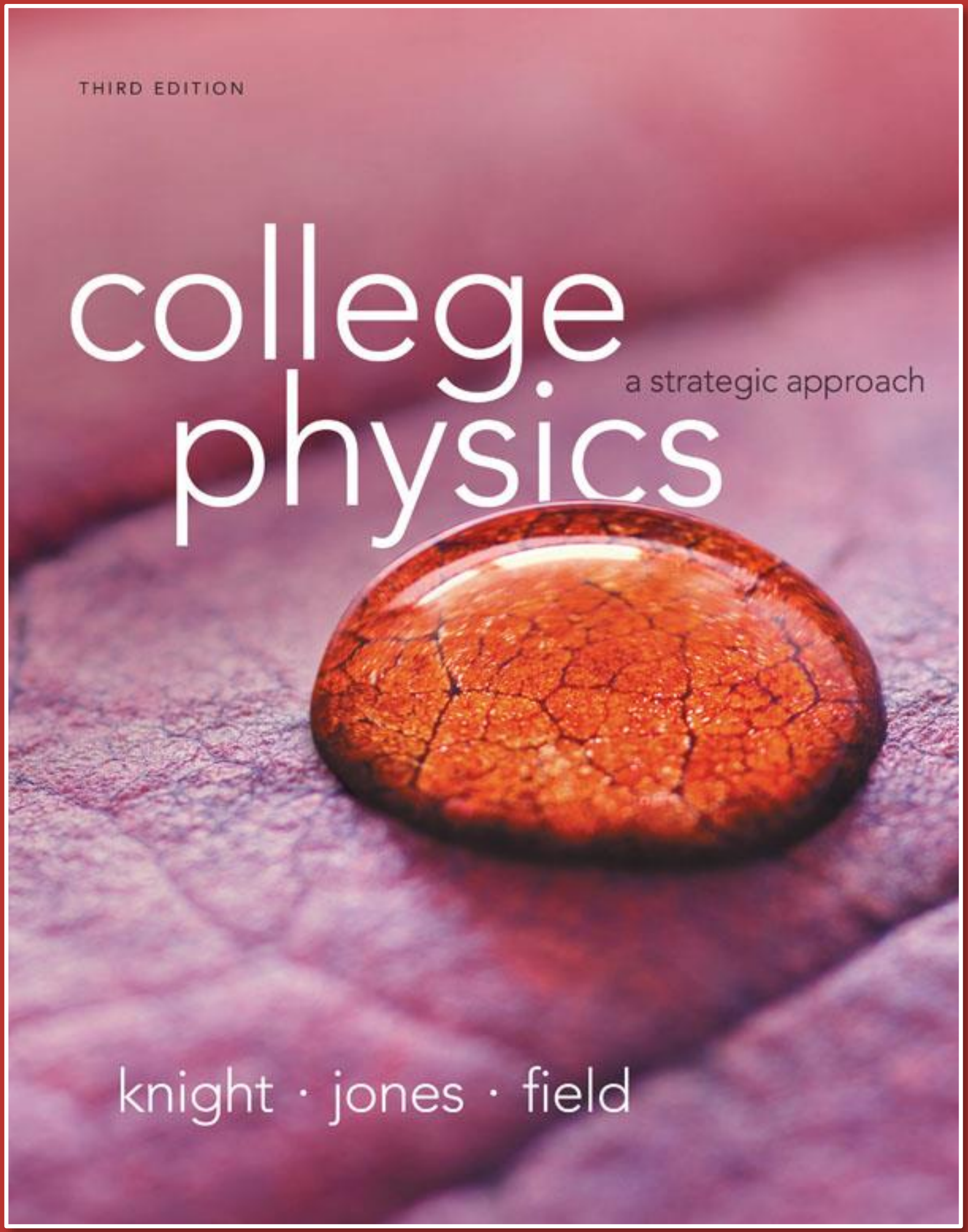


Lecture Presentation

Chapter 7

Rotational Motion

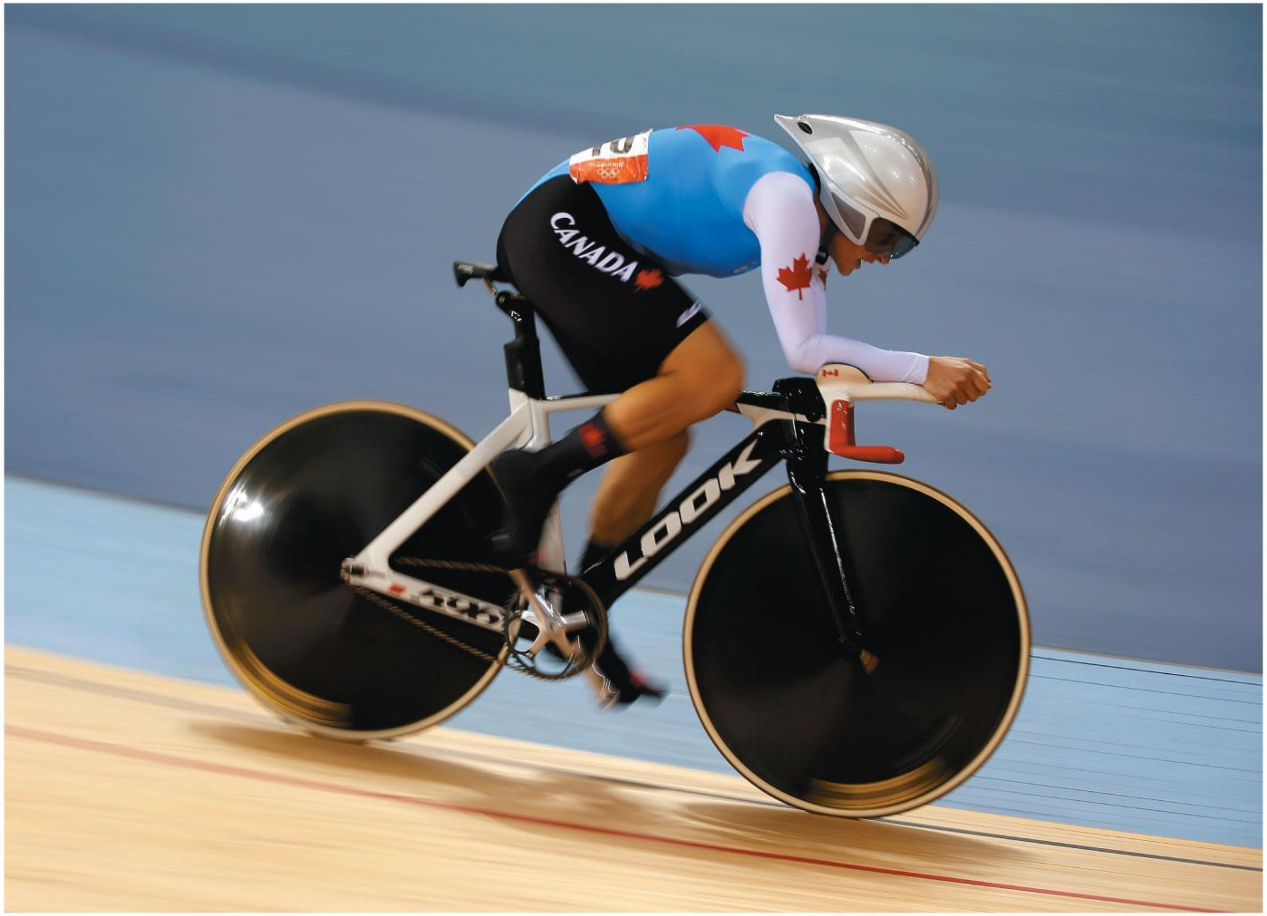
Suggested Videos for Chapter 7

- **Prelecture Videos**
  - *Describing Rotational Motion*
  - *Moment of Inertia and Center of Gravity*
  - *Newton's Second Law for Rotation*
- **Class Videos**
  - *Torque*
  - *Torques and Moment Arms*
  - *Walking on a Tightrope*
- **Video Tutor Solutions**
  - *Rotational Motion*
- **Video Tutor Demos**
  - *Walking the UC Plank*
  - *Balancing a Meter Stick*

Suggested Simulations for Chapter 7

- **ActivPhysics**
  - *7.1, 7.6–7.10*
- **PhETs**
  - *Ladybug Revolution*
  - *Torque*

Chapter 7 Rotational Motion



Chapter Goal: To understand the physics of rotating objects.

Chapter 7 Preview
Looking Ahead: Rotational Kinematics

- The spinning roulette wheel isn't going anywhere, but it is moving. This is **rotational motion**.

- You'll learn about angular velocity and other quantities we use to describe rotational motion.

Chapter 7 Preview
Looking Ahead: Torque

- To start something moving, apply a force. To start something rotating, apply a **torque**, as the sailor is doing to the wheel.

- You'll see that torque depends on how hard you push and also on *where* you push. A push far from the axle gives a large torque.

Chapter 7 Preview
Looking Ahead: Rotational Dynamics

- The girl pushes on the outside edge of the merry-go-round, gradually increasing its rotation rate.

- You'll learn a version of Newton's second law for rotational motion and use it to solve problems.

Chapter 7 Preview
Looking Ahead

Rotational Kinematics

The spinning roulette wheel isn't going anywhere, but it is moving. This is **rotational motion.**

You'll learn about angular velocity and other quantities we use to describe rotational motion.

Torque

To start something moving, apply a force. To start something rotating, apply a **torque,** as this sailor is doing to the wheel.

You'll see that torque depends on how hard you push and also on *where* you push. A push far from the axle gives a large torque.

Rotational Dynamics

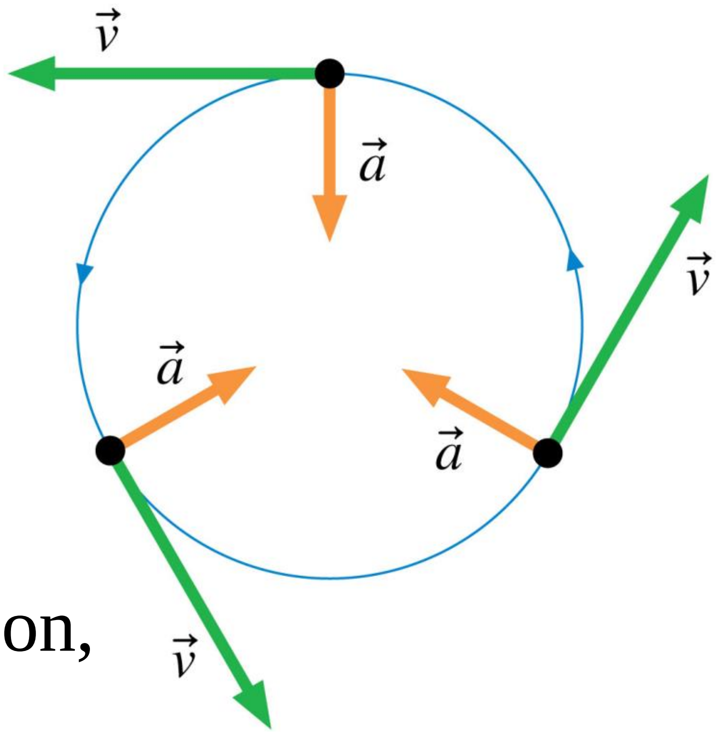
The girl pushes on the outside edge of the merry-go-round, gradually increasing its rotation rate.

You'll learn a version of Newton's second law for rotational motion and use it to solve problems.

Text p. 189

Chapter 7 Preview
Looking Back: Circular Motion

- In Chapter 6, you learned to describe *circular* motion in terms of period, frequency, velocity, and centripetal acceleration.
- In this chapter, you'll learn to use angular velocity, angular acceleration, and other quantities that describe *rotational* motion.



Chapter 7 Preview
Stop to Think

As an audio CD plays, the frequency at which the disk spins changes. At 210 rpm, the speed of a point on the outside edge of the disk is 1.3 m/s. At 420 rpm, the speed of a point on the outside edge is

A. 1.3 m/s
B. 2.6 m/s
C. 3.9 m/s
D. 5.2 m/s

Reading Question 7.1

If an object is rotating clockwise, this corresponds to a _______ angular velocity.

A. Positive
B. Negative

Reading Question 7.1

If an object is rotating clockwise, this corresponds to a _______ angular velocity.

A. Positive

✔ B. Negative

Reading Question 7.2

The angular displacement of a rotating object is measured in

A. Degrees.
B. Radians.
C. Degrees per second.
D. Radians per second.

Reading Question 7.2

The angular displacement of a rotating object is measured in

A. Degrees.
B. Radians.
C. Degrees per second.
D. Radians per second.

Reading Question 7.3

Moment of inertia is

A. The rotational equivalent of mass.
B. The time at which inertia occurs.
C. The point at which all forces appear to act.
D. An alternative term for *moment arm.*

Reading Question 7.3

Moment of inertia is

✔ A. The rotational equivalent of mass.

B. The time at which inertia occurs.

C. The point at which all forces appear to act.

D. An alternative term for *moment arm.*

Reading Question 7.4

Which factor does the torque on an object *not* depend on?

A. The magnitude of the applied force
B. The object's angular velocity
C. The angle at which the force is applied
D. The distance from the axis to the point at which the force is applied

Reading Question 7.4

Which factor does the torque on an object *not* depend on?

A. The magnitude of the applied force
✔ B. The object's angular velocity
C. The angle at which the force is applied
D. The distance from the axis to the point at which the force is applied

Reading Question 7.5

A net torque applied to an object causes

A. A linear acceleration of the object.
B. The object to rotate at a constant rate.
C. The angular velocity of the object to change.
D. The moment of inertia of the object to change.

Reading Question 7.5

A net torque applied to an object causes

A. A linear acceleration of the object.
B. The object to rotate at a constant rate.
✔ C. The angular velocity of the object to change.
D. The moment of inertia of the object to change.

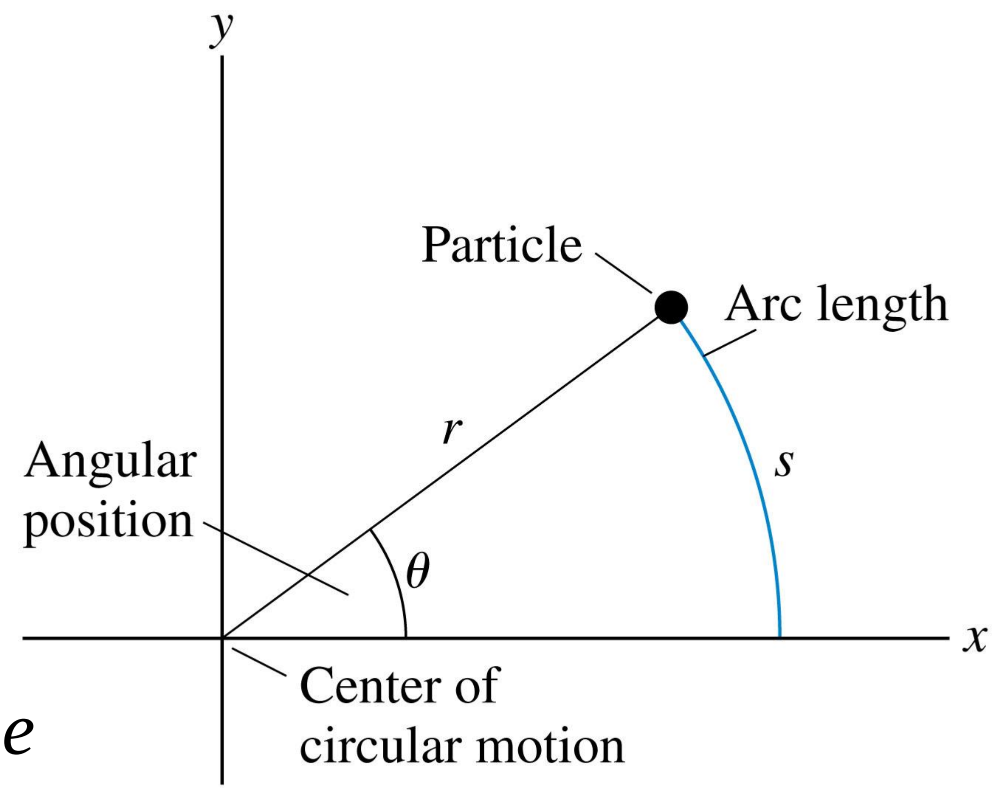
Section 7.1 Describing Circular and Rotational Motion

Describing Circular and Rotational Motion

- **Rotational motion** is the motion of objects that spin about an axis.

Angular Position

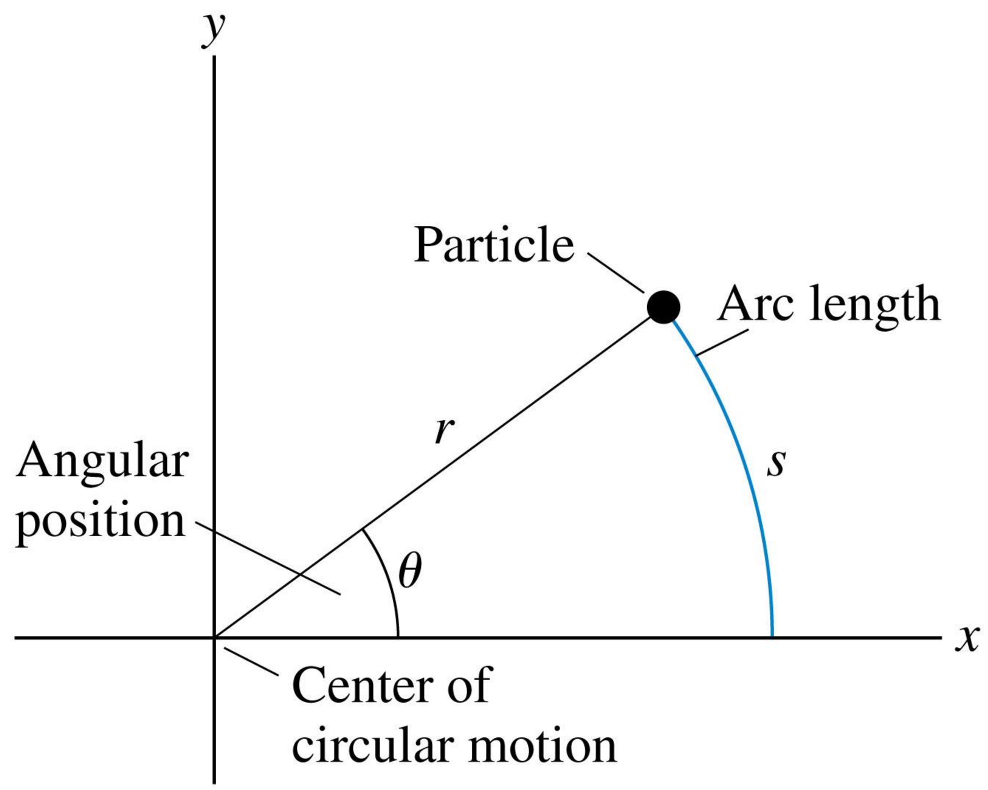
- We use the angle θ from the positive x-axis to describe the particle's location.
- Angle θ is the **angular position** of the particle.
- θ is positive when measured *counterclockwise* from the positive x-axis.



- An angle measured *clockwise* from the positive x-axis has a negative value.

Angular Position

- We measure angle θ in the angular unit of ***radians***, not degrees.
- The radian is abbreviated "rad."
- The **arc length**, s, is the distance that the particle has traveled along its circular path.



Angular Position

- We define the particle's angle θ in terms of arc length and radius of the circle:

$$\theta\ (\text{radians}) = \frac{s}{r}$$

$$s = r\theta$$

Angular Position

- One *revolution* (rev) is when a particle travels all the way around the circle.
- The angle of the full circle is

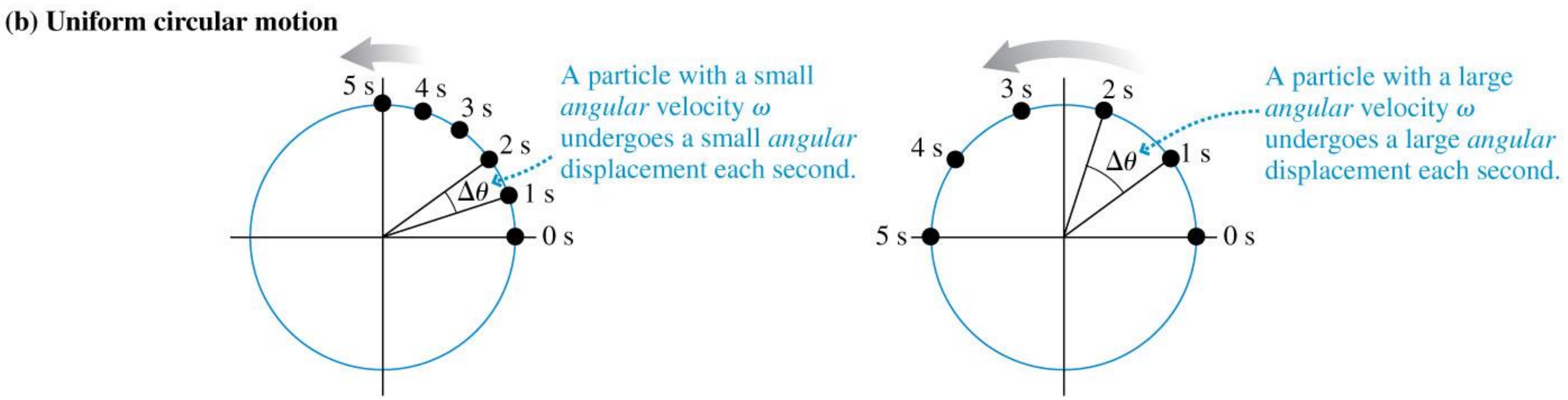
$$\theta_{\text{full circle}} = \frac{s}{r} = \frac{2\pi r}{r} = 2\pi \text{ rad}$$

Angular Displacement and Angular Velocity

- For *linear* motion, a particle with a larger velocity undergoes a greater displacement

Angular Displacement and Angular Velocity

- For uniform *circular* motion, a particle with a larger **angular velocity** will undergo a greater **angular displacement $\Delta\theta$.**



- **Angular velocity** is the angular displacement through which the particle moves each second.

Angular Displacement and Angular Velocity

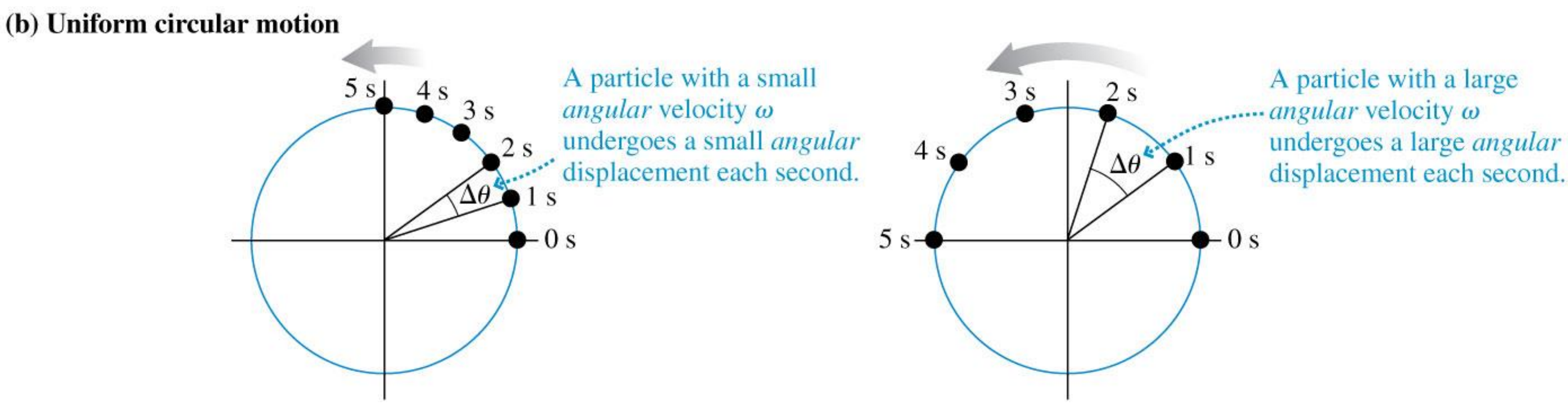
$$\omega = \frac{\text{angular displacement}}{\text{time interval}} = \frac{\Delta\theta}{\Delta t}$$

Angular velocity of a particle in uniform circular motion

- **The angular velocity $\omega = \Delta\theta/\Delta t$ is constant for a particle moving with uniform circular motion.**

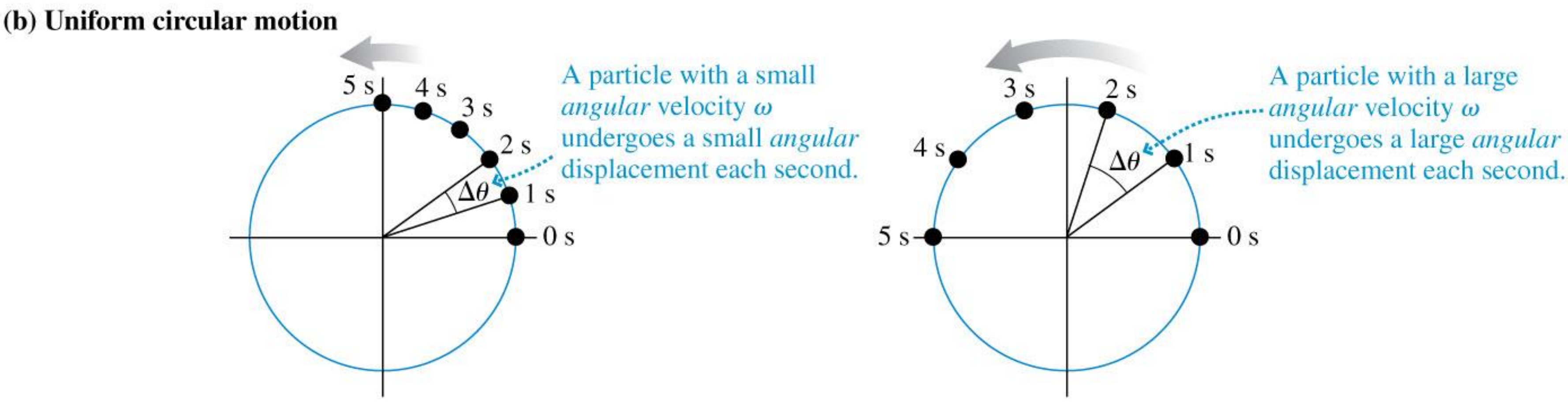
Example 7.1 Comparing angular velocities

Find the angular velocities of the two particles in Figure 7.2b.



PREPARE For uniform circular motion, we can use any angular displacement $\Delta\theta$, as long as we use the corresponding time interval Δt. For each particle, we'll choose the angular displacement corresponding to the motion from $t = 0$ s to $t = 5$ s.

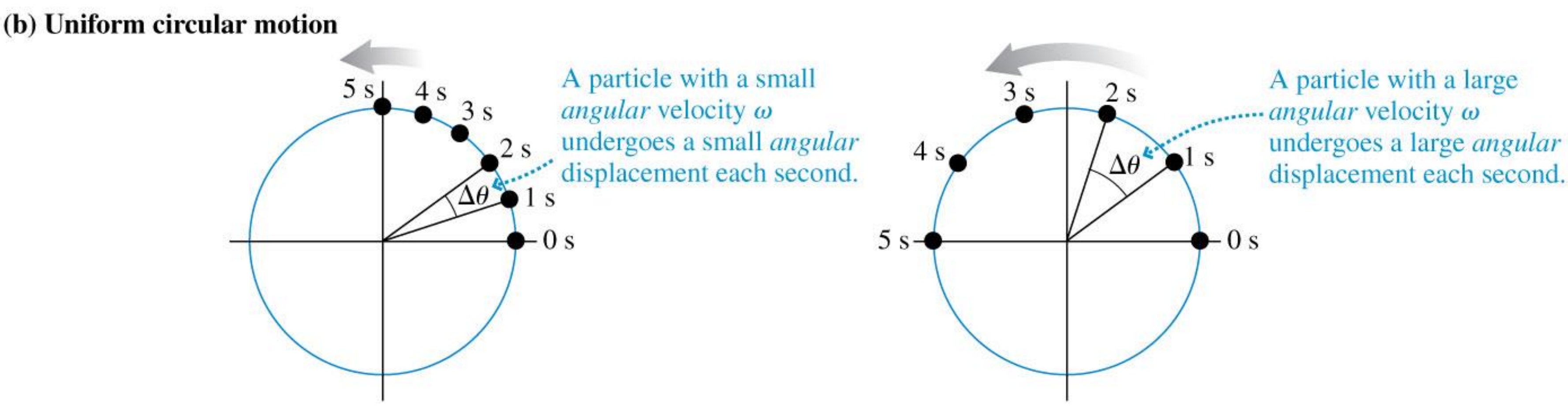
Example 7.1 Comparing angular velocities (cont.)



SOLVE The particle on the left travels one-quarter of a full circle during the 5 s time interval. We learned earlier that a full circle corresponds to an angle of 2π rad, so the angular displacement for this particle is $\Delta\theta = (2\pi \text{ rad})/4 = \pi/2$ rad. Thus its angular velocity is

$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{\pi/2 \text{ rad}}{5 \text{ s}} = 0.314 \text{ rad/s}$$

Example 7.1 Comparing angular velocities (cont.)

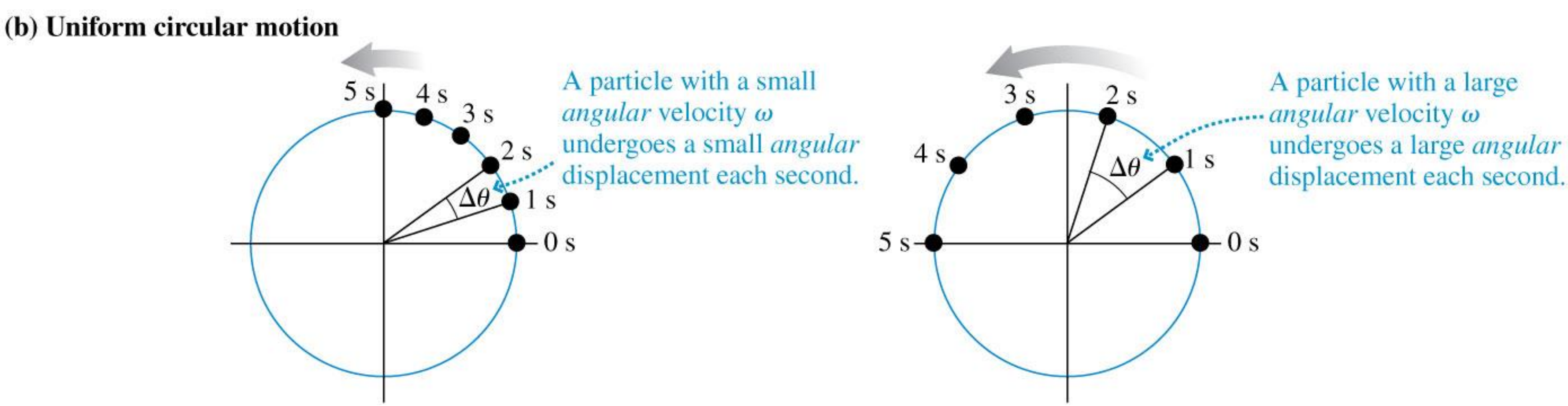


The particle on the right travels halfway around the circle, or π rad, in the 5 s interval. Its angular velocity is

$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{\pi \text{ rad}}{5 \text{ s}} = 0.628 \text{ rad/s}$$

Example 7.1 Comparing angular velocities (cont.)

ASSESS The speed of the second particle is double that of the first, as it should be. We should also check the scale of the answers. The angular velocity of the particle on the right is 0.628 rad/s, meaning that the particle travels through an angle of 0.628 rad each second. Because 1 rad ≈ 60°, 0.628 rad is roughly 35°. In Figure 7.2b, the particle on the right appears to move through an angle of about this size during each 1 s time interval, so our answer is reasonable.



Angular Displacement and Angular Velocity

- The linear displacement during a time interval is

$$x_f - x_i = \Delta x = v_x \, \Delta t$$

- Similarly, the angular displacement for uniform circular motion is

$$\theta_f - \theta_i = \Delta\theta = \omega \, \Delta t$$

Angular displacement for uniform circular motion

Angular Displacement and Angular Velocity

- **Angular speed** is the absolute value of the angular velocity.
- The angular speed is related to the period T:

$$\omega = \frac{2\pi \text{ rad}}{T}$$

- Frequency (in rev/s) $f = 1/T$:

$$\omega = (2\pi \text{ rad})f$$

QuickCheck 7.1

A ball rolls around a circular track with an angular velocity of 4π rad/s. What is the period of the motion?

A. $\frac{1}{2}$ s

B. 1 s

C. 2 s

D. $\frac{1}{2\pi}$ s

E. $\frac{1}{4\pi}$ s

QuickCheck 7.1

A ball rolls around a circular track with an angular velocity of 4π rad/s. What is the period of the motion?

✔ A. $\frac{1}{2}$ s $\quad T = \dfrac{2\pi}{\omega}$

B. 1 s

C. 2 s

D. $\dfrac{1}{2\pi}$ s

E. $\dfrac{1}{4\pi}$ s

Example 7.3 Rotations in a car engine

The crankshaft in your car engine is turning at 3000 rpm. What is the shaft's angular speed?

PREPARE We'll need to convert rpm to rev/s and then use Equation 7.6.

SOLVE We convert rpm to rev/s by

$$\left(3000\ \frac{\text{rev}}{\text{min}}\right)\left(\frac{1\ \text{min}}{60\ \text{s}}\right) = 50.0\ \text{rev/s}$$

Thus the crankshaft's angular speed is

$$\omega = (2\pi\ \text{rad})f = (2\pi\ \text{rad})(50.0\ \text{rev/s}) = 314\ \text{rad/s}$$

Angular-Position and Angular-Velocity Graphs

- We construct **angular position-versus-time graphs** using the change in angular *position* for each second.
- **Angular velocity-versus-time graphs** can be created by finding the slope of the corresponding angular position-versus-time graph.

Relating Speed and Angular Speed

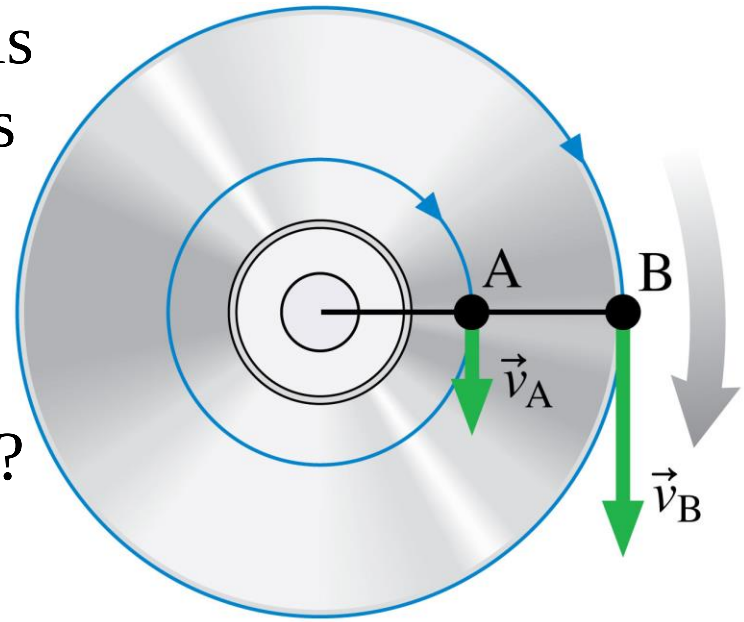
- Speed v and angular speed ω are related by

$$v = \omega r$$

Relationship between speed and angular speed

- **Angular speed ω must be in units of rad/s.**

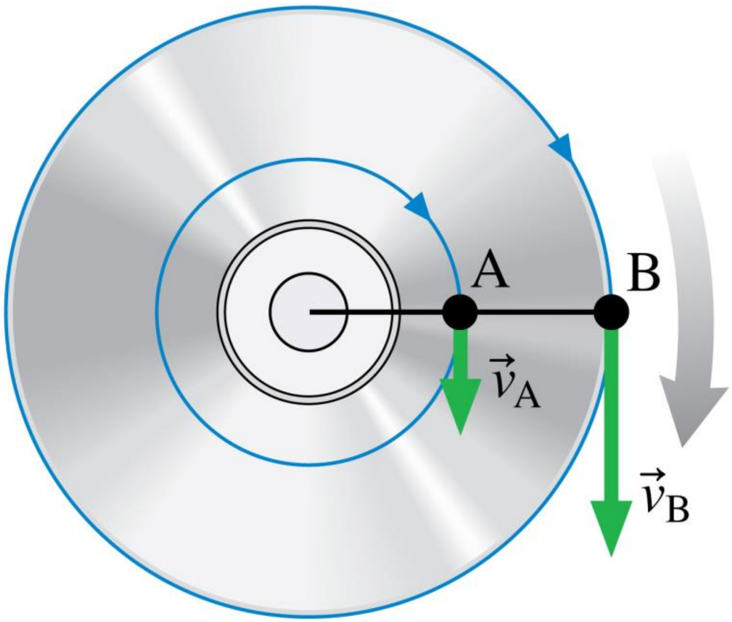
Example 7.5 Finding the speed at two points on a CD



The diameter of an audio compact disk is 12.0 cm. When the disk is spinning at its maximum rate of 540 rpm, what is the speed of a point (a) at a distance 3.0 cm from the center and (b) at the outside edge of the disk, 6.0 cm from the center?

PREPARE Consider two points A and B on the rotating compact disk in FIGURE 7.7. During one period T, the disk rotates once, and both points rotate through the same angle, 2π rad. Thus the angular speed, $\omega = 2\pi/T$, is the same for these two points; in fact, it is the same for all points on the disk.

Example 7.5 Finding the speed at two points on a CD (cont.)



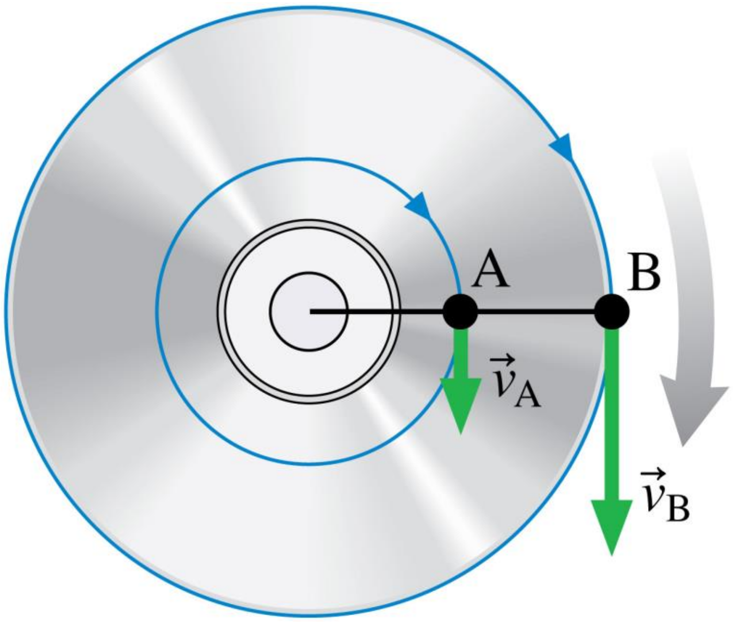
But as they go around one time, the two points move different *distances*. The outer point B goes around a larger circle. The two points thus have different *speeds*. We can solve this problem by first finding the angular speed of the disk and then computing the speeds at the two points.

Example 7.5 Finding the speed at two points on a CD (cont.)

SOLVE We first convert the frequency of the disk to rev/s:

$$f = \left(540\,\frac{\text{rev}}{\text{min}}\right) \times \left(\frac{1\ \text{min}}{60\ \text{s}}\right)$$

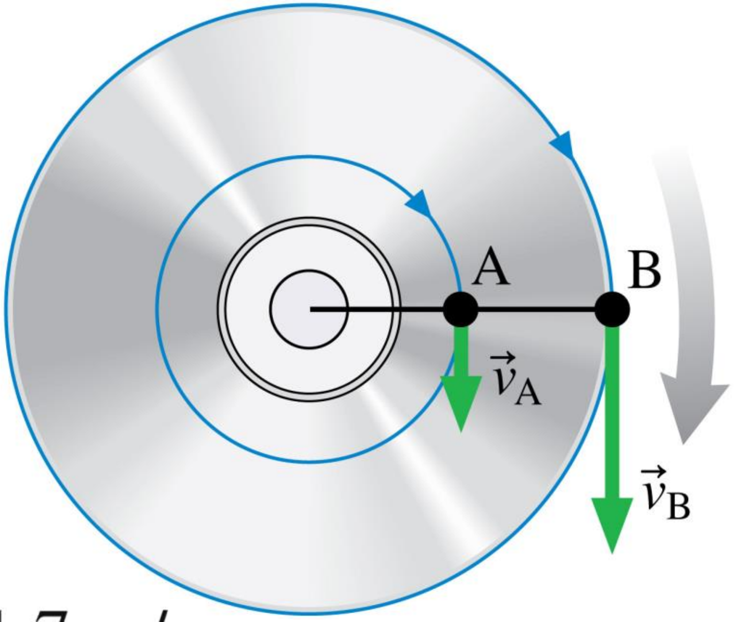
$$= 9.00\ \text{rev/s}$$



We then compute the angular speed using Equation 7.6:

$$\omega = (2\pi\ \text{rad})(9.00\ \text{rev/s}) = 56.5\ \text{rad/s}$$

Example 7.5 Finding the speed at two points on a CD (cont.)



We can now use Equation 7.7 to compute the speeds of points on the disk. At point A, r = 3.0 cm = 0.030 m, so the speed is

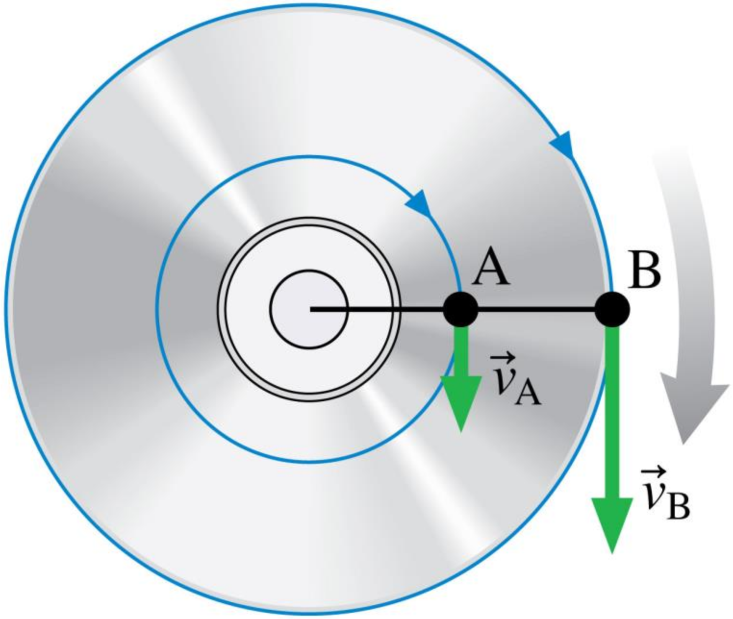
$$v_A = \omega r = (56.5 \text{ rad/s})(0.030 \text{ m}) = 1.7 \text{ m/s}$$

At point B, r = 6.0 cm = 0.060 m, so the speed at the outside edge is

$$v_B = \omega r = (56.5 \text{ rad/s})(0.060 \text{ m}) = 3.4 \text{ m/s}$$

Example 7.5 Finding the speed at two points on a CD (cont.)

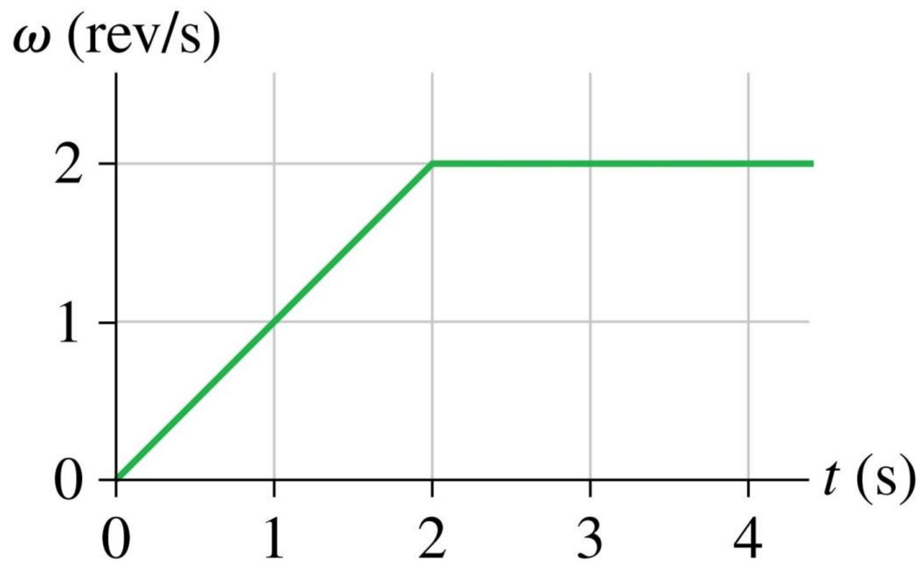
ASSESS The speeds are a few meters per second, which seems reasonable. The point farther from the center is moving at a higher speed, as we expected.



QuickCheck 7.7

This is the angular velocity graph of a wheel. How many revolutions does the wheel make in the first 4 s?

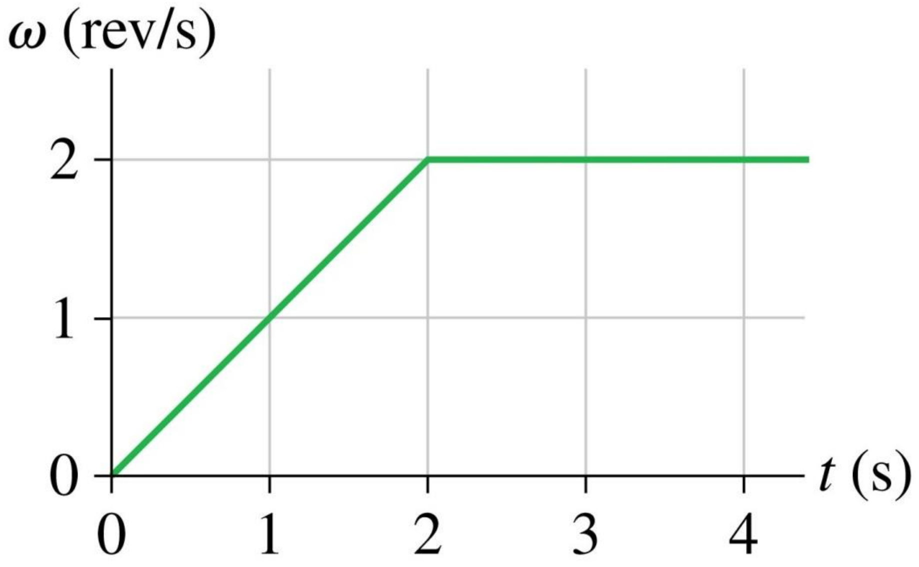
A. 1
B. 2
C. 4
D. 6
E. 8



QuickCheck 7.7

This is the angular velocity graph of a wheel. How many revolutions does the wheel make in the first 4 s?

A. 1
B. 2
C. 4
✔ D. 6
E. 8



$\Delta\theta$ = area under the angular velocity curve

QuickCheck 7.9

Starting from rest, a wheel with constant angular acceleration turns through an angle of 25 rad in a time t. Through what angle will it have turned after time $2t$?

A. 25 rad
B. 50 rad
C. 75 rad
D. 100 rad
E. 200 rad

QuickCheck 7.9

Starting from rest, a wheel with constant angular acceleration turns through an angle of 25 rad in a time *t*. Through what angle will it have turned after time 2*t*?

A. 25 rad

B. 50 rad

C. 75 rad

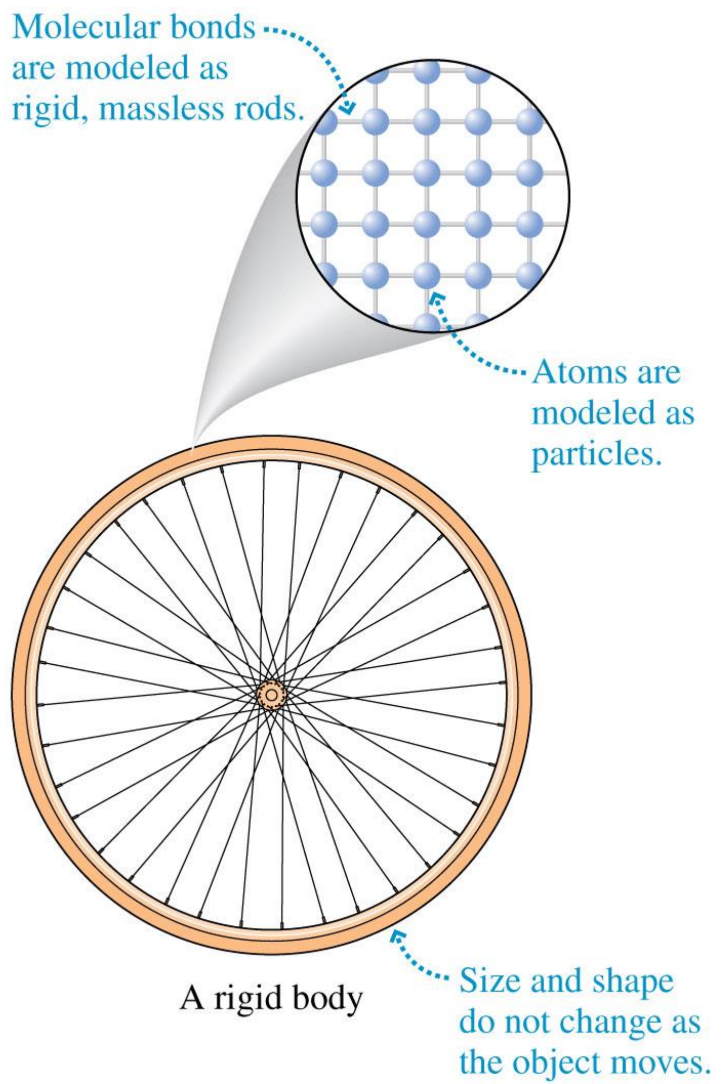
✔ D. 100 rad $\Delta\theta \propto (\Delta t)^2$

E. 200 rad

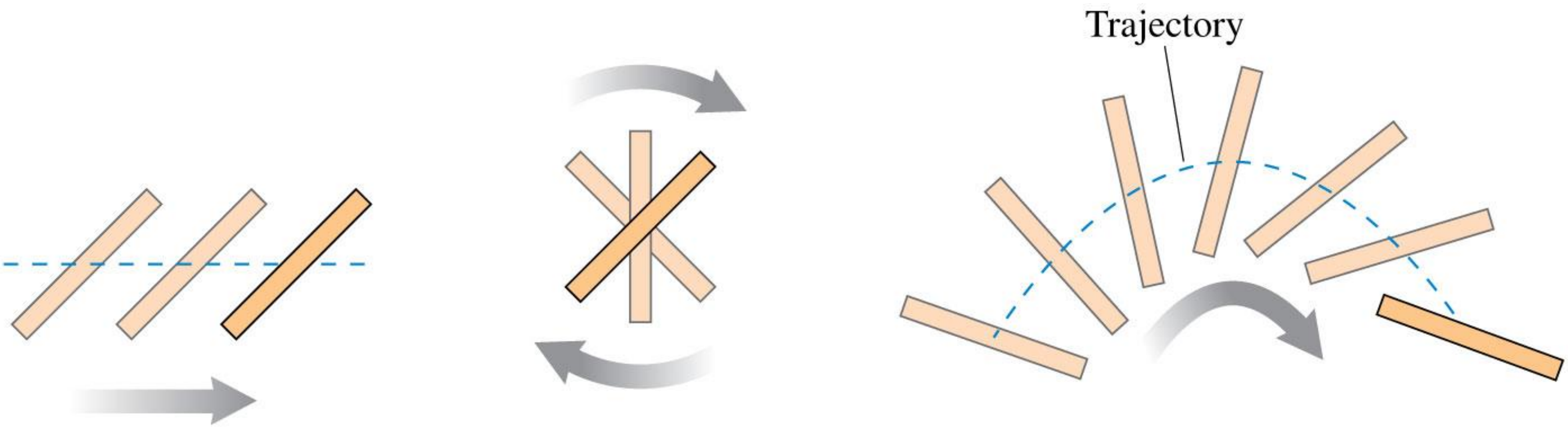
Section 7.2 The Rotation of a Rigid Body

The Rotation of a Rigid Body

- A **rigid body** is an extended object whose size and shape do not change as it moves.
- The **rigid-body model** is a good approximation for many real objects.



The Rotation of a Rigid Body



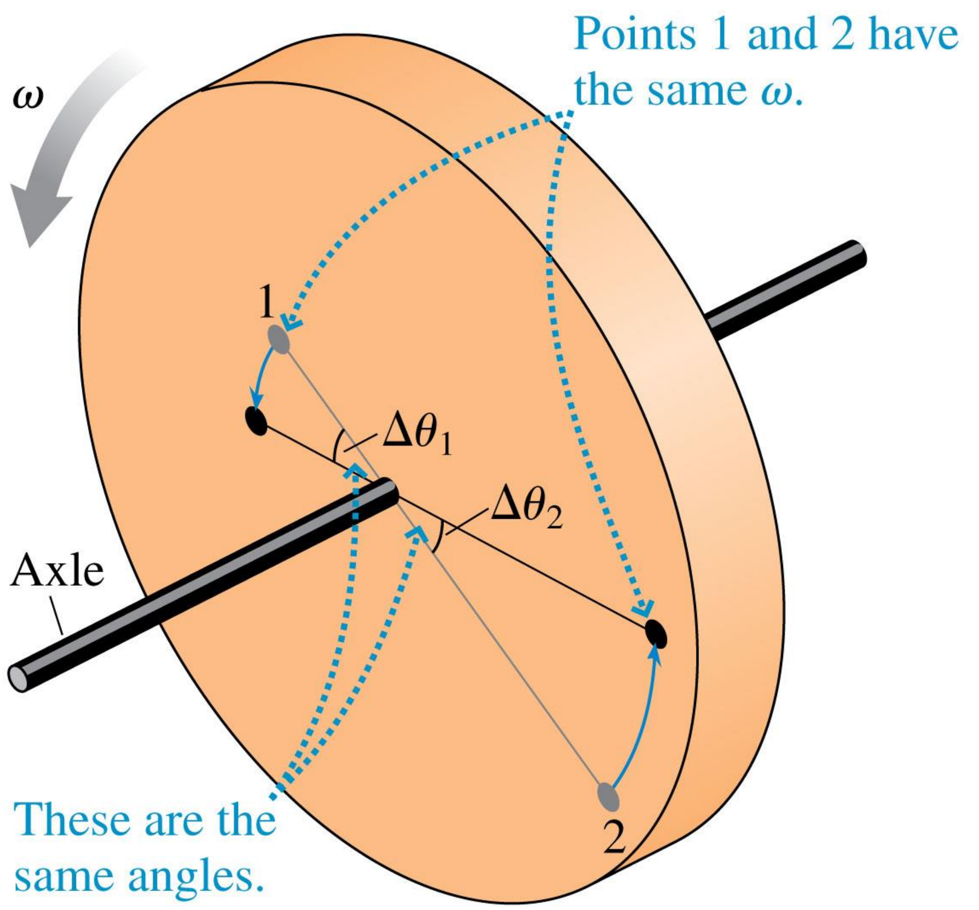
Translational motion: The object as a whole moves along a trajectory but does not rotate.

Rotational motion: The object rotates about a fixed point. Every point on the object moves in a circle.

Combination motion: An object rotates as it moves along a trajectory.

Rotational Motion of a Rigid Body

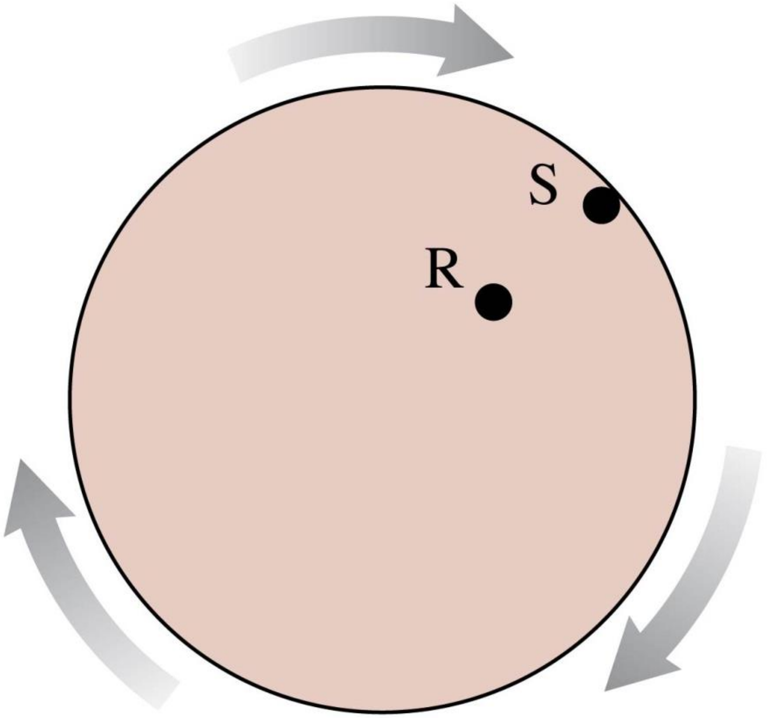
- **Every point on a rotating body has the same angular velocity.**
- Two points on the object at different distances from the axis of rotation will have different *speeds*.



QuickCheck 7.2

Rasheed and Sofia are riding a merry-go-round that is spinning steadily. Sofia is twice as far from the axis as is Rasheed. Sofia's angular velocity is _______ that of Rasheed.

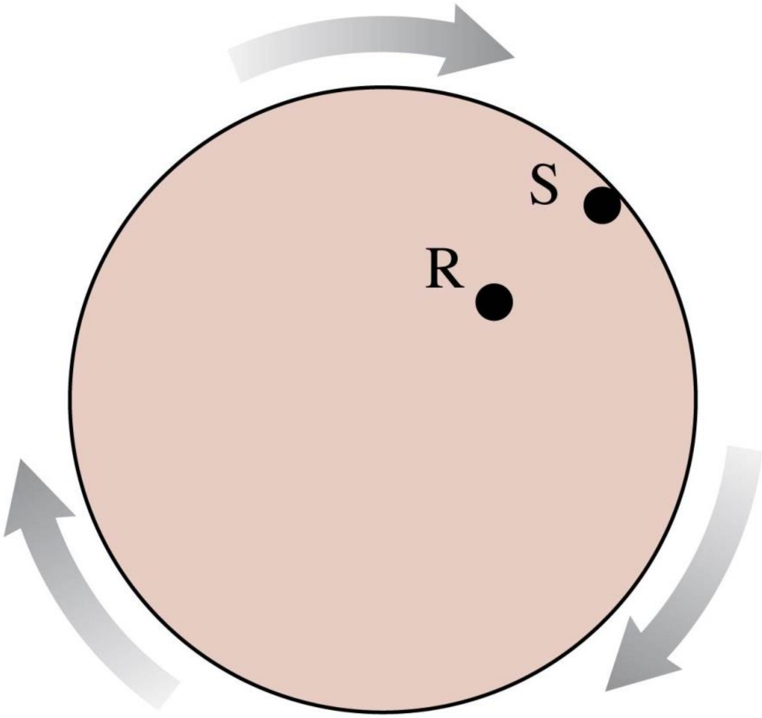
A. Half
B. The same as
C. Twice
D. Four times
E. We can't say without knowing their radii.



QuickCheck 7.2

Rasheed and Sofia are riding a merry-go-round that is spinning steadily. Sofia is twice as far from the axis as is Rasheed. Sofia's angular velocity is _______ that of Rasheed.

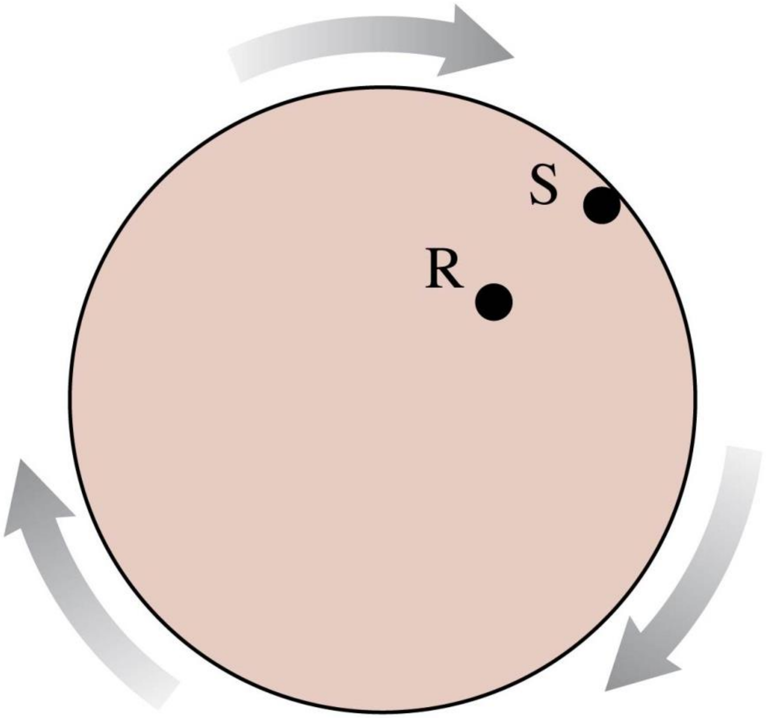
A. Half
✔ B. The same as
C. Twice
D. Four times
E. We can't say without knowing their radii.



QuickCheck 7.3

Rasheed and Sofia are riding a merry-go-round that is spinning steadily. Sofia is twice as far from the axis as is Rasheed. Sofia's speed is _______ that of Rasheed.

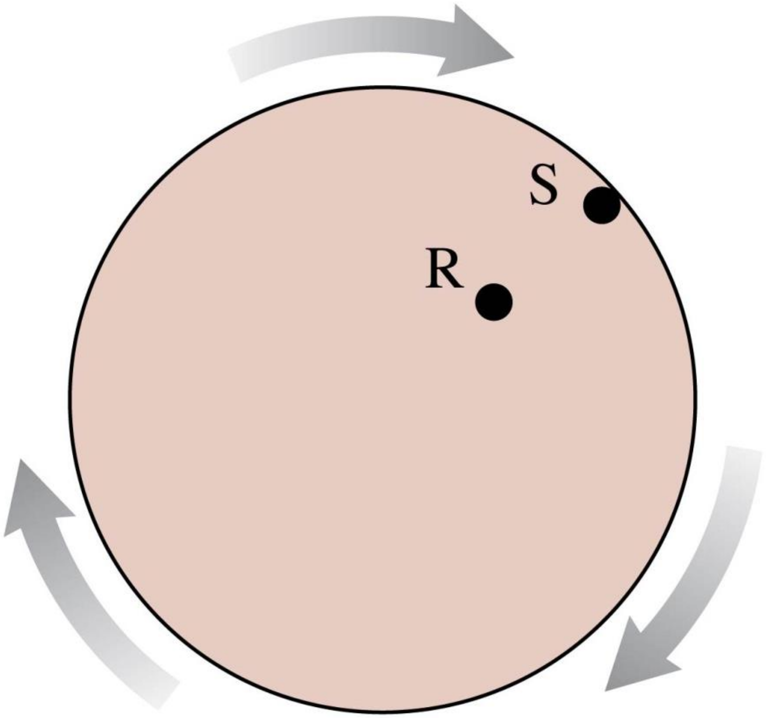
A. Half
B. The same as
C. Twice
D. Four times
E. We can't say without knowing their radii.



QuickCheck 7.3

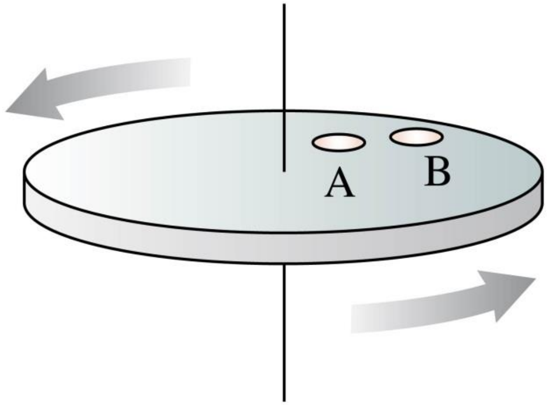
Rasheed and Sofia are riding a merry-go-round that is spinning steadily. Sofia is twice as far from the axis as is Rasheed. Sofia's speed is _______ that of Rasheed.

A. Half
B. The same as
C. Twice $v = \omega r$
D. Four times
E. We can't say without knowing their radii.



QuickCheck 7.4

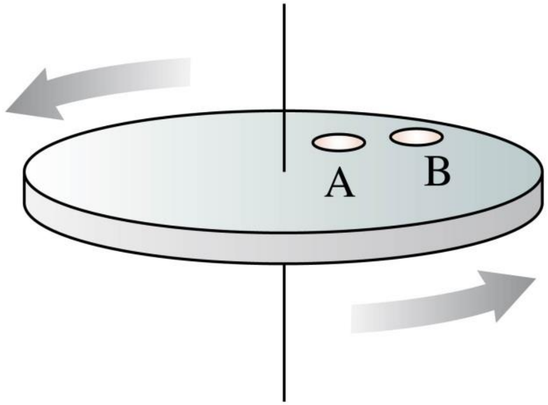
Two coins rotate on a turntable. Coin B is twice as far from the axis as coin A.



A. The angular velocity of A is twice that of B
B. The angular velocity of A equals that of B
C. The angular velocity of A is half that of B

QuickCheck 7.4

Two coins rotate on a turntable. Coin B is twice as far from the axis as coin A.



A. The angular velocity of A is twice that of B

✔ B. The angular velocity of A equals that of B

C. The angular velocity of A is half that of B

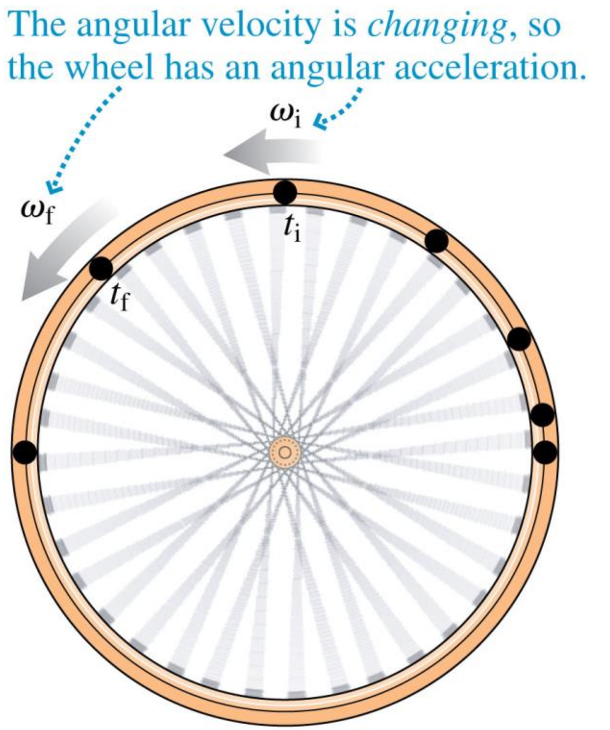
Angular Acceleration

- **Angular acceleration** is defined as:

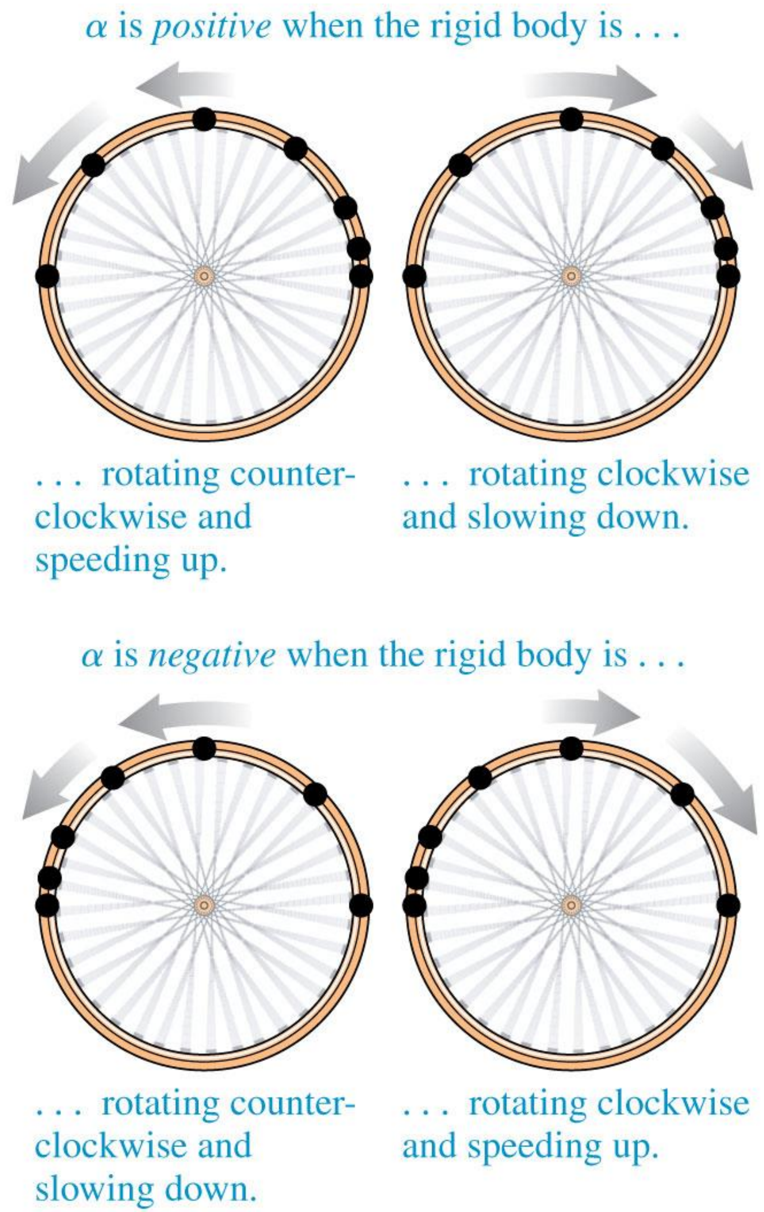
$$\alpha = \frac{\text{change in angular velocity}}{\text{time interval}} = \frac{\Delta\omega}{\Delta t}$$

Angular acceleration for a particle in nonuniform circular motion

- The units of angular acceleration are rad/s^2.

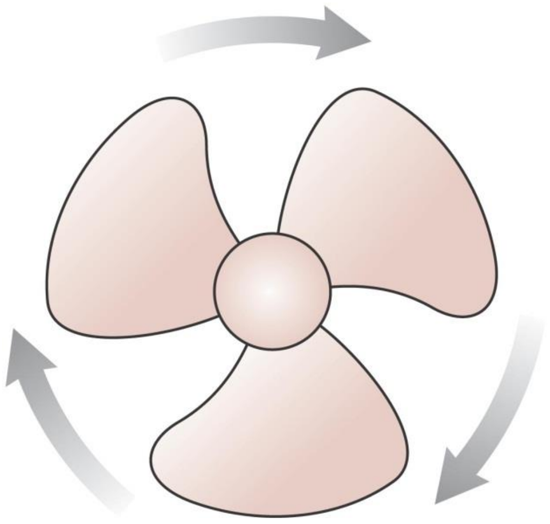


Angular Acceleration



QuickCheck 7.5

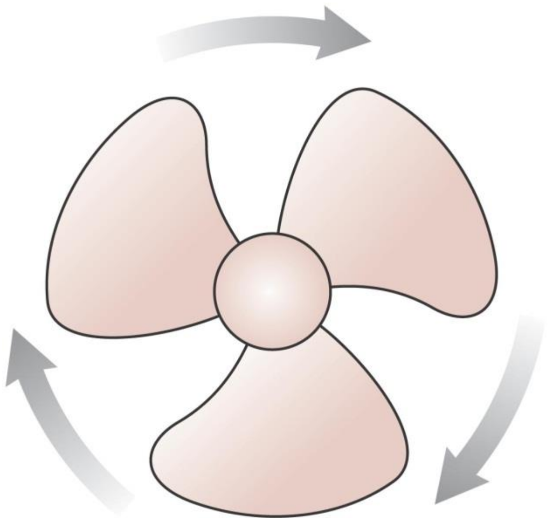
The fan blade is slowing down. What are the signs of ω and α?



A. ω is positive and α is positive.
B. ω is positive and α is negative.
C. ω is negative and α is positive.
D. ω is negative and α is negative.
E. ω is positive and α is zero.

QuickCheck 7.5

The fan blade is slowing down. What are the signs of ω and α?



A. ω is positive and α is positive.

B. ω is positive and α is negative.

✔ C. ω is negative and α is positive.

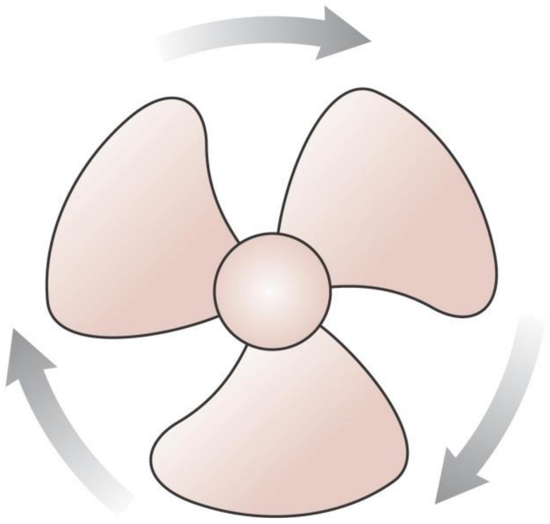
D. ω is negative and α is negative.

E. ω is positive and α is zero.

"Slowing down" means that ω and α have opposite signs, not that α is negative.

QuickCheck 7.6

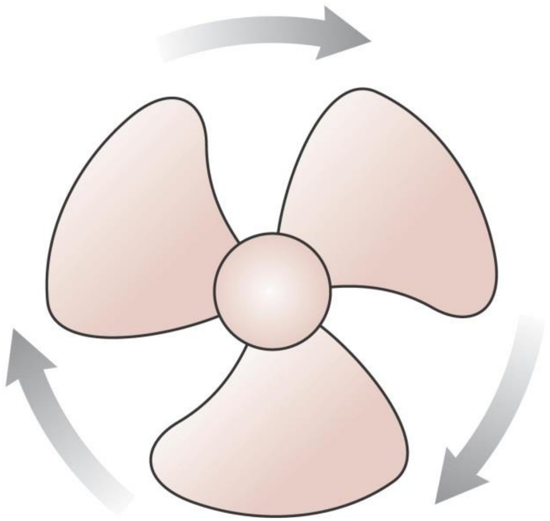
The fan blade is speeding up. What are the signs of ω and α?



A. ω is positive and α is positive.
B. ω is positive and α is negative.
C. ω is negative and α is positive.
D. ω is negative and α is negative.

QuickCheck 7.6

The fan blade is speeding up. What are the signs of ω and α?



A. ω is positive and α is positive.

B. ω is positive and α is negative.

C. ω is negative and α is positive.

✔ D. ω is negative and α is negative.

Example Problem

A high-speed drill rotating counterclockwise takes 2.5 s to speed up to 2400 rpm.

A. What is the drill's angular acceleration?
B. How many revolutions does it make as it reaches top speed?

Linear and Circular Motion

SYNTHESIS 7.1 **Linear and circular motion**

The variables and equations for linear motion have analogs for circular motion.

		Linear motion	Circular motion	
Variables	Position (m)	x	θ	Angle (rad)
	Velocity (m/s)	$v_x = \dfrac{\Delta x}{\Delta t}$	$\omega = \dfrac{\Delta \theta}{\Delta t}$	Angular velocity (rad/s)
	Acceleration (m/s^2)	$a_x = \dfrac{\Delta v_x}{\Delta t}$	$\alpha = \dfrac{\Delta \omega}{\Delta t}$	Angular acceleration (rad/s^2)
Equations	Constant velocity	$\Delta x = v\,\Delta t$	$\Delta \theta = \omega\,\Delta t$	Constant angular velocity
	Constant acceleration	$\Delta v = a\,\Delta t$	$\Delta \omega = \alpha\,\Delta t$	Constant angular acceleration
		$\Delta x = v\,\Delta t + \frac{1}{2}a(\Delta t)^2$	$\Delta \theta = \omega_i\,\Delta t + \frac{1}{2}\alpha(\Delta t)^2$	

Text: p. 196

QuickCheck 7.8

Starting from rest, a wheel with constant angular acceleration spins up to 25 rpm in a time t. What will its angular velocity be after time $2t$?

A. 25 rpm
B. 50 rpm
C. 75 rpm
D. 100 rpm
E. 200 rpm

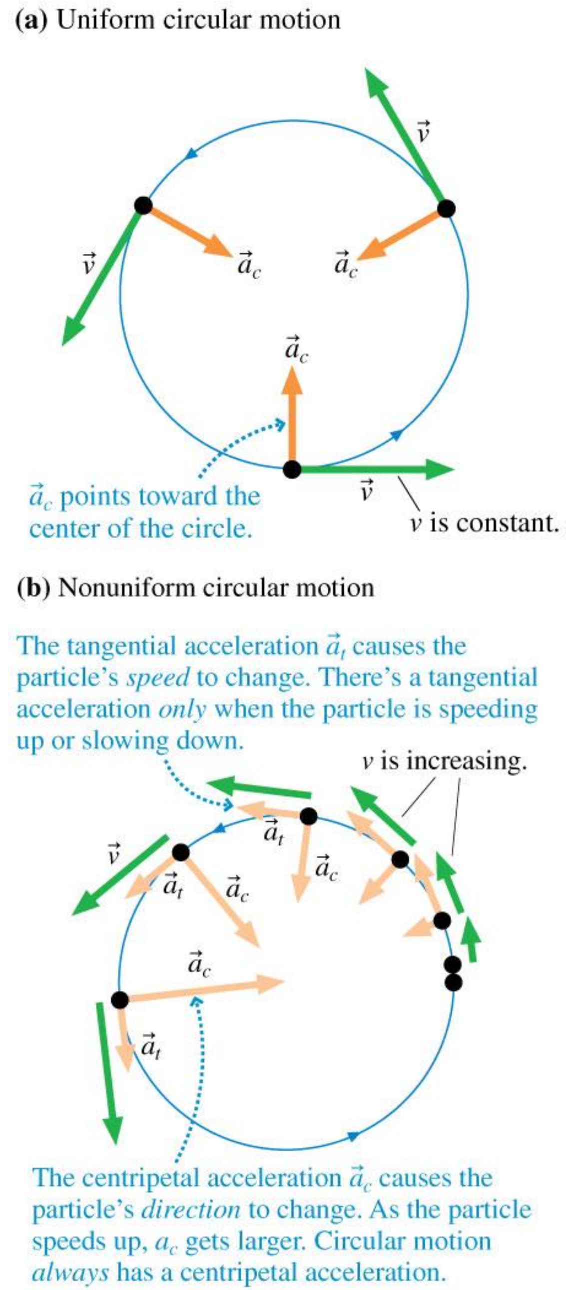
QuickCheck 7.8

Starting from rest, a wheel with constant angular acceleration spins up to 25 rpm in a time *t*. What will its angular velocity be after time 2*t*?

A. 25 rpm
B. 50 rpm $\Delta\omega \propto \Delta t$
C. 75 rpm
D. 100 rpm
E. 200 rpm

Tangential Acceleration

- **Tangential acceleration** is the component of acceleration directed tangentially to the circle.
- The tangential acceleration measures the rate at which the particle's speed around the circle increases.



Tangential Acceleration

- We can relate tangential acceleration to the angular acceleration by $v = \omega r$.

$$a_t = \frac{\Delta v}{\Delta t} = \frac{\Delta(\omega r)}{\Delta t} = \frac{\Delta \omega}{\Delta t} r$$

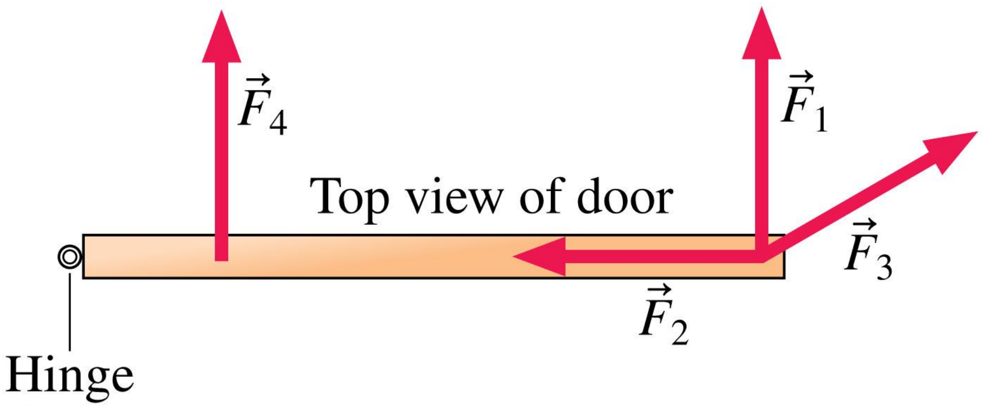
$$a_t = \alpha r$$

Relationship between tangential and angular acceleration

Section 7.3 Torque

Torque

- Forces with equal strength will have different effects on a swinging door.
- The ability of a force to cause rotation depends on
  - The magnitude F of the force.
  - The distance r from the pivot—the axis about which the object can rotate—to the point at which force is applied.
  - The angle at which force is applied.



Torque

- **Torque (τ) is the rotational equivalent of force.**

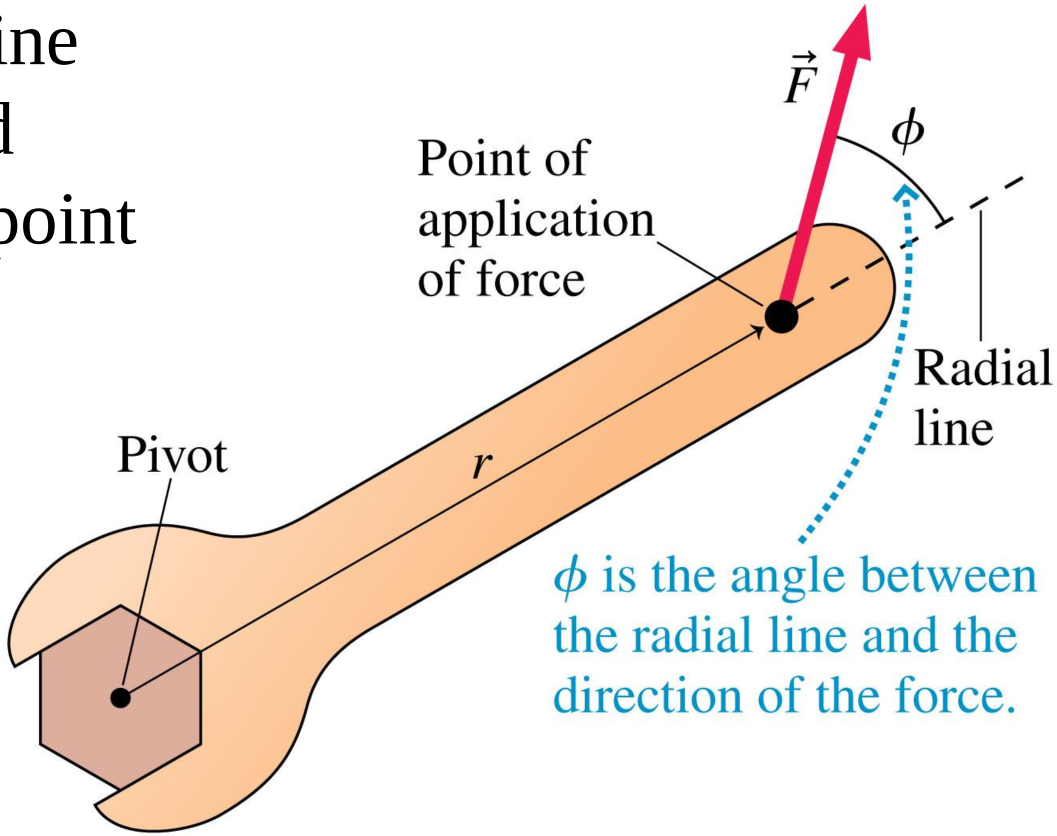
$$\tau = rF_{\perp}$$

Torque due to a force with perpendicular component $F_{\perp}$ acting at a distance r from the pivot

- Torque units are newton-meters, abbreviated N · m.

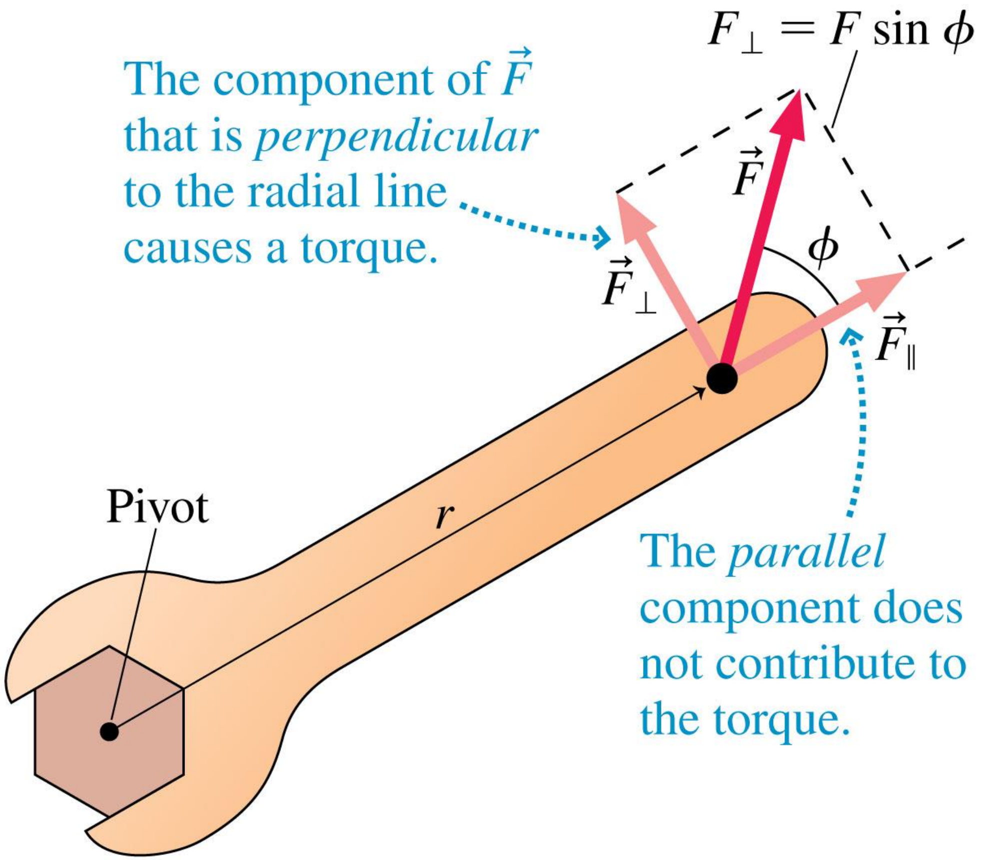
Torque

- The **radial line** is the line starting at the pivot and extending through the point where force is applied.
- The angle ϕ is measured from the radial line to the direction of the force.



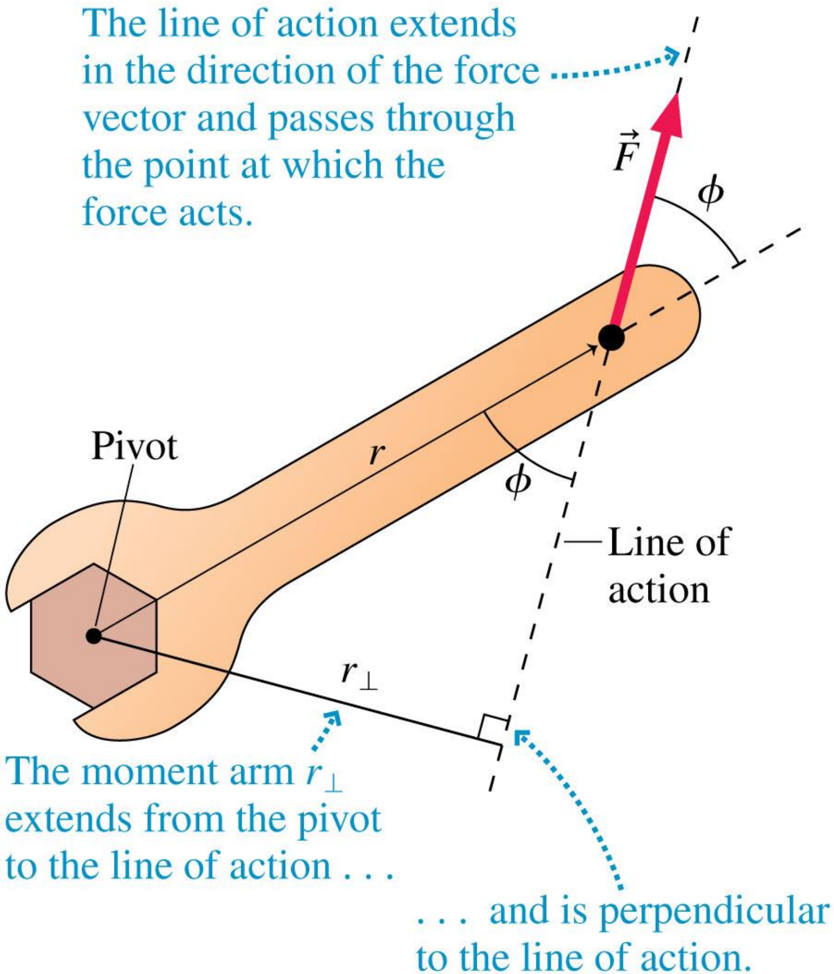
Torque

- The **radial line** is the line starting at the pivot and extending through the point where force is applied.
- The angle ϕ is measured from the radial line to the direction of the force.
- Torque is dependent on the perpendicular component of the force being applied.



Torque

- An alternate way to calculate torque is in terms of the moment arm.
- The **moment arm** (or lever arm) is the perpendicular distance from the *line of action* to the pivot.
- The *line of action* is the line that is in the direction of the force and passes through the point at which the force acts.



Torque

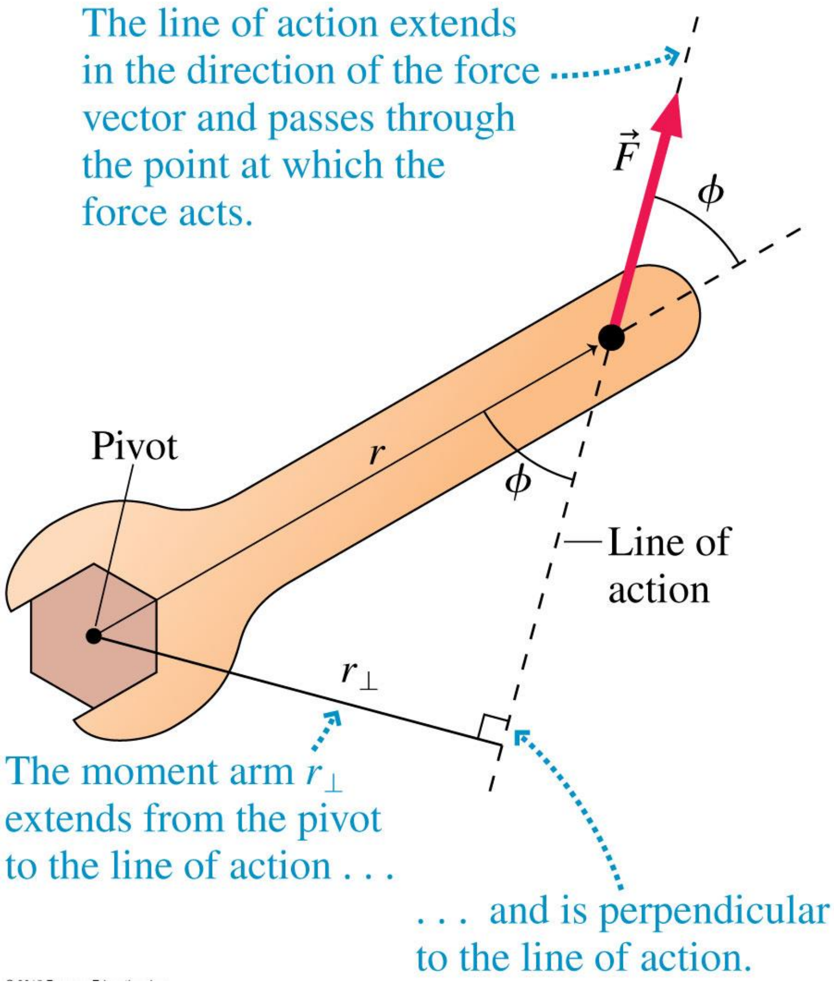
- The equivalent expression for torque is

$$\tau = r_{\perp} F$$

Torque due to a force F with moment arm $r_{\perp}$

- For both methods for calculating torque, the resulting expression is the same:

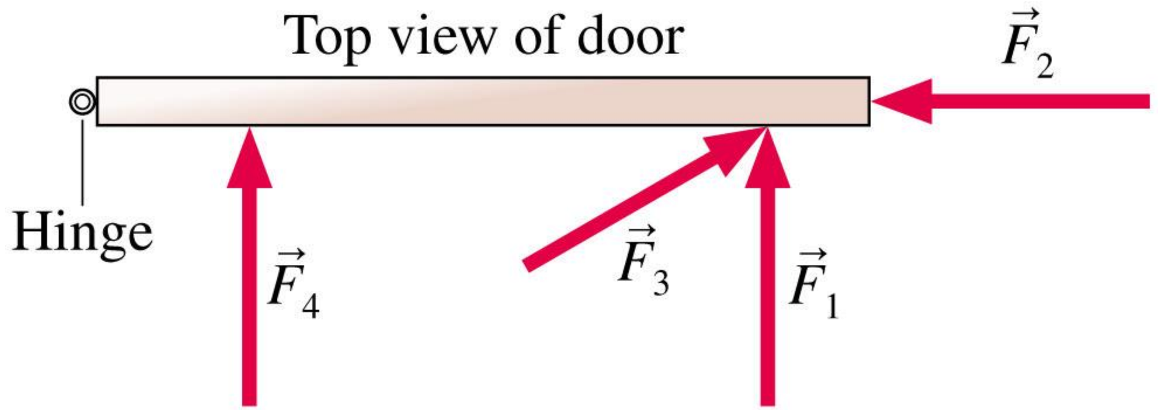
$$\tau = rF\sin\phi$$



QuickCheck 7.10

The four forces shown have the same strength. Which force would be most effective in opening the door?

A. Force F_1
B. Force F_2
C. Force F_3
D. Force F_4
E. Either F_1 or F_3



QuickCheck 7.10

The four forces shown have the same strength. Which force would be most effective in opening the door?

✔ A. Force F_1
B. Force F_2
C. Force F_3
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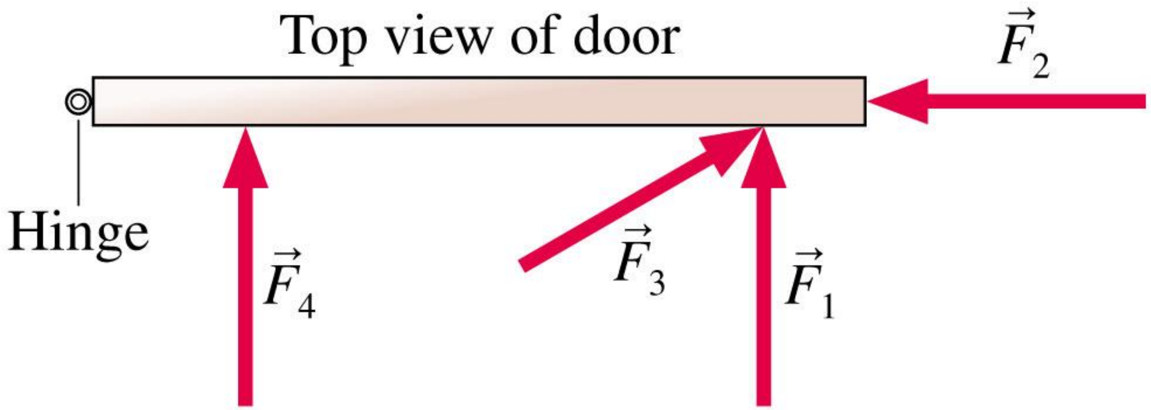
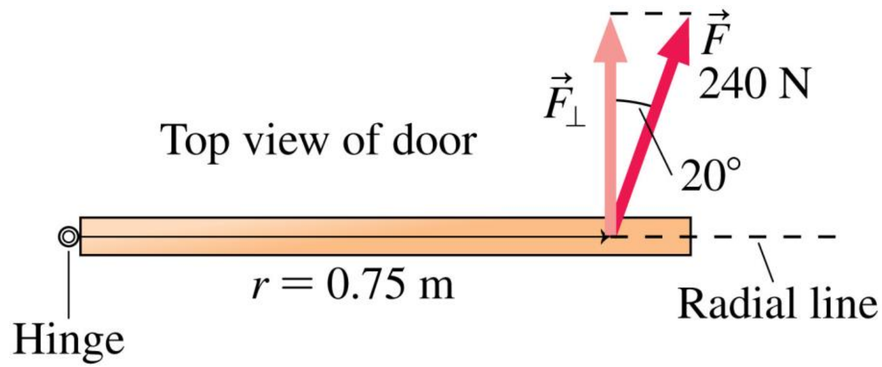


Your intuition likely led you to choose F_1. The reason is that F_1 exerts the largest *torque* about the hinge.

Example 7.9 Torque in opening a door

Ryan is trying to open a stuck door. He pushes it at a point 0.75 m from the hinges with a 240 N force directed 20° away from being perpendicular to the door. There's a natural pivot point, the hinges. What torque does Ryan exert? How could he exert more torque?

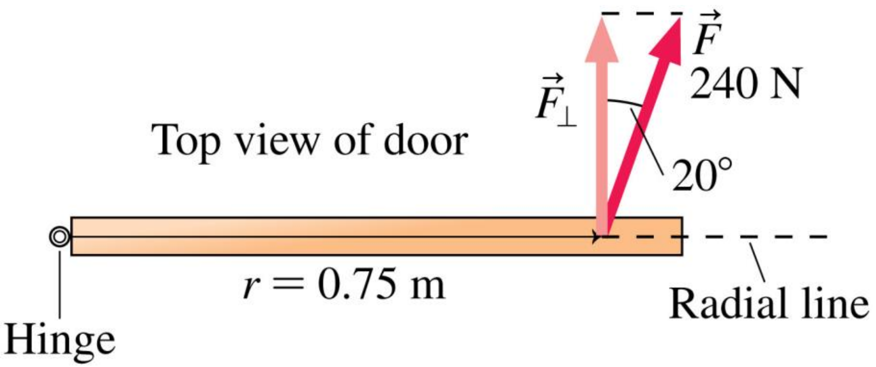


PREPARE In FIGURE 7.20 the radial line is shown drawn from the pivot—the hinge—through the point at which the force $\vec{F}$ is applied. We see that the component of $\vec{F}$ that is perpendicular to the radial line is $F_\perp$ = F cos 20° = 226 N. The distance from the hinge to the point at which the force is applied is r = 0.75 m.

Example 7.9 Torque in opening a door (cont.)

SOLVE We can find the torque on the door from Equation 7.10:

$$\tau = rF_{\perp} = (0.75 \text{ m})(226 \text{ N})$$
$$= 170 \text{ N} \cdot \text{m}$$



The torque depends on how hard Ryan pushes, where he pushes, and at what angle. If he wants to exert more torque, he could push at a point a bit farther out from the hinge, or he could push exactly perpendicular to the door. Or he could simply push harder!

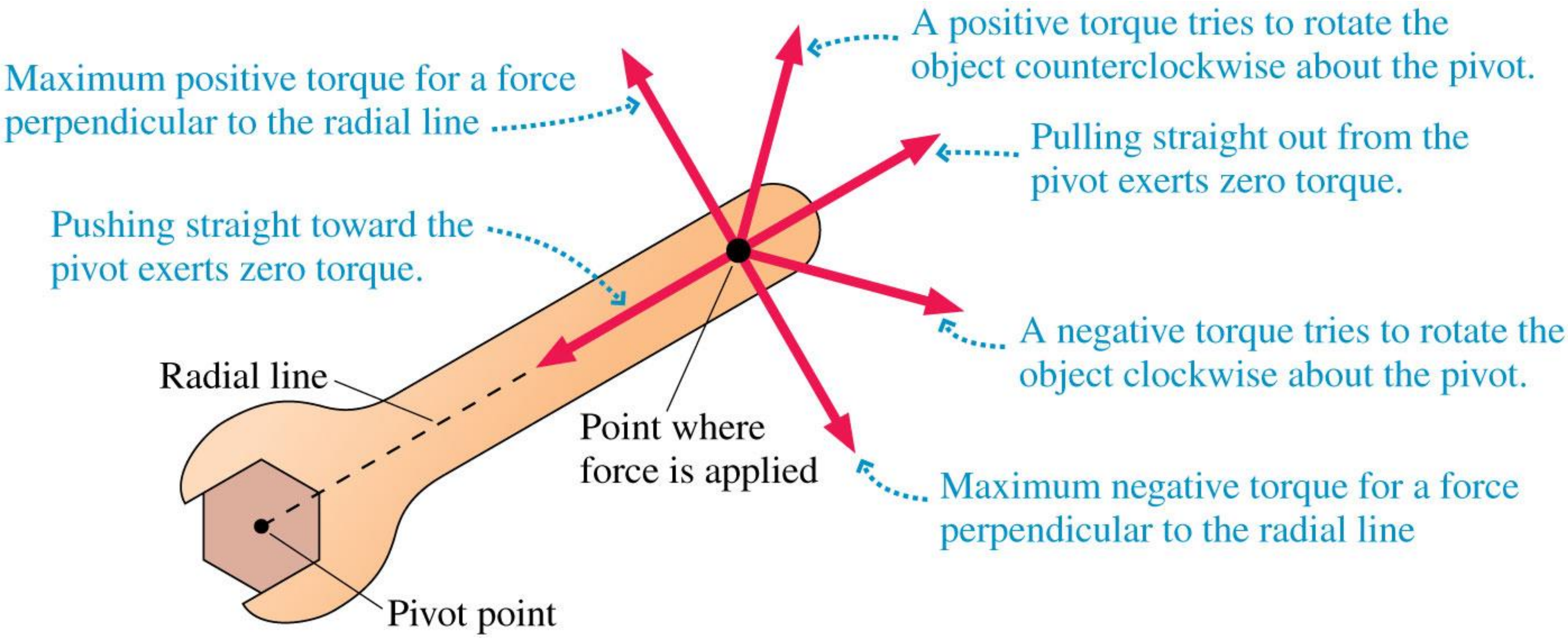
ASSESS As you'll see by doing more problems, 170 N · m is a significant torque, but this makes sense if you are trying to free a stuck door.

Example Problem

Revolutionaries attempt to pull down a statue of the Great Leader by pulling on a rope tied to the top of its head. The statue is 17 m tall, and they pull with a force of 4200 N at an angle of 65° to the horizontal. What is the torque they exert on the statue? If they are standing to the right of the statue, is the torque positive or negative?

Torque

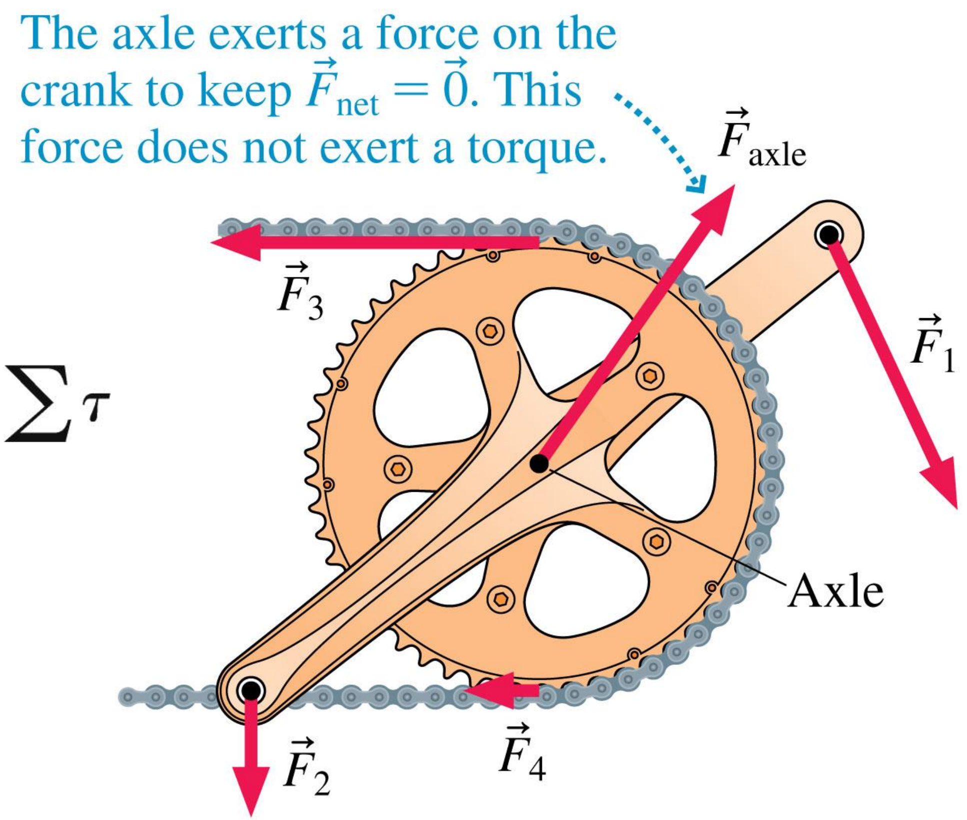
- **A torque that tends to rotate the object in a counter-clockwise direction is positive, while a torque that tends to rotate the object in a clockwise direction is negative.**



Net Torque

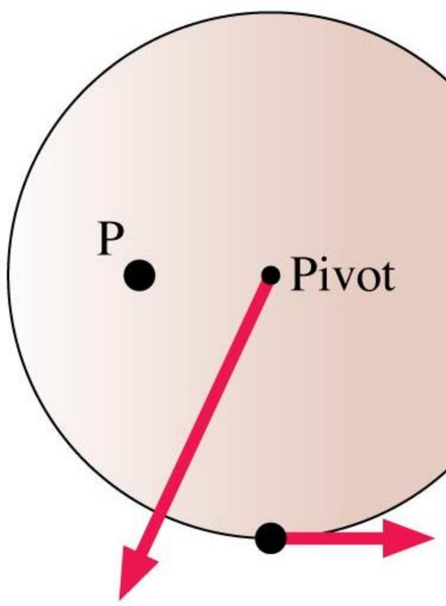
- The *net* torque is the sum of the torques due to the *applied* forces:

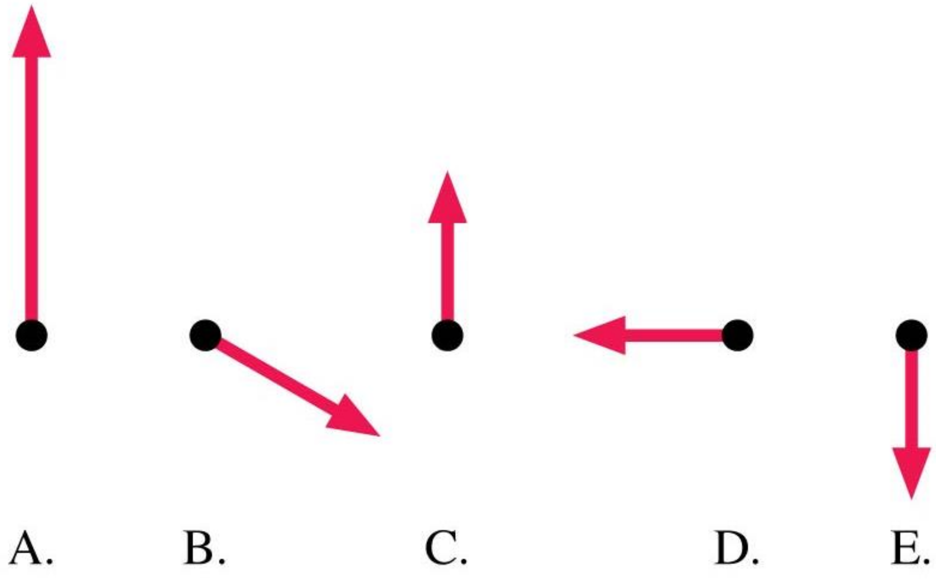
$$\tau_{\text{net}} = \tau_1 + \tau_2 + \tau_3 + \tau_4 + \cdots = \sum \tau$$



QuickCheck 7.11

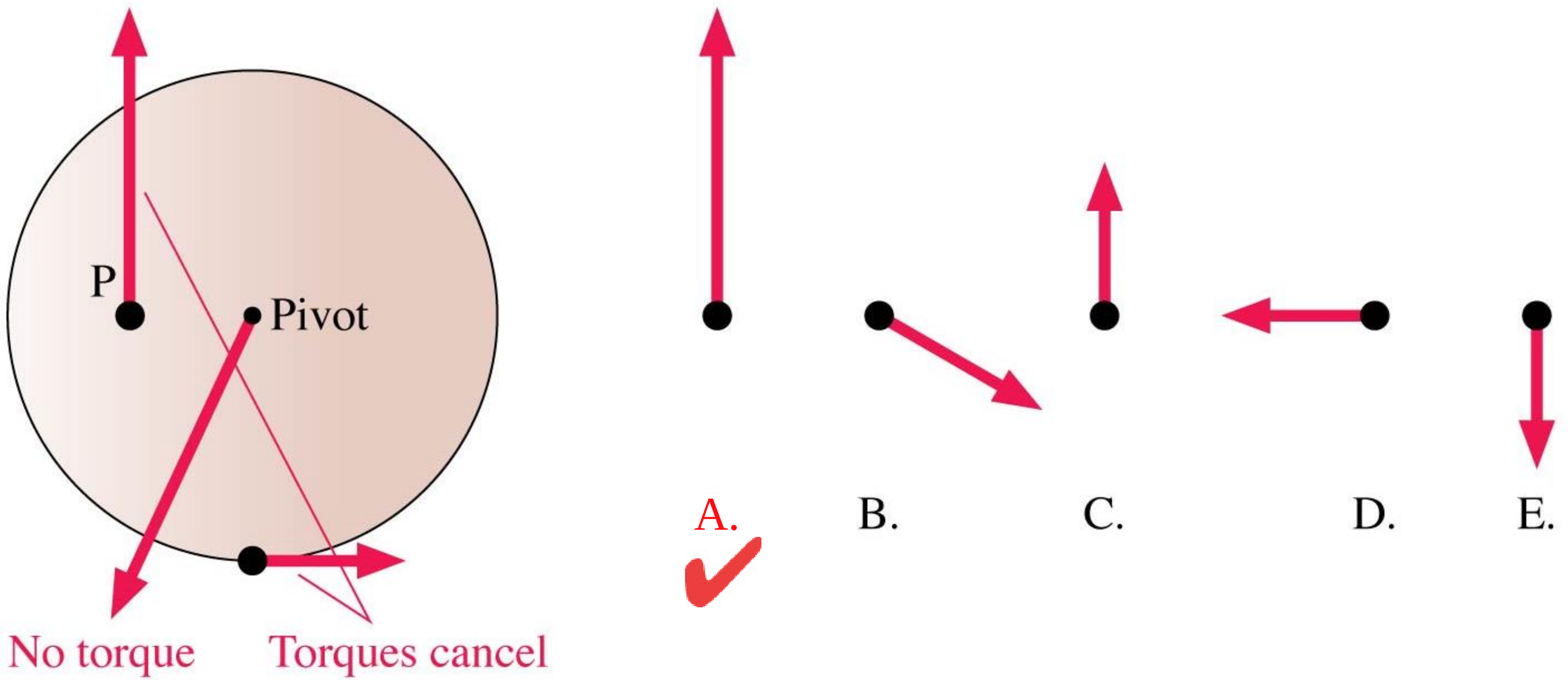
Which third force on the wheel, applied at point P, will make the net torque zero?





QuickCheck 7.11

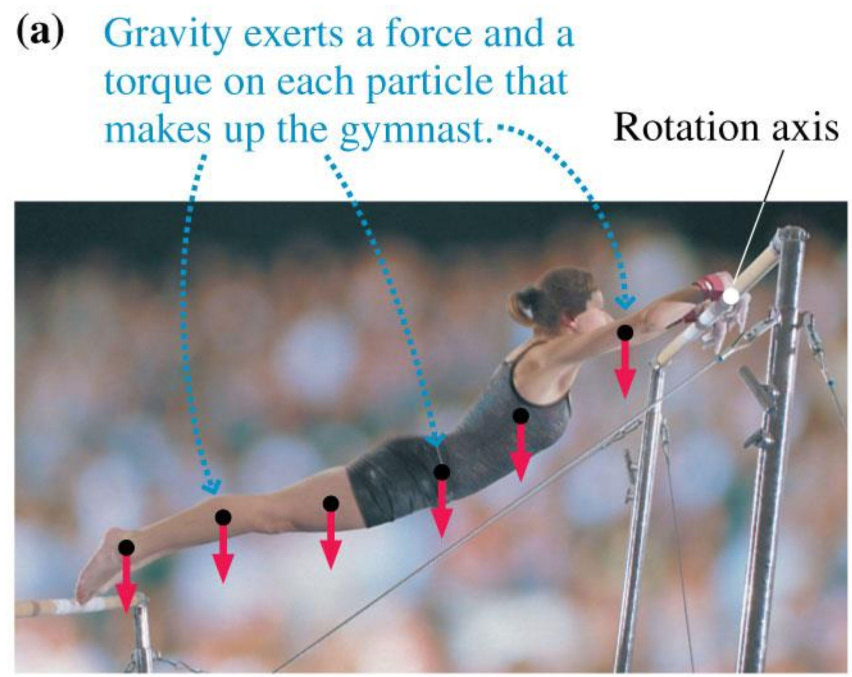
Which third force on the wheel, applied at point P, will make the net torque zero?



Section 7.4 Gravitational Torque and the Center of Gravity

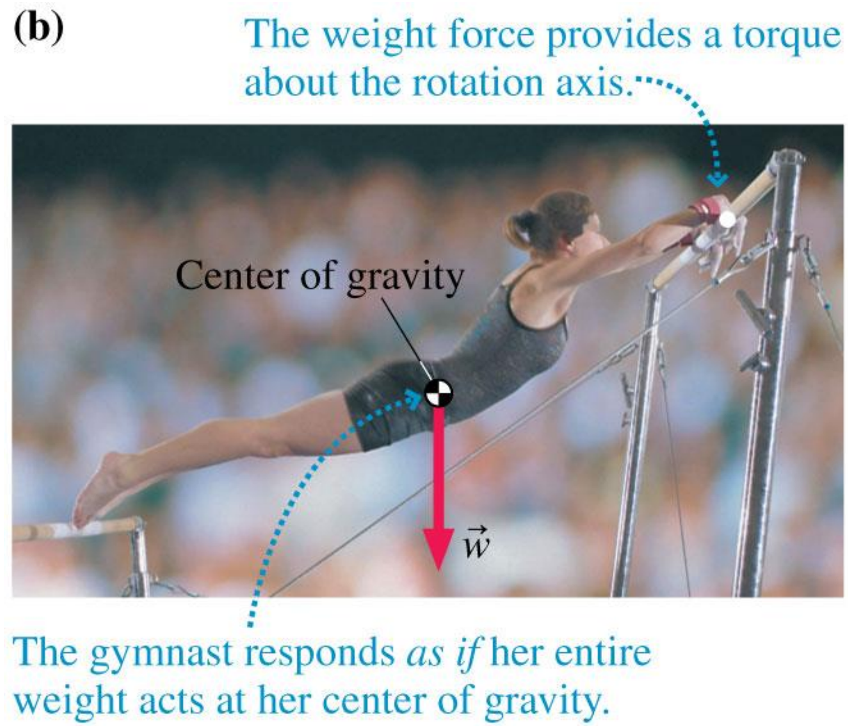
Gravitational Torque and the Center of Gravity

- Gravity pulls downward on *every particle* that makes up an object (like the gymnast).
- Each particle experiences a torque due to the force of gravity.



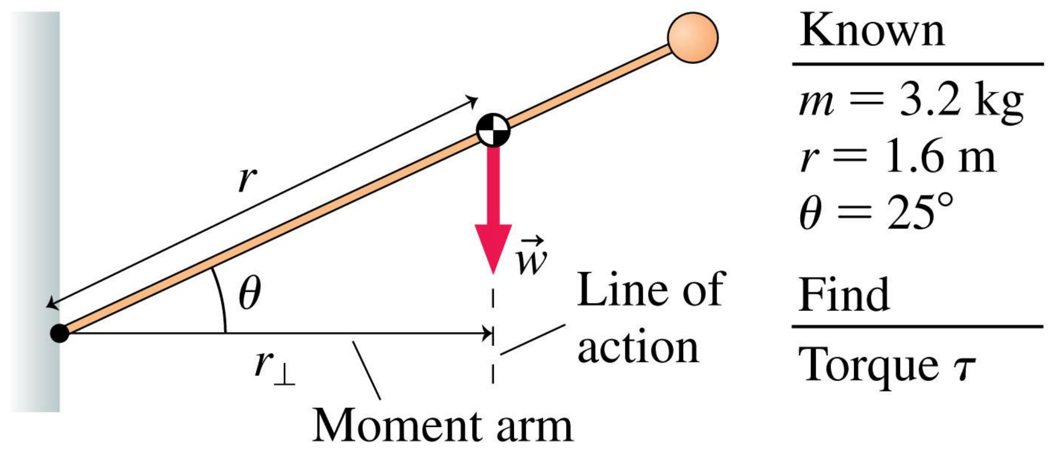
Gravitational Torque and the Center of Gravity

- The gravitational torque can be calculated by assuming that the net force of gravity (the object's weight) acts as a single point.
- That single point is called the **center of gravity**.



Example 7.12 The torque on a flagpole

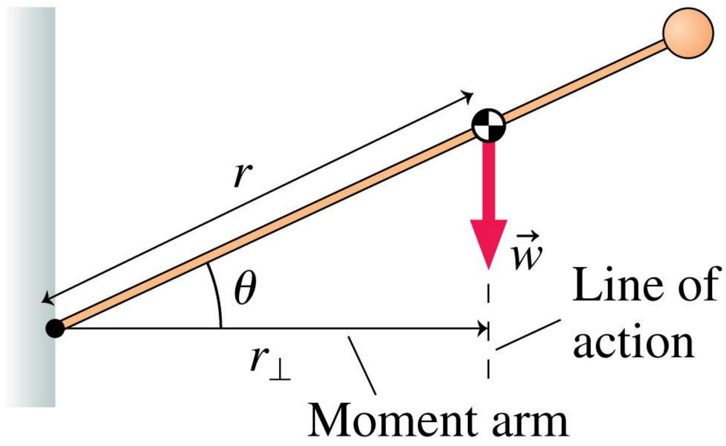
A 3.2 kg flagpole extends from a wall at an angle of 25° from the horizontal. Its center of gravity is 1.6 m from the point where the pole is attached to the wall. What is the gravitational torque on the flagpole about the point of attachment?



PREPARE FIGURE 7.26 shows the situation. For the purpose of calculating torque, we can consider the entire weight of the pole as acting at the center of gravity. Because the moment arm $r_{\perp}$ is simple to visualize here, we'll use Equation 7.11 for the torque.

Example 7.12 The torque on a flagpole (cont.)

SOLVE From Figure 7.26, we see that the moment arm is $r_\perp = (1.6\text{ m})\cos 25° = 1.45\text{ m}$. Thus the gravitational torque on the flagpole, about the point where it attaches to the wall, is



Known

$m = 3.2$ kg
$r = 1.6$ m
$\theta = 25°$

Find

Torque τ

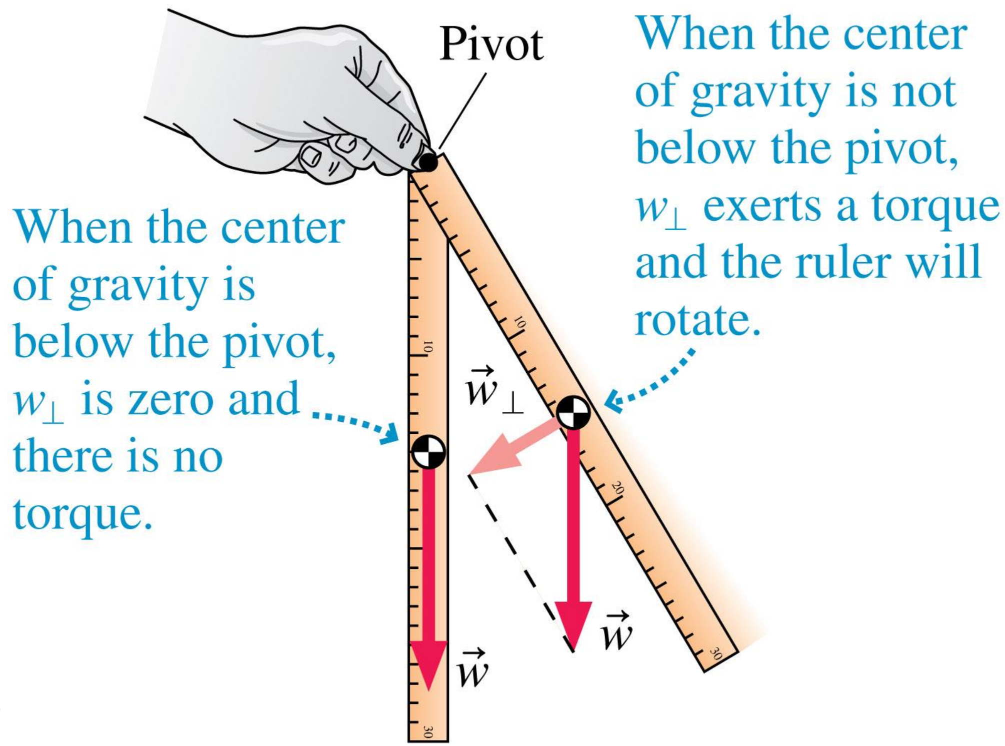
$$\tau = -r_\perp w = -r_\perp mg = -(1.45\text{ m})(3.2\text{ kg})(9.8\text{ m/s}^2) = -45\text{ N}\cdot\text{m}$$

We inserted the minus sign because the torque tries to rotate the pole in a clockwise direction.

ASSESS If the pole were attached to the wall by a hinge, the gravitational torque would cause the pole to fall. However, the actual rigid connection provides a counteracting (positive) torque to the pole that prevents this. The net torque is zero.

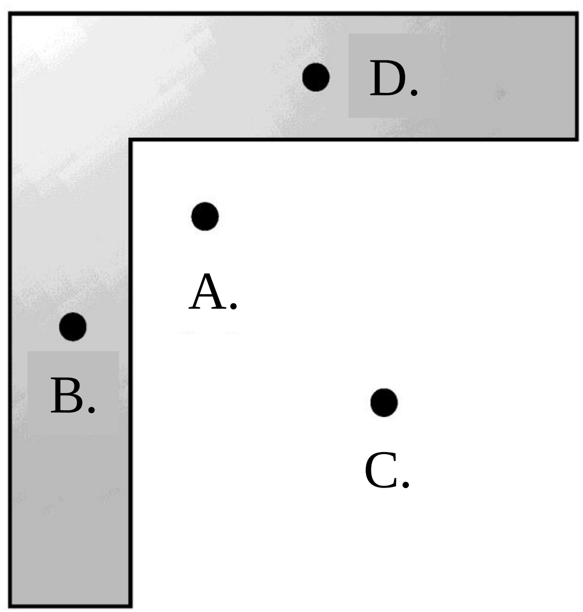
Gravitational Torque and the Center of Gravity

- **An object that is free to rotate about a pivot will come to rest with the center of gravity below the pivot point.**
- If you hold a ruler by one end and allow it to rotate, it will stop rotating when the center of gravity is directly above or below the pivot point. There is no torque acting at these positions.



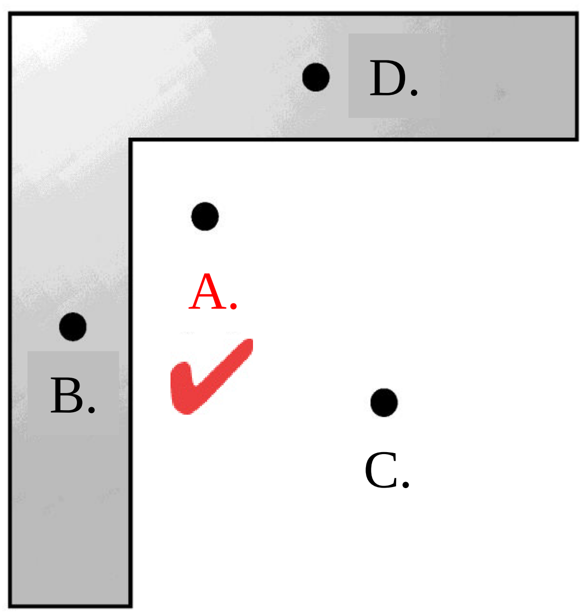
QuickCheck 7.12

Which point could be the center of gravity of this L-shaped piece?



QuickCheck 7.12

Which point could be the center of gravity of this L-shaped piece?

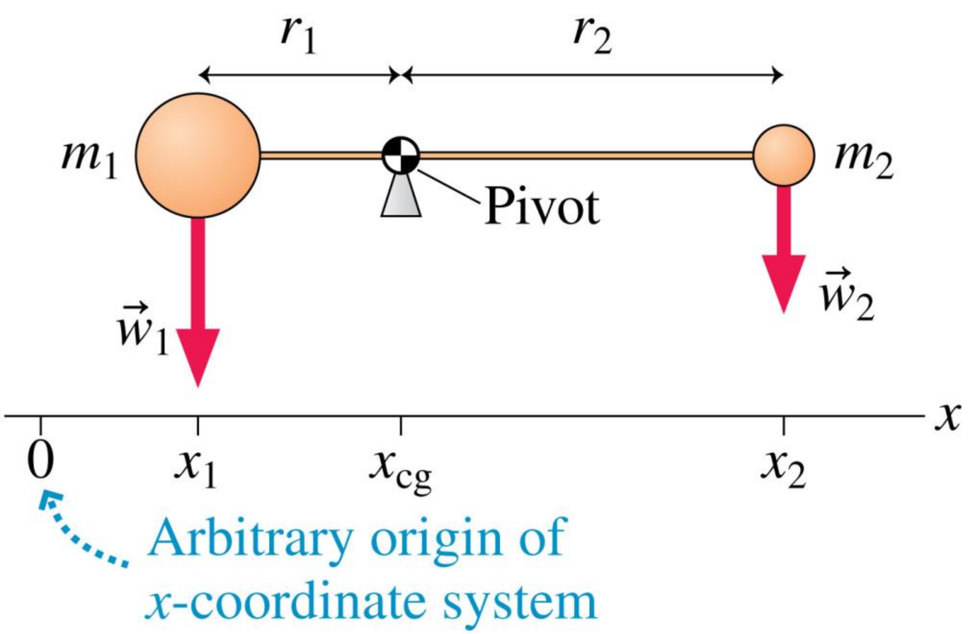


Calculating the Position of the Center of Gravity

- **The torque due to gravity when the pivot is *at* the center of gravity is zero.**
- We can use this to find an expression for the position of the center of gravity.

Calculating the Position of the Center of Gravity

- For the dumbbell to balance, the pivot must be at the center of gravity.
- We calculate the torque on either side of the pivot, which is located at the position x_{cg}.



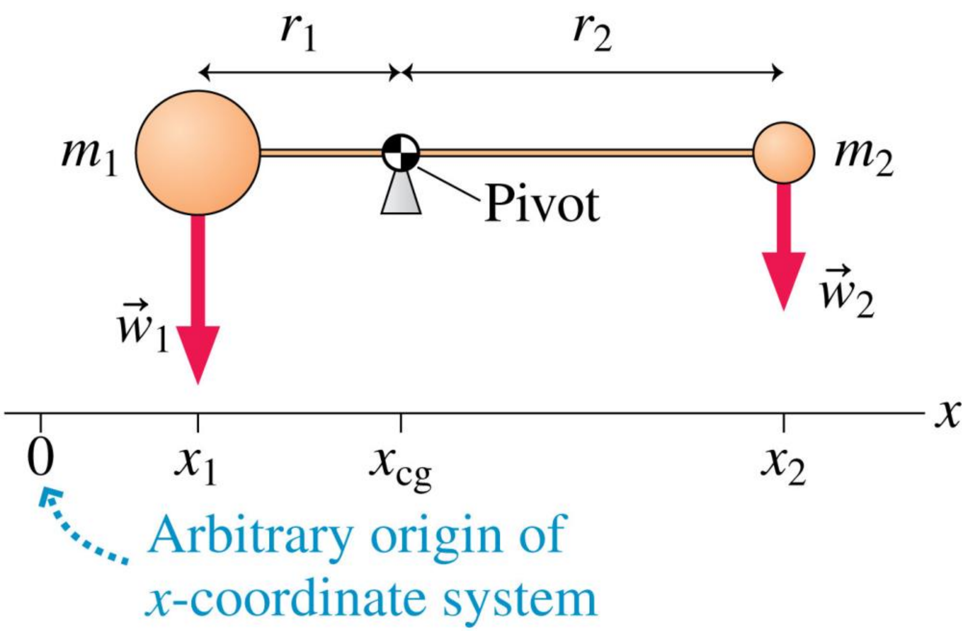
Calculating the Position of the Center of Gravity

- The torque due to the weight on the left side of the pivot is

$$\tau_1 = r_1 w_1 = (x_{cg} - x_1)m_1 g$$

- The torque due to the weight on the right side of the pivot is

$$\tau_2 = -r_2 w_2 = -(x_2 - x_{cg})m_2 g$$



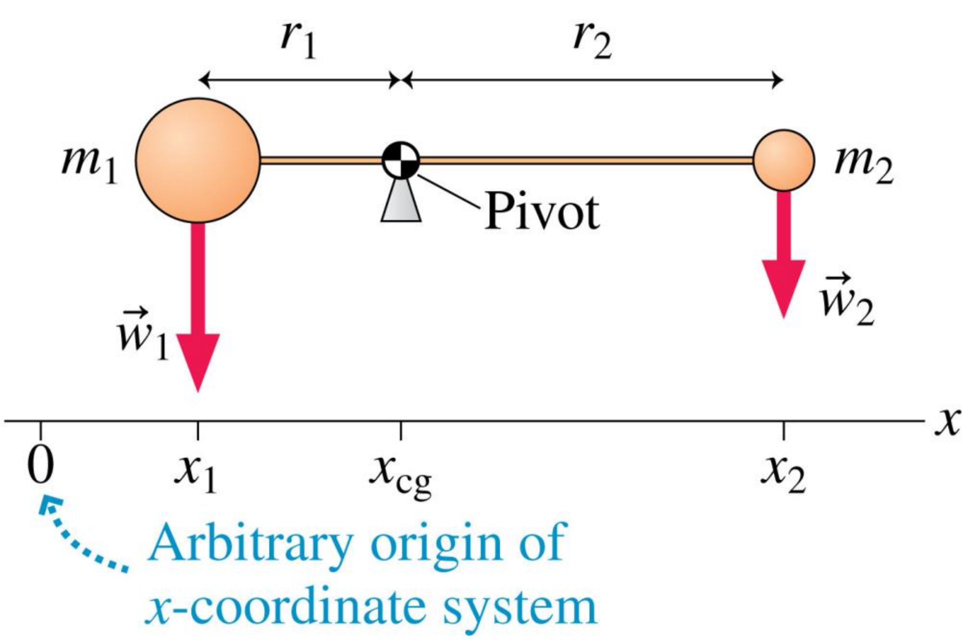
Calculating the Position of the Center of Gravity

- The total torque is

$$\tau_{\text{net}} = 0 = \tau_1 + \tau_2 = (x_{\text{cg}} - x_1)m_1 g - (x_2 - x_{\text{cg}})m_2 g$$

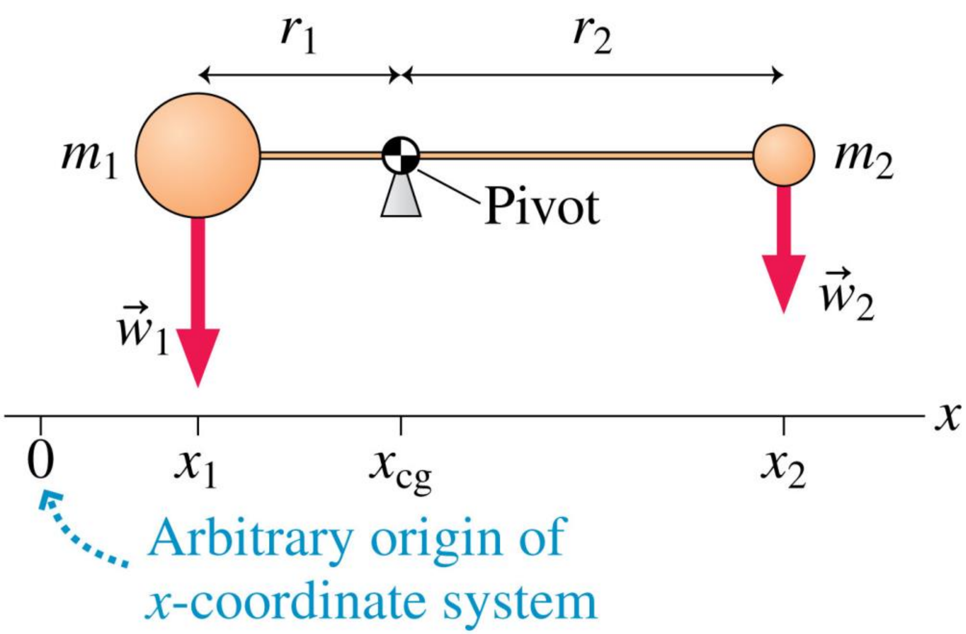
- The location of the center of gravity is

$$x_{\text{cg}} = \frac{x_1 m_1 + x_2 m_2}{m_1 + m_2}$$



Calculating the Position of the Center of Gravity

- Because the center of gravity depends on distance and mass from the pivot point, objects with large masses count more heavily.
- **The center of gravity tends to lie closer to the heavier objects or particles that make up the object.**



Calculating the Position of the Center of Gravity

TACTICS BOX 7.1 **Finding the center of gravity**

1. Choose an origin for your coordinate system. You can choose any convenient point as the origin.
2. Determine the coordinates $(x_1, y_1), (x_2, y_2), (x_3, y_3), \ldots$ for the particles of masses $m_1, m_2, m_3, \ldots$, respectively.
3. The x-coordinate of the center of gravity is

$$x_{cg} = \frac{x_1 m_1 + x_2 m_2 + x_3 m_3 + \cdots}{m_1 + m_2 + m_3 + \cdots} \quad (7.15)$$

4. Similarly, the y-coordinate of the center of gravity is

$$y_{cg} = \frac{y_1 m_1 + y_2 m_2 + y_3 m_3 + \cdots}{m_1 + m_2 + m_3 + \cdots} \quad (7.16)$$

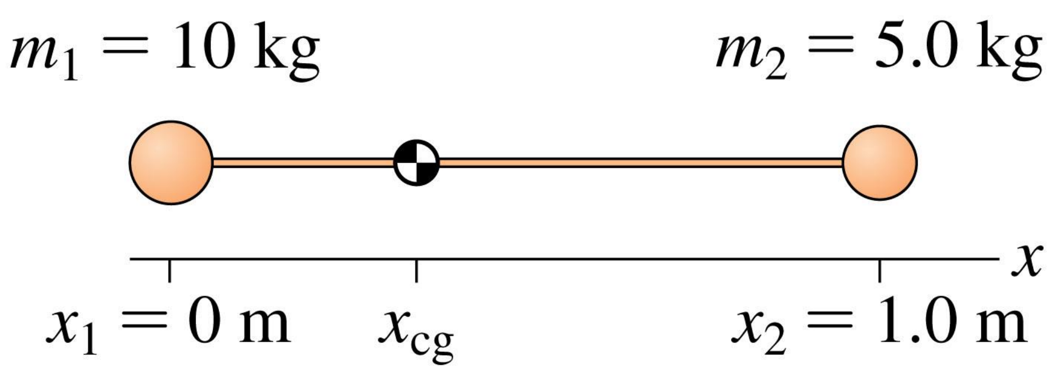
Exercises 18–21

Text: p. 204

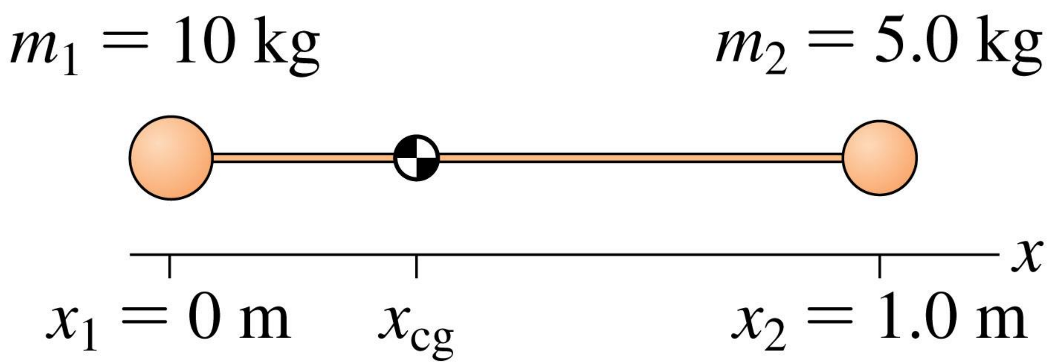
Example 7.13 Where should the dumbbell be lifted?

A 1.0-m-long dumbbell has a 10 kg mass on the left and a 5.0 kg mass on the right. Find the position of the center of gravity, the point where the dumbbell should be lifted in order to remain balanced.

PREPARE First we sketch the situation as in FIGURE 7.30.

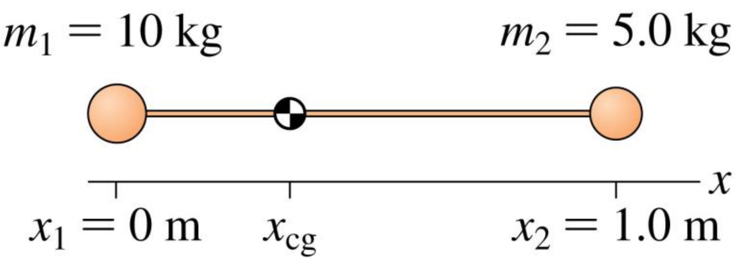


Example 7.13 Where should the dumbbell be lifted? (cont.)



Next, we can use the steps from Tactics Box 7.1 to find the center of gravity. Let's choose the origin to be at the position of the 10 kg mass on the left, making $x_1 = 0$ m and $x^2 = 1.0$ m. Because the dumbbell masses lie on the x-axis, the y-coordinate of the center of gravity must also lie on the x-axis. Thus we only need to solve for the x-coordinate of the center of gravity.

Example 7.13 Where should the dumbbell be lifted? (cont.)



SOLVE The *x*-coordinate of the center of gravity is found from Equation 7.15:

$$x_{cg} = \frac{x_1 m_1 + x_2 m_2}{m_1 + m_2} = \frac{(0 \text{ m})(10 \text{ kg}) + (1.0 \text{ m})(5.0 \text{ kg})}{10 \text{ kg} + 5.0 \text{ kg}}$$

$$= 0.33 \text{ m}$$

The center of gravity is 0.33 m from the 10 kg mass or, equivalently, 0.17 m left of the center of the bar.

ASSESS The position of the center of gravity is closer to the larger mass. This agrees with our general statement that the center of gravity tends to lie closer to the heavier particles.

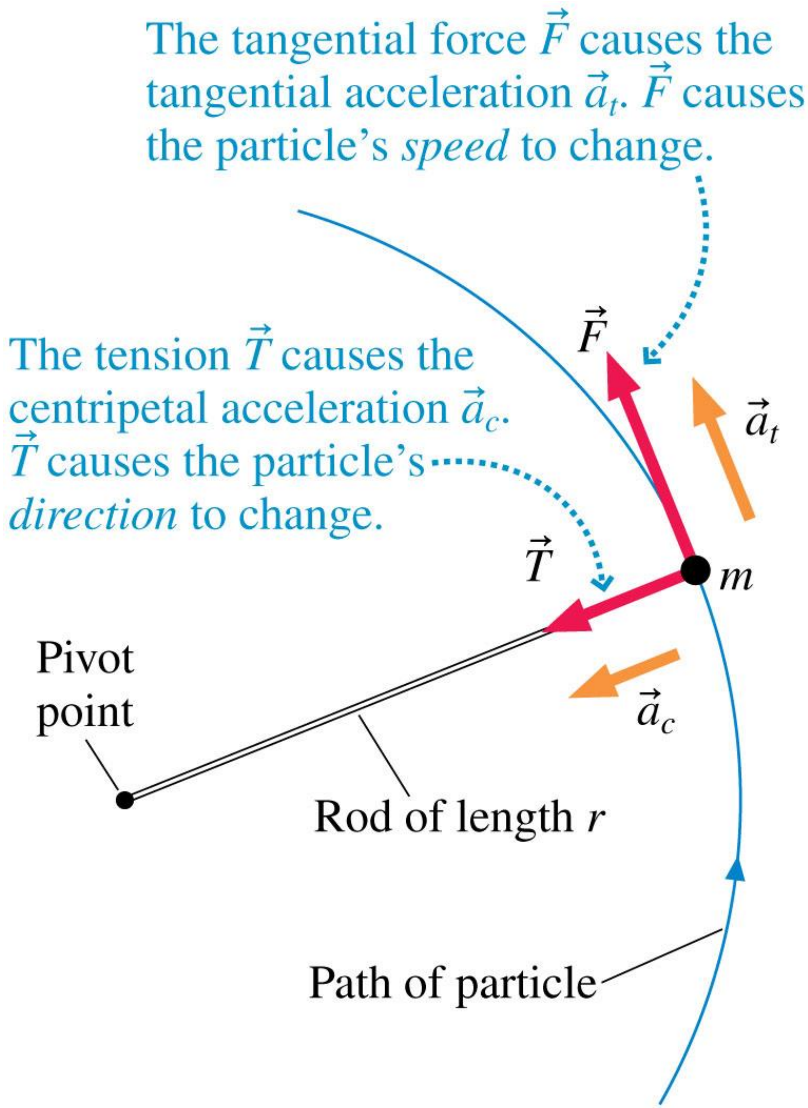
Section 7.5 Rotational Dynamics and Moment of Inertia

Rotational Dynamics and Moment of Inertia

- **A torque causes an angular acceleration.**
- The tangential and angular accelerations are

$$a_t = \frac{F}{m}$$

$$\alpha = \frac{F}{mr}$$



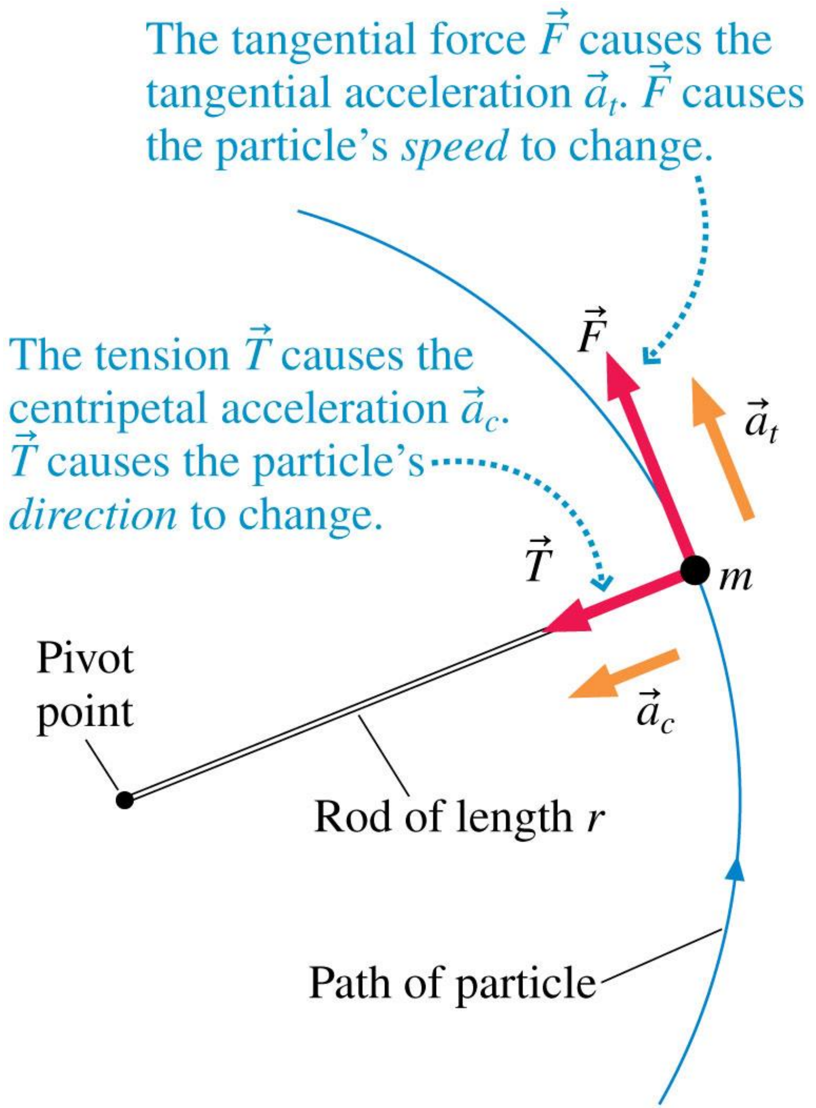
Rotational Dynamics and Moment of Inertia

- We compare with torque:

$$\tau = rF$$

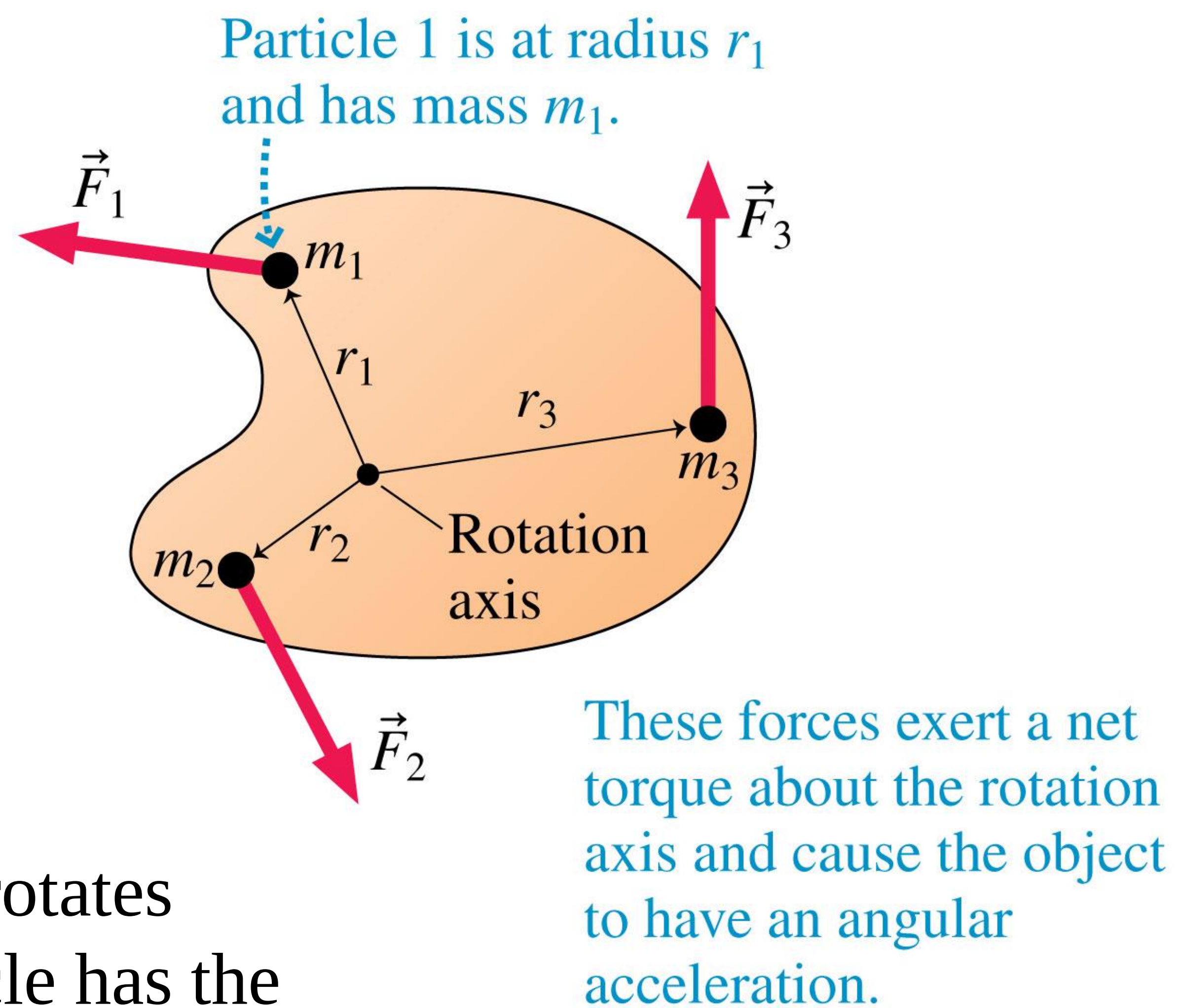
- We find the relationship with angular acceleration:

$$\alpha = \frac{\tau}{mr^2}$$



Newton's Second Law for Rotational Motion

- For a rigid body rotating about a fixed axis, we can think of the object as consisting of multiple particles.
- We can calculate the torque on each particle.
- Because the object rotates together, each particle has the *same* angular acceleration.



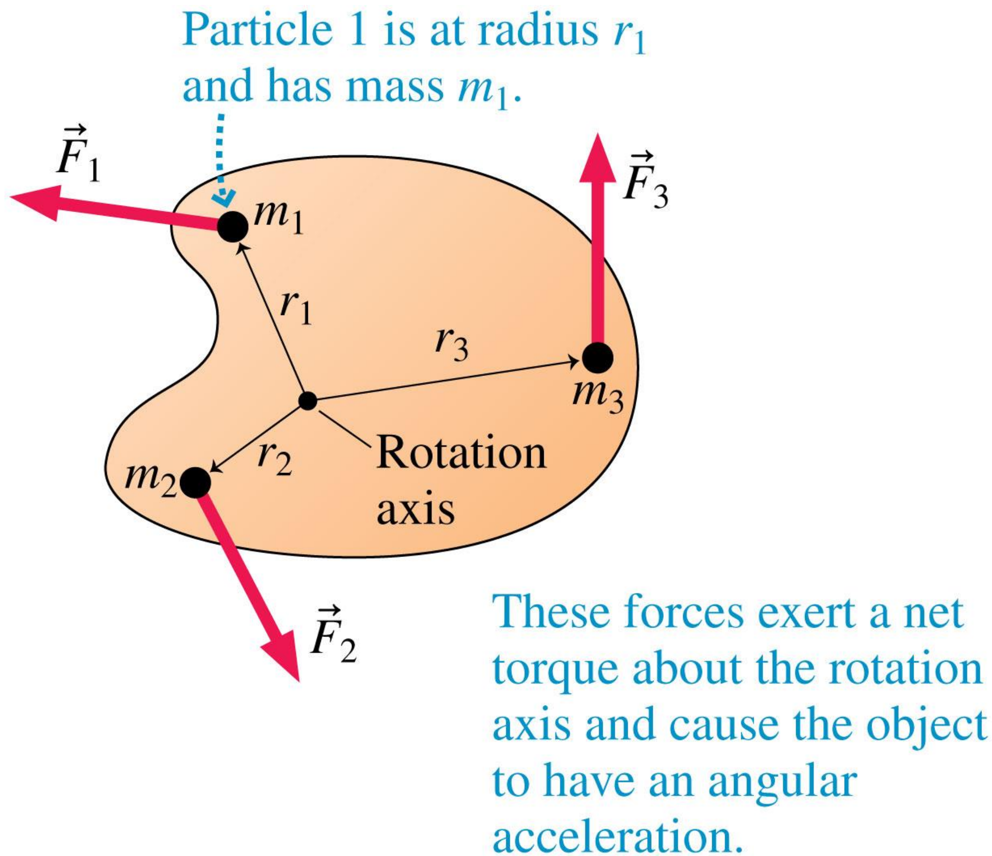
Newton's Second Law for Rotational Motion

- The torque for each "particle" is

$$\tau_1 = m_1 r_1{}^2 \alpha$$

$$\tau_2 = m_2 r_2{}^2 \alpha$$

$$\tau_3 = m_3 r_3{}^2 \alpha$$



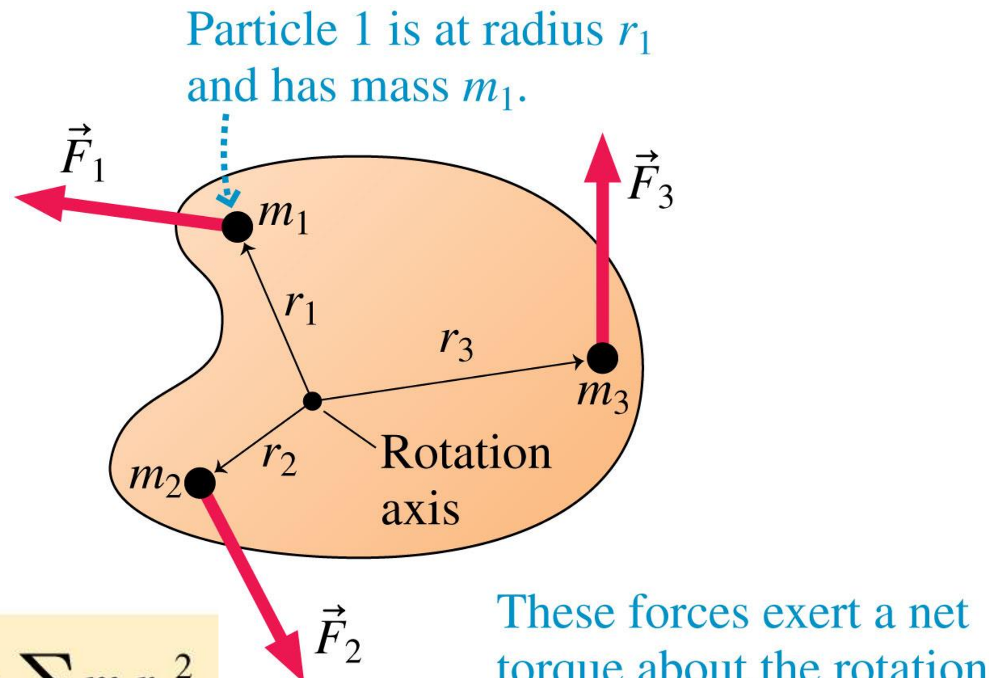
- The *net* torque is

$$\tau_{\text{net}} = \tau_1 + \tau_2 + \tau_3 + \cdots = m_1 r_1{}^2 \alpha + m_2 r_2{}^2 \alpha + m_3 r_3{}^2 \alpha + \cdots$$

$$= \alpha \left(m_1 r_1{}^2 + m_2 r_2{}^2 + m_3 r_3{}^2 + \cdots \right) = \alpha \sum m_i r_i{}^2$$

Newton's Second Law for Rotational Motion

- The quantity Σmr^2 in Equation 7.20, which is the proportionality constant between angular acceleration and net torque, is called the object's **moment of inertia I:**



$$I = m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + \cdots = \sum m_i r_i^2$$

Moment of inertia of a collection of particles

- The units of moment of inertia are kg · m^2.
- The moment of inertia *depends on the axis of rotation*.

Newton's Second Law for Rotational Motion

Newton's second law for rotation An object that experiences a net torque τ_{net} about the axis of rotation undergoes an angular acceleration

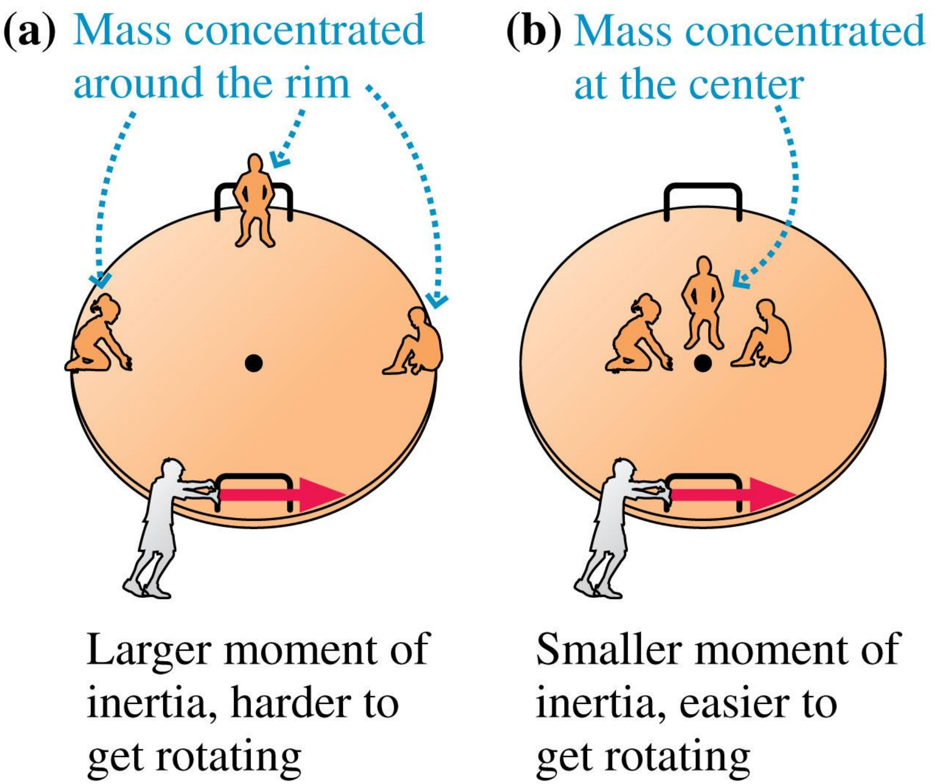
$$\alpha = \frac{\tau_{\text{net}}}{I}$$

where I is the moment of inertia of the object *about the rotation axis*.

A net torque is the cause of angular acceleration.

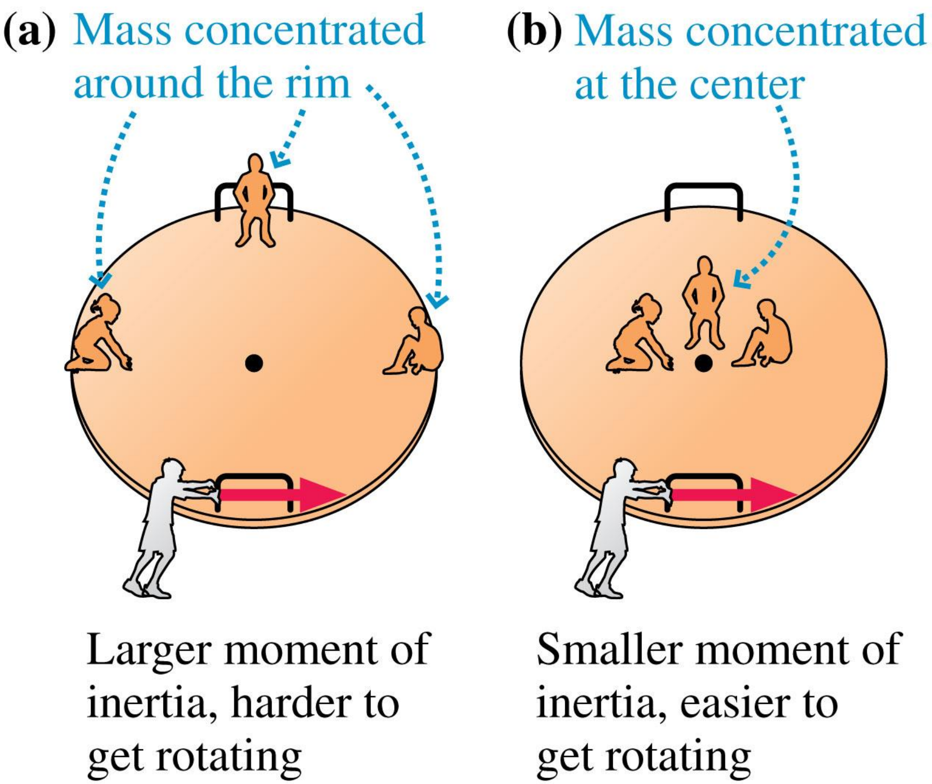
Interpreting the Moment of Inertia

- **The moment of inertia is the rotational equivalent of mass.**
- An object's moment of inertia depends not only on the object's mass but also on *how the mass is distributed* around the rotation axis.



Interpreting the Moment of Inertia

- **The moment of inertia is the rotational equivalent of mass.**
- It is more difficult to spin the merry-go-round when people sit far from the center because it has a higher inertia than when people sit close to the center.



Interpreting the Moment of Inertia

SYNTHESIS 7.2 **Linear and rotational dynamics**

The variables for linear dynamics have analogs for rotational dynamics. Newton's second law for rotational dynamics is expressed in terms of these variables.

	Linear dynamics	Rotational dynamics
Variables	Net force (N) $\vec{F}_{\text{net}}$ Mass (kg) m Acceleration (m/s^2) $\vec{a}$	τ_{net} Net torque (N · m) I Moment of inertia (kg · m^2) α Angular acceleration (rad/s^2)
Newton's second law	Acceleration is caused by forces. $\vec{a} = \dfrac{\vec{F}_{\text{net}}}{m}$ The larger the mass, the smaller the acceleration.	Angular acceleration is caused by torques. $\alpha = \dfrac{\tau_{\text{net}}}{I}$ The larger the moment of inertia, the smaller the angular acceleration.

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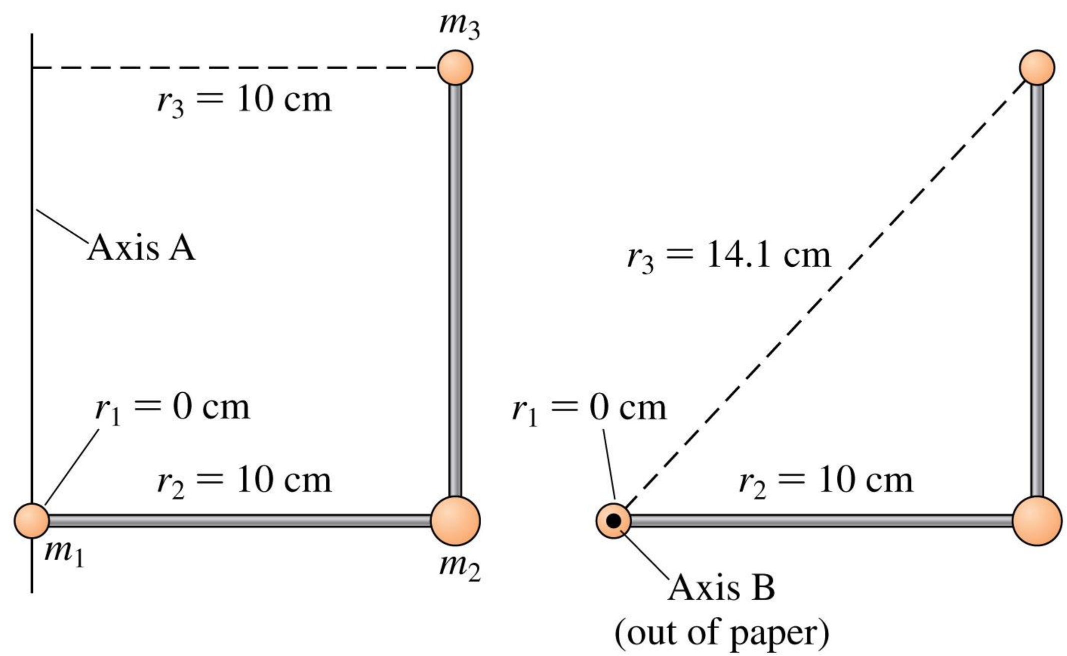
Example 7.15 Calculating the moment of inertia



Your friend is creating an abstract sculpture that consists of three small, heavy spheres attached by very lightweight 10-cm-long rods as shown in FIGURE 7.36. The spheres have masses $m_1 = 1.0$ kg, $m_2 = 1.5$ kg, and $m_3 = 1.0$ kg. What is the object's moment of inertia if it is rotated about axis A? About axis B?

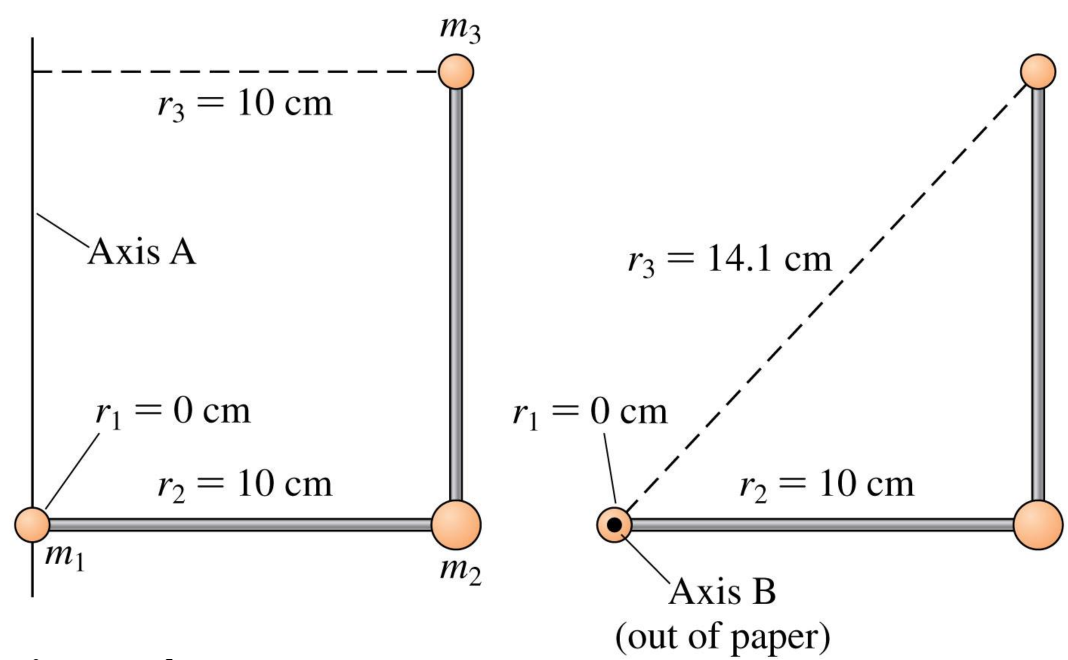
PREPARE We'll use Equation 7.21 for the moment of inertia:

$$I = m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2$$

In this expression, r_1, r_2, and r_3 are the distances of each particle from the axis of rotation, so they depend on the axis chosen.

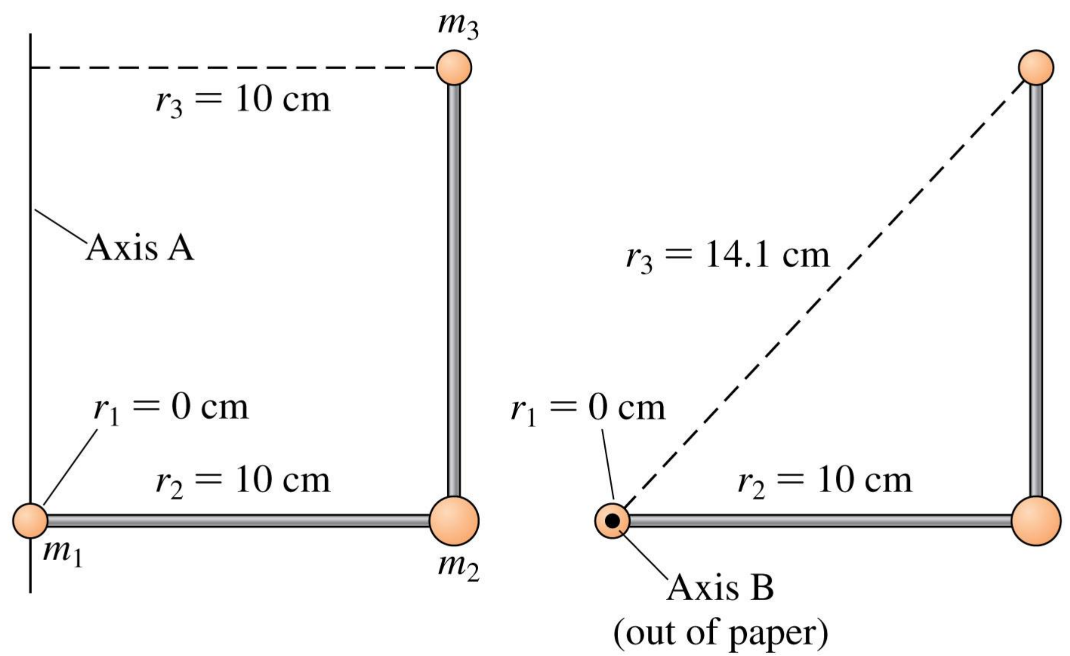
Example 7.15 Calculating the moment of inertia (cont.)

Particle 1 lies on both axes, so r_1 = 0 cm in both cases. Particle 2 lies 10 cm (0.10 m) from both axes. Particle 3 is 10 cm from axis A but farther from axis B. We can find r_3 for axis B by using the Pythagorean theorem, which gives r_3 = 14.1 cm. These distances are indicated in the figure.



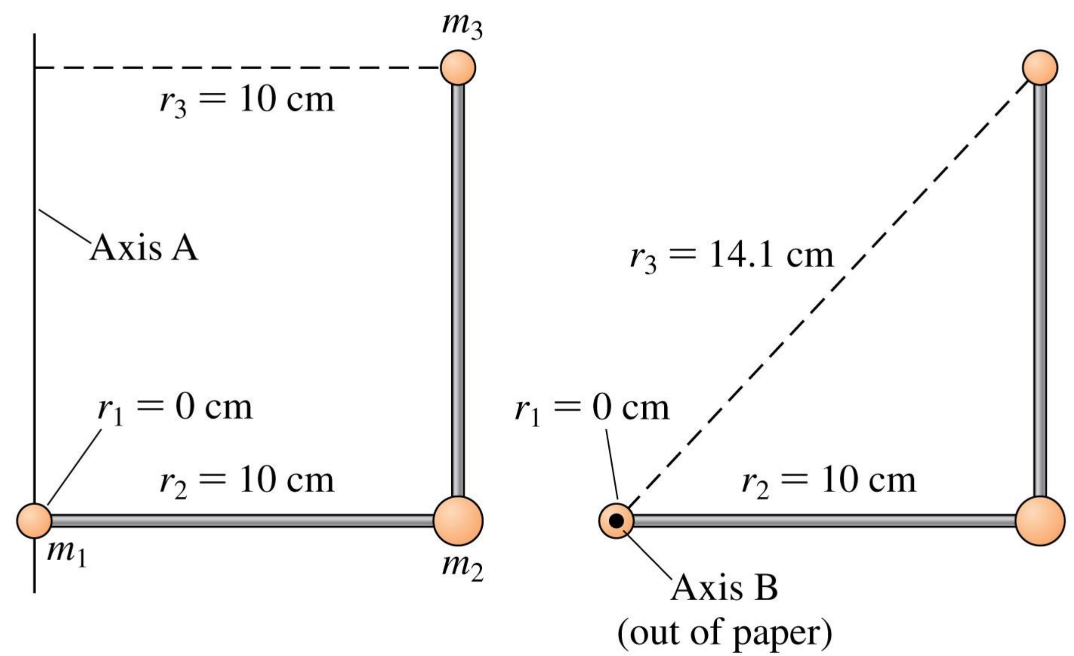
Example 7.15 Calculating the moment of inertia (cont.)

SOLVE For each axis, we can prepare a table of the values of r, m, and mr^2 for each particle, then add the values of mr^2. For axis A we have



Particle	r	m	mr^2
1	0 m	1.0 kg	$0\ \text{kg}\cdot\text{m}^2$
2	0.10 m	1.5 kg	$0.015\ \text{kg}\cdot\text{m}^2$
3	0.10 m	1.0 kg	$0.010\ \text{kg}\cdot\text{m}^2$
			$I_A = 0.025\ \text{kg}\cdot\text{m}^2$

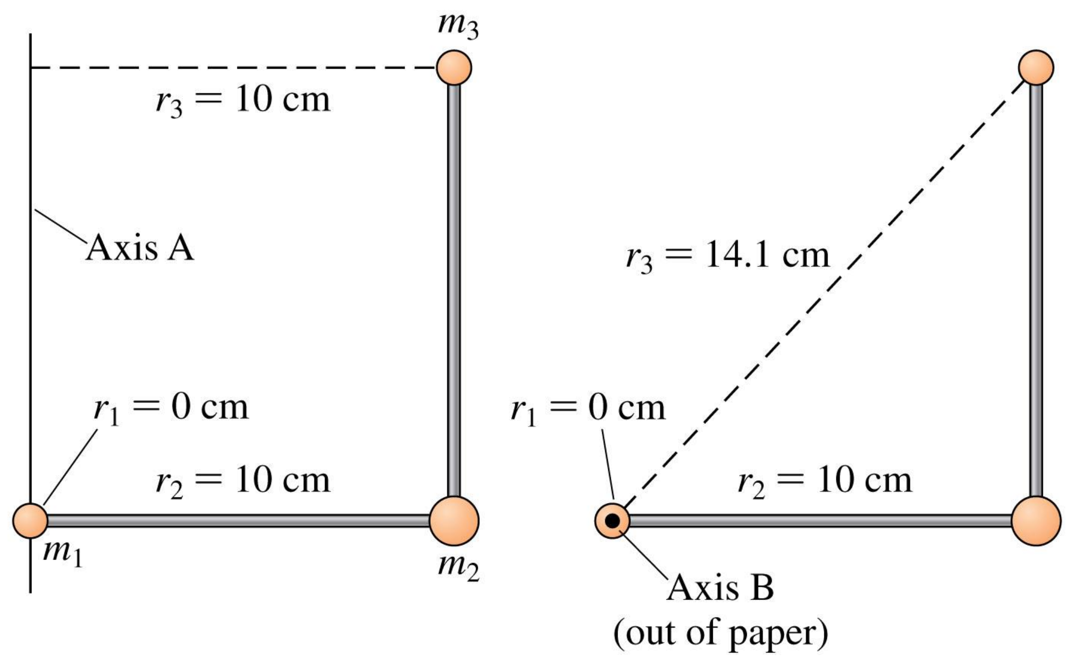
Example 7.15 Calculating the moment of inertia (cont.)



For axis B we have

Particle	r	m	mr^2
1	0 m	1.0 kg	$0\ \text{kg} \cdot \text{m}^2$
2	0.10 m	1.5 kg	$0.015\ \text{kg} \cdot \text{m}^2$
3	0.141 m	1.0 kg	$0.020\ \text{kg} \cdot \text{m}^2$
			$I_B = 0.035\ \text{kg} \cdot \text{m}^2$

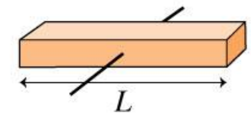
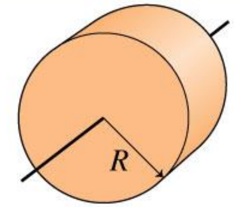
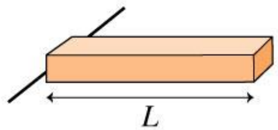
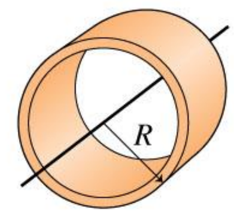
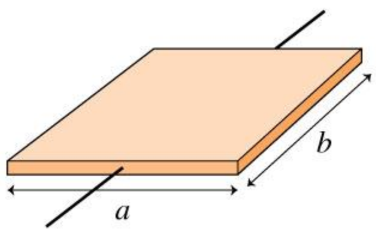
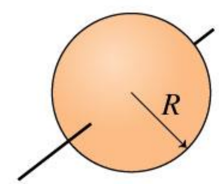
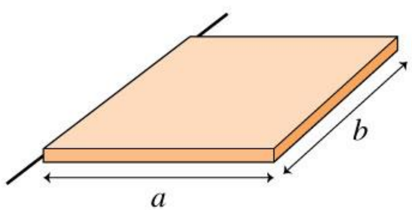
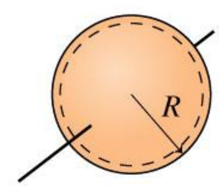
Example 7.15 Calculating the moment of inertia (cont.)



ASSESS We've already noted that the moment of inertia of an object is higher when its mass is distributed farther from the axis of rotation. Here, m_3 is farther from axis B than from axis A, leading to a higher moment of inertia about that axis.

The Moments of Inertia of Common Shapes

TABLE 7.1 Moments of inertia of objects with uniform density and total mass M

Object and axis	Picture	I	Object and axis	Picture	I
Thin rod (of any cross section), about center		$\frac{1}{12}ML^2$	Cylinder or disk, about center		$\frac{1}{2}MR^2$
Thin rod (of any cross section), about end		$\frac{1}{3}ML^2$	Cylindrical hoop, about center		MR^2
Plane or slab, about center		$\frac{1}{12}Ma^2$	Solid sphere, about diameter		$\frac{2}{5}MR^2$
Plane or slab, about edge		$\frac{1}{3}Ma^2$	Spherical shell, about diameter		$\frac{2}{3}MR^2$

Try It Yourself: Hammering Home Inertia

Most of the mass of a hammer is in its head, so the hammer's moment of inertia is large when calculated about an axis passing through the end of the handle (far from the head), but small when calculated about an axis passing through the head itself. You can *feel* this difference by attempting to wave a hammer back and forth about the handle end and the head end. It's much harder to do about the handle end because the large moment of inertia keeps the angular acceleration small.

Section 7.6 Using Newton's Second Law for Rotation

Using Newton's Second Law for Rotation

PROBLEM-SOLVING STRATEGY 7.1 **Rotational dynamics problems**

We can use a problem-solving strategy for rotational dynamics that is very similar to the strategy for linear dynamics in Chapter 5.

PREPARE Model the object as a simple shape. Draw a pictorial representation to clarify the situation, define coordinates and symbols, and list known information.

- Identify the axis about which the object rotates.
- Identify the forces and determine their distance from the axis.
- Calculate the torques caused by the forces, and find the signs of the torques.

SOLVE The mathematical representation is based on Newton's second law for rotational motion:

$$\tau_{\text{net}} = I\alpha \qquad \text{or} \qquad \alpha = \frac{\tau_{\text{net}}}{I}$$

- Find the moment of inertia either by direct calculation using Equation 7.21 or from Table 7.1 for common shapes of objects.
- Use rotational kinematics to find angular positions and velocities.

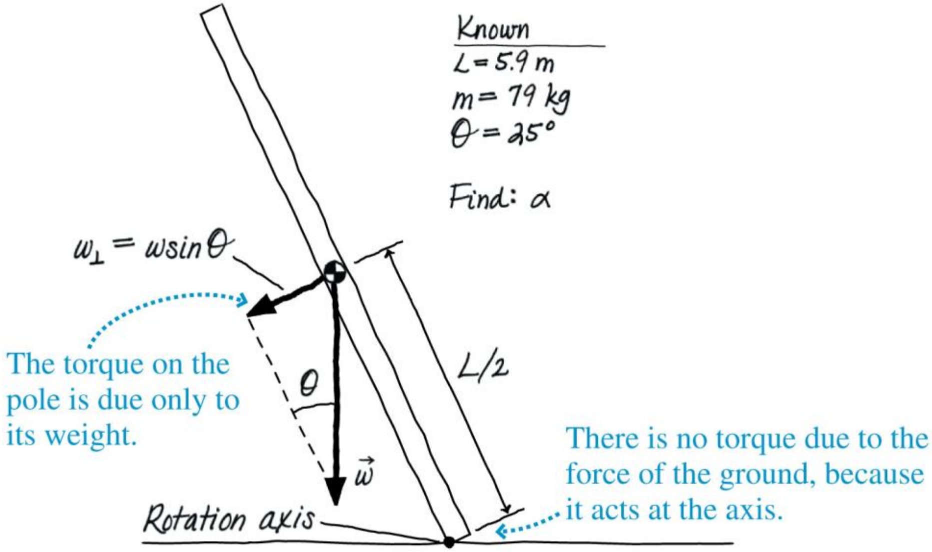
ASSESS Check that your result has the correct units, is reasonable, and answers the question.

Exercise 31

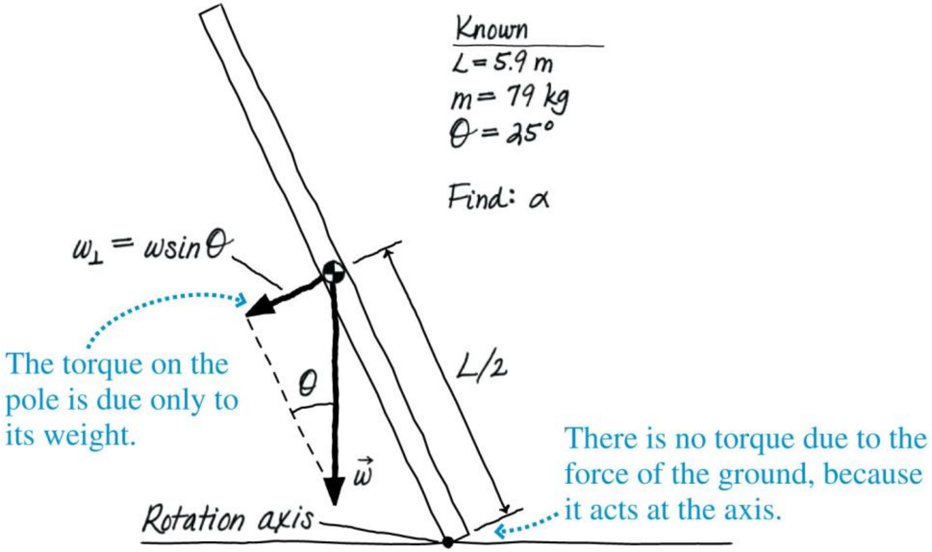
Text: p. 211

Example 7.16 Angular acceleration of a falling pole

In the caber toss, a contest of strength and skill that is part of Scottish games, contestants toss a heavy uniform pole, landing it on its end. A 5.9-m-tall pole with a mass of 79 kg has just landed on its end. It is tipped by 25° from the vertical and is starting to rotate about the end that touches the ground. Estimate the angular acceleration.

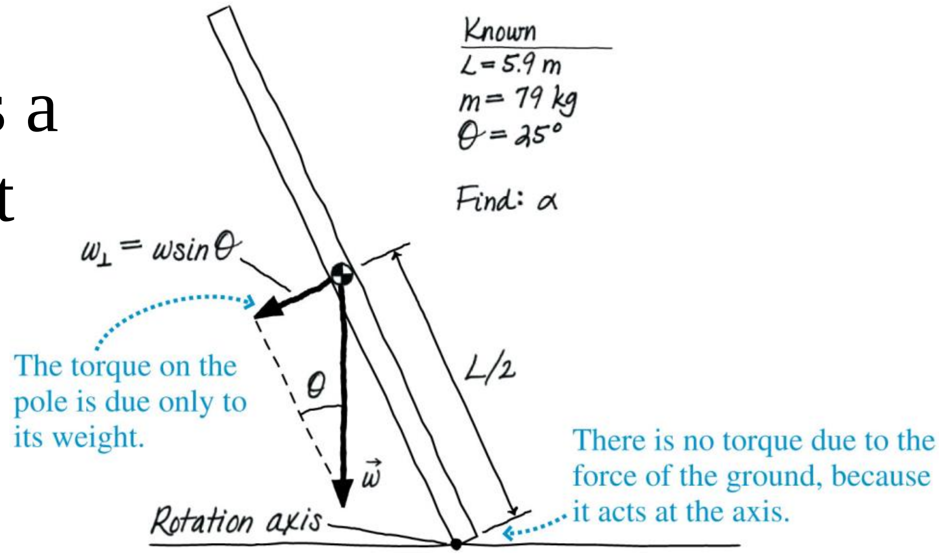


Example 7.16 Angular acceleration of a falling pole (cont.)



PREPARE The situation is shown in FIGURE 7.37, where we define our symbols and list the known information. Two forces are acting on the pole: the pole's weight $\vec{w}$, which acts at the center of gravity, and the force of the ground on the pole (not shown). This second force exerts no torque because it acts at the axis of rotation. The torque on the pole is thus due only to gravity. From the figure we see that this torque tends to rotate the pole in a counterclockwise direction, so the torque is positive.

Example 7.16 Angular acceleration of a falling pole (cont.)

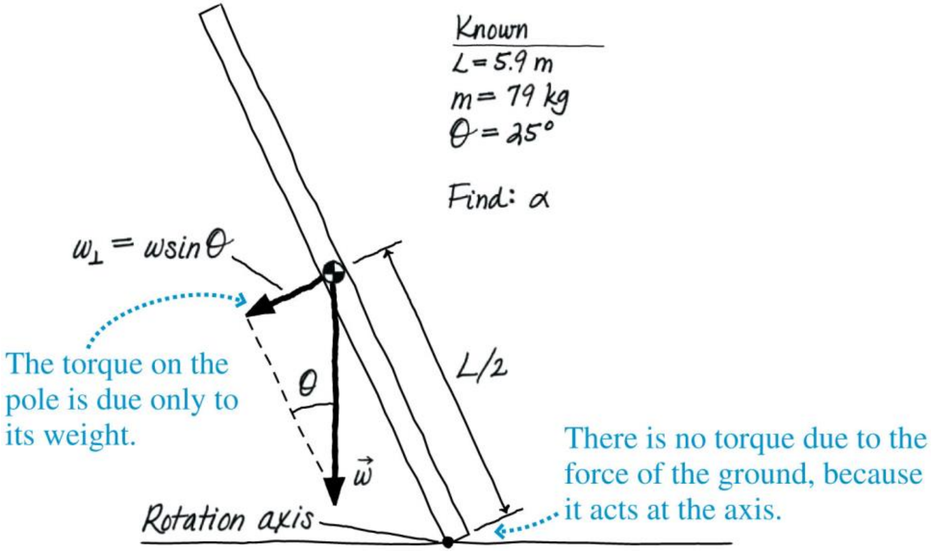


SOLVE We'll model the pole as a uniform thin rod rotating about one end. Its center of gravity is at its center, a distance $L/2$ from the axis. You can see from the figure that the perpendicular component of $\vec{w}$ is $w_\perp = w \sin\theta$. Thus the torque due to gravity is

$$\tau_{\text{net}} = \left(\frac{L}{2}\right) w_\perp = \left(\frac{L}{2}\right) w \sin\theta = \frac{mgL}{2} \sin\theta$$

Example 7.16 Angular acceleration of a falling pole (cont.)

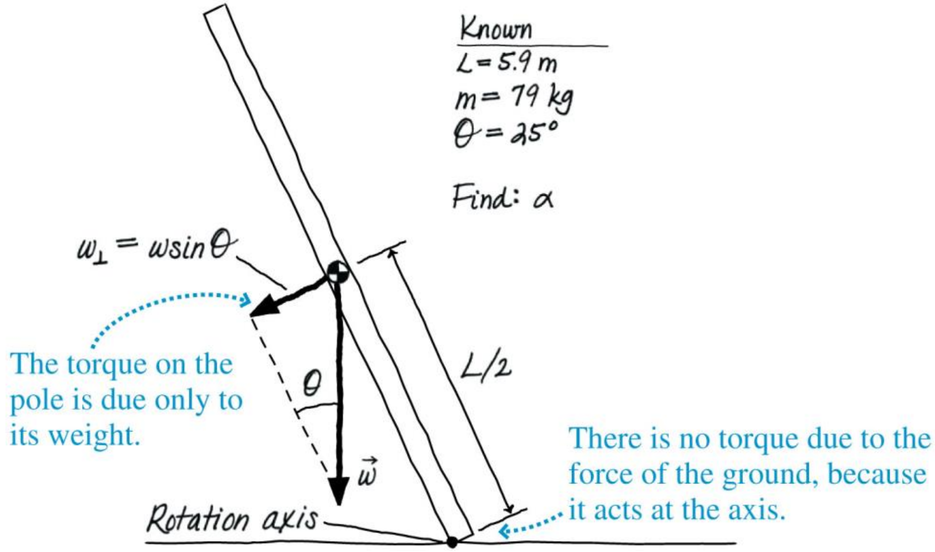
From Table 7.1, the moment of inertia of a thin rod rotated about its end is $I = \frac{1}{3}mL^2$. Thus, from Newton's second law for rotational motion, the angular acceleration is



$$\alpha = \frac{\tau_{\text{net}}}{I} = \frac{\frac{1}{2}mgL\sin\theta}{\frac{1}{3}mL^2} = \frac{3g\sin\theta}{2L}$$

$$= \frac{3(9.8\ \text{m/s}^2)\sin 25°}{2(5.9\ \text{m})} = 1.1\ \text{rad/s}^2$$

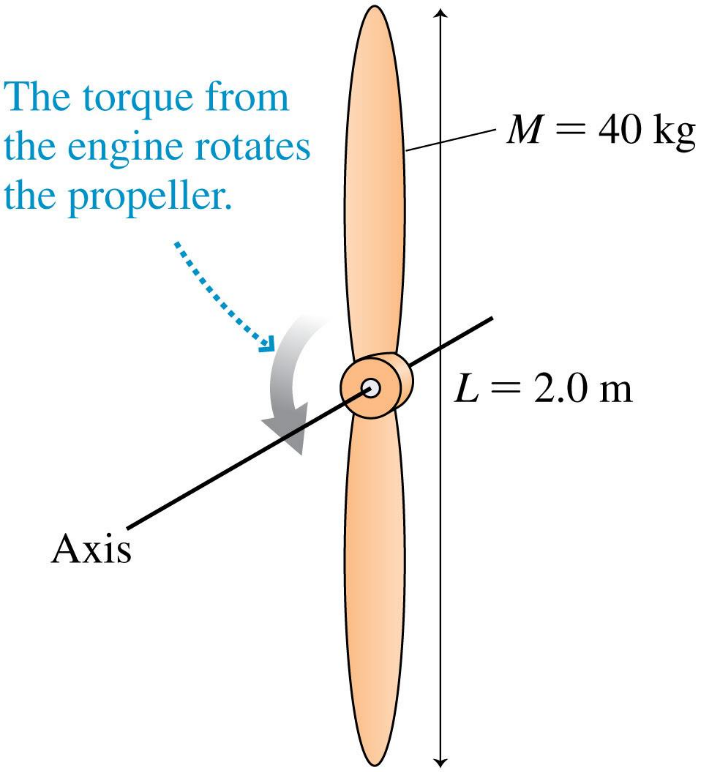
Example 7.16 Angular acceleration of a falling pole (cont.)



ASSESS The final result for the angular acceleration did not depend on the mass, as we might expect given the analogy with free-fall problems. And the final value for the angular acceleration is quite modest. This is reasonable: You can see that the angular acceleration is inversely proportional to the length of the pole, and it's a long pole. The modest value of angular acceleration is fortunate—the caber is pretty heavy, and folks need some time to get out of the way when it topples!

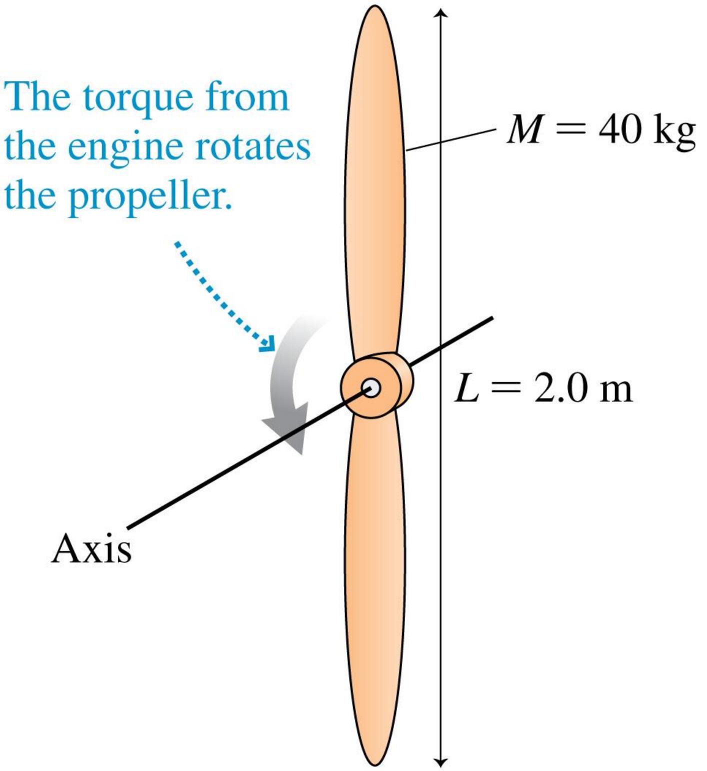
Example 7.18 Starting an airplane engine

The engine in a small air-plane is specified to have a torque of 500 N · m. This engine drives a 2.0-m-long, 40 kg single-blade propeller. On start-up, how long does it take the propeller to reach 2000 rpm?



Example 7.18 Starting an airplane engine (cont.)

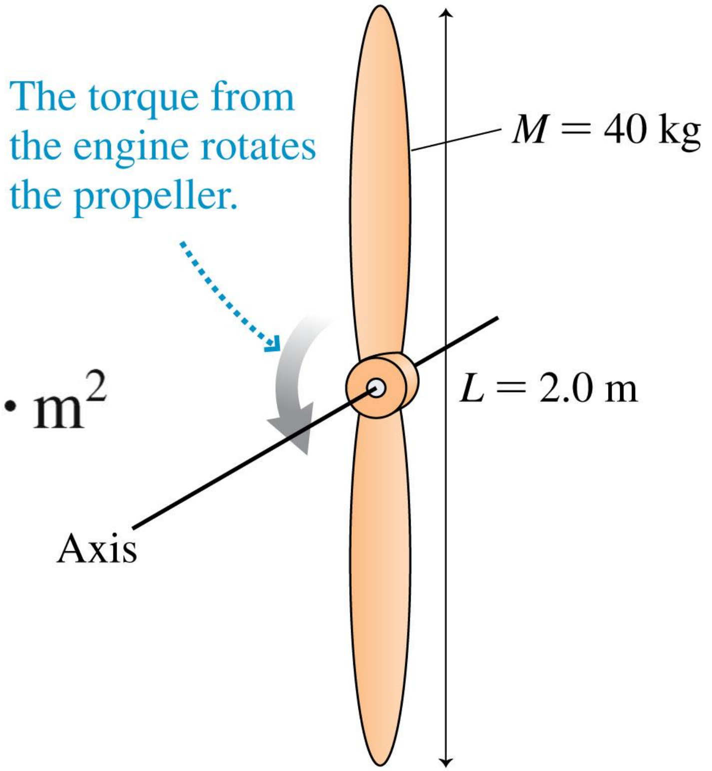
PREPARE The propeller can be modeled as a rod that rotates about its center. The engine exerts a torque on the propeller. FIGURE 7.38 shows the propeller and the rotation axis.



Example 7.18 Starting an airplane engine (cont.)

SOLVE The moment of inertia of a rod rotating about its center is found in Table 7.1:

$$I = \tfrac{1}{12}ML^2 = \tfrac{1}{12}(40\ \text{kg})(2.0\ \text{m})^2 = 13.3\ \text{kg}\cdot\text{m}^2$$



The 500 N · m torque of the engine causes an angular acceleration of

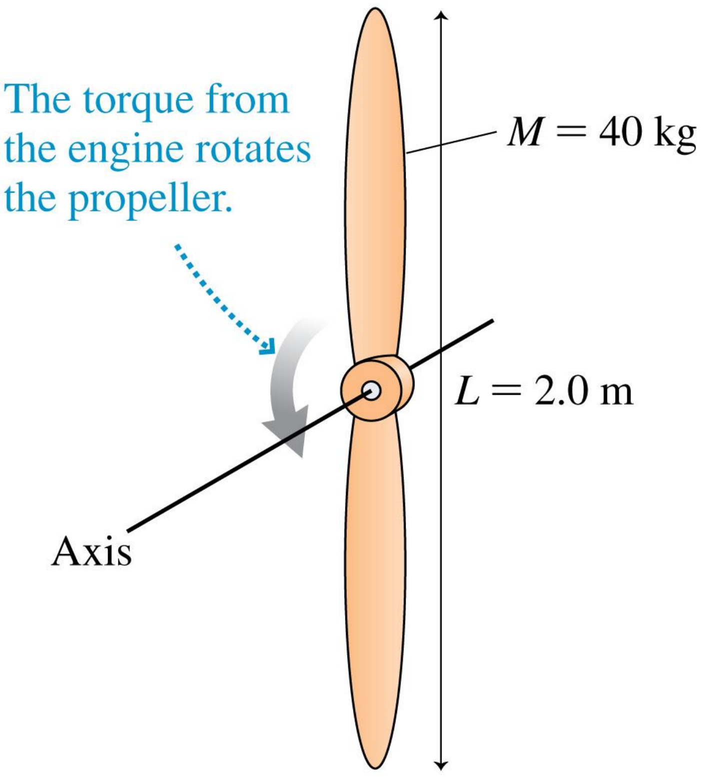
$$\alpha = \frac{\tau}{I} = \frac{500\ \text{N}\cdot\text{m}}{13.3\ \text{kg}\cdot\text{m}^2} = 37.5\ \text{rad/s}^2$$

Example 7.18 Starting an airplane engine (cont.)

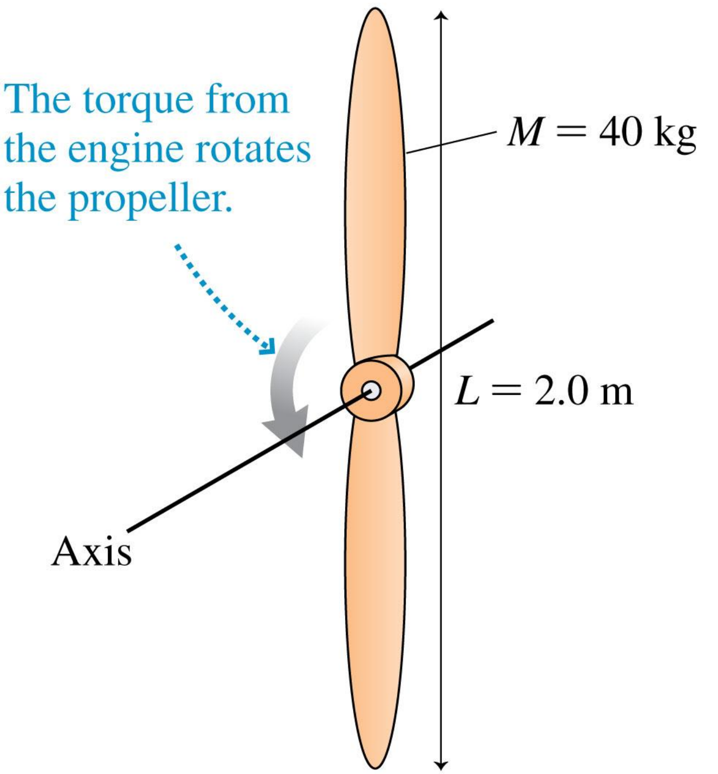
The time needed to reach ω_f = 2000 rpm = 33.3 rev/s = 209 rad/s is

$$\Delta t = \frac{\Delta\omega}{\alpha} = \frac{\omega_f - \omega_i}{\alpha}$$

$$= \frac{209\ \text{rad/s} - 0\ \text{rad/s}}{37.5\ \text{rad/s}^2} = 5.6\ \text{s}$$



Example 7.18 Starting an airplane engine (cont.)



ASSESS We've assumed a constant angular acceleration, which is reasonable for the first few seconds while the propeller is still turning slowly. Eventually, air resistance and friction will cause opposing torques and the angular acceleration will decrease. At full speed, the negative torque due to air resistance and friction cancels the torque of the engine. Then $\tau_{\text{net}} = 0$ and the propeller turns at constant angular velocity with no angular acceleration.

Example Problem

A baseball bat has a mass of 0.82 kg and is 0.86 m long. It's held vertically and then allowed to fall. What is the bat's angular acceleration when it has reached 20° from the vertical? (Model the bat as a uniform cylinder).

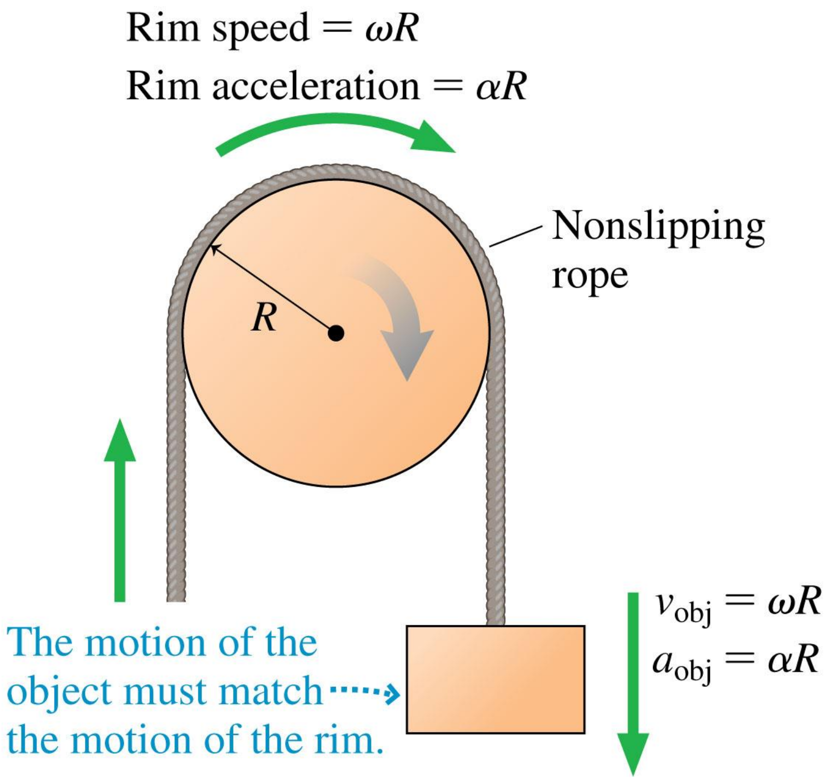
Constraints Due to Ropes and Pulleys

- If the pulley turns *without the rope slipping on it* then the rope's speed must exactly match the speed of the rim of the pulley.
- The attached object must have the same speed and acceleration as the rope.

$$v_{obj} = \omega R$$

$$a_{obj} = \alpha R$$

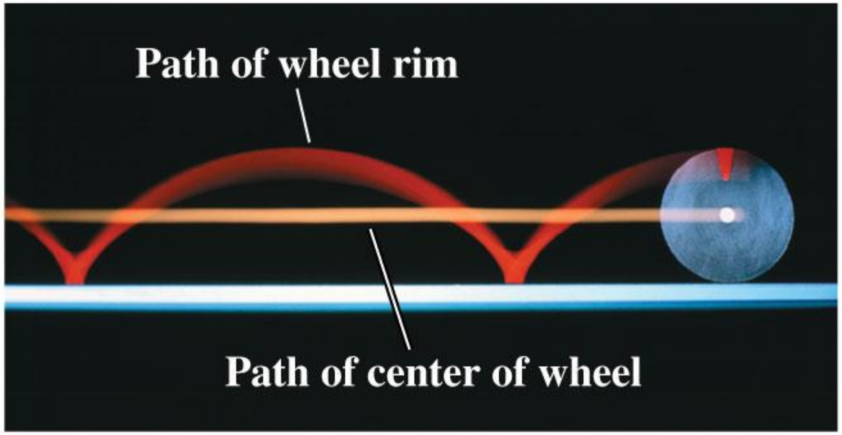
Motion constraints for an object connected to a pulley of radius R by a nonslipping rope



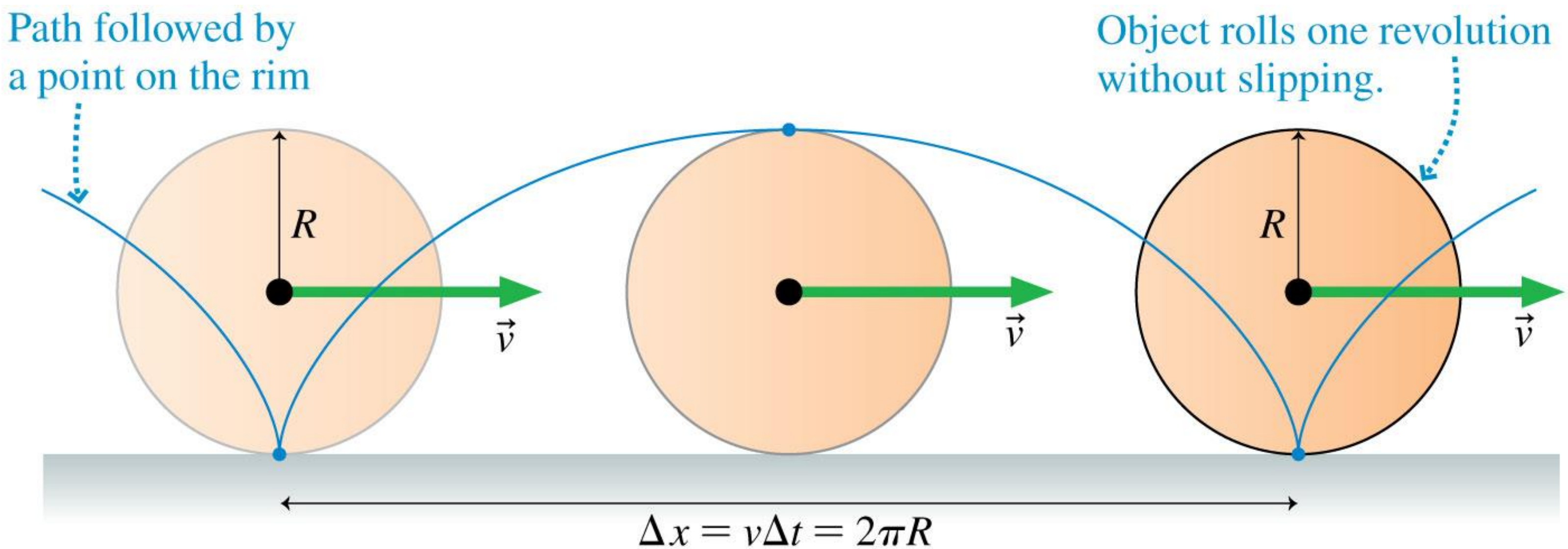
Section 7.7 Rolling Motion

Rolling Motion

- Rolling is a *combination motion* in which an object rotates about an axis that is moving along a straight-line trajectory.

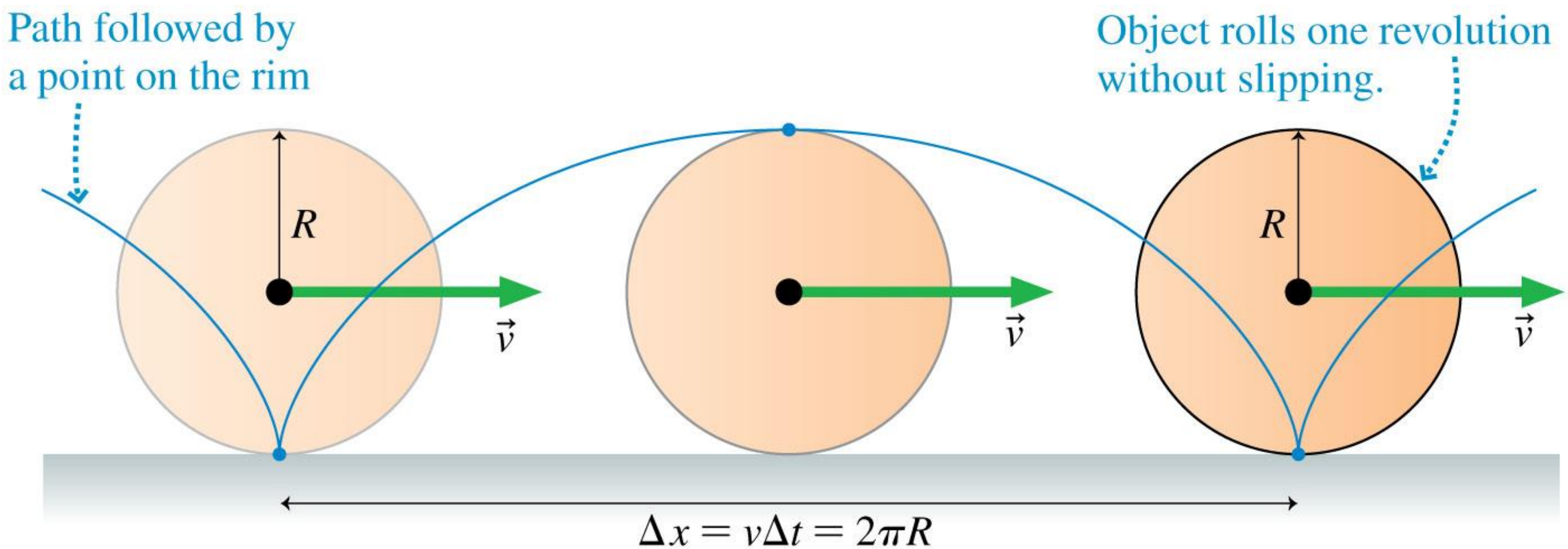


Rolling Motion



- The figure above shows exactly one revolution for a wheel or sphere that rolls forward *without slipping*.
- The overall position is measured at the object's center.

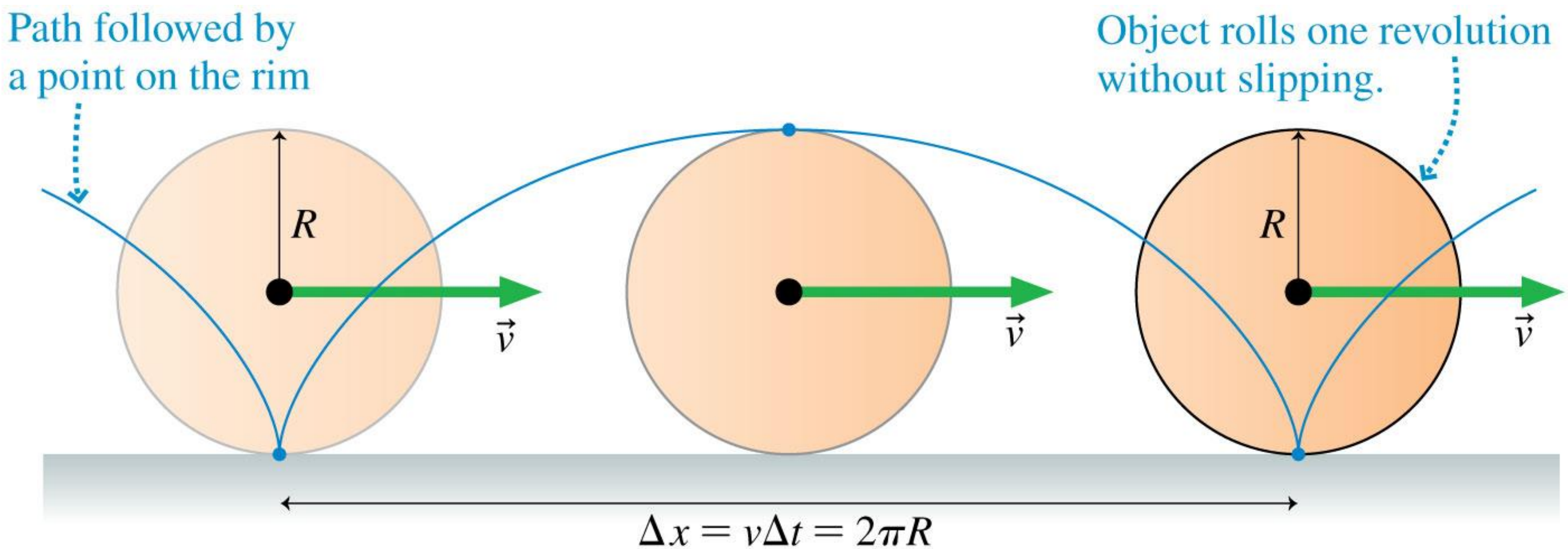
Rolling Motion



- In one revolution, the center moves forward by exactly one circumference ($\Delta x = 2\pi R$).

$$v = \frac{\Delta x}{T} = \frac{2\pi R}{T}$$

Rolling Motion

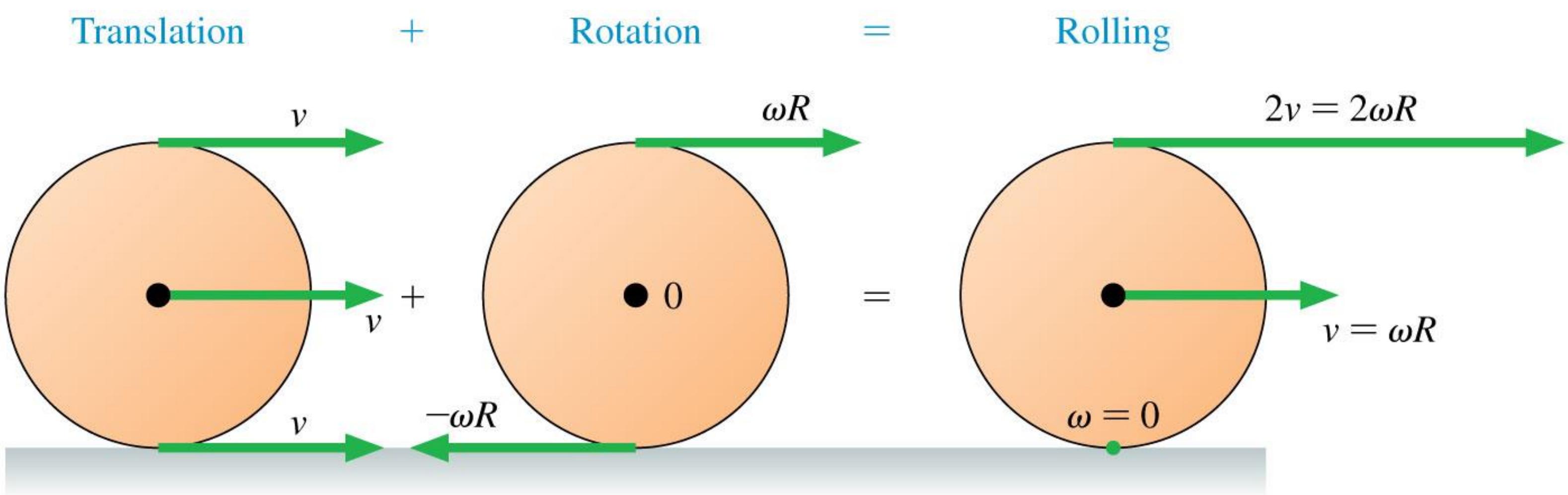


- Since $2\pi/T$ is the angular velocity, we find

$$v = \omega R$$

- This is the **rolling constraint**, the basic link between translation and rotation for objects that roll without slipping.

Rolling Motion



- The point at the bottom of the wheel has a translational velocity and a rotational velocity in opposite directions, which cancel each other.
- **The point on the bottom of a rolling object is instantaneously at rest.**
- This is the idea behind "rolling without slipping."

Example 7.20 Rotating your tires

The diameter of your tires is 0.60 m. You take a 60 mile trip at a speed of 45 mph.

a. During this trip, what was your tires' angular speed?

b. How many times did they revolve?

Example 7.20 Rotating your tires (cont.)

PREPARE The angular speed is related to the speed of a wheel's center by Equation 7.25: $v = \omega R$. Because the center of the wheel turns on an axle fixed to the car, the speed v of the wheel's center is the same as that of the car. We prepare by converting the car's speed to SI units:

$$v = (45 \text{ mph}) \times \left(0.447 \frac{\text{m/s}}{\text{mph}}\right) = 20 \text{ m/s}$$

Once we know the angular speed, we can find the number of times the tires turned from the rotational-kinematic equation $\Delta\theta = \omega\, \Delta t$. We'll need to find the time traveled Δt from $v = \Delta x/\Delta t$.

Example 7.20 Rotating your tires (cont.)

SOLVE a. From Equation 7.25 we have

$$\omega = \frac{v}{R} = \frac{20\text{ m/s}}{0.30\text{ m}} = 67\text{ rad/s}$$

b. The time of the trip is

$$\Delta t = \frac{\Delta x}{v} = \frac{60\text{ mi}}{45\text{ mi/h}} = 1.33\text{ h} \times \frac{3600\text{ s}}{1\text{ h}} = 4800\text{ s}$$

Example 7.20 Rotating your tires (cont.)

Thus the total angle through which the tires turn is

$$\Delta\theta = \omega\,\Delta t = (67\text{ rad/s})(4800\text{ s}) = 3.2 \times 10^5\text{ rad}$$

Because each turn of the wheel is 2π rad, the number of turns is

$$\frac{3.2 \times 10^5\text{ rad}}{2\pi\text{ rad}} = 51{,}000\text{ turns}$$

Example 7.20 Rotating your tires (cont.)

ASSESS You probably know from seeing tires on passing cars that a tire rotates several times a second at 45 mph. Because there are 3600 s in an hour, and your 60 mile trip at 45 mph is going to take over an hour—say, ≈ 5000 s—you would expect the tire to make many thousands of revolutions. So 51,000 turns seems to be a reasonable answer. You can see that your tires rotate roughly a thousand times per mile. During the lifetime of a tire, about 50,000 miles, it will rotate about 50 million times!

Summary: General Principles

Newton's Second Law for Rotational Motion

If a net torque τ_{net} acts on an object, the object will experience an angular acceleration given by $\alpha = \tau_{net}/I$, where I is the object's moment of inertia about the rotation axis.

This law is analogous to Newton's second law for linear motion, $\vec{a} = \vec{F}_{net}/m$.

Text: p. 217

Summary: Important Concepts

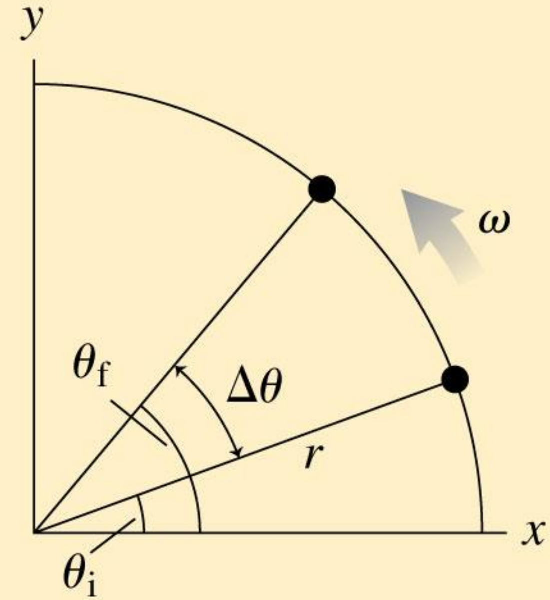
Describing circular motion

We define new variables for circular motion. By convention, counterclockwise is positive.

Angular displacement: $\Delta\theta = \theta_f - \theta_i$

Angular velocity: $\omega = \dfrac{\Delta\theta}{\Delta t}$

Angular acceleration: $\alpha = \dfrac{\Delta\omega}{\Delta t}$



Angles are measured in radians:

$$1 \text{ rev} = 360° = 2\pi \text{ rad}$$

The angular velocity depends on the frequency and period:

$$\omega = \frac{2\pi}{T} = 2\pi f$$

Text: p. 217

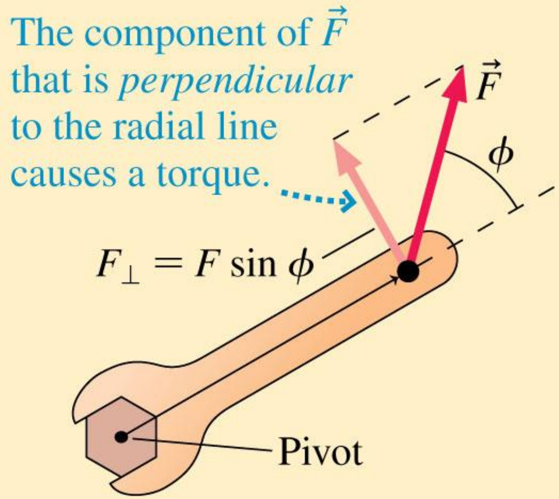
Summary: Important Concepts

Torque

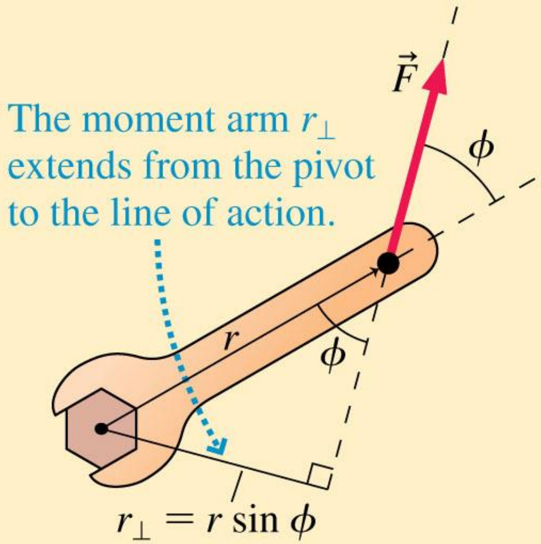
A force causes an object to undergo a linear acceleration, a torque causes an object to undergo an angular acceleration.

There are two interpretations of torque:

Interpretation 1: $\tau = rF_{\perp}$



Interpretation 2: $\tau = r_{\perp} F$



Both interpretations give the same expression for the magnitude of the torque:

$$\tau = rF \sin \phi$$

Text: p. 217

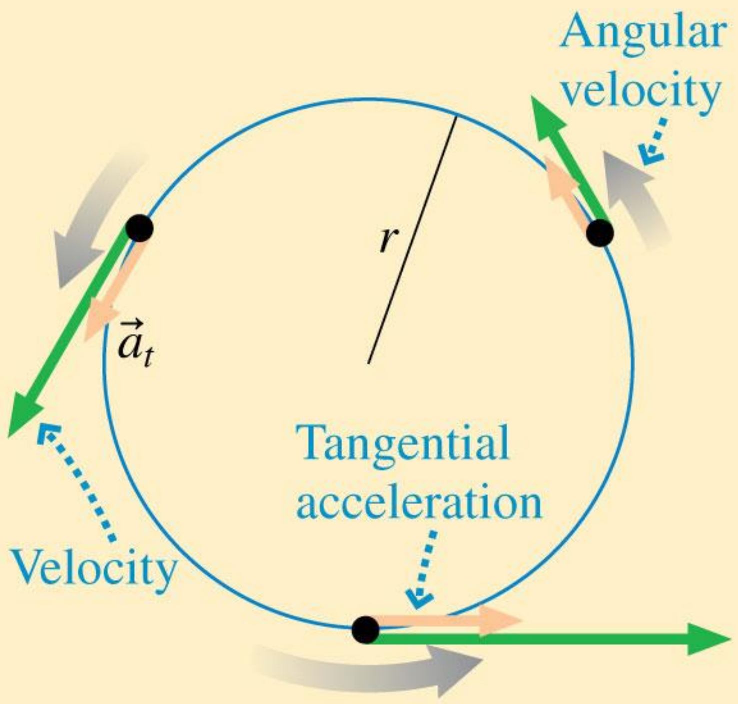
Summary: Important Concepts

Relating linear and circular motion quantities

Linear and angular speeds are related by: $v = \omega r$

If the particle's speed is increasing, it will also have a tangential acceleration $\vec{a}_t$ directed tangent to the circle and an angular acceleration α.

Angular and tangential accelerations are related by: $a_t = \alpha r$



Text: p. 217

Summary: Important Concepts

The **moment of inertia** is the rotational equivalent of mass. For an object made up of particles of masses $m_1, m_2, \ldots$ at distances $r_1, r_2, \ldots$ from the axis, the moment of inertia is

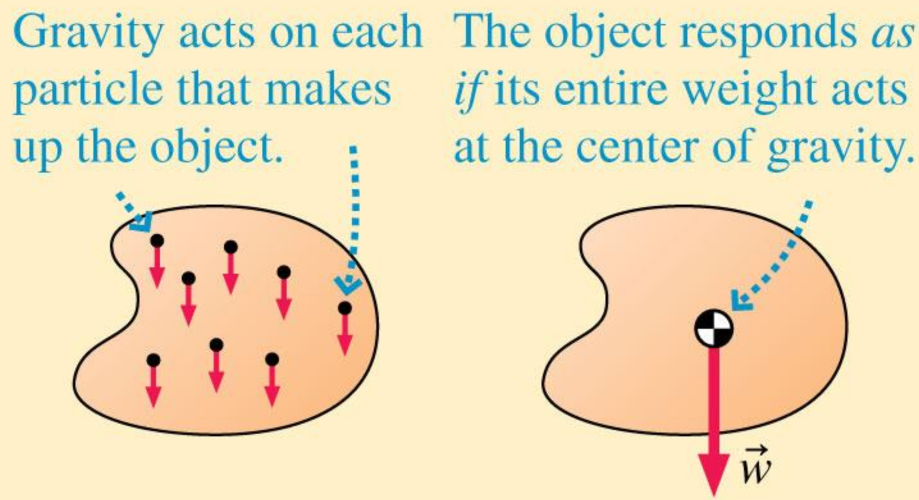
$$I = m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + \cdots = mr^2$$

Text: p. 217

Summary: Important Concepts

Center of gravity

The **center of gravity** of an object is the point at which gravity can be considered to be acting.



The **position of the center of gravity** depends on the distance $x_1, x_2, \ldots$ of each particle of mass $m_1, m_2, \ldots$ from the origin:

$$x_{cg} = \frac{x_1 m_1 + x_2 m_2 + x_3 m_3 + \cdots}{m_1 + m_2 + m_3 + \cdots}$$

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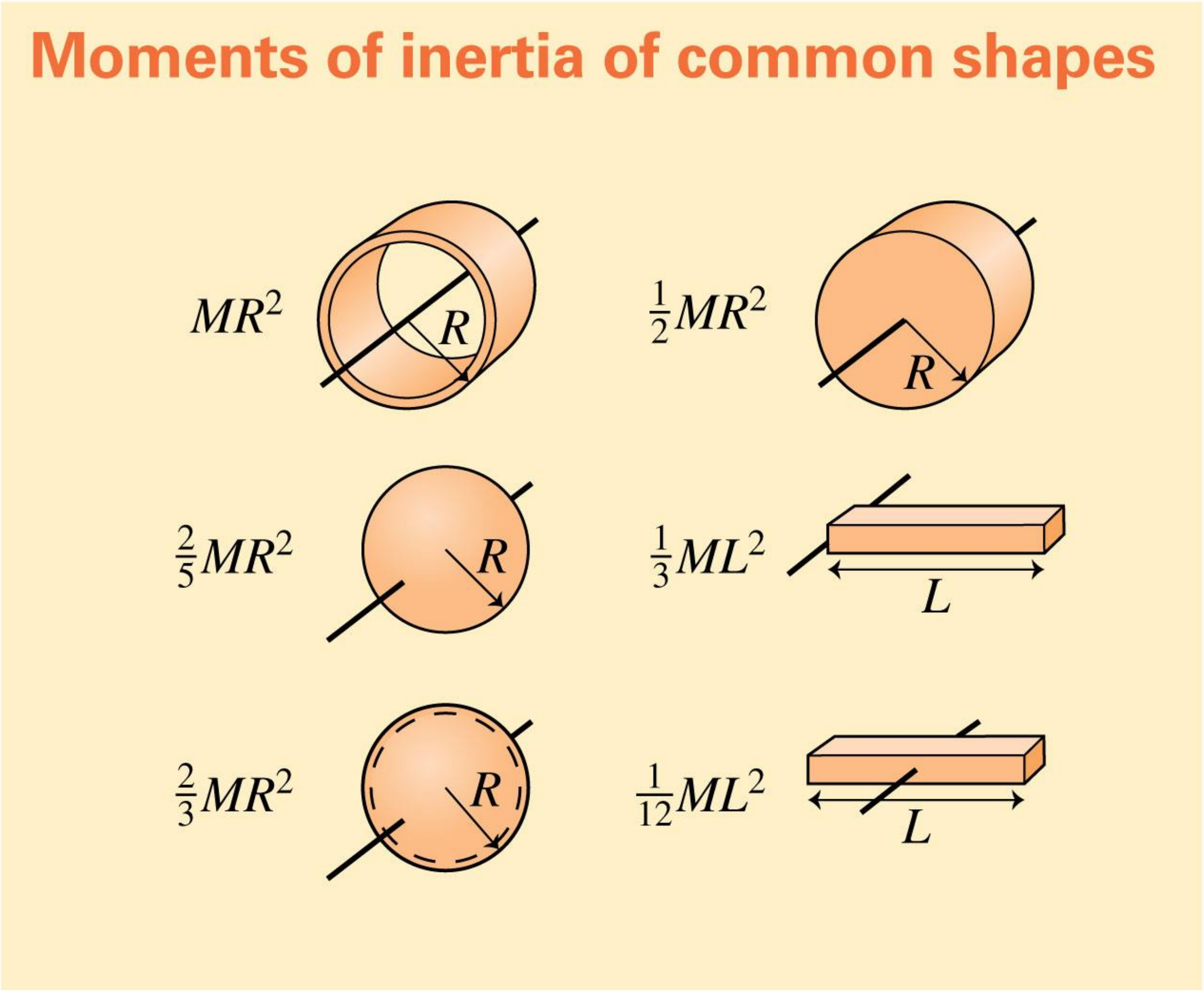
Summary: Applications



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Summary: Applications

Rotation about a fixed axis

When a net torque is applied to an object that rotates about a fixed axis, the object will undergo an **angular acceleration** given by

$$\alpha = \frac{\tau_{\text{net}}}{I}$$

If a rope unwinds from a pulley of radius R, the linear motion of an object tied to the rope is related to the angular motion of the pulley by

$$a_{\text{obj}} = \alpha R \qquad v_{\text{obj}} = \omega R$$

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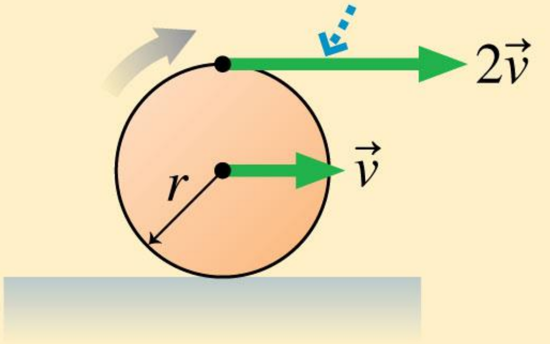
Summary: Applications

Rolling motion

For an object that rolls without slipping,

$$v = \omega R$$

The velocity of a point at the top of the object is twice that of the center.



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Summary

GENERAL PRINCIPLES

Newton's Second Law for Rotational Motion

If a net torque τ_{net} acts on an object, the object will experience an angular acceleration given by $\alpha = \tau_{\text{net}}/I$, where I is the object's moment of inertia about the rotation axis.

This law is analogous to Newton's second law for linear motion, $\vec{a} = \vec{F}_{\text{net}}/m$.

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Summary

IMPORTANT CONCEPTS

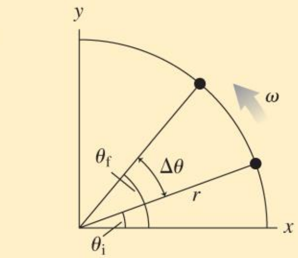
Describing circular motion

We define new variables for circular motion. By convention, counterclockwise is positive.

Angular displacement: $\Delta\theta = \theta_f - \theta_i$

Angular velocity: $\omega = \dfrac{\Delta\theta}{\Delta t}$

Angular acceleration: $\alpha = \dfrac{\Delta\omega}{\Delta t}$



Angles are measured in radians:

$1 \text{ rev} = 360° = 2\pi \text{ rad}$

The angular velocity depends on the frequency and period:

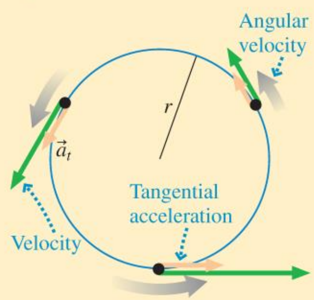
$$\omega = \frac{2\pi}{T} = 2\pi f$$

Relating linear and circular motion quantities

Linear and angular speeds are related by: $v = \omega r$

If the particle's speed is increasing, it will also have a tangential acceleration $\vec{a}_t$ directed tangent to the circle and an angular acceleration α.

Angular and tangential accelerations are related by: $a_t = \alpha r$



The **moment of inertia** is the rotational equivalent of mass. For an object made up of particles of masses $m_1, m_2, \ldots$ at distances $r_1, r_2, \ldots$ from the axis, the moment of inertia is

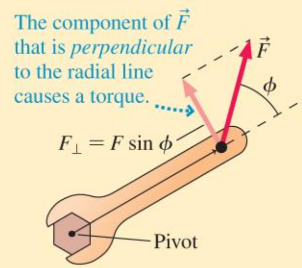
$$I = m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + \cdots = \sum mr^2$$

Torque

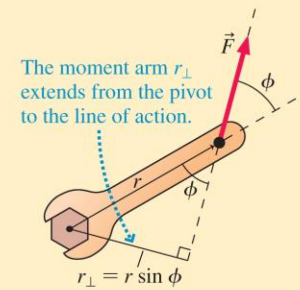
A force causes an object to undergo a linear acceleration, a torque causes an object to undergo an angular acceleration.

There are two interpretations of torque:

Interpretation 1: $\tau = rF_\perp$



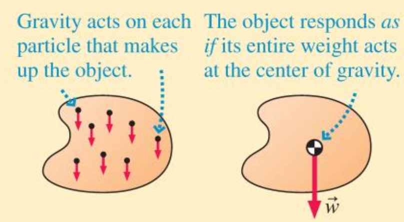
Interpretation 2: $\tau = r_\perp F$



Both interpretations give the same expression for the magnitude of the torque: $\tau = rF \sin\phi$

Center of gravity

The **center of gravity** of an object is the point at which gravity can be considered to be acting.



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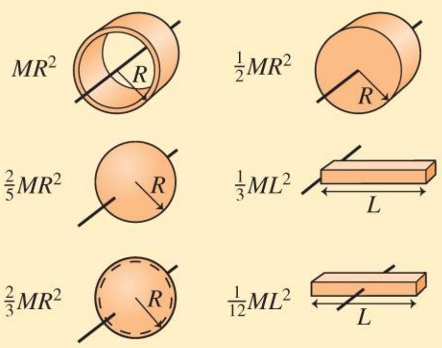
Summary

APPLICATIONS

Moments of inertia of common shapes



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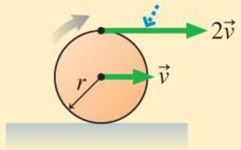

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