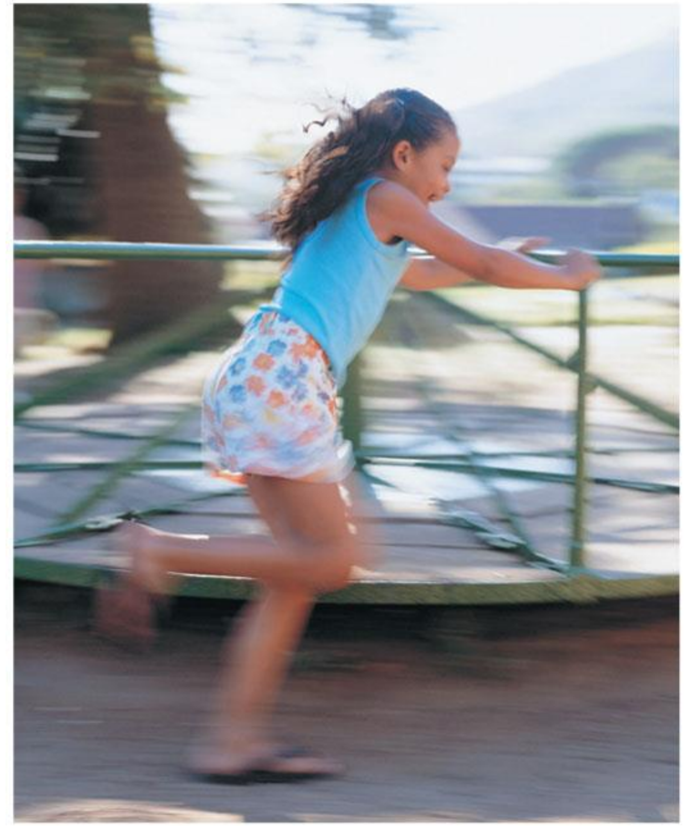
Example Problem

Two pucks of equal mass 100 g collide on an air hockey table. Neglect friction. Prior to the collision, puck 1 travels in a direction that can be considered the $+x$-axis at 1 m/s, and puck 2 travels in the $-y$-direction at 2 m/s prior to the collision. After the collision, puck 2 travels 30 degrees above the $+x$-direction (between $+x$ and $+y$) at 0.8 m/s. What is the velocity (direction and speed) of puck 1 after the collision? How does the final kinetic energy compare to the initial kinetic energy?

Section 9.7 Angular Momentum

Angular Momentum

- Momentum is not conserved for a spinning object because the direction of motion keeps changing.
- Still, if it weren't for friction, a spinning bicycle wheel would keep turning.
- The quantity that expresses this idea for circular motion is called *angular momentum.*



Angular Momentum

- We can calculate the **angular momentum** *L*.
- Newton's second law gives the angular acceleration:

$$\alpha = \frac{\tau_{\text{net}}}{I}$$

- We also know that the angular acceleration is defined as

$$\alpha = \frac{\Delta\omega}{\Delta t}$$

- If we set those equations equal and rearrange, we find

$$\tau_{\text{net}}\,\Delta t = I\,\Delta\omega$$

Angular Momentum

- For linear motion, the impulse-momentum theorem is written

$$\vec{F}_{\text{net}}\,\Delta t = m\,\Delta\vec{v} = \Delta\vec{p}$$

- The quantity $I\omega$ is the rotational equivalent of linear momentum, so it is reasonable to define angular momentum L as

$$L = I\omega$$

Angular momentum of an object with moment of inertia I rotating at angular velocity ω

- The SI units of angular momentum are $\text{kg} \cdot \text{m}^2/\text{s}$.

Angular Momentum

TABLE 9.2 Rotational and linear dynamics

Rotational dynamics	**Linear dynamics**
Torque τ_{net}	Force $\vec{F}_{\text{net}}$
Moment of inertia I	Mass m
Angular velocity ω	Velocity $\vec{v}$
Angular momentum $L = I\omega$	Linear momentum $\vec{p} = m\vec{v}$

Conservation of Angular Momentum

- Angular momentum can be written

$$\tau_{\text{net}}\,\Delta t = \Delta L$$

- If the net external torque on an object is zero, then the change in angular momentum is zero as well:

Law of conservation of angular momentum The angular momentum of a rotating object subject to no net external torque ($\tau_{\text{net}} = 0$) is a constant. The final angular momentum L_f is equal to the initial angular momentum L_i.

Conservation of Angular Momentum

- The **total angular momentum** is the sum of the angular momenta of all the objects in the system.
- If no net external torque acts on the system, then the law of conservation of angular momentum is written

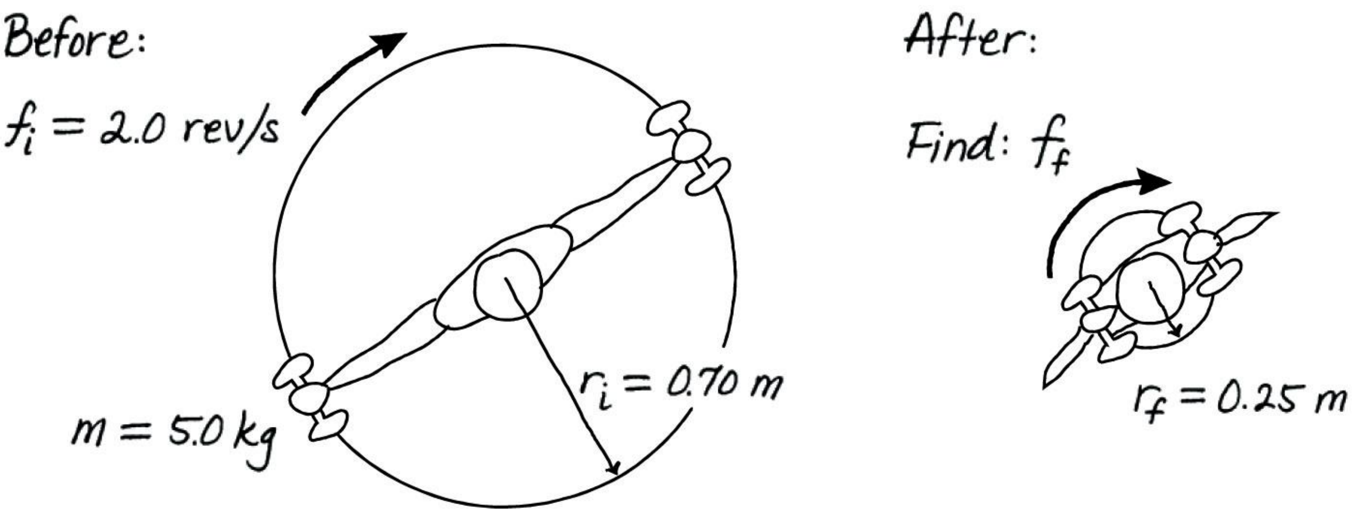
Final (f) and initial (i) moment of inertia and angular velocity of object 1

$$\underbrace{(I_1)_f(\omega_1)_f} + \underbrace{(I_2)_f(\omega_2)_f} + \cdots = \underbrace{(I_1)_i(\omega_1)_i} + \underbrace{(I_2)_i(\omega_2)_i} + \cdots$$

Final (f) and initial (i) moment of inertia and angular velocity of object 2

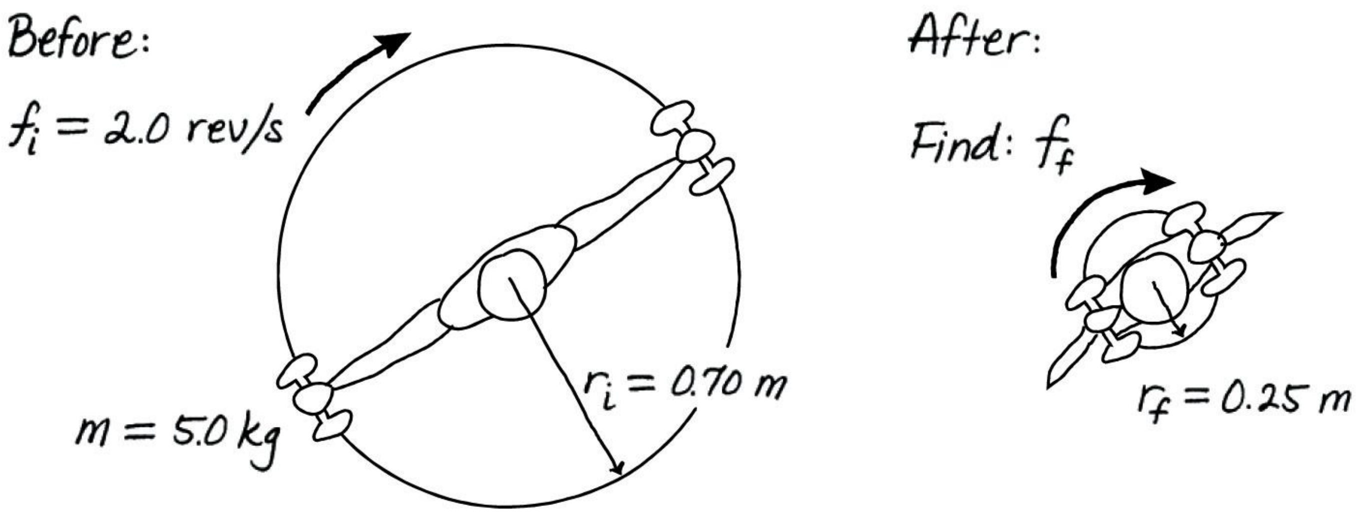
Example 9.11 Analyzing a spinning ice skater

An ice skater spins around on the tips of his blades while holding a 5.0 kg weight in each hand. He begins with his arms straight out from his body and his hands 140 cm apart. While spinning at 2.0 rev/s, he pulls the weights in and holds them 50 cm apart against his shoulders. If we neglect the mass of the skater, how fast is he spinning after pulling the weights in?

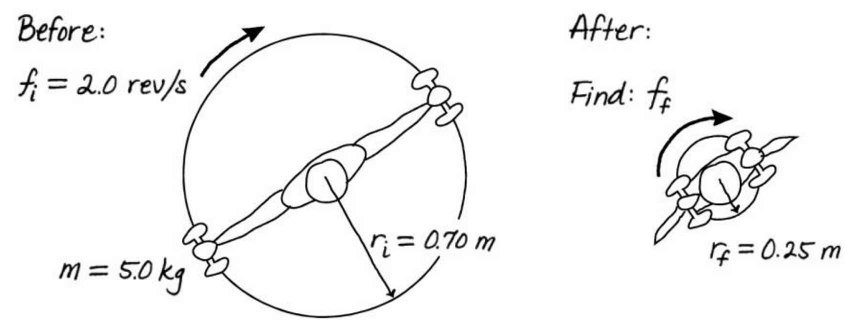


Example 9.11 Analyzing a spinning ice skater (cont.)

PREPARE There is no external torque acting on the system consisting of the skater and the weights, so their angular momentum is conserved. FIGURE 9.29 shows a before-and-after visual overview, as seen from above.



Example 9.11 Analyzing a spinning ice skater (cont.)



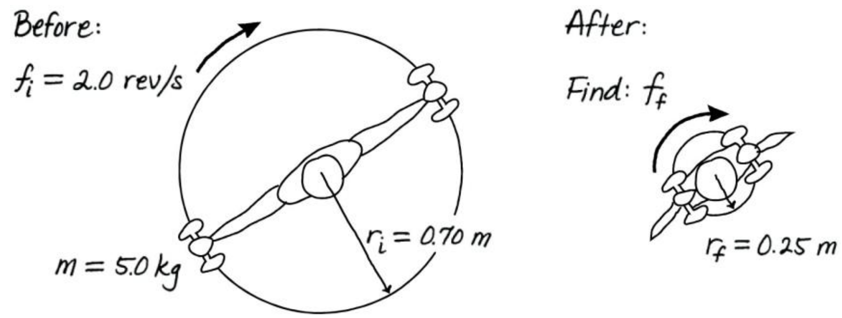
SOLVE The two weights have the same mass, move in circles with the same radius, and have the same angular velocity. Thus the total angular momentum is twice that of one weight. The mathematical statement of angular momentum conservation, $I_f \, \omega_f = I_i \omega_i$, is

There are two weights.

$$\underbrace{(2\,mr_f^{\,2})}_{I_f}\omega_f = \underbrace{(2\,mr_i^{\,2})}_{I_i}\omega_i$$

Example 9.11 Analyzing a spinning ice skater (cont.)

Because the angular velocity is related to the rotation frequency f by $\omega = 2\pi f$, this equation simplifies to

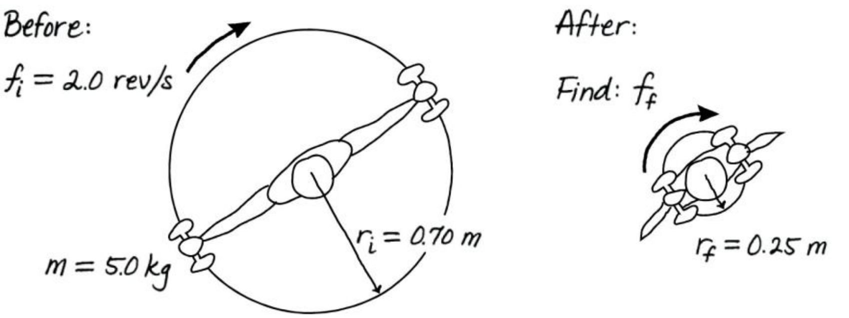


$$f_f = \left(\frac{r_i}{r_f}\right)^2 f_i$$

When he pulls the weights in, his rotation frequency increases to

$$f_f = \left(\frac{0.70\text{ m}}{0.25\text{ m}}\right)^2 \times 2.0\text{ rev/s} = 16\text{ rev/s}$$

Example 9.11 Analyzing a spinning ice skater (cont.)



ASSESS Pulling in the weights increases the skater's spin from 2 rev/s to 16 rev/s. This is somewhat high because we neglected the mass of the skater, but it illustrates how skaters do "spin up" by pulling their mass in toward the rotation axis.

Example Problem

Bicycle riders can stay upright because a torque is required to change the direction of the angular momentum of the spinning wheels. A bike with wheels with a radius of 33 cm and a mass of 1.5 kg (each) travels at a speed of 10 mph. What is the angular momentum of the bike? Treat the wheels of the bike as though all the mass is at the rim.

Summary: General Principles

Conservation Laws

When a quantity *before* an interaction is the same *after* the interaction, we say that the quantity is **conserved.**

Conservation of momentum

The total momentum $\vec{P} = \vec{p}_1 + \vec{p}_2 + \cdots$ of an **isolated system**—one on which no net force acts—is a constant. Thus

$$\vec{P}_\text{f} = \vec{P}_\text{i}$$

Conservation of angular momentum

The angular momentum L of a rotating object or system of objects subject to zero net external torque is a constant. Thus

$$L_\text{f} = L_\text{i}$$

This can be written in terms of the initial and final moments of inertia I and angular velocities ω as

$$(I_1)_\text{f}(\omega_1)_\text{f} + (I_2)_\text{f}(\omega_2)_\text{f} + \cdots = (I_1)_\text{i}(\omega_1)_\text{i} + (I_2)_\text{i}(\omega_2)_\text{i} + \cdots$$

Text: p. 275

Summary: General Principles

Solving Momentum Conservation Problems

PREPARE Choose an isolated system or a system that is isolated during at least part of the problem. Draw a visual overview of the system before and after the interaction.

SOLVE Write the law of conservation of momentum in terms of vector components:

$$(p_{1x})_f + (p_{2x})_f + \cdots = (p_{1x})_i + (p_{2x})_i + \cdots$$
$$(p_{1y})_f + (p_{2y})_f + \cdots = (p_{1y})_i + (p_{2y})_i + \cdots$$

In terms of masses and velocities, this is

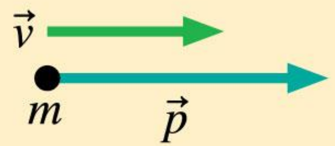
$$m_1(v_{1x})_f + m_2(v_{2x})_f + \cdots = m_1(v_{1x})_i + m_2(v_{2x})_i + \cdots$$
$$m_1(v_{1y})_f + m_2(v_{2y})_f + \cdots = m_1(v_{1y})_i + m_2(v_{2y})_i + \cdots$$

ASSESS Is the result reasonable?

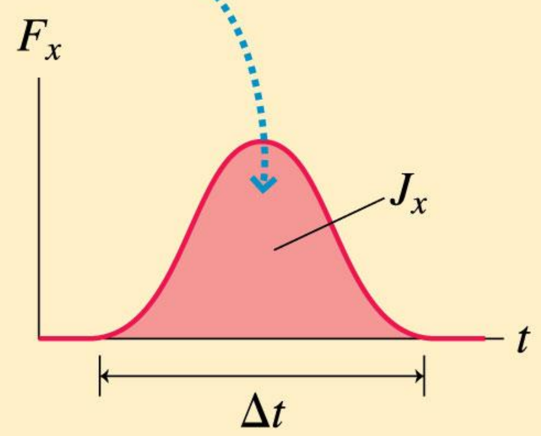
Text: p. 275

Summary: Important Concepts

Momentum $\vec{p} = m\vec{v}$



Impulse J_x = area under force curve



Impulse and momentum are related by the **impulse-momentum theorem**

$$\Delta p_x = J_x$$

This is an alternative statement of Newton's second law.

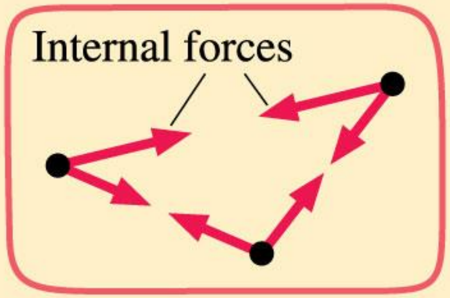
Angular momentum $L = I\omega$ is the rotational analog of linear momentum $\vec{p} = m\vec{v}$.

Text: p. 275

Summary: Important Concepts

System A group of interacting particles

Isolated system A system on which the net external force is zero

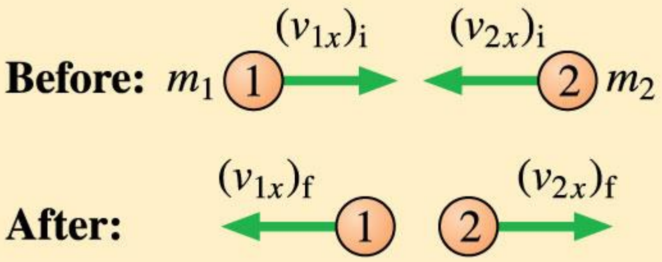


Text: p. 275

Summary: Important Concepts

Before-and-after visual overview

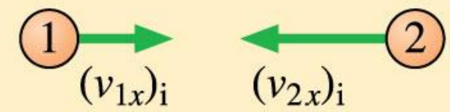
- Define the system.
- Use two drawings to show the system *before* and *after* the interaction.
- List known information and identify what you are trying to find.



Text: p. 275

Summary: Applications

Collisions Two or more particles come together. In a perfectly inelastic collision, they stick together and move with a common final velocity.

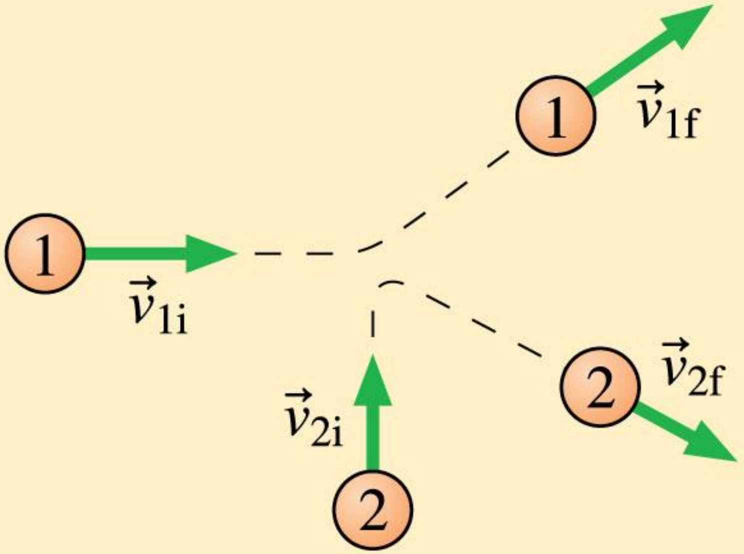


Explosions Two or more particles move away from each other.

Text: p. 275

Summary: Applications

Two dimensions Both the *x*- and *y*-components of the total momentum $\vec{P}$ must be conserved, giving two simultaneous equations.



Text: p. 275

Summary

GENERAL PRINCIPLES

Conservation Laws

When a quantity *before* an interaction is the same *after* the interaction, we say that the quantity is **conserved.**

Conservation of momentum

The total momentum $\vec{P} = \vec{p}_1 + \vec{p}_2 + \cdots$ of an **isolated system**—one on which no net force acts—is a constant. Thus

$$\vec{P}_f = \vec{P}_i$$

Conservation of angular momentum

The angular momentum L of a rotating object or system of objects subject to zero net external torque is a constant. Thus

$$L_f = L_i$$

This can be written in terms of the initial and final moments of inertia I and angular velocities ω as

$$(I_1)_f(\omega_1)_f + (I_2)_f(\omega_2)_f + \cdots = (I_1)_i(\omega_1)_i + (I_2)_i(\omega_2)_i + \cdots$$

Solving Momentum Conservation Problems

PREPARE Choose an isolated system or a system that is isolated during at least part of the problem. Draw a visual overview of the system before and after the interaction.

SOLVE Write the law of conservation of momentum in terms of vector components:

$$(p_{1x})_f + (p_{2x})_f + \cdots = (p_{1x})_i + (p_{2x})_i + \cdots$$
$$(p_{1y})_f + (p_{2y})_f + \cdots = (p_{1y})_i + (p_{2y})_i + \cdots$$

In terms of masses and velocities, this is

$$m_1(v_{1x})_f + m_2(v_{2x})_f + \cdots = m_1(v_{1x})_i + m_2(v_{2x})_i + \cdots$$
$$m_1(v_{1y})_f + m_2(v_{2y})_f + \cdots = m_1(v_{1y})_i + m_2(v_{2y})_i + \cdots$$

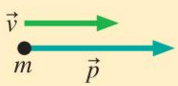
ASSESS Is the result reasonable?

Text: p. 275

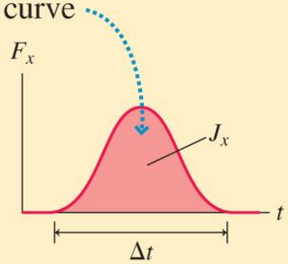
Summary

IMPORTANT CONCEPTS

Momentum $\vec{p} = m\vec{v}$



Impulse J_x = area under force curve



Impulse and momentum are related by the **impulse-momentum theorem**

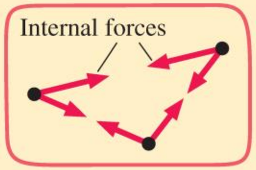
$$\Delta p_x = J_x$$

This is an alternative statement of Newton's second law.

Angular momentum $L = I\omega$ is the rotational analog of linear momentum $\vec{p} = m\vec{v}$.

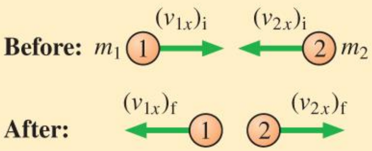
System A group of interacting particles

Isolated system A system on which the net external force is zero



Before-and-after visual overview

- Define the system.
- Use two drawings to show the system *before* and *after* the interaction.
- List known information and identify what you are trying to find.

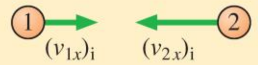


Text: p. 275

Summary

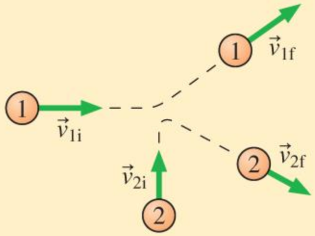
APPLICATIONS

Collisions Two or more particles come together. In a perfectly inelastic collision, they stick together and move with a common final velocity.



Explosions Two or more particles move away from each other.

Two dimensions Both the *x*- and *y*-components of the total momentum $\vec{P}$ must be conserved, giving two simultaneous equations.



Text: p. 275