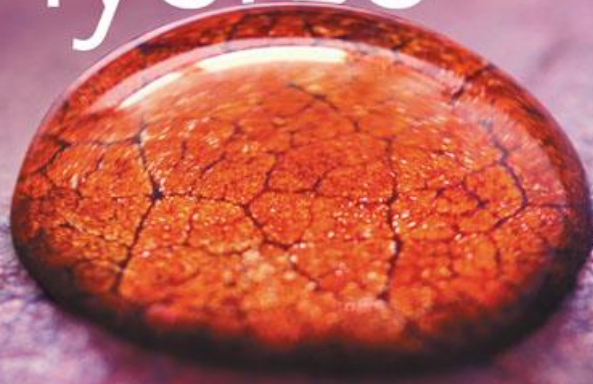


THIRD EDITION

college
physics a strategic approach



knight · jones · field

Lecture Presentation

Chapter 8

Equilibrium and Elasticity

Suggested Videos for Chapter 8

- **Prelecture Videos**

- *Static Equilibrium*
- *Elasticity*

- **Class Videos**

- *Center of Gravity and Stability*

- **Video Tutor Solutions**

- *Equilibrium and Elasticity*

- **Video Tutor Demos**

- *Balancing a Meter Stick*

Suggested Simulations for Chapter 8

- **ActivPhysics**

- 7.2–7.5

- **PhETs**

- *Molecular Motors*
- *Masses & Springs*
- *Stretching DNA*

Chapter 8 Equilibrium and Elasticity



Chapter Goal: To learn about the static equilibrium of extended objects, and the basic properties of springs and elastic materials.

Chapter 8 Preview

Looking Ahead: Static Equilibrium

- As the cyclist balances on his back tire, the net force *and* the net torque on him must be zero.



- You'll learn to analyze objects that are in **static equilibrium**.

Chapter 8 Preview

Looking Ahead: Springs

- When a rider takes a seat, the spring is compressed and exerts a **restoring force**, pushing upward.



- You'll learn to solve problems involving stretched and compressed springs.

Chapter 8 Preview

Looking Ahead: Properties of Materials

- All materials have some “give”; if you pull on them, they stretch, and at some point they break.



- Is this spider silk as strong as steel? You’ll learn to think about what the question means, and how to answer it.

Chapter 8 Preview

Looking Ahead

Static Equilibrium

As this cyclist balances on his back tire, the net force *and* the net torque on him must be zero.



You'll learn to analyze objects that are in **static equilibrium**.

Springs

When a rider takes a seat, the spring is compressed and exerts a **restoring force**, pushing upward.



You'll learn to solve problems involving stretched and compressed springs.

Properties of Materials

All materials have some “give”; if you pull on them, they stretch, and at some point they break.



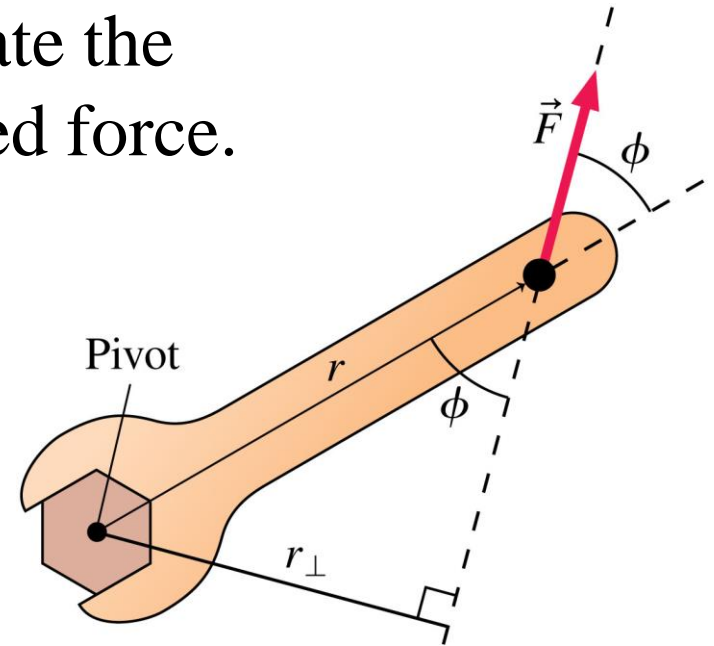
Is this spider silk as strong as steel? You'll learn to think about what the question means, and how to answer it.

Text: p. 225

Chapter 8 Preview

Looking Back: Torque

- In Chapter 7, you learned to calculate the torque on an object due to an applied force.
- In this chapter, we'll extend our analysis to consider objects with many forces—and many torques—that act on them.



Chapter 8 Preview

Stop to Think

An old-fashioned tire swing exerts a force on the branch and a torque about the point where the branch meets the trunk. If you hang the swing closer to the trunk, this will _____ the force and _____ the torque.

- A. Increase, increase
- B. Not change, increase
- C. Not change, not change
- D. Not change, decrease
- E. Decrease, not change
- F. Decrease, decrease



Reading Question 8.1

An object is in equilibrium if

A. $\vec{F}_{\text{net}} = \vec{0}$

B. $\tau_{\text{net}} = 0$

C. Either A or B

D. Both A and B

Reading Question 8.1

An object is in equilibrium if

A. $\vec{F}_{\text{net}} = \vec{0}$

B. $\tau_{\text{net}} = 0$

C. Either A or B

 D. Both A and B

Reading Question 8.2

If you are performing the weightlifting exercise known as the strict curl, the tension in your biceps tendon is

- A. Larger than the weight you are lifting.
- B. Equal to the weight you are lifting.
- C. Smaller than the weight you are lifting.

Reading Question 8.2

If you are performing the weightlifting exercise known as the strict curl, the tension in your biceps tendon is

- ✓ A. Larger than the weight you are lifting.
- B. Equal to the weight you are lifting.
- C. Smaller than the weight you are lifting.

Reading Question 8.3

An object will be stable if

- A. Its center of gravity is below its highest point.
- B. Its center of gravity lies over its base of support.
- C. Its center of gravity lies outside its base of support.
- D. The height of its center of gravity is less than $1/2$ its total height.

Reading Question 8.3

An object will be stable if

- A. Its center of gravity is below its highest point.
- ✓ B. Its center of gravity lies over its base of support.
- C. Its center of gravity lies outside its base of support.
- D. The height of its center of gravity is less than $1/2$ its total height.


Reading Question 8.4

Hooke's law describes the force of

- A. Gravity.
- B. A spring.
- C. Collisions.
- D. Tension.
- E. None of the above.

Reading Question 8.4

Hooke's law describes the force of

- A. Gravity.
-  B. A spring.
- C. Collisions.
- D. Tension.
- E. None of the above.


Reading Question 8.5

A very rigid material—one that stretches or compresses only slightly under large forces—has a large value of

- A. Tensile strength.
- B. Elastic limit.
- C. Density.
- D. Young's modulus.

Reading Question 8.5

A very rigid material—one that stretches or compresses only slightly under large forces—has a large value of

- A. Tensile strength.
- B. Elastic limit.
- C. Density.
-  D. Young's modulus.

Section 8.1 Torque and Static Equilibrium

Torque and Static Equilibrium

(a) When the net force on a particle is zero, the particle is in static equilibrium.

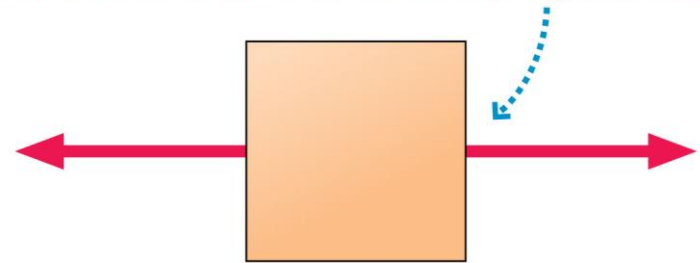


- An object at rest is in *static equilibrium*.
- As long as the object can be modeled as a *particle*, static equilibrium is achieved when the net force on the particle is zero.

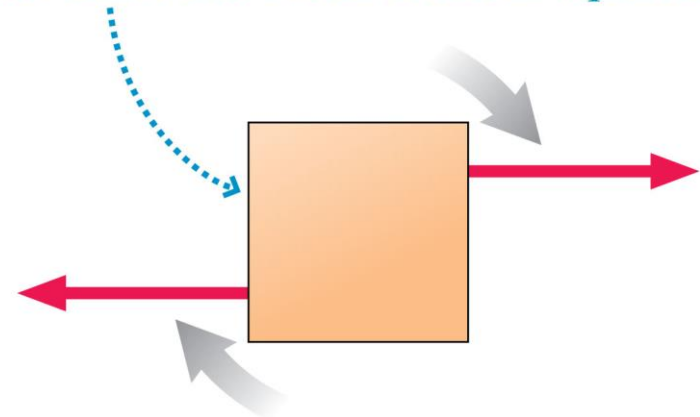
Torque and Static Equilibrium

- For extended objects that can rotate, we must consider the net *torque*, too.
- When the net force and the net torque are zero, the block is in static equilibrium.
- When the net force is zero, but the net torque is *not* zero, the object is *not* in static equilibrium.

(b) Both the net force and the net torque are zero, so the block is in static equilibrium.



(c) The net force is still zero, but the net torque is *not* zero. The block is not in equilibrium.



Torque and Static Equilibrium

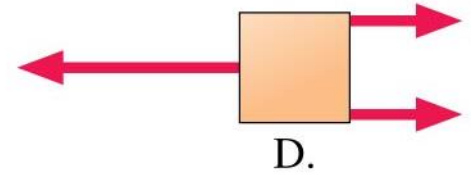
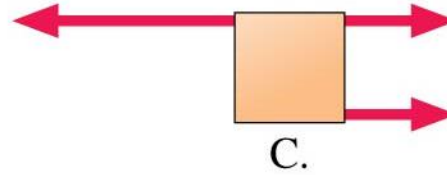
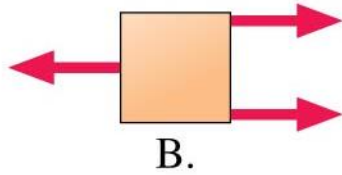
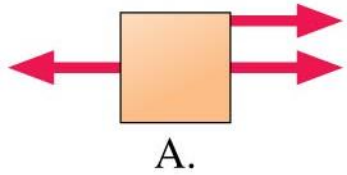
- There are two conditions for static equilibrium on an extended object:
 - The net force on the object must be zero.
 - The net torque on the object must be zero.

$$\left. \begin{array}{l} \sum F_x = 0 \\ \sum F_y = 0 \end{array} \right\} \text{No net force}$$
$$\sum \tau = 0 \quad \left. \vphantom{\sum F_x} \right\} \text{No net torque}$$

Conditions for static equilibrium of an extended object

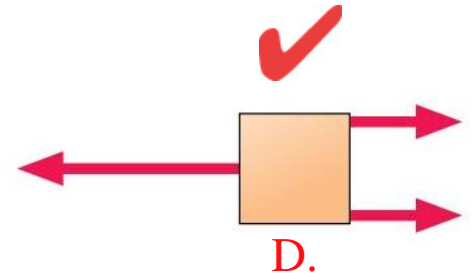
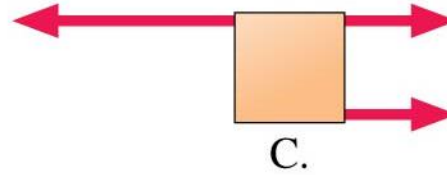
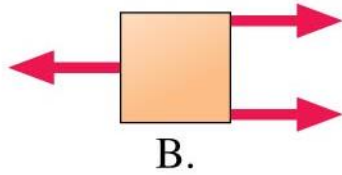
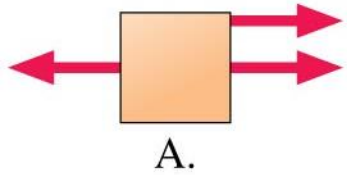
QuickCheck 8.1

Which object is in static equilibrium?



QuickCheck 8.1

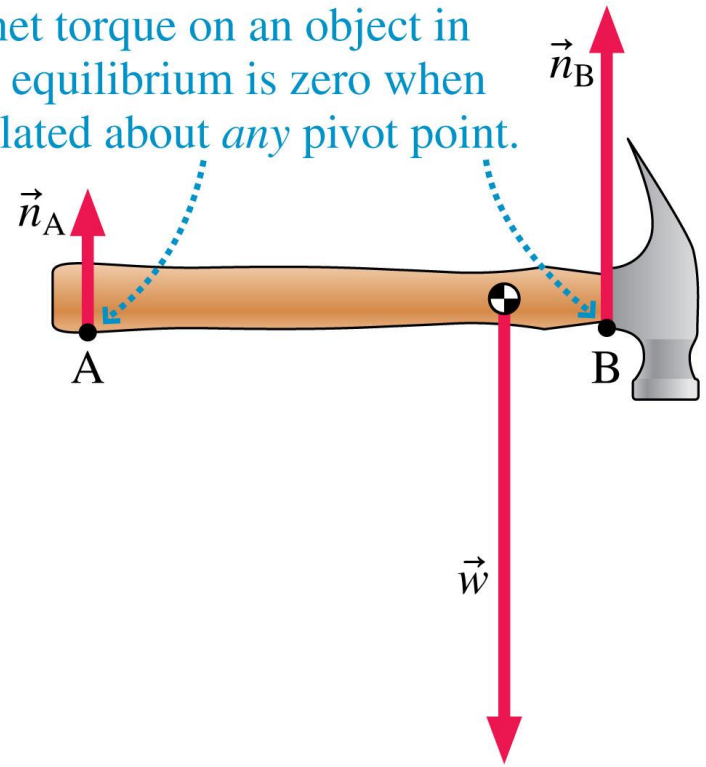
Which object is in static equilibrium?



Choosing the Pivot Point

- For an object in static equilibrium, the net torque about *every* point must be zero.
- You can choose *any* point you wish as a pivot point for calculating torque.

The net torque on an object in static equilibrium is zero when calculated about *any* pivot point.

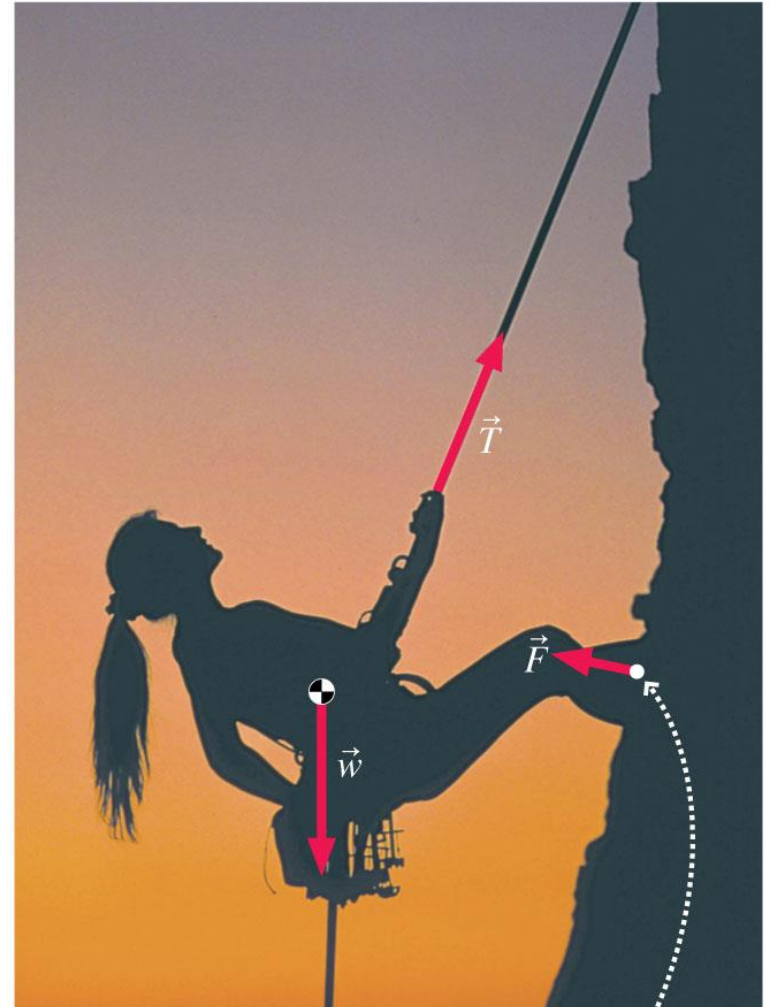


Choosing the Pivot Point

- Any pivot point will work, but some pivot points can simplify calculations.
- There is a “natural” axis of rotation for many situations. A natural axis is an axis about which rotation *would* occur if the object were not in static equilibrium.

Choosing the Pivot Point

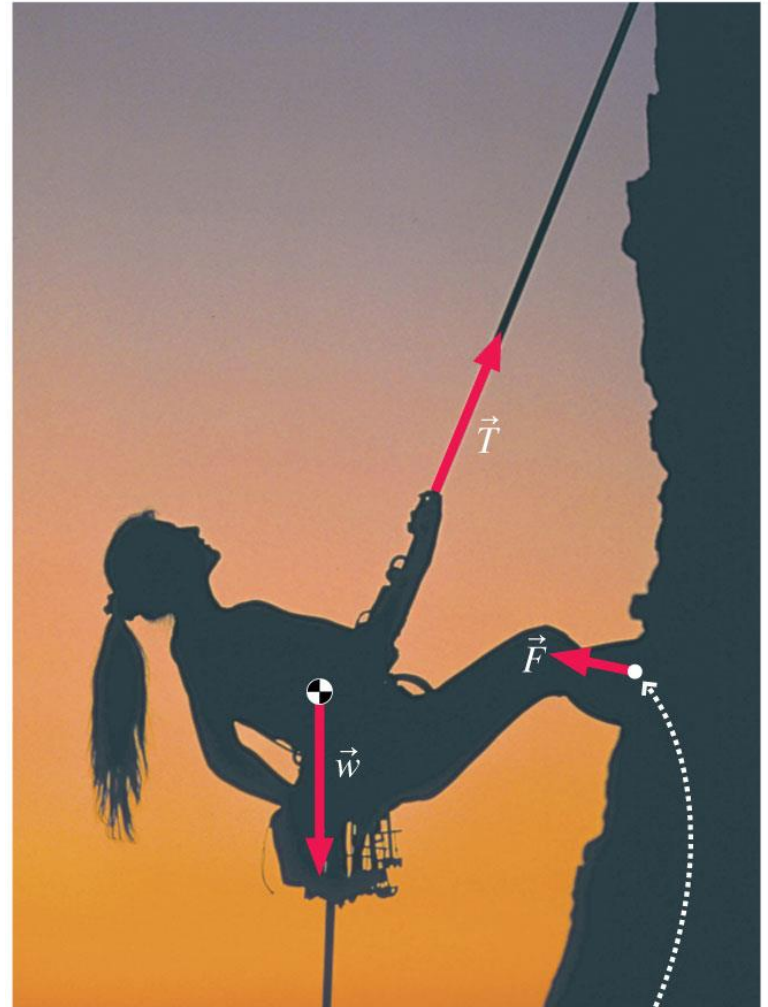
- The solution is simplified if you choose the pivot point to be the location where the forces are poorly specified.
- For the woman on the rock wall, the force of the wall on her feet is a mix of normal and frictional forces, and the direction is not well known.



The torque due to \vec{F} about this point is zero.
This makes this point a good choice as the pivot.

Choosing the Pivot Point

- Choosing the point where the woman's foot contacts the wall as the pivot point eliminates the torque due to the force of the wall on her foot.
- The other forces and their directions are well known. It is straightforward to calculate the torque due to those forces (weight and tension).



The torque due to \vec{F} about this point is zero. This makes this point a good choice as the pivot.

Choosing the Pivot Point

PROBLEM-SOLVING STRATEGY 8.1

Static equilibrium problems



If an object is in static equilibrium, we can use the fact that there is no net force and no net torque as a basis for solving problems.

PREPARE Model the object as a simple shape. Draw a visual overview that shows all forces and distances. List known information.

- Pick an axis or pivot about which the torques will be calculated.
- Determine the torque about this pivot point due to each force acting on the object. The torques due to any forces acting *at* the pivot are zero.
- Determine the sign of each torque about this pivot point.

SOLVE The mathematical steps are based on the conditions:

$$\vec{F}_{\text{net}} = \vec{0} \quad \text{and} \quad \tau_{\text{net}} = 0$$

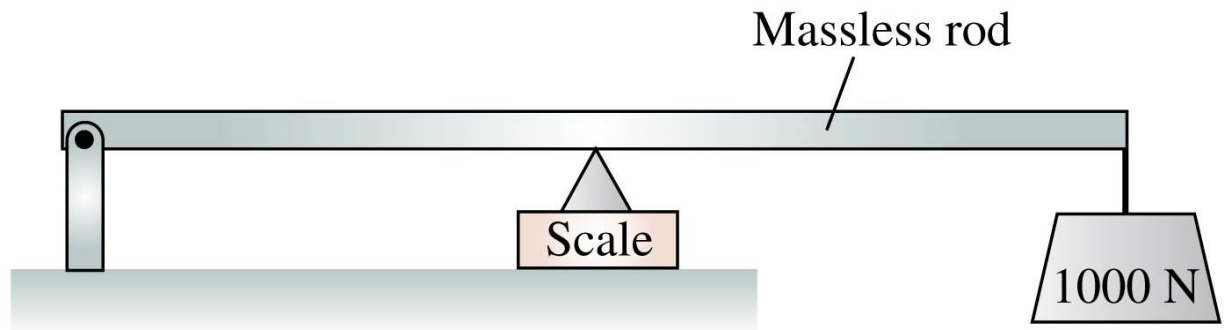
- Write equations for $\sum F_x = 0$, $\sum F_y = 0$, and $\sum \tau = 0$.
- Solve the resulting equations.

ASSESS Check that your result is reasonable and answers the question.

QuickCheck 8.2

What does the scale read?

- A. 500 N
- B. 1000 N
- C. 2000 N
- D. 4000 N

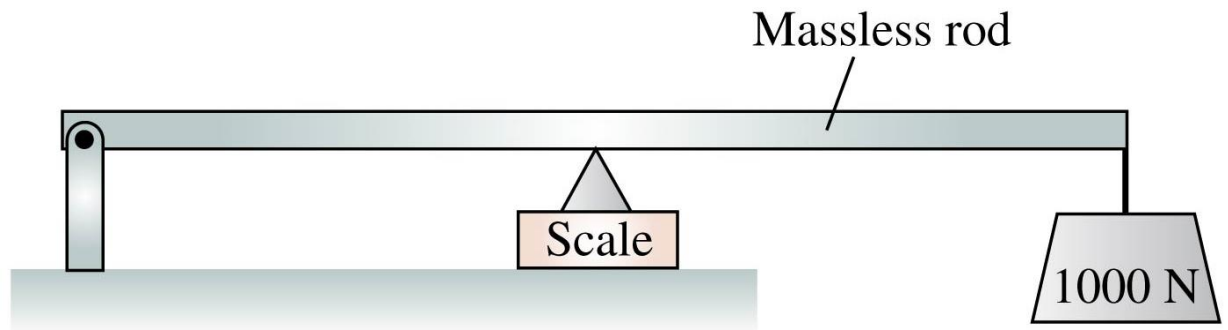


Answering this requires reasoning, not calculating.

QuickCheck 8.2

What does the scale read?

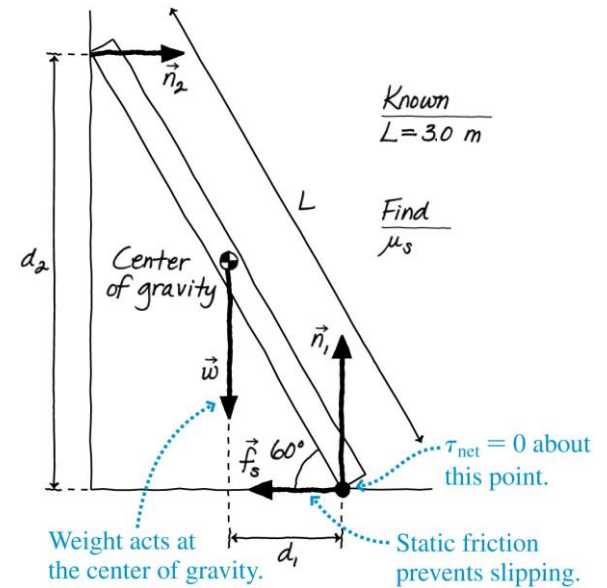
- A. 500 N
- B. 1000 N
- C. 2000 N
- D. 4000 N



Answering this requires reasoning, not calculating.

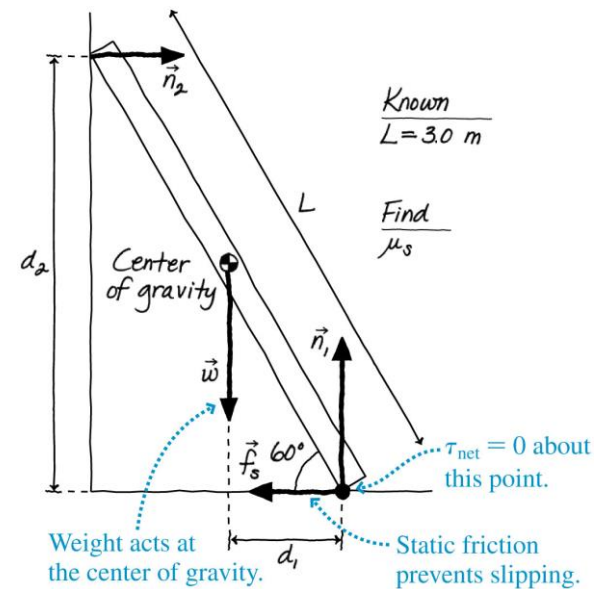
Example 8.4 Will the ladder slip?

A 3.0-m-long ladder leans against a wall at an angle of 60° with respect to the floor. What is the minimum value of μ_s , the coefficient of static friction with the ground, that will prevent the ladder from slipping? Assume that friction between the ladder and the wall is negligible.



Example 8.4 Will the ladder slip? (cont.)

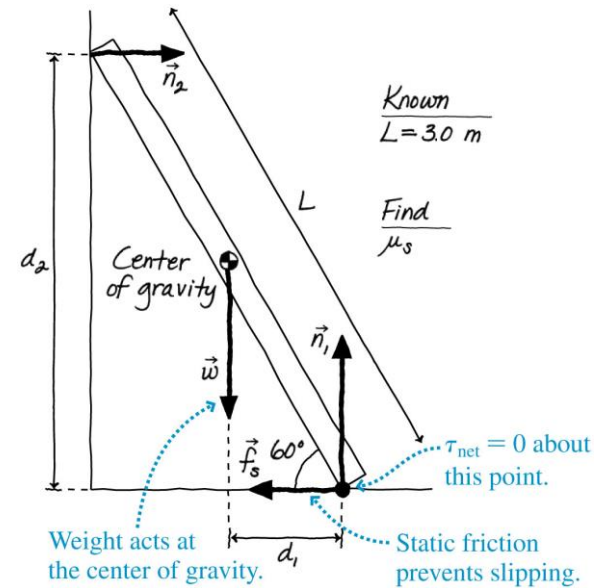
PREPARE The ladder is a rigid rod of length L . To not slip, both the net force and net torque on the ladder must be zero. FIGURE 8.9 on the next page shows the ladder and the forces acting on it. We are asked to find the necessary coefficient of static friction.



Example 8.4 Will the ladder slip? (cont.)

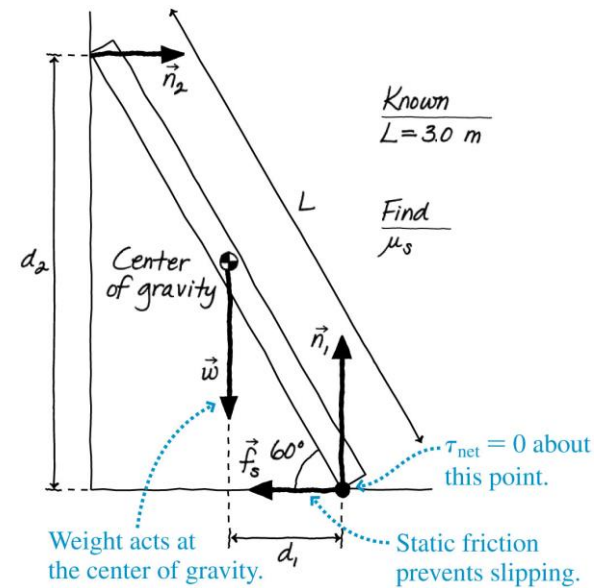
First, we'll solve for the magnitudes of the static friction force and the normal force. Then we can use these values to determine the necessary value of the coefficient of friction.

These forces both act at the bottom corner of the ladder, so even though we are interested in these forces, this is a good choice for the pivot point because two of the forces that act provide no torque, which simplifies the solution.



Example 8.4 Will the ladder slip? (cont.)

With this choice of pivot, the weight of the ladder, acting at the center of gravity, exerts torque $d_1 w$ and the force of the wall exerts torque $-d_2 n_2$. The signs are based on the observation that \vec{w} would cause the ladder to rotate counterclockwise, while \vec{n}_2 would cause it to rotate clockwise.



Example 8.4 Will the ladder slip? (cont.)

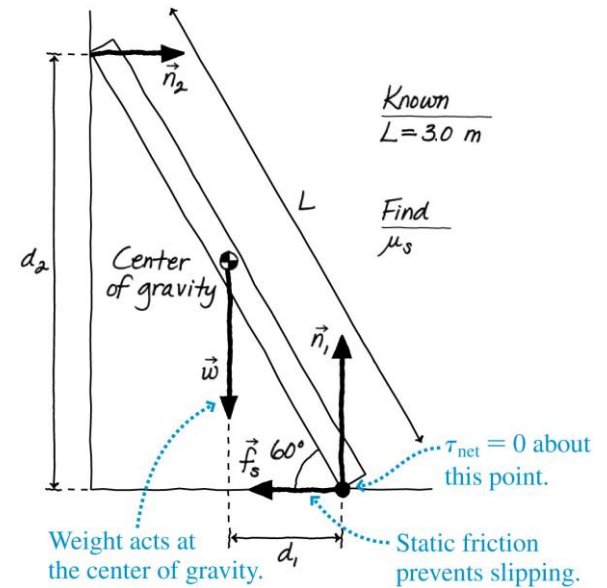
SOLVE The x - and y -components of $\vec{F}_{\text{net}} = \vec{0}$ are

$$\sum F_x = n_2 - f_s = 0$$

$$\sum F_y = n_1 - w = n_1 - Mg = 0$$

The torque about the bottom corner is

$$\tau_{\text{net}} = d_1 w - d_2 n_2 = \frac{1}{2}(L \cos 60^\circ)Mg - (L \sin 60^\circ)n_2 = 0$$



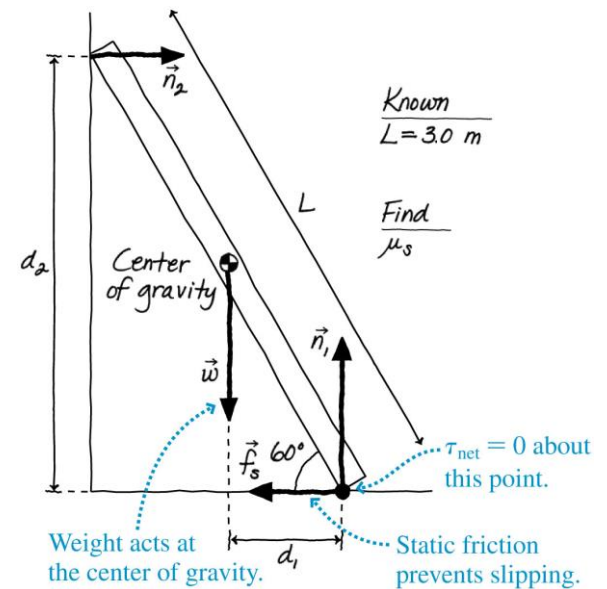
Example 8.4 Will the ladder slip? (cont.)

Altogether, we have three equations with the three unknowns n_1 , n_2 , and f_s . If we solve the third equation for n_2 ,

$$n_2 = \frac{\frac{1}{2}(L \cos 60^\circ)Mg}{L \sin 60^\circ} = \frac{Mg}{2 \tan 60^\circ}$$

we can then substitute this into the first equation to find

$$f_s = \frac{Mg}{2 \tan 60^\circ}$$



Example 8.4 Will the ladder slip? (cont.)

Our model of static friction is

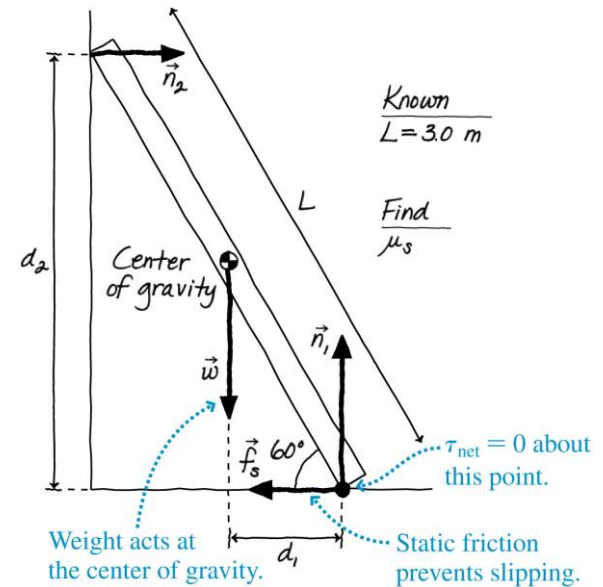
$f_s \leq f_{s \max} = \mu_s n_1$. We can find n_1 from the second equation: $n_1 = Mg$. From this, the model of friction tells us that

$$f_s \leq \mu_s Mg$$

Comparing these two expressions for f_s , we see that μ_s must obey

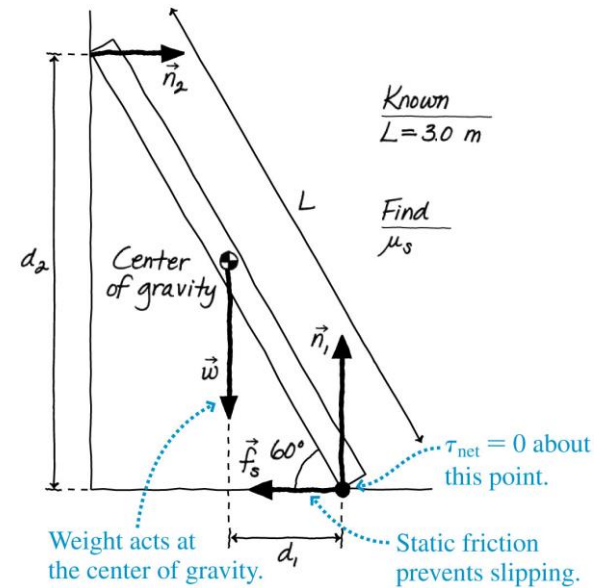
$$\mu_s \geq \frac{1}{2 \tan 60^\circ} = 0.29$$

Thus the minimum value of the coefficient of static friction is 0.29.



Example 8.4 Will the ladder slip? (cont.)

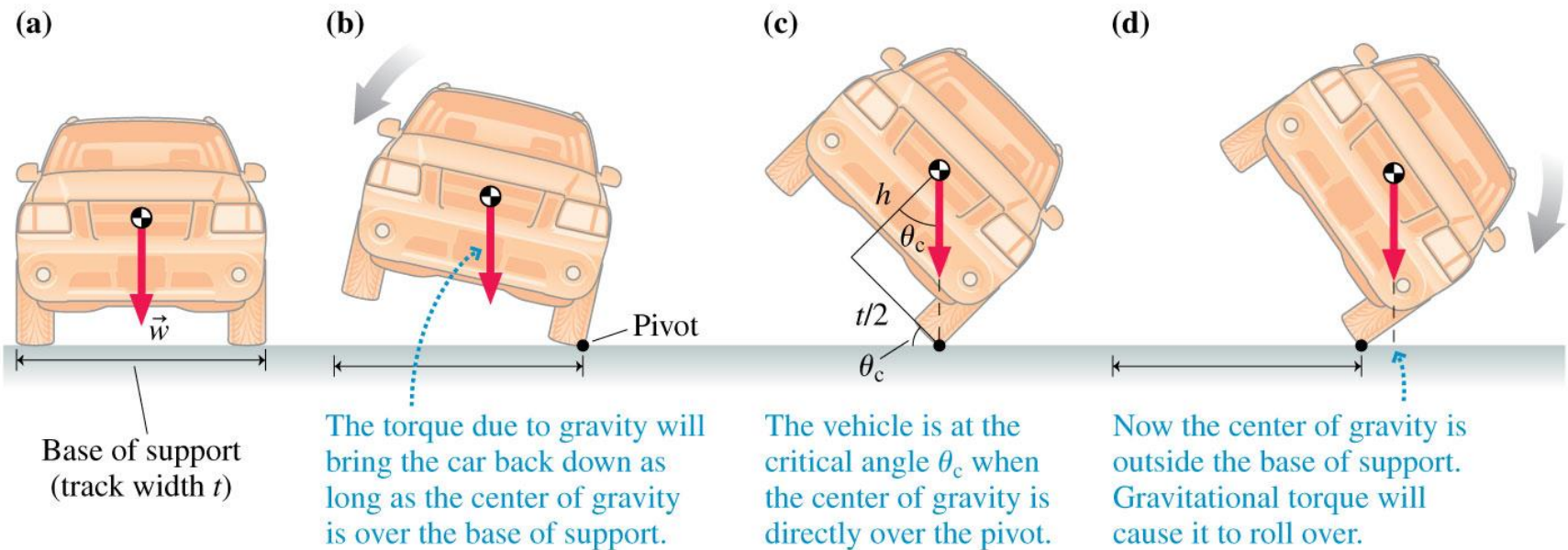
ASSESS You know from experience that you can lean a ladder or other object against a wall if the ground is “rough,” but it slips if the surface is too smooth. 0.29 is a “medium” value for the coefficient of static friction, which is reasonable.



Section 8.2 Stability and Balance

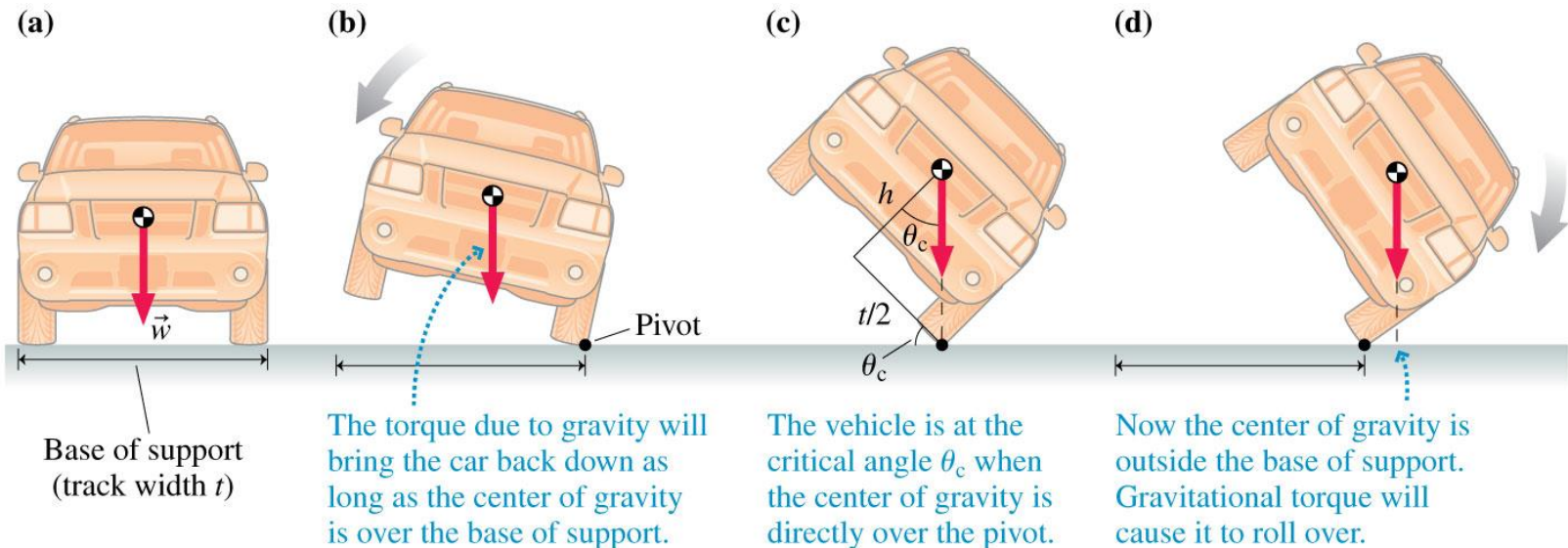
Stability and Balance

- An extended object has a *base of support* on which it rests when in static equilibrium.
- **A wider base of support and/or a lower center of gravity improves stability.**



Stability and Balance

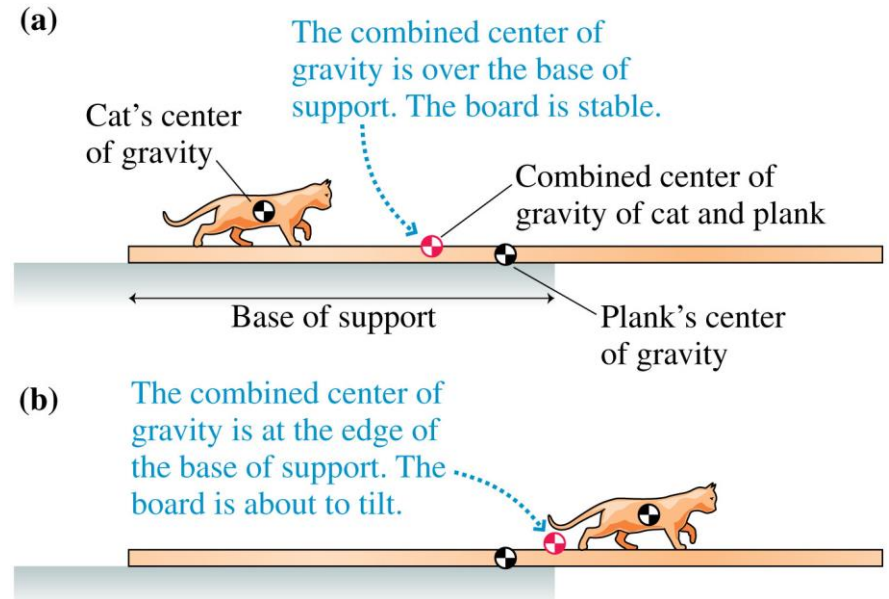
- As long as the object's center of gravity remains over the base of support, torque due to gravity will rotate the object back toward its stable equilibrium position. The object is **stable**.
- If the object's center of gravity moves outside the base of support, the object is **unstable**.



Conceptual Example 8.5 How far to walk the plank?

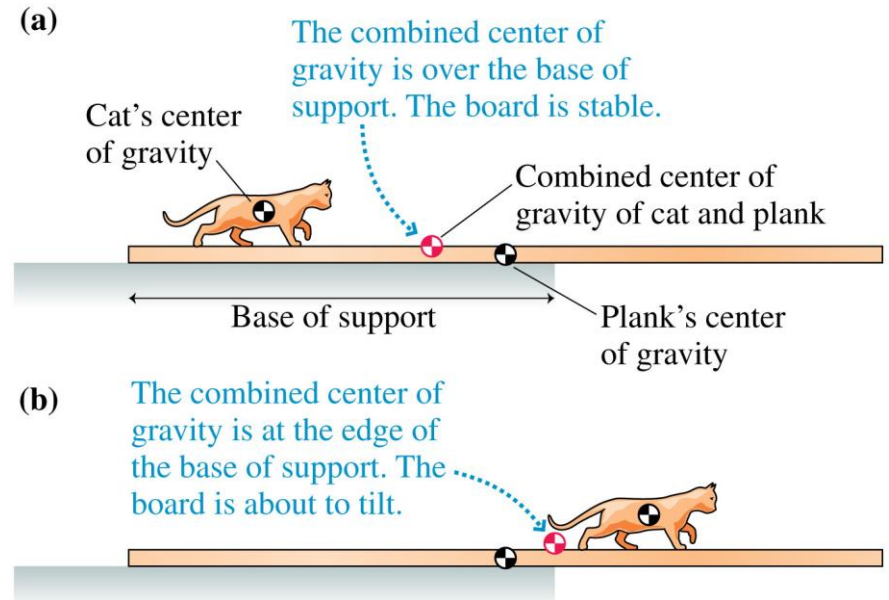
A cat walks along a plank that extends out from a table. If the cat walks too far out on the plank, the plank will begin to tilt. What determines when this happens?

REASON An object is stable if its center of gravity lies over its base of support, and unstable otherwise. Let's take the cat and the plank to be one combined object whose center of gravity lies along a line between the cat's center of gravity and that of the plank.



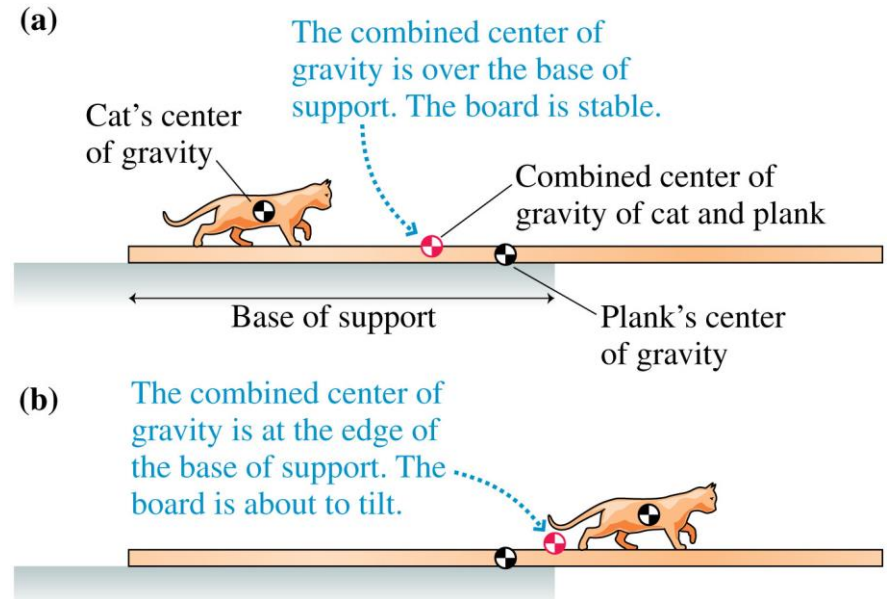
Conceptual Example 8.5 How far to walk the plank? (cont.)

In FIGURE 8.12a, when the cat is near the left end of the plank, the combined center of gravity is over the base of support and the plank is stable. As the cat moves to the right, he reaches a point where the combined center of gravity is directly over the edge of the table, as shown in FIGURE 8.12b. If the cat takes one more step, the cat and plank will become unstable and the plank will begin to tilt.



Conceptual Example 8.5 How far to walk the plank? (cont.)

ASSESS Because the plank's center of gravity must be to the left of the edge for it to be stable by itself, the cat can actually walk a short distance out onto the unsupported part of the plank before it starts to tilt. The heavier the plank is, the farther the cat can walk.



Example Problem

A 2-m-long board weighing 50 N extends out over the edge of a table, with 40% of the board's length off the table. How far beyond the table edge can a 25-N cat walk before the board begins to tilt?

Stability and Balance of the Human Body

- As a human moves, the body's center of gravity is constantly changing.
- To maintain stability, people unconsciously adjust the positions of their arms and legs to keep their center of gravity over their base of support.



Stability and Balance of the Human Body

- When the woman stands on her tiptoes, she leans forward, readjusting her center of gravity to be over the balls of her feet (her base of support).



Try It Yourself: Impossible Balance

Stand facing a wall with your toes touching the base of the wall. Now rise onto your tiptoes. You will not be able to do so without falling backward. As we see from Figure 8.13b, your body has to lean forward to stand on tiptoes. With the wall in your way, you cannot lean enough to maintain your balance, and you will begin to topple backward.

Try It Yourself: Balancing a Soda Can

Try to balance a soda can—full or empty—on the narrow bevel at the bottom. It can't be done because, either full or empty, the center of gravity is near the center of the can. If the can is tilted enough to sit on the bevel, the center of gravity lies far outside this small base of support.

But if you put about 2 ounces (60 ml) of water in an empty can, the center of gravity will be right over the bevel and the can will balance.



Section 8.3 Springs and Hooke's Law

Springs and Hooke's Law

- We have assumed that objects in equilibrium maintain their shapes as forces and torques are applied to them.
- This is an oversimplification; every solid object stretches, compresses, or deforms when a force acts on it.

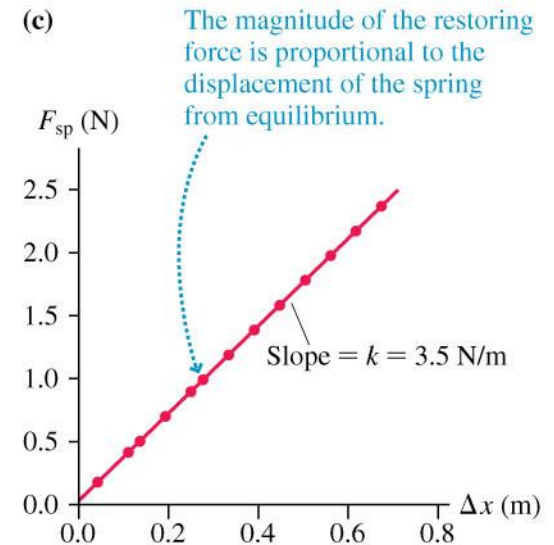
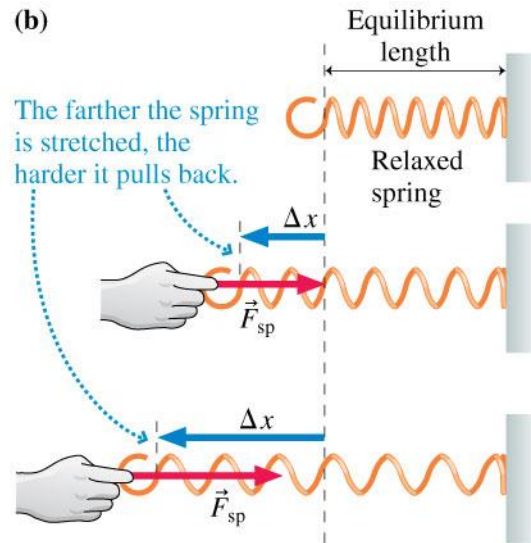
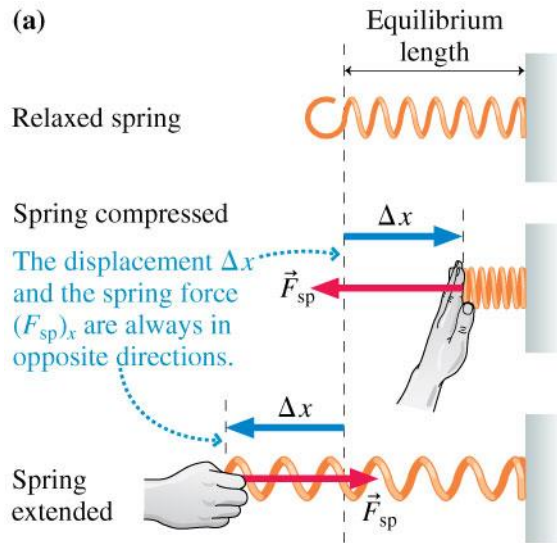


Springs and Hooke's Law

- A **restoring force** is a force that restores a system to an equilibrium position.
- Systems that exhibit restoring forces are called **elastic**.
- Springs and rubber bands are basic examples of **elasticity**.

Springs and Hooke's Law

- The spring force is *proportional* to the displacement of the end of the spring.

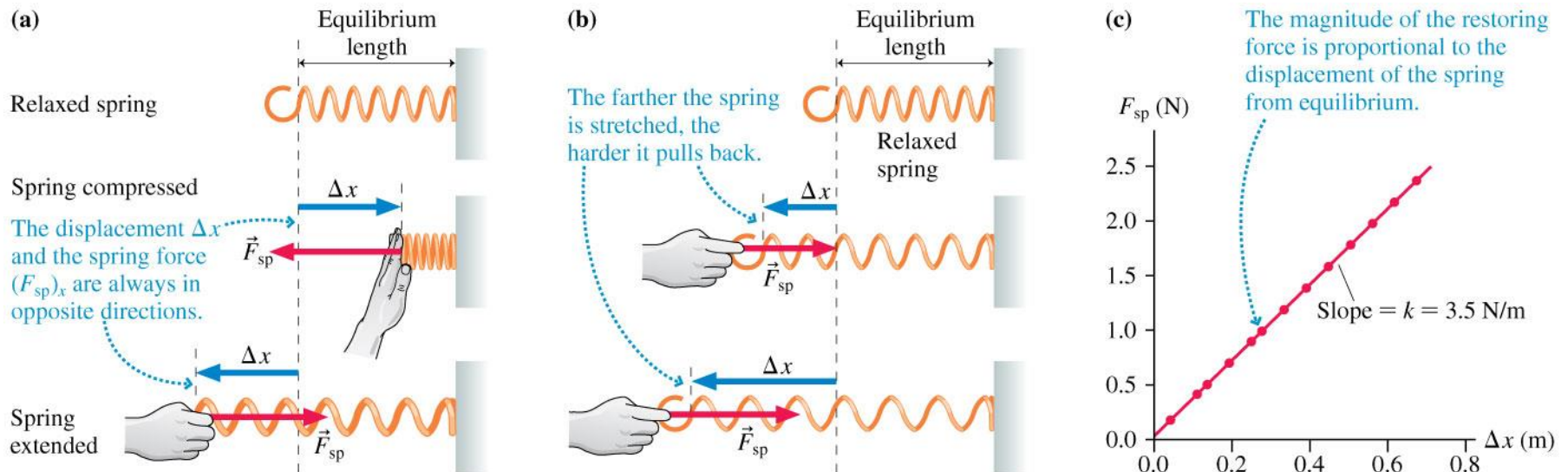


Springs and Hooke's Law

- The spring force and the displacement of the end of the spring have a *linear relationship*.

$$F_{\text{sp}} = k \Delta x$$

- The slope k of the line is called the **spring constant** and has units of N/m.



Springs and Hooke's Law

- **Hooke's law** describes the most general form of the relationship between the restoring force and the displacement of the end of a spring.

The diagram shows the equation $(F_{\text{sp}})_x = -k\Delta x$ with several annotations. A dotted arrow points from the text 'x-component of the restoring force of the spring (N)' to the term $(F_{\text{sp}})_x$. Another dotted arrow points from 'Displacement of the end of the spring (m)' to Δx . A third dotted arrow points from 'Spring constant (N/m)' to k . A fourth dotted arrow points from the negative sign to the text 'The negative sign says the restoring force and the displacement are in *opposite* directions.'

x-component of the restoring force of the spring (N) $(F_{\text{sp}})_x = -k\Delta x$ Displacement of the end of the spring (m) Spring constant (N/m)

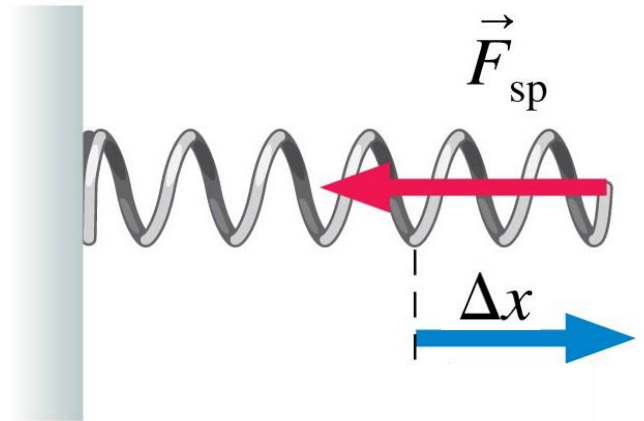
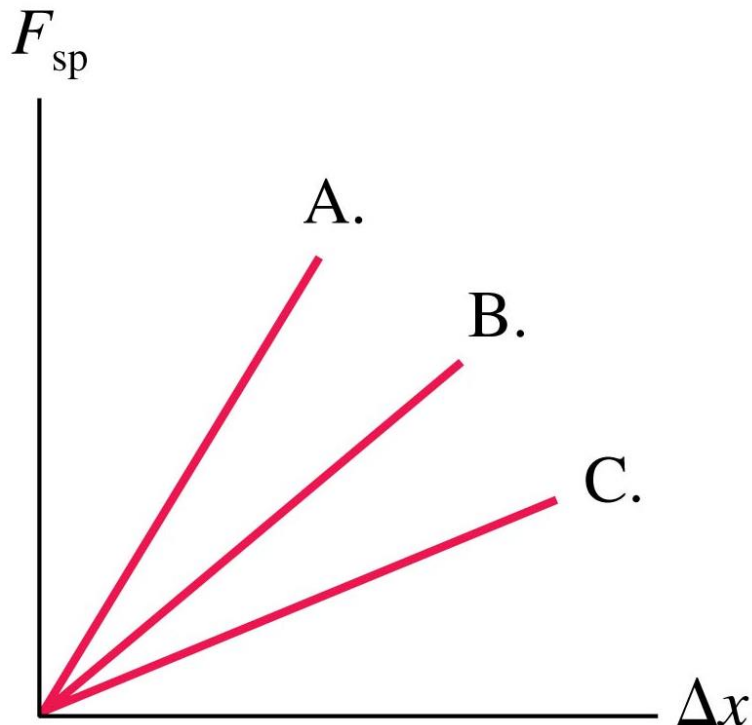
The negative sign says the restoring force and the displacement are in *opposite* directions.

- For motion in the vertical (y) direction, Hooke's law is

$$(F_{\text{sp}})_y = -k\Delta y$$

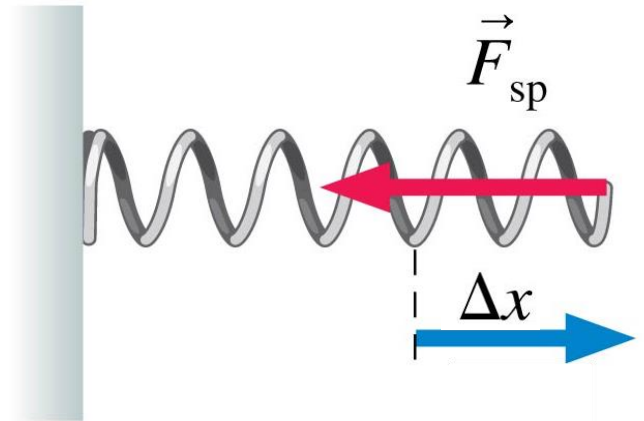
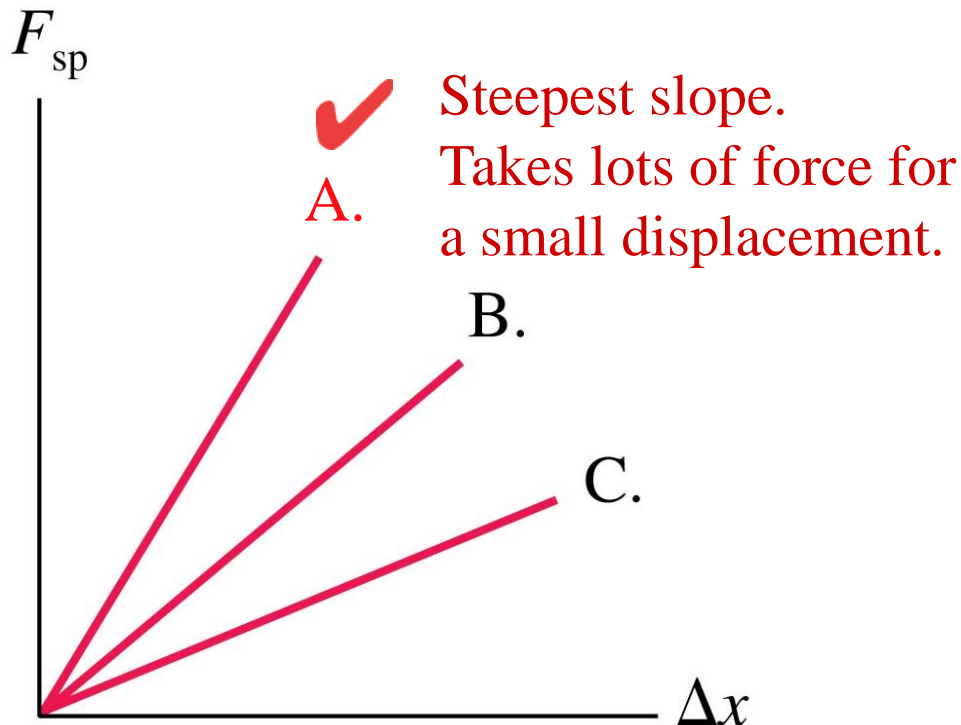
QuickCheck 8.3

The restoring force of three springs is measured as they are stretched. Which spring has the largest spring constant?



QuickCheck 8.3

The restoring force of three springs is measured as they are stretched. Which spring has the largest spring constant?



Example 8.6 Weighing a fish

A scale used to weigh fish consists of a spring hung from a support. The spring's equilibrium length is 10.0 cm. When a 4.0 kg fish is suspended from the end of the spring, it stretches to a length of 12.4 cm.

- What is the spring constant k for this spring?
- If an 8.0 kg fish is suspended from the spring, what will be the length of the spring?

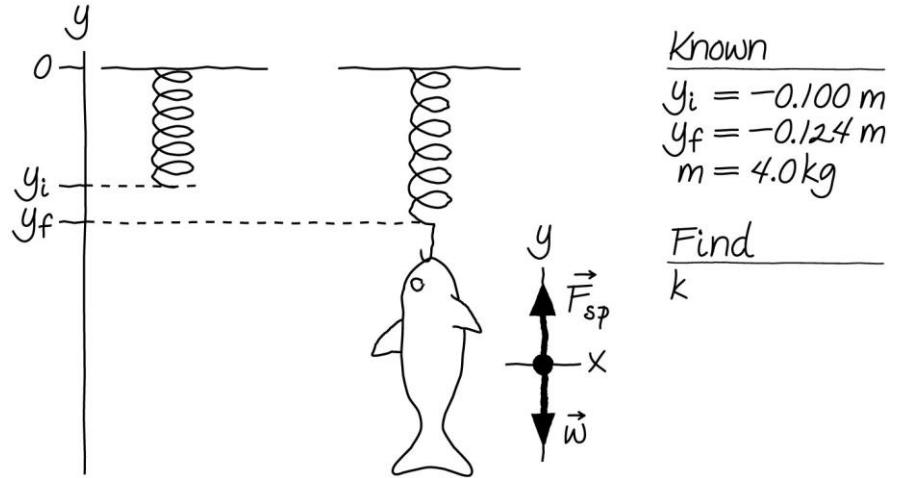
PREPARE The visual overview in FIGURE 8.15 shows the details for the first part of the problem. The fish hangs in static equilibrium, so the net force in the y -direction and the net torque must be zero.

Example 8.6 Weighing a fish (cont.)

SOLVE a. Because the fish is in static equilibrium, we have

$$\begin{aligned}\sum F_y &= (F_{\text{sp}})_y + w_y \\ &= -k \Delta y - mg = 0\end{aligned}$$

so that $k = -mg/\Delta y$. (The net torque is zero because the fish's center of gravity comes to rest directly under the pivot point of the hook.)



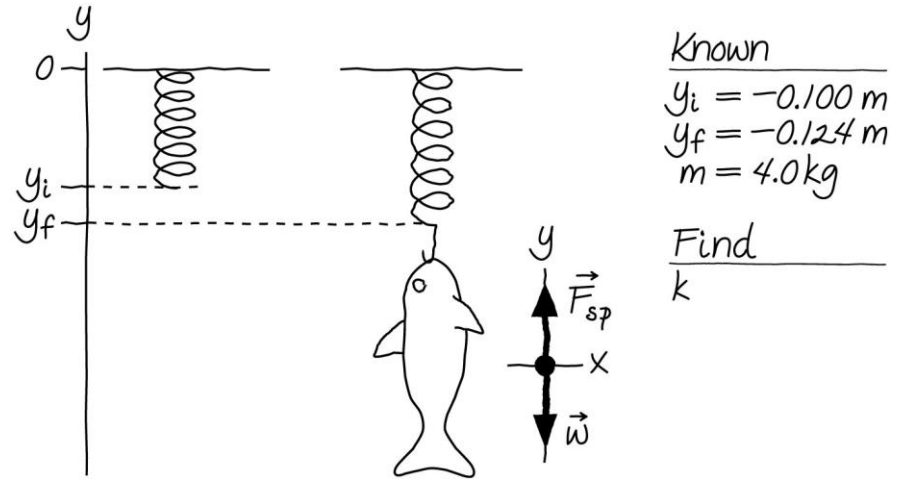
Example 8.6 Weighing a fish (cont.)

From Figure 8.15, the displacement of the spring from equilibrium is

$\Delta y = y_f - y_i = (-0.124 \text{ m}) - (-0.100 \text{ m}) = -0.024 \text{ m}$. This displacement is negative

because the fish moves in the $-y$ -direction. We can now solve for the spring constant:

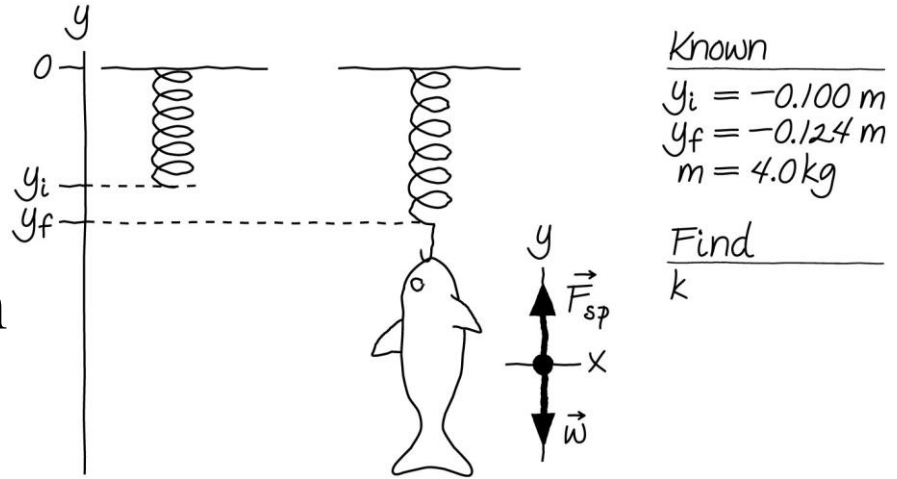
$$k = -\frac{mg}{\Delta y} = -\frac{(4.0 \text{ kg})(9.8 \text{ m/s}^2)}{-0.024 \text{ m}} = 1600 \text{ N/m}$$



Example 8.6 Weighing a fish (cont.)

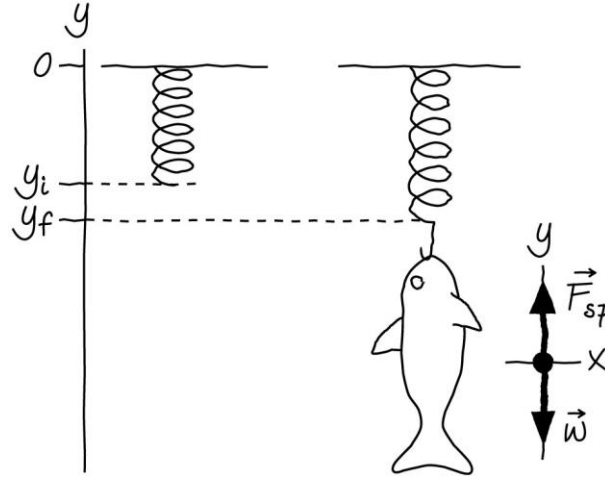
b. The restoring force is proportional to the displacement of the spring from its equilibrium length. If we double the mass (and thus the weight) of the fish, the displacement

of the end of the spring will double as well, to $\Delta y = -0.048$ m. Thus the spring will be 0.048 m longer, so its new length is 0.100 m + 0.048 m = 0.148 m = 14.8 cm.



Example 8.6 Weighing a fish (cont.)

ASSESS The spring doesn't stretch very much when a 4.0 kg mass is hung from it. A large spring constant of 1600 N/m thus seems reasonable for this stiff spring.



Known
 $y_i = -0.100 \text{ m}$
 $y_f = -0.124 \text{ m}$
 $m = 4.0 \text{ kg}$

Find
 k

Example Problem

A 20-cm-long spring is attached to a wall. When pulled horizontally with a force of 100 N, the spring stretches to a length of 22 cm. What is the value of the spring constant?

Example Problem

A 20-cm-long spring is attached to a wall. When pulled horizontally with a force of 100 N, the spring stretches to a length of 22 cm. The same spring is now suspended from a hook and a 10.2-kg block is attached to the bottom end. How long is the stretched spring?

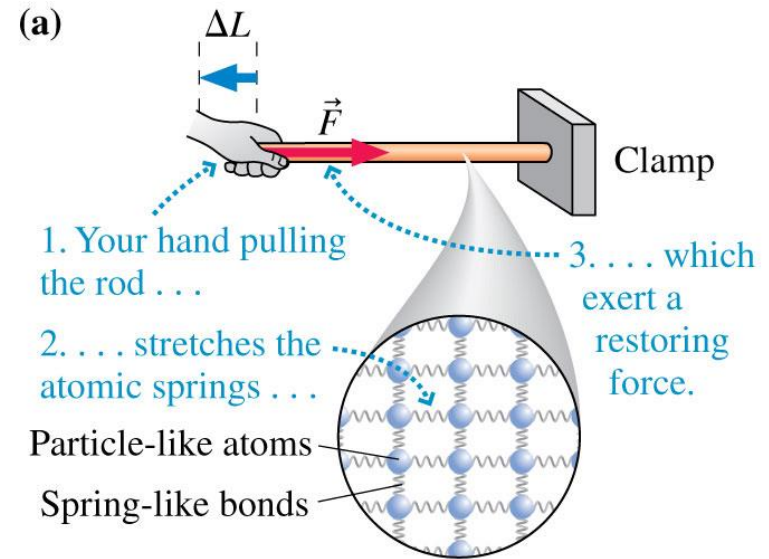
Example Problem

A spring with spring constant $k = 125 \text{ N/m}$ is used to pull a 25 N wooden block horizontally across a tabletop. The coefficient of friction between the block and the table is $\mu_k = 0.20$. By how much does this spring stretch from its equilibrium length?

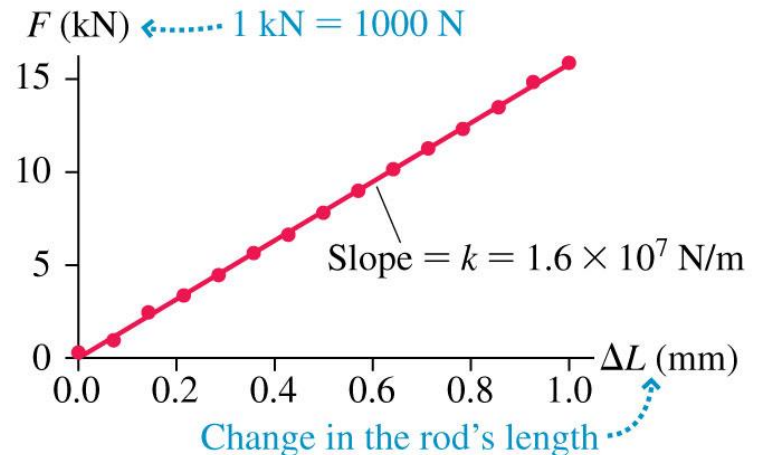
Section 8.4 Stretching and Compressing Materials

Stretching and Compressing Materials

- We can model most solid materials as being made up of particle-like atoms connected by spring-like bonds.
- Pulling on a steel rod will slightly stretch the bonds between particles, and the rod will stretch.



(b) Data for a 1.0-m-long, 1.0-cm-diameter steel rod

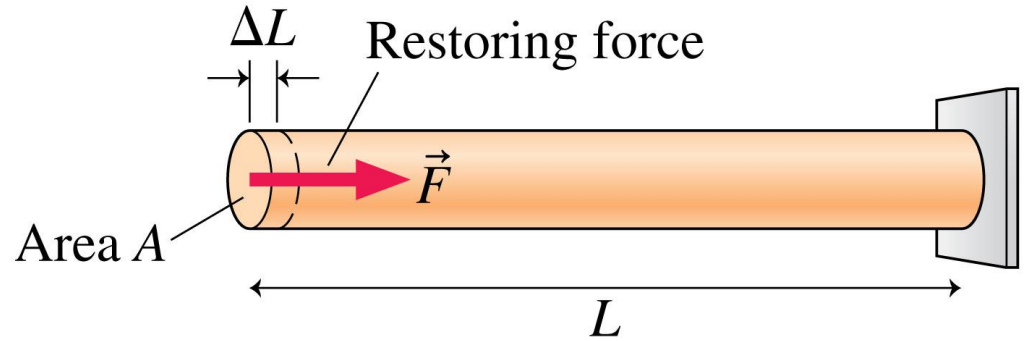


Stretching and Compressing Materials

- Steel is elastic, but under normal forces it experiences only small changes in dimension. Materials of this sort are called **rigid**.
- Rubber bands and other materials that can be stretched easily or show large deformations with small forces are called **pliant**.

Stretching and Compressing Materials

- For a rod, the spring constant depends on the cross-sectional area A , the length of the rod, L , and the material from which it is made.

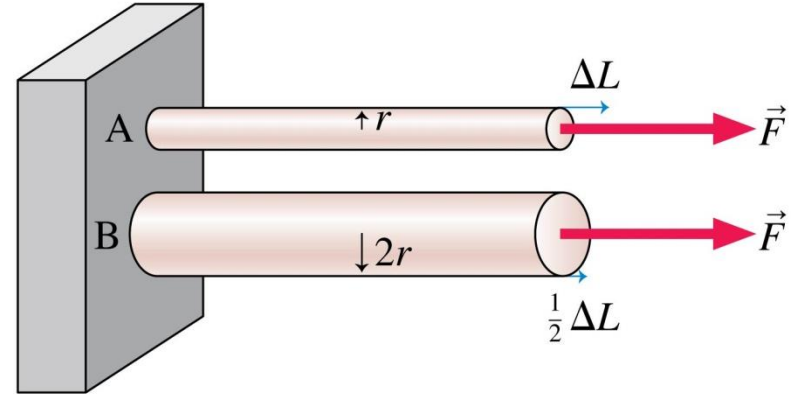


$$k = \frac{YA}{L}$$

- The constant Y is called **Young's modulus** and is a property of the *material* from which the rod is made.

QuickCheck 8.4

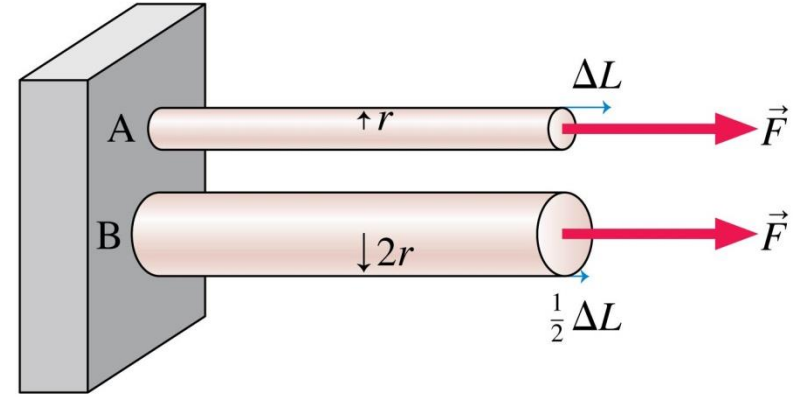
Bars A and B are attached to a wall on the left and pulled with equal forces to the right. Bar B, with twice the radius, is stretched half as far as bar A. Which has the larger value of Young's modulus Y ?



- A. $Y_A > Y_B$
- B. $Y_A = Y_B$
- C. $Y_A < Y_B$
- D. Not enough information to tell

QuickCheck 8.4

Bars A and B are attached to a wall on the left and pulled with equal forces to the right. Bar B, with twice the radius, is stretched half as far as bar A. Which has the larger value of Young's modulus Y ?



- ✓ A. $Y_A > Y_B$
- B. $Y_A = Y_B$
- C. $Y_A < Y_B$
- D. Not enough information to tell
- $\frac{F}{A} = Y \frac{\Delta L}{L}$
- Area of B increases by 4. If $Y_B = Y_A$, stretch would be only $\Delta L/4$. Stretch of $\Delta L/2$ means B is “softer” than A.

Stretching and Compressing Materials

TABLE 8.1 Young's modulus for rigid materials

Material	Young's modulus (10^{10} N/m²)
Cast iron	20
Steel	20
Silicon	13
Copper	11
Aluminum	7
Glass	7
Concrete	3
Wood (Douglas Fir)	1

Stretching and Compressing Materials

- The restoring force can be written in terms of the change in length ΔL :

$$F = \frac{YA}{L} \Delta L$$

- It is useful to rearrange the equation in terms of two ratios, the *stress* and the *strain*:

The ratio of force to cross-section area is called **stress**.

$$\frac{F}{A} = Y \left(\frac{\Delta L}{L} \right)$$

The ratio of the change in length to the original length is called **strain**.

- The unit of stress is N/m^2
- If stress is due to stretching, we call it **tensile stress**.

Example 8.8 Finding the stretch of a wire

A *Foucault pendulum* in a physics department (used to prove that the earth rotates) consists of a 120 kg steel ball that swings at the end of a 6.0-m-long steel cable. The cable has a diameter of 2.5 mm. When the ball was first hung from the cable, by how much did the cable stretch?



Example 8.8 Finding the stretch of a wire (cont.)

PREPARE The amount by which the cable stretches depends on the elasticity of the steel cable. Young's modulus for steel is given in Table 8.1 as $Y = 20 \times 10^{10} \text{ N/m}^2$.



Example 8.8 Finding the stretch of a wire (cont.)

SOLVE Equation 8.6 relates the stretch of the cable ΔL to the restoring force F and to the properties of the cable. Rearranging terms, we find that the cable stretches by

$$\Delta L = \frac{LF}{AY}$$

The cross-section area of the cable is

$$A = \pi r^2 = \pi(0.00125 \text{ m})^2 = 4.91 \times 10^{-6} \text{ m}^2$$

Example 8.8 Finding the stretch of a wire (cont.)

The restoring force of the cable is equal to the ball's weight:

$$F = w = mg = (120 \text{ kg})(9.8 \text{ m/s}^2) = 1180 \text{ N}$$

The change in length is thus

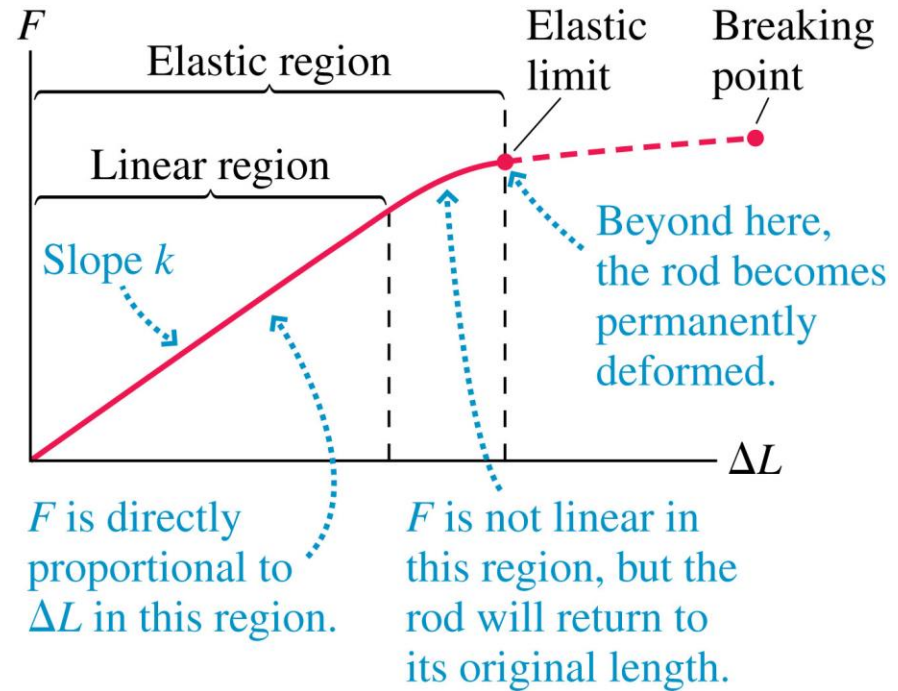
$$\begin{aligned}\Delta L &= \frac{(6.0 \text{ m})(1180 \text{ N})}{(4.91 \times 10^{-6} \text{ m}^2)(20 \times 10^{10} \text{ N/m}^2)} \\ &= 0.0072 \text{ m} = 7.2 \text{ mm}\end{aligned}$$

Example 8.8 Finding the stretch of a wire (cont.)

ASSESS If you've ever strung a guitar with steel strings, you know that the strings stretch several millimeters with the force you can apply by turning the tuning pegs. So a stretch of 7 mm under a 120 kg load seems reasonable.

Beyond the Elastic Limit

- As long as the stretch stays within the linear region, a solid rod acts like a spring and obeys Hooke's law.
- As long as the stretch is less than the **elastic limit**, the rod returns to its initial length when force is removed.
- The elastic limit is the end of the **elastic region**.



Beyond the Elastic Limit

- For a rod or cable of a particular material, there is an *ultimate stress*.
- The ultimate stress, or **tensile strength**, is the largest stress the material can sustain before breaking.

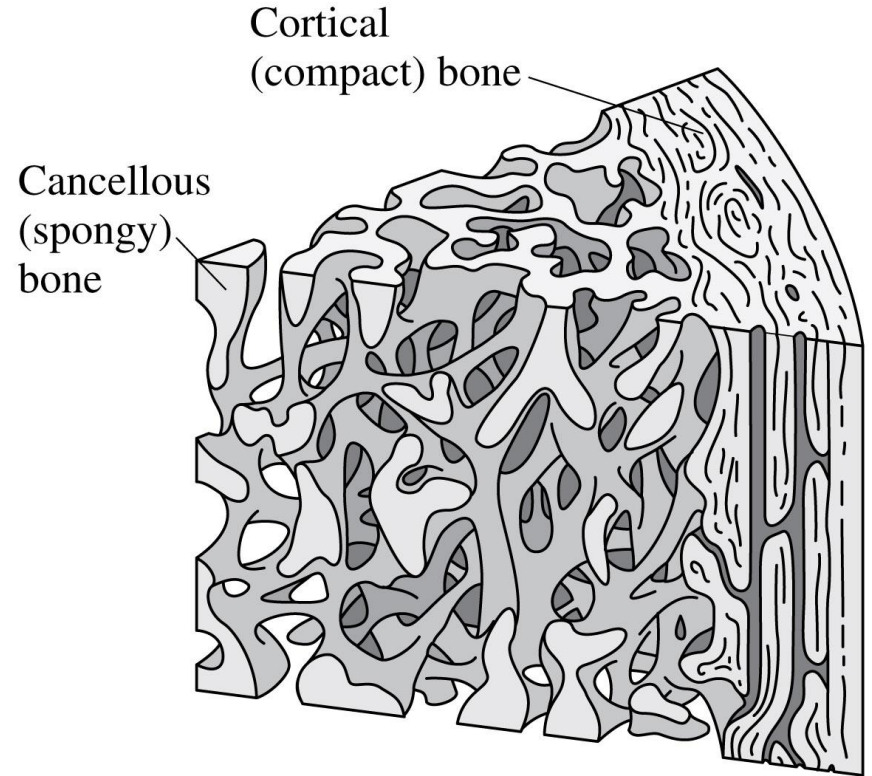
Largest stress that can be sustained (N/m²)→ Tensile strength = $\frac{F_{\max}}{A}$

.....→ Largest force that can be sustained (N)

.....→ Cross-section area (m²)

Biological Materials

- Most bones in your body are made of two different kinds of bony material: dense and rigid cortical (or compact bone) on the outside, and porous, flexible cancellous (or spongy) bone on the inside.



Biological Materials

TABLE 8.3 Young's modulus for biological materials

Material	Young's modulus (10^{10} N/m²)
Tooth enamel	6
Cortical bone	1.6
Cancellous bone	0.02–0.3
Spider silk	0.2
Tendon	0.15
Cartilage	0.0001
Blood vessel (aorta)	0.00005

Biological Materials

TABLE 8.4 Tensile strengths of biological materials

Material	Tensile strength (N/m²)
Cancellous bone	5×10^6
Cortical bone	100×10^6
Tendon	100×10^6
Spider silk	1000×10^6

Summary: General Principles

Static Equilibrium

An object in **static equilibrium** must have no net force on it and no net torque.

Mathematically, we express this as

$$\sum F_x = 0$$

$$\sum F_y = 0$$

$$\sum \tau = 0$$

Since the net torque is zero about *any* point, the pivot point for calculating the torque can be chosen at any convenient location.

Text: p. 240

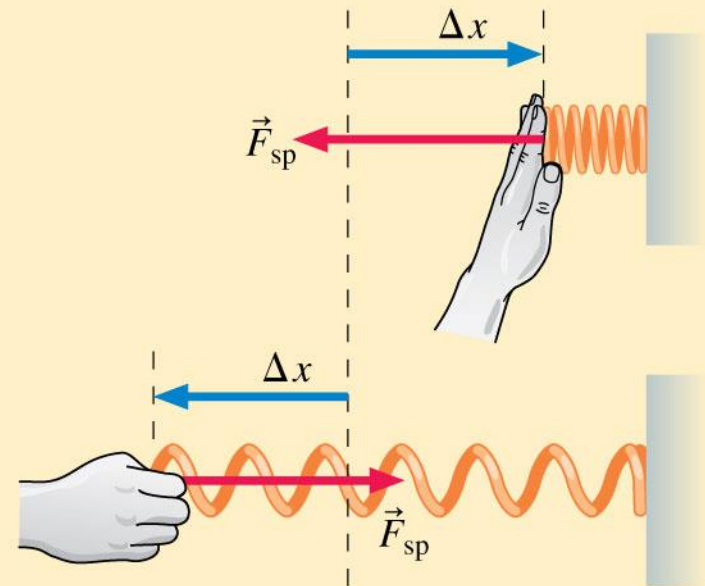
Summary: General Principles

Springs and Hooke's Law

When a spring is stretched or compressed, it exerts a force proportional to the change Δx in its length but in the opposite direction. This is known as **Hooke's law**:

$$(F_{\text{sp}})_x = -k \Delta x$$

The constant of proportionality k is called the **spring constant**. It is larger for a “stiff” spring.



Text: p. 240

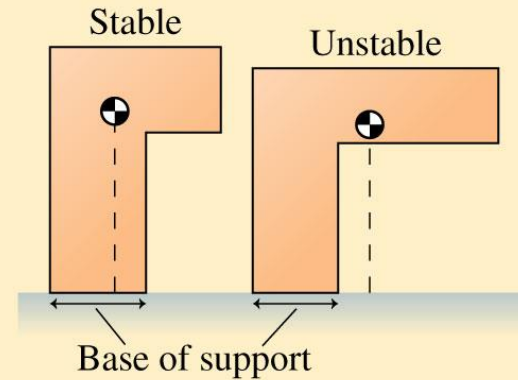
Summary: Important Concepts

Stability

An object is **stable** if its center of gravity is over its base of support; otherwise, it is **unstable**.

If an object is tipped, it will reach the limit of its stability when its center of gravity is over the edge of the base. This defines the **critical angle** θ_c .

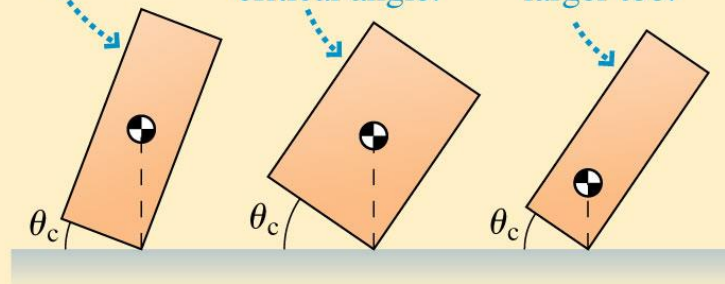
Greater stability is possible with a lower center of gravity or a broader base of support.



This object is at its critical angle.

This object has a wider base of support and hence a larger critical angle.

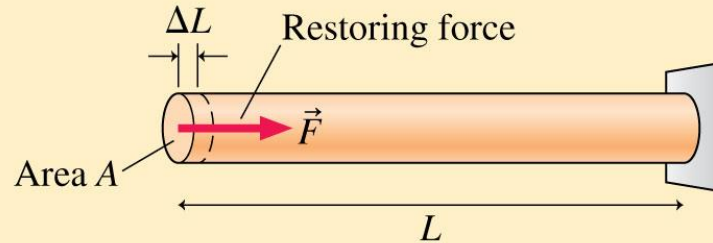
This object has a lower center of gravity, so its critical angle is larger too.



Text: p. 240

Summary: Important Concepts

Elastic materials and Young's modulus



A solid rod illustrates how materials respond when stretched or compressed.

Stress is the restoring force of the rod divided by its cross-section area. $\left(\frac{F}{A}\right) = Y \left(\frac{\Delta L}{L}\right)$ **Strain** is the fractional change in the rod's length.

Young's modulus

This equation can also be written as

This is the "spring constant" k for the rod. $F = \left(\frac{YA}{L}\right)\Delta L$

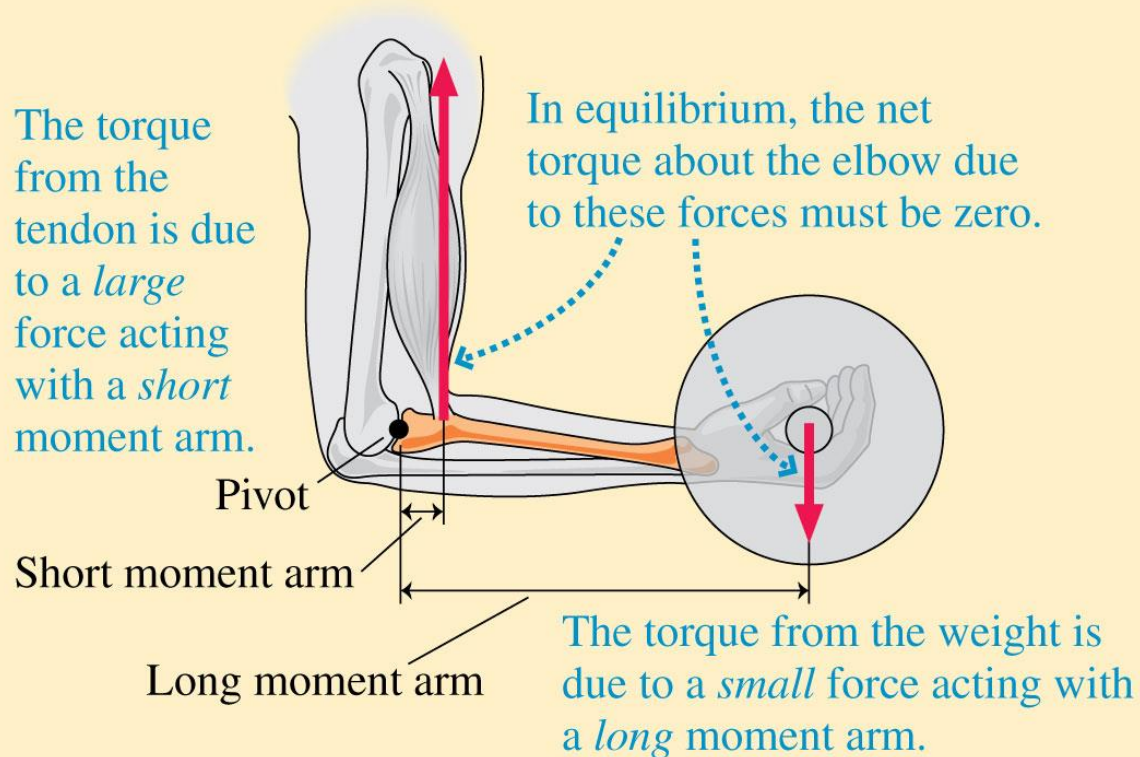
showing that a rod obeys Hooke's law and acts like a very stiff spring.

Text: p. 240

Summary: Applications

Forces in the body

Muscles and tendons apply the forces and torques needed to maintain static equilibrium. These forces may be quite large.

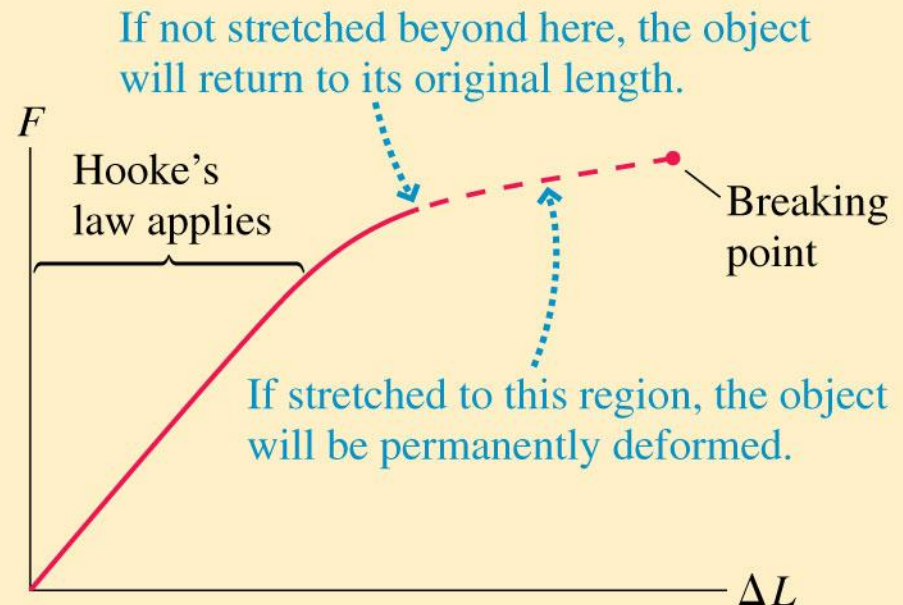


Summary: Applications

The elastic limit and beyond

If a rod or other object is not stretched too far, when released it will return to its original shape.

If stretched too far, an object will permanently deform and finally break. The stress at which an object breaks is its **tensile stress**.



Text: p. 240

Summary

GENERAL PRINCIPLES

Static Equilibrium

An object in **static equilibrium** must have no net force on it and no net torque.

Mathematically, we express this as

$$\sum F_x = 0$$

$$\sum F_y = 0$$

$$\sum \tau = 0$$

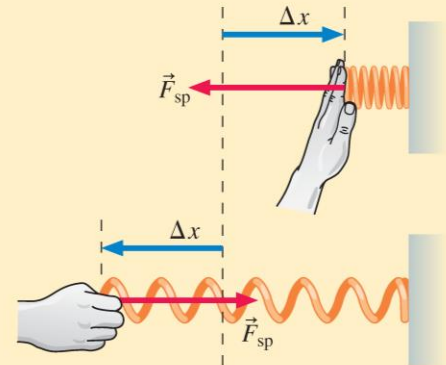
Since the net torque is zero about *any* point, the pivot point for calculating the torque can be chosen at any convenient location.

Springs and Hooke's Law

When a spring is stretched or compressed, it exerts a force proportional to the change Δx in its length but in the opposite direction. This is known as **Hooke's law**:

$$(F_{\text{sp}})_x = -k \Delta x$$

The constant of proportionality k is called the **spring constant**. It is larger for a “stiff” spring.



Text: p. 240

Summary

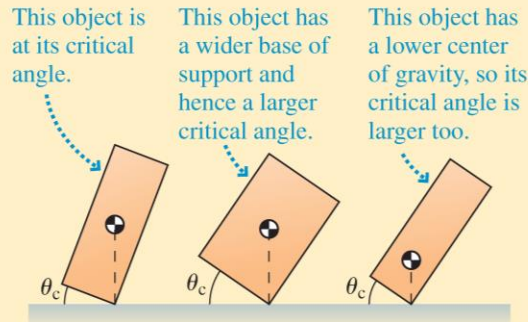
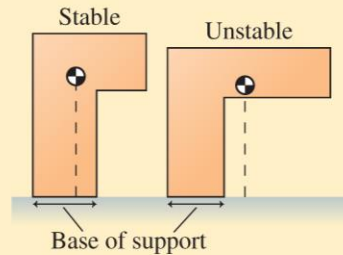
IMPORTANT CONCEPTS

Stability

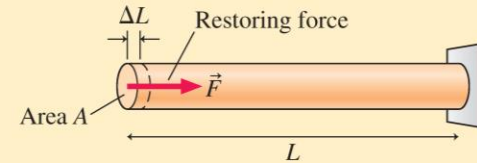
An object is **stable** if its center of gravity is over its base of support; otherwise, it is **unstable**.

If an object is tipped, it will reach the limit of its stability when its center of gravity is over the edge of the base. This defines the **critical angle** θ_c .

Greater stability is possible with a lower center of gravity or a broader base of support.



Elastic materials and Young's modulus



A solid rod illustrates how materials respond when stretched or compressed.

Stress is the restoring force of the rod divided by its cross-section area. $\left(\frac{F}{A}\right) = Y \left(\frac{\Delta L}{L}\right)$ **Strain** is the fractional change in the rod's length.

Young's modulus

This equation can also be written as

This is the "spring constant" k for the rod. $F = \left(\frac{YA}{L}\right) \Delta L$

showing that a rod obeys Hooke's law and acts like a very stiff spring.

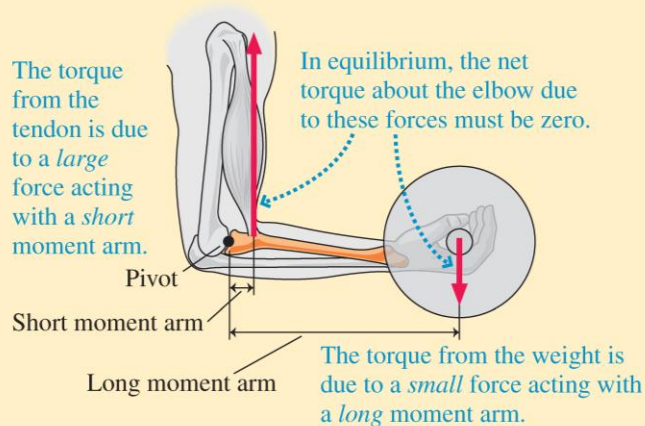
Text: p. 240

Summary

APPLICATIONS

Forces in the body

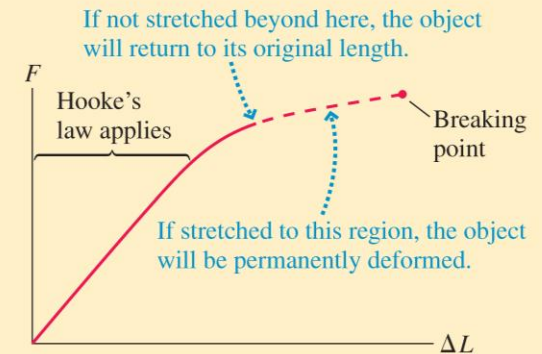
Muscles and tendons apply the forces and torques needed to maintain static equilibrium. These forces may be quite large.



The elastic limit and beyond

If a rod or other object is not stretched too far, when released it will return to its original shape.

If stretched too far, an object will permanently deform and finally break. The stress at which an object breaks is its **tensile stress**.



Text: p. 240