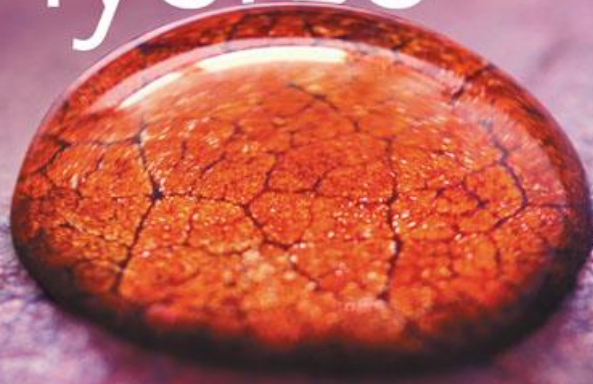


THIRD EDITION

college physics

a strategic approach



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Lecture Presentation

Chapter 6

Circular Motion, Orbits, and Gravity

Suggested Videos for Chapter 6

- **Prelecture Videos**

- *Forces and Apparent Forces*
- *Solving Circular Motion Problems*
- *Orbits and Gravity*

- **Class Videos**

- *Forces in Circular Motion*
- *Car Driving over a Rise*

- **Video Tutor Solutions**

- *Circular Motion, Orbits, and Gravity*

- **Video Tutor Demos**

- *Ball Leaves Circular Track*

Suggested Simulations for Chapter 6

- **ActivPhysics**

- 4.2–4.6

- **PhETs**

- *Ladybug Motion 2D*
- *Ladybug Revolution*
- *Gravity Force Lab*
- *My Solar System*

Chapter 6 Circular Motion, Orbits, and Gravity



Chapter Goal: To learn about motion in a circle, including orbital motion under the influence of a gravitational force.

Chapter 6 Preview

Looking Ahead: Circular Motion

- An object moving in a circle has an acceleration toward the center, so there must be a net force toward the center as well.



- How much force does it take to swing the girl in a circle? You'll learn how to solve such problems.

Chapter 6 Preview

Looking Ahead: Apparent Forces

- The riders feel pushed out. This isn't a real force, though it is often called centrifugal force; it's an **apparent force**.



- This apparent force makes the riders “feel heavy.” You’ll learn to calculate their apparent weight.

Chapter 6 Preview

Looking Ahead: Gravity and Orbits

- The space station appears to float in space, but gravity is pulling down on it quite forcefully.



- You'll learn **Newton's law of gravity**, and you'll see how the force of gravity keeps the station in orbit.

Chapter 6 Preview

Looking Ahead

Circular Motion

An object moving in a circle has an acceleration toward the center, so there must be a net force toward the center as well.



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Gravity and Orbits

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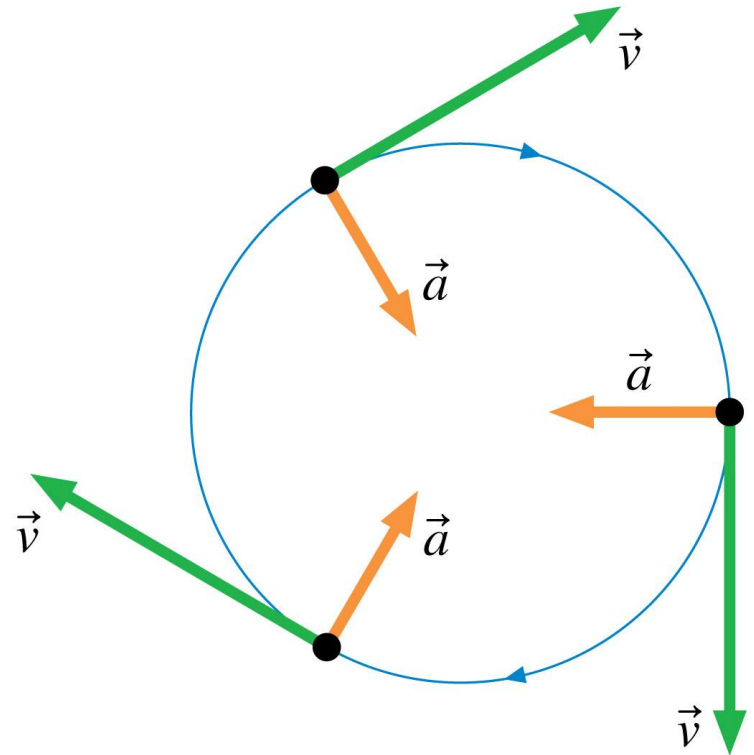
You'll learn **Newton's law of gravity**, and you'll see how the force of gravity keeps the station in orbit.

Text: p. 160

Chapter 6 Preview

Looking Back: Centripetal Acceleration

- In Section 3.8, you learned that an object moving in a circle at a constant speed experiences an acceleration directed toward the center of the circle.



- In this chapter, you'll learn how to extend Newton's second law, which relates acceleration to the forces that cause it, to this type of acceleration.

Chapter 6 Preview

Stop to Think

A softball pitcher is throwing a pitch. At the instant shown, the ball is moving in a circular arc at a steady speed. At this instant, the acceleration is

- A. Directed up.
- B. Directed down.
- C. Directed left.
- D. Directed right.
- E. Zero.



Reading Question 6.1

For uniform circular motion, the acceleration

- A. Is parallel to the velocity.
- B. Is directed toward the center of the circle.
- C. Is larger for a larger orbit at the same speed.
- D. Is always due to gravity.
- E. Is always negative.

Reading Question 6.1

For uniform circular motion, the acceleration

- A. Is parallel to the velocity.
- ✓ B. Is directed toward the center of the circle.
- C. Is larger for a larger orbit at the same speed.
- D. Is always due to gravity.
- E. Is always negative.

Reading Question 6.2

When a car turns a corner on a level road, which force provides the necessary centripetal acceleration?

- A. Friction
- B. Normal force
- C. Gravity
- D. Tension
- E. Air resistance

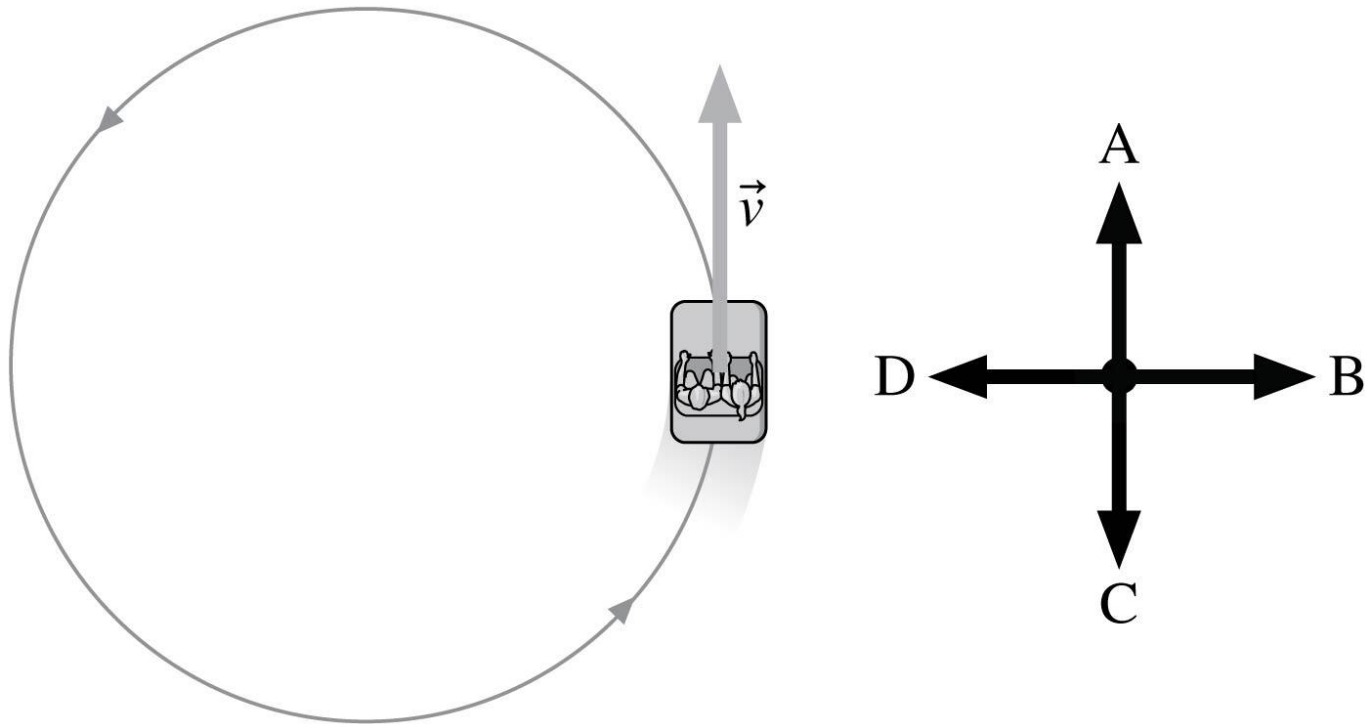
Reading Question 6.2

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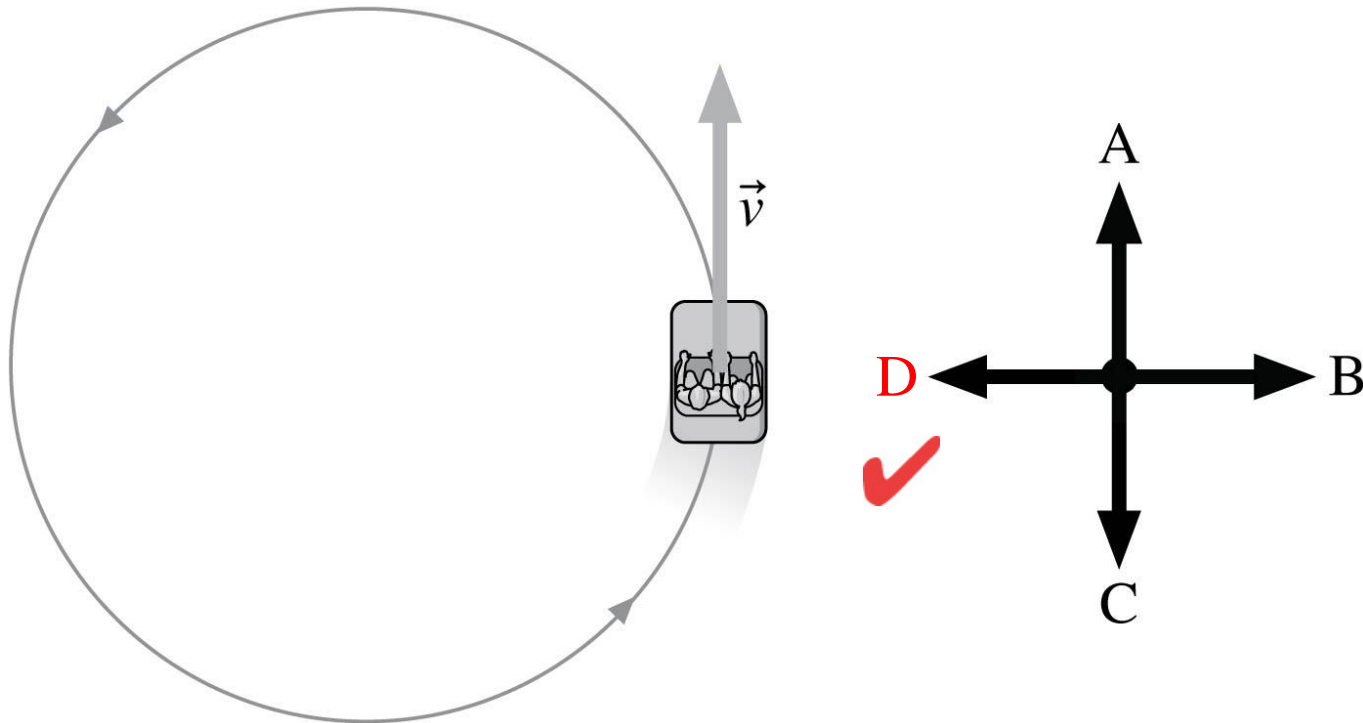
Reading Question 6.3

A passenger on a carnival ride rides in a car that spins in a horizontal circle as shown at right. At the instant shown, which arrow gives the direction of the net force on one of the riders?



Reading Question 6.3

A passenger on a carnival ride rides in a car that spins in a horizontal circle as shown at right. At the instant shown, which arrow gives the direction of the net force on one of the riders?



Reading Question 6.4

How many times per day does the International Space Station—or any satellite in a similar low orbit—go around the earth?

- A. 1
- B. 3
- C. 5
- D. 15

Reading Question 6.4

How many times per day does the International Space Station—or any satellite in a similar low orbit—go around the earth?

A. 1

B. 3

C. 5

 D. 15


Reading Question 6.5

Newton's law of gravity describes the gravitational force between

- A. The earth and the moon.
- B. The earth and the sun.
- C. The sun and the planets.
- D. A person and the earth.
- E. All of the above.

Reading Question 6.5

Newton's law of gravity describes the gravitational force between

- A. The earth and the moon.
- B. The earth and the sun.
- C. The sun and the planets.
- D. A person and the earth.
-  E. All of the above.

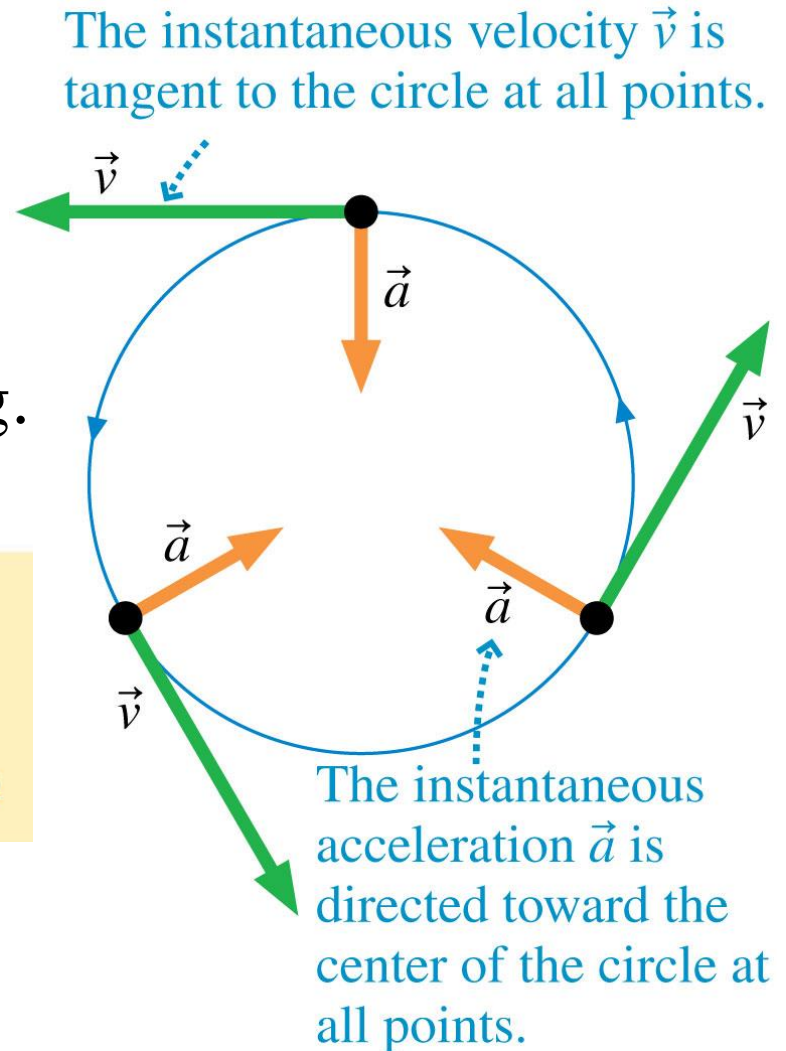
Section 6.1 Uniform Circular Motion

Velocity and Acceleration in Uniform Circular Motion

- Although the *speed* of a particle in uniform circular motion is constant, its *velocity* is not constant because the *direction* of the motion is always changing.

$$a = \frac{v^2}{r}$$

Centripetal acceleration for uniform circular motion




QuickCheck 6.2

A ball at the end of a string is being swung in a horizontal circle. The ball is accelerating because

- A. The speed is changing.
- B. The direction is changing.
- C. The speed and the direction are changing.
- D. The ball is not accelerating.

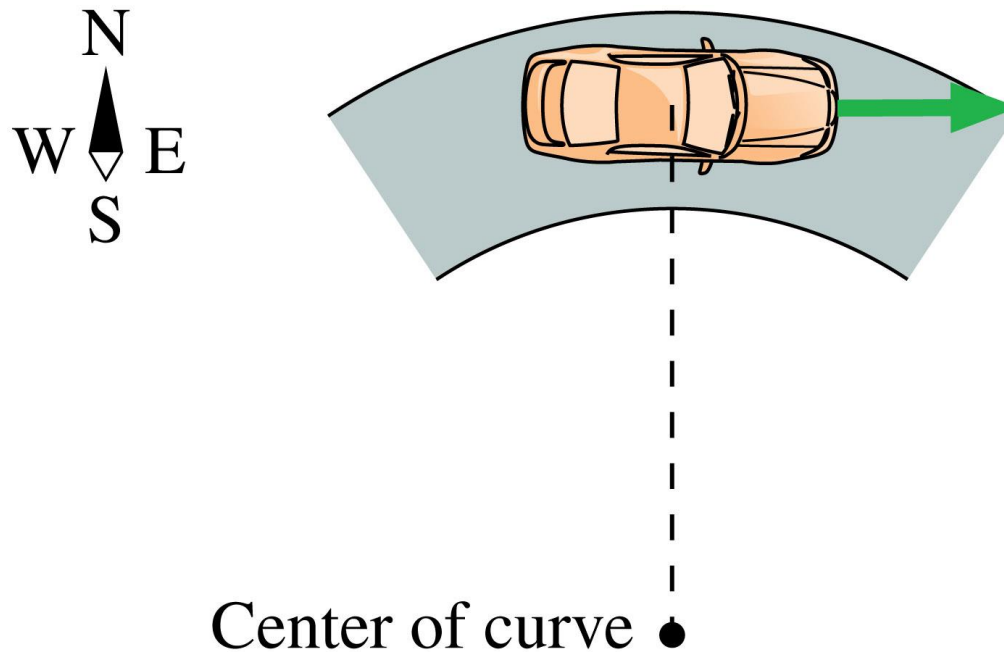
QuickCheck 6.2

A ball at the end of a string is being swung in a horizontal circle. The ball is accelerating because

- A. The speed is changing.
-  B. The direction is changing.
- C. The speed and the direction are changing.
- D. The ball is not accelerating.

Conceptual Example 6.1 Velocity and acceleration in uniform circular motion

A car is turning a tight corner at a constant speed. A top view of the motion is shown in FIGURE 6.2. The velocity vector for the car points to the east at the instant shown. What is the direction of the acceleration?



Conceptual Example 6.1 Velocity and acceleration in uniform circular motion (cont.)

REASON The curve that the car is following is a segment of a circle, so this is an example of uniform circular motion. For uniform circular motion, the acceleration is directed toward the center of the circle, which is to the south.

ASSESS This acceleration is due to a change in direction, not a change in speed. And this matches your experience in a car: If you turn the wheel to the right—as the driver of this car is doing—your car then *changes* its motion toward the right, in the direction of the center of the circle.

QuickCheck 6.3

A ball at the end of a string is being swung in a horizontal circle. What is the direction of the acceleration of the ball?

- A. Tangent to the circle, in the direction of the ball's motion
- B. Toward the center of the circle

QuickCheck 6.3

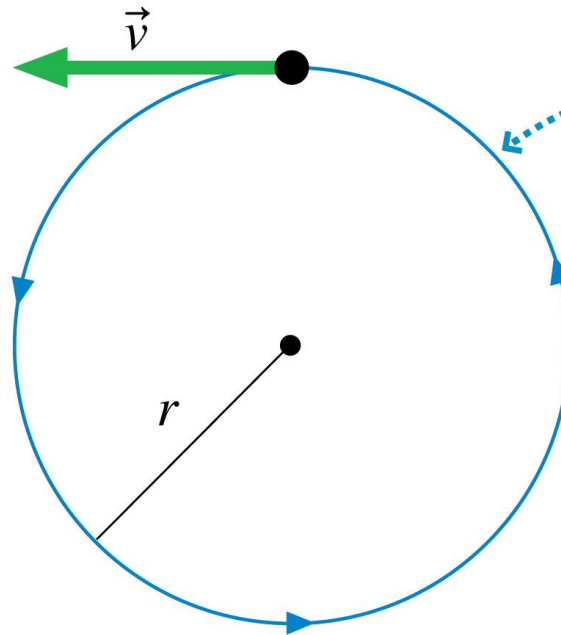
A ball at the end of a string is being swung in a horizontal circle. What is the direction of the acceleration of the ball?

A. Tangent to the circle, in the direction of the ball's motion

 B. Toward the center of the circle

Period, Frequency, and Speed

- The time interval it takes an object to go around a circle one time is called the **period** of the motion.



In one period T , the object travels around the circumference of the circle, a distance of $2\pi r$.

- We can specify circular motion by its frequency, the number of revolutions per second:

$$f = \frac{1}{T}$$

- The SI unit of frequency is inverse seconds, or s^{-1} .

Period, Frequency, and Speed

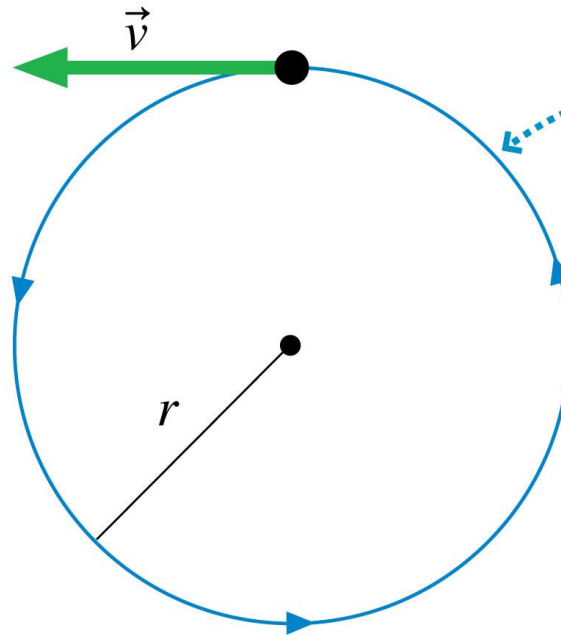
$$v = \frac{2\pi r}{T}$$

- Given Equation 6.2 relating frequency and period, we can also write this equation as

$$v = 2\pi fr$$

- We can combine this with the expression for centripetal acceleration:

$$a = \frac{v^2}{r} = (2\pi f)^2 r = \left(\frac{2\pi}{T}\right)^2 r$$



In one period T , the object travels around the circumference of the circle, a distance of $2\pi r$.

Example 6.2 Spinning some tunes

An audio CD has a diameter of 120 mm and spins at up to 540 rpm. When a CD is spinning at its maximum rate, how much time is required for one revolution? If a speck of dust rides on the outside edge of the disk, how fast is it moving? What is the acceleration?

PREPARE Before we get started, we need to do some unit conversions. The diameter of a CD is given as 120 mm, which is 0.12 m. The radius is 0.060 m. The frequency is given in rpm; we need to convert this to s^{-1} :

$$f = 540 \frac{\text{rev}}{\text{min}} \times \frac{1 \text{ min}}{60 \text{ s}} = 9.0 \frac{\text{rev}}{\text{s}} = 9.0 \text{ s}^{-1}$$

Example 6.2 Spinning some tunes (cont.)

SOLVE The time for one revolution is the period. This is given by Equation 6.2:

$$T = \frac{1}{f} = \frac{1}{9.0 \text{ s}^{-1}} = 0.11 \text{ s}$$

The dust speck is moving in a circle of radius 0.0060 m at a frequency of 9.0 s^{-1} . We can use Equation 6.4 to find the speed:

$$v = 2\pi fr = 2\pi(9.0 \text{ s}^{-1})(0.060 \text{ m}) = 3.4 \text{ m/s}$$

We can then use Equation 6.5 to find the acceleration:

$$a = (2\pi f)^2 r = (2\pi(9.0 \text{ s}^{-1}))^2(0.060 \text{ m}) = 190 \text{ m/s}^2$$

Example 6.2 Spinning some tunes (cont.)

ASSESS If you've watched a CD spin, you know that it takes much less than a second to go around, so the value for the period seems reasonable. The speed we calculate for the dust speck is nearly 8 mph, but for a point on the edge of the CD to go around so many times in a second, it must be moving pretty fast. And we'd expect that such a high speed in a small circle would lead to a very large acceleration.

Example Problem

A hard drive disk rotates at 7200 rpm. The disk has a diameter of 5.1 in (13 cm). What is the speed of a point 6.0 cm from the center axle? What is the acceleration of this point on the disk?

Section 6.2 Dynamics of Uniform Circular Motion

Dynamics of Uniform Circular Motion

- Riders traveling around on a circular carnival ride are accelerating, as we have seen:

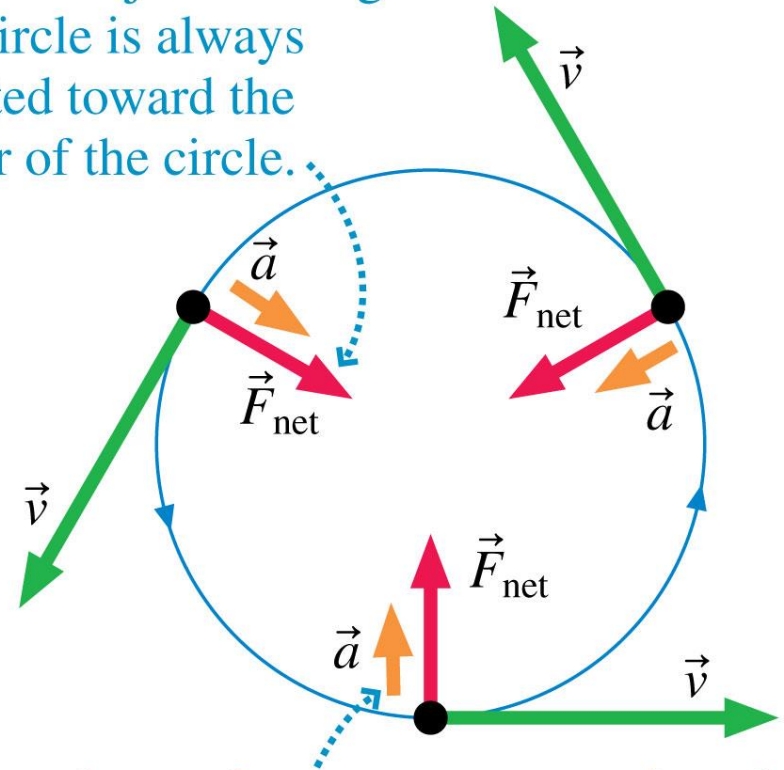
$$\vec{F}_{\text{net}} = m\vec{a} = \left(\frac{mv^2}{r}, \text{ toward center of circle} \right)$$

Net force producing the centripetal acceleration of uniform circular motion

Dynamics of Uniform Circular Motion

- A particle of mass m moving at constant speed v around a circle of radius r must always have a net force of magnitude mv^2/r pointing toward the center of the circle.
- This is **not** a new kind of force: The net force is due to one or more of our familiar forces such as tension, friction, or the normal force.

The net force required to keep the object moving in a circle is always directed toward the center of the circle.



The net force causes a centripetal acceleration.


QuickCheck 6.4

A ball at the end of a string is being swung in a horizontal circle. What force is producing the centripetal acceleration of the ball?

- A. Gravity
- B. Air resistance
- C. Normal force
- D. Tension in the string

QuickCheck 6.4

A ball at the end of a string is being swung in a horizontal circle. What force is producing the centripetal acceleration of the ball?

- A. Gravity
- B. Air resistance
- C. Normal force
-  D. Tension in the string


QuickCheck 6.5

A ball at the end of a string is being swung in a horizontal circle. What is the direction of the net force on the ball?

- A. Tangent to the circle
- B. Toward the center of the circle
- C. There is no net force.

QuickCheck 6.5

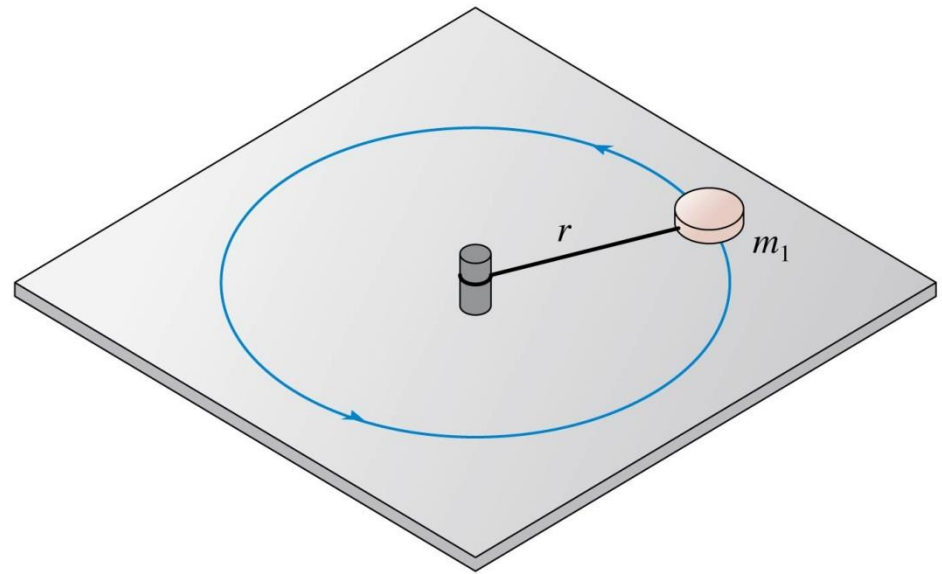
A ball at the end of a string is being swung in a horizontal circle. What is the direction of the net force on the ball?

- A. Tangent to the circle
-  B. **Toward the center of the circle**
- C. There is no net force.

QuickCheck 6.6

An ice hockey puck is tied by a string to a stake in the ice. The puck is then swung in a circle. What force is producing the centripetal acceleration of the puck?

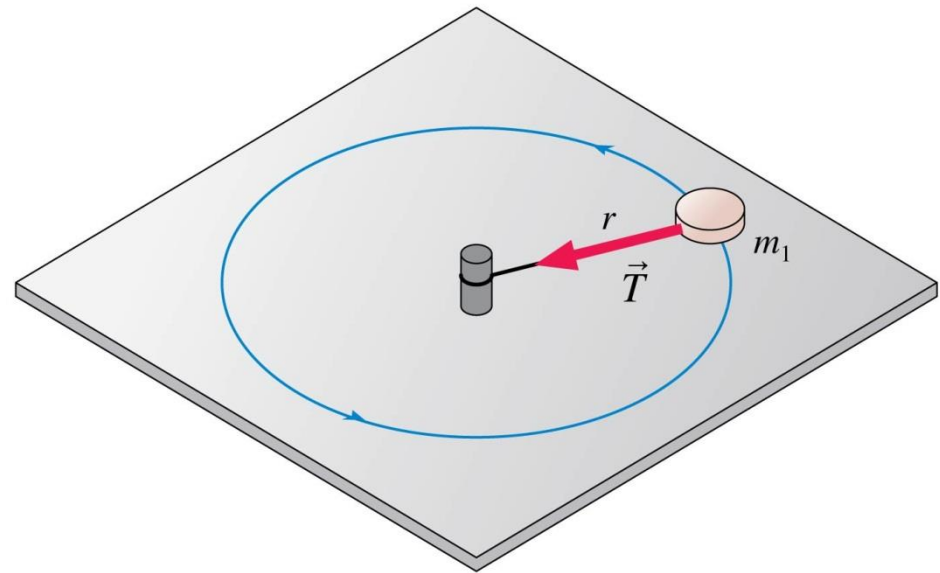
- A. Gravity
- B. Air resistance
- C. Friction
- D. Normal force
- E. Tension in the string



QuickCheck 6.6

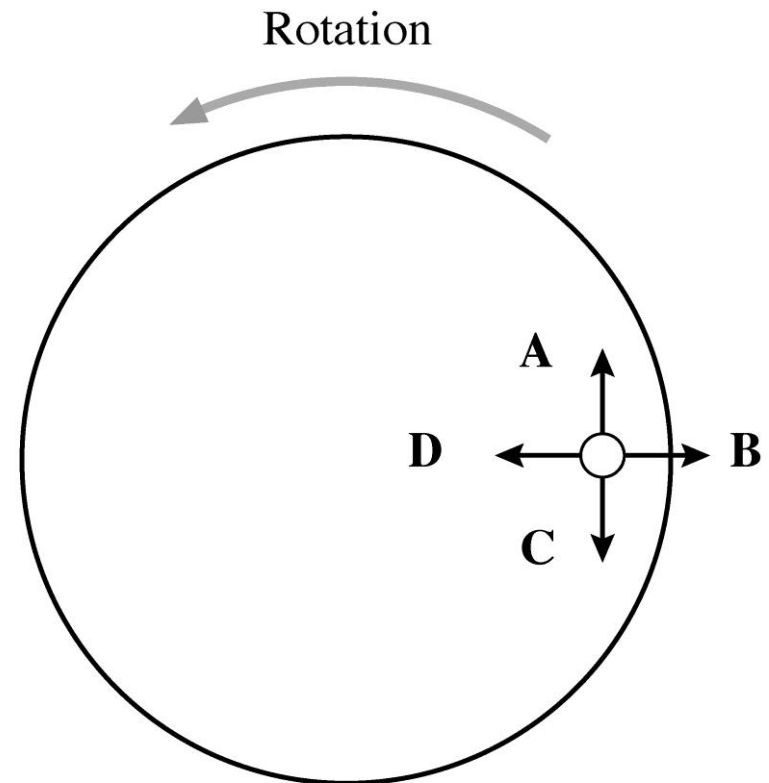
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- A. Gravity
- B. Air resistance
- C. Friction
- D. Normal force
- ✓ E. Tension in the string



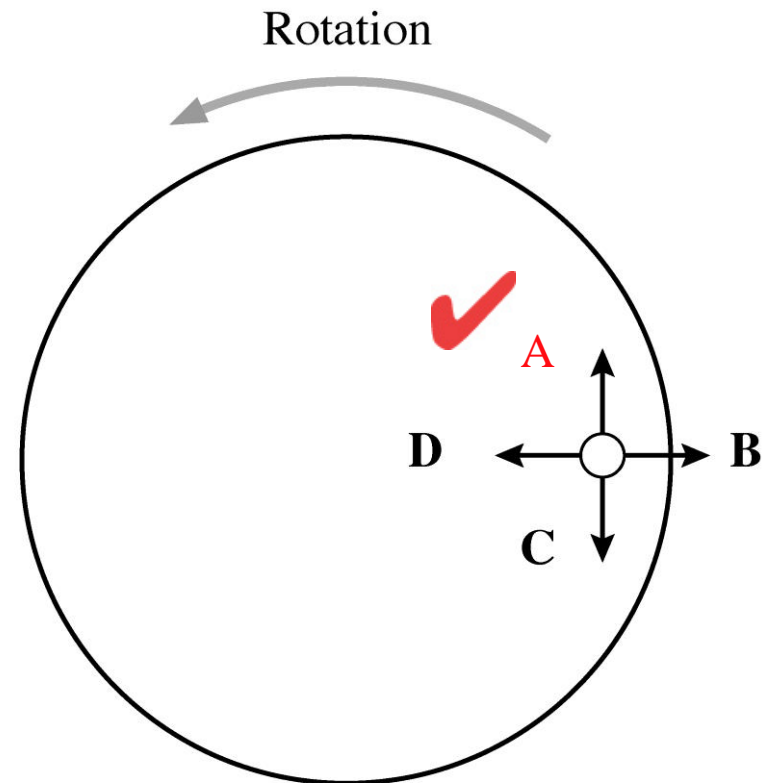
QuickCheck 6.7

A coin is rotating on a turntable; it moves without sliding. At the instant shown in the figure, which arrow gives the direction of the coin's velocity?



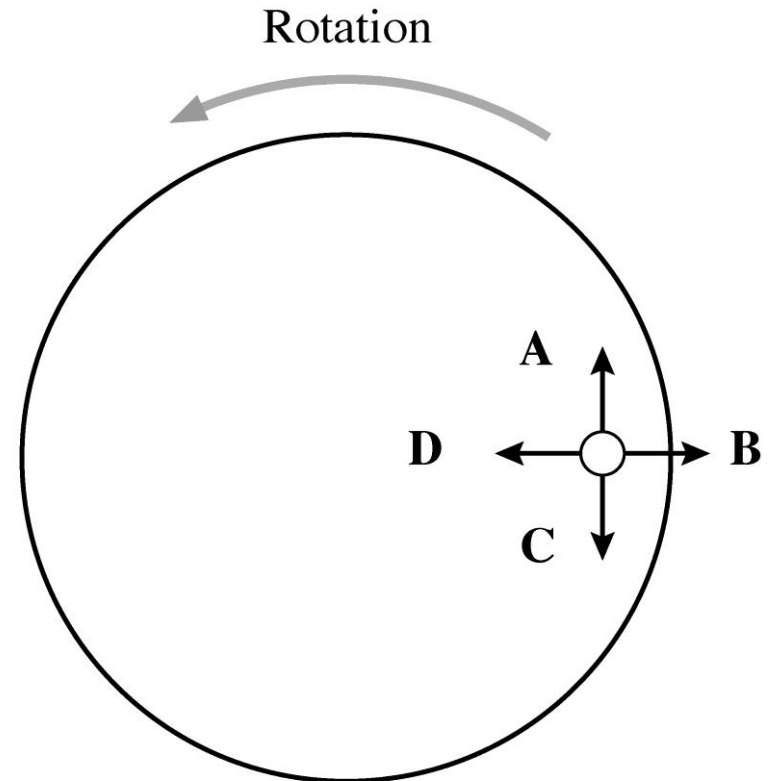
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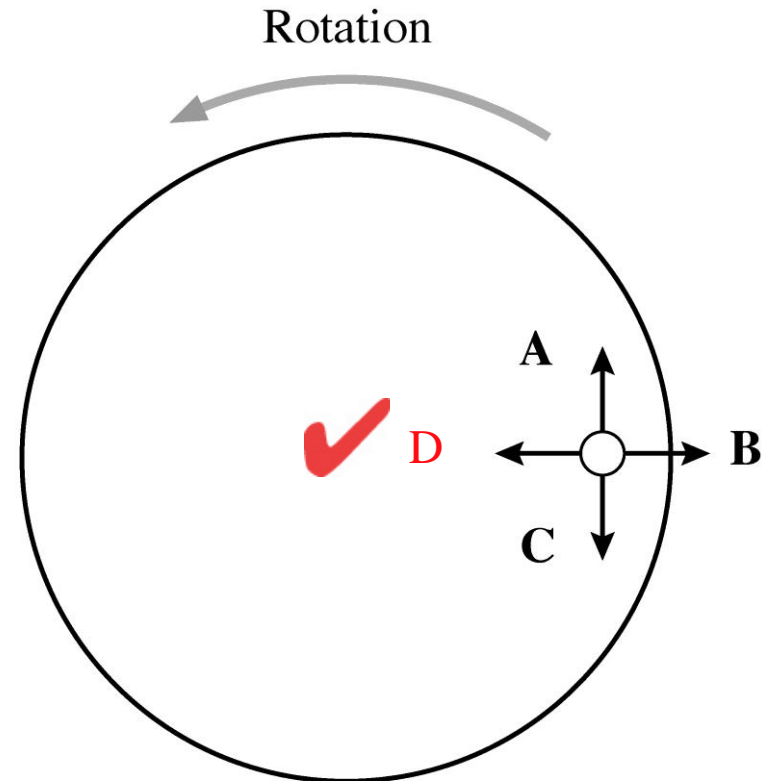
QuickCheck 6.8

A coin is rotating on a turntable; it moves without sliding. At the instant shown in the figure, which arrow gives the direction of the frictional force on the coin?



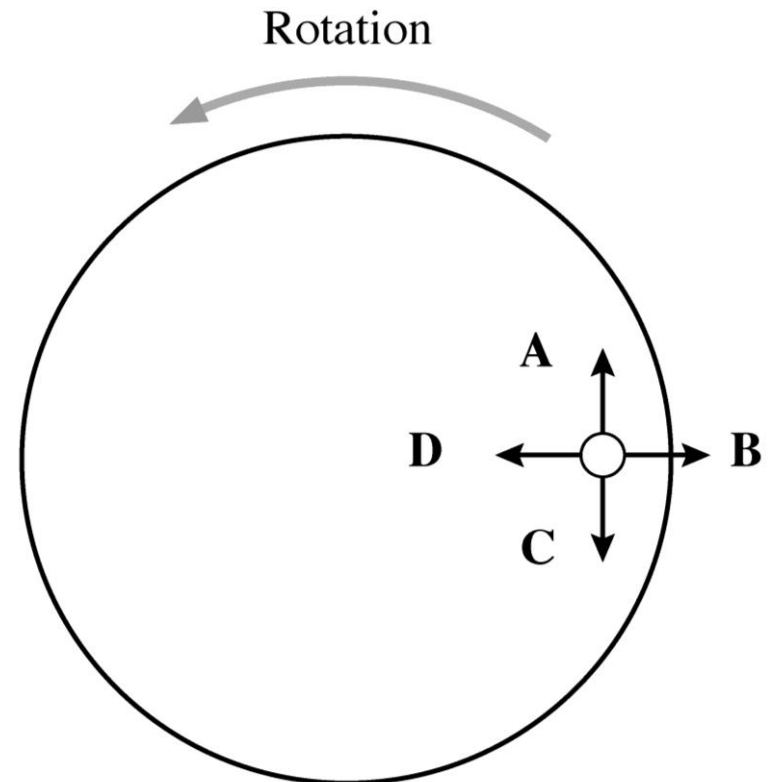
QuickCheck 6.8

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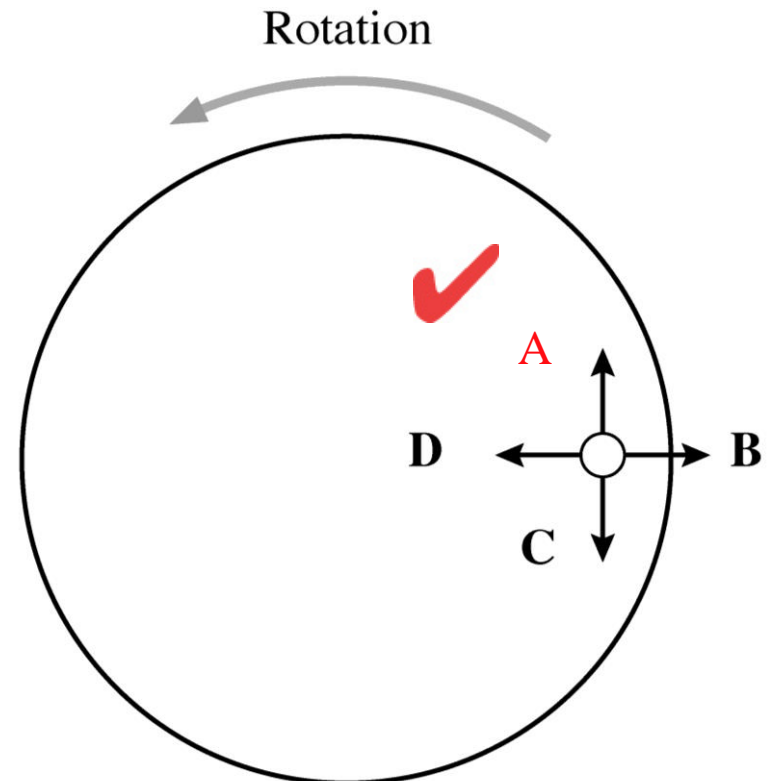
QuickCheck 6.9

A coin is rotating on a turntable; it moves without sliding. At the instant shown, suppose the frictional force disappeared. In what direction would the coin move?



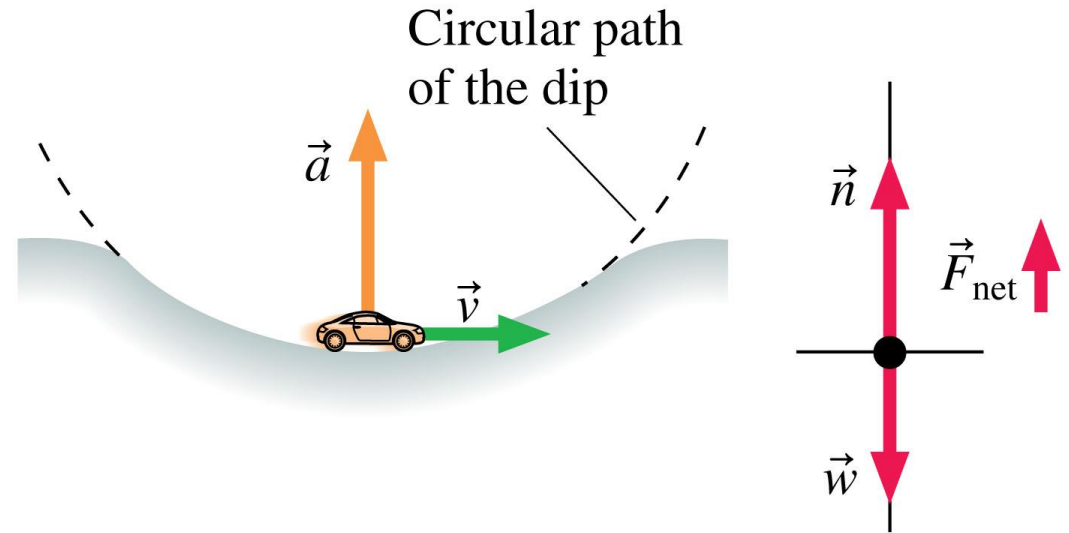
QuickCheck 6.9

A coin is rotating on a turntable; it moves without sliding. At the instant shown, suppose the frictional force disappeared. In what direction would the coin move?



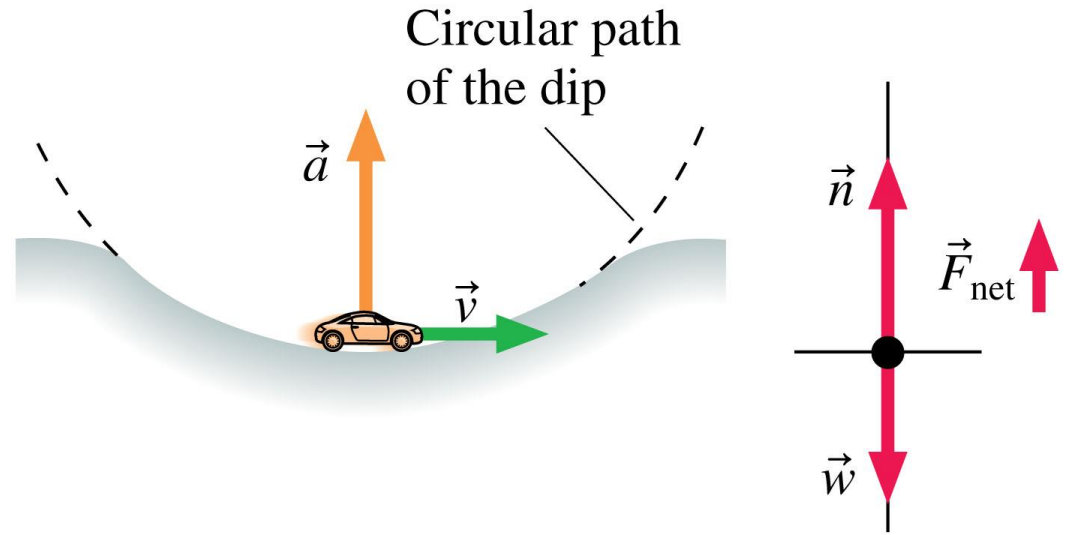
Conceptual Example 6.4 Forces on a car, part I

Engineers design curves on roads to be segments of circles. They also design dips and peaks in roads to be segments of circles with a radius that depends on expected speeds and other factors. A car is moving at a constant speed and goes into a dip in the road. At the very bottom of the dip, is the normal force of the road on the car greater than, less than, or equal to the car's weight?



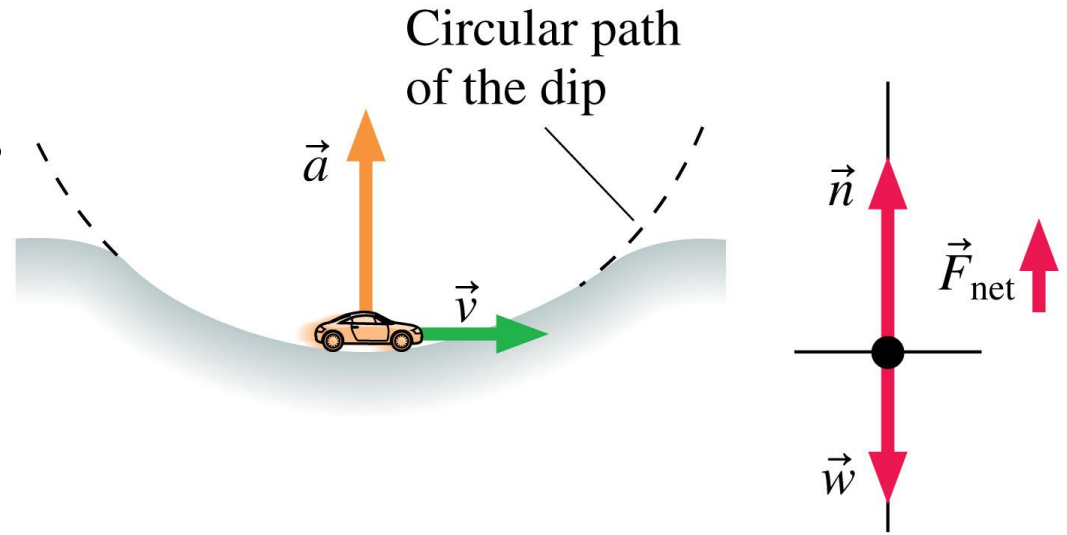
Conceptual Example 6.4 Forces on a car, part I (cont.)

REASON FIGURE 6.6 shows a visual overview of the situation. The car is accelerating, even though it is moving at a constant speed, because its direction is changing. When the car is at the bottom of the dip, the center of its circular path is directly above it and so its acceleration vector points straight up. The free-body diagram of Figure 6.6 identifies the only two forces acting on the car as the normal force, pointing upward, and its weight, pointing downward. Which is larger: n or w ?



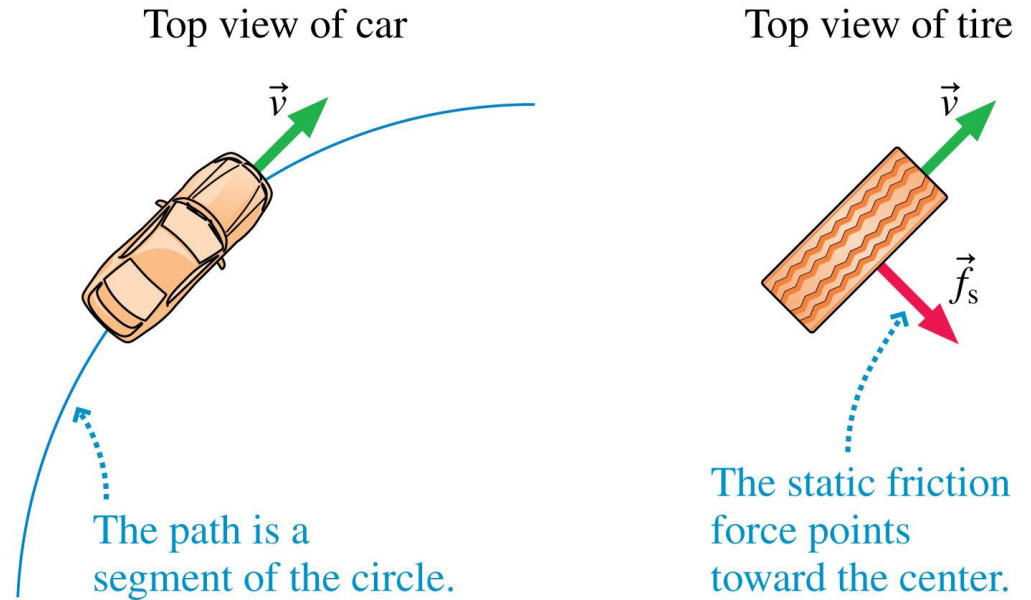
Conceptual Example 6.4 Forces on a car, part I (cont.)

Because \vec{a} points upward, by Newton's second law there must be a net force on the car that also points upward. In order for this to be the case, the free-body diagram shows that the magnitude of the normal force must be *greater* than the weight.



Conceptual Example 6.5 Forces on a car, part II

A car is turning a corner at a constant speed, following a segment of a circle. What force provides the necessary centripetal acceleration?



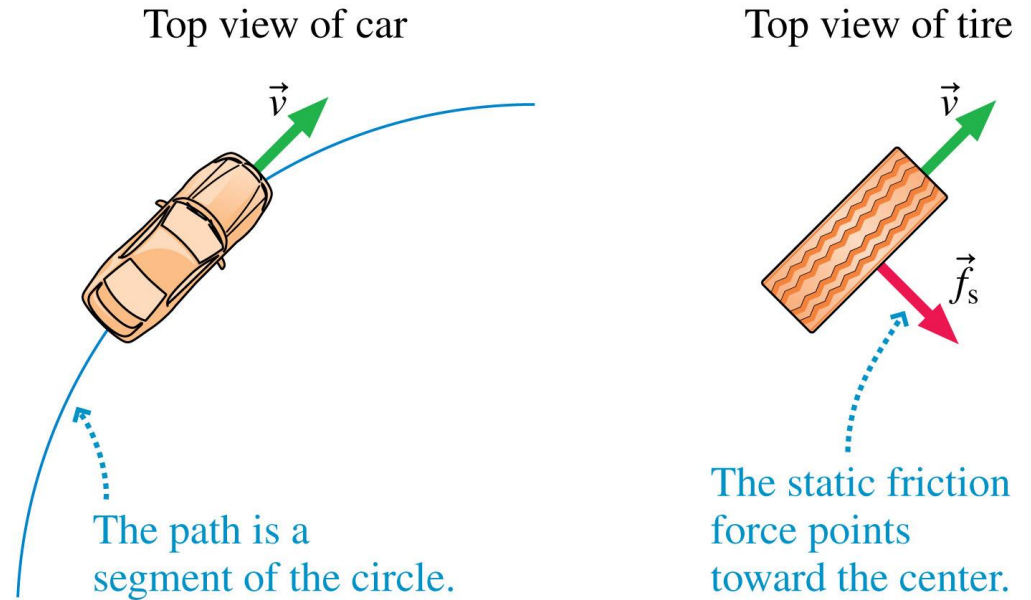
REASON The car moves along a circular arc at a constant speed—uniform circular motion—for the quarter-circle necessary to complete the turn. We know that the acceleration is directed toward the center of the circle. What force or forces can we identify that provide this acceleration?

Conceptual Example 6.5 Forces on a car, part II (cont.)

Imagine you are driving a car on a frictionless road, such as a very icy road. You would not be able to turn a corner.

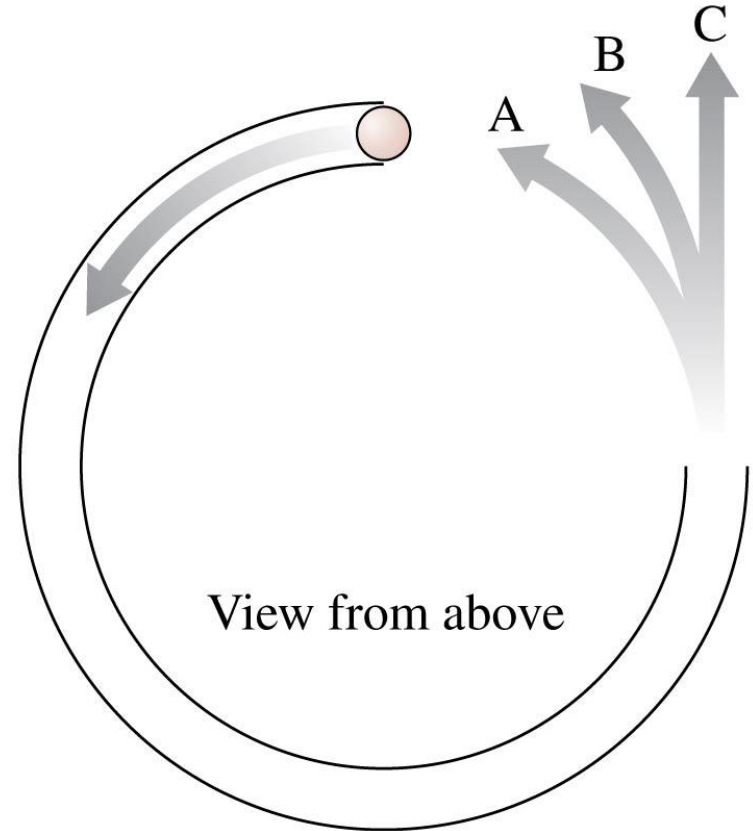
Turning the steering wheel would be of no use. The car would slide straight ahead, in accordance with both

Newton's first law and the experience of anyone who has ever driven on ice! So it must be *friction* that causes the car to turn. The top view of the tire in FIGURE 6.7 shows the force on one of the car's tires as it turns a corner. It must be a *static* friction force, not kinetic, because the tires are not skidding: The points where the tires touch the road are not moving relative to the surface. If you skid, your car won't turn the corner—it will continue in a straight line!



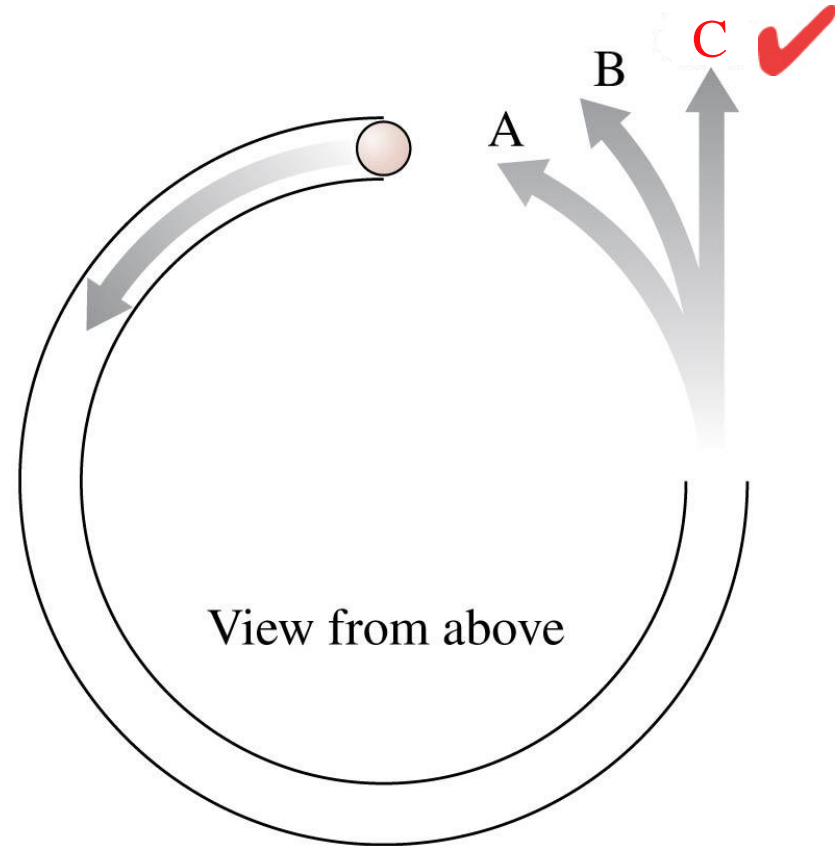
QuickCheck 6.1

A hollow tube lies flat on a table.
A ball is shot through the tube.
As the ball emerges from the other end,
which path does it follow?



QuickCheck 6.1

A hollow tube lies flat on a table.
A ball is shot through the tube.
As the ball emerges from the other end, which path does it follow?



Problem-Solving Strategy 6.1

Circular Dynamics Problems

Circular motion involves an acceleration and thus a net force. We can therefore use techniques very similar to those we've already seen for other Newton's second-law problems.

PREPARE Begin your visual overview with a pictorial representation in which you sketch the motion, define symbols, define axes, and identify what the problem is trying to find. There are two common situations:

- If the motion is in a horizontal plane, like a tabletop, draw the free-body diagram with the circle viewed edge-on, the x -axis pointing toward the center of the circle, and the y -axis perpendicular to the plane of the circle.
- If the motion is in a vertical plane, like a Ferris wheel, draw the free-body diagram with the circle viewed face-on, the x -axis pointing toward the center of the circle, and the y -axis tangent to the circle.

Text: p. 165

Example Problem

In the track and field event known as the hammer throw, an athlete spins a heavy mass in a circle at the end of a chain. Once the mass gets moving at a good clip, the athlete lets go of the chain. The mass flies off in a parabolic arc; the winner is the one who gets the maximum distance. For male athletes, the “hammer” is a mass of 7.3 kg at the end of a 1.2-m chain. A world-class thrower can get the hammer up to a speed of 29 m/s. If an athlete swings the mass in a horizontal circle centered on the handle he uses to hold the chain, what is the tension in the chain?

Problem-Solving Strategy 6.1

Circular Dynamics Problems (cont.)

SOLVE Newton's second law for uniform circular motion, $\vec{F}_{\text{net}} = (mv^2/r, \text{toward center of circle})$, is a vector equation. Some forces act in the plane of the circle, some act perpendicular to the circle, and some may have components in both directions. In the coordinate system described above, with the x -axis pointing toward the center of the circle, Newton's second law is

$$\sum F_x = \frac{mv^2}{r} \quad \text{and} \quad \sum F_y = 0$$

That is, the net force toward the center of the circle has magnitude mv^2/r while the net force perpendicular to the circle is zero. The components of the forces are found directly from the free-body diagram. Depending on the problem, either:

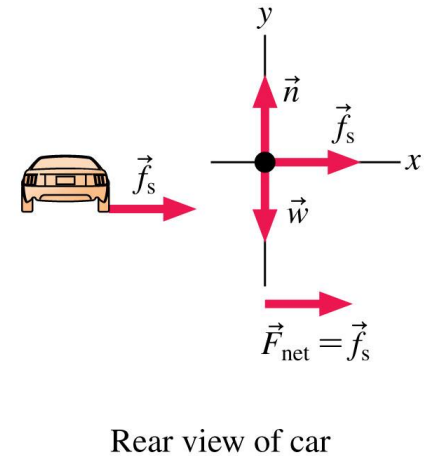
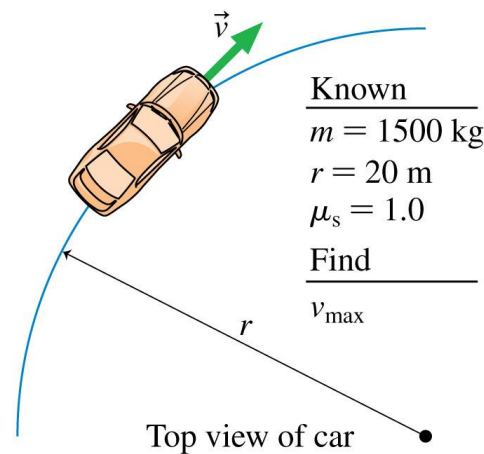
- Use the net force to determine the speed v , then use circular kinematics to find frequencies or other details of the motion.
- Use circular kinematics to determine the speed v , then solve for unknown forces.

ASSESS Make sure your net force points toward the center of the circle. Check that your result has the correct units, is reasonable, and answers the question.

Text: p. 165

Example 6.7 Finding the maximum speed to turn a corner

What is the maximum speed with which a 1500 kg car can make a turn around a curve of radius 20 m on a level (unbanked) road without sliding? (This radius turn is about what you might expect at a major intersection in a city.)



PREPARE The car moves along a circular arc at a constant speed—uniform circular motion—during the turn. The direction of the net force—and thus the static friction force—must point in the direction of the acceleration. The free-body diagram shows the static friction force pointing toward the center of the circle. Because the motion is in a horizontal plane, we've again chosen an x -axis toward the center of the circle and a y -axis perpendicular to the plane of motion.

Example 6.7 Finding the maximum speed to turn a corner (cont.)

SOLVE The only force in the x -direction, toward the center of the circle, is static friction. Newton's second law along the x -axis is

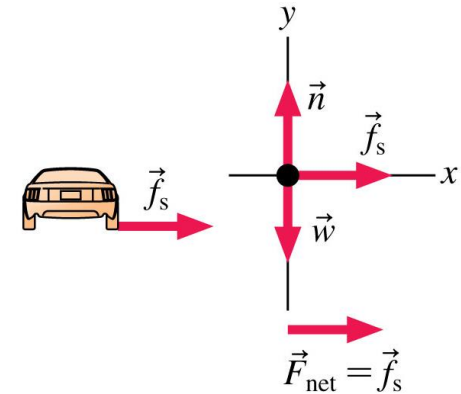
$$\sum F_x = f_s = \frac{mv^2}{r}$$

The only difference between this example and the preceding one is that the tension force toward the center has been replaced by a static friction force toward the center.

Newton's second law in the y -direction is

$$\sum F_y = n - w = ma_y = 0$$

so that $n = w = mg$.



Rear view of car

Example 6.7 Finding the maximum speed to turn a corner (cont.)

The net force toward the center of the circle is the force of static friction. Recall from Equation 5.7 in Chapter 5 that static friction has a maximum possible value:

$$f_{s \max} = \mu_s n = \mu_s mg$$

Because the static friction force has a maximum value, there will be a maximum speed at which a car can turn without sliding. This speed is reached when the static friction force reaches its maximum value $f_{s \max} = \mu_s mg$. If the car enters the curve at a speed higher than the maximum, static friction cannot provide the necessary centripetal acceleration and the car will slide.

Thus the maximum speed occurs at the maximum value of the force of static friction, or when

$$f_{s \max} = \frac{mv_{\max}^2}{r}$$

Example 6.7 Finding the maximum speed to turn a corner (cont.)

Using the known value of $f_{s \max}$, we find

$$\frac{mv_{\max}^2}{r} = f_{s \max} = \mu_s mg$$

Rearranging, we get

$$v_{\max}^2 = \mu_s gr$$

For rubber tires on pavement, we find from Table 5.2 that $\mu_s = 1.0$. We then have

$$v_{\max} = \sqrt{\mu_s gr} = \sqrt{(1.0)(9.8 \text{ m/s}^2)(20 \text{ m})} = 14 \text{ m/s}$$

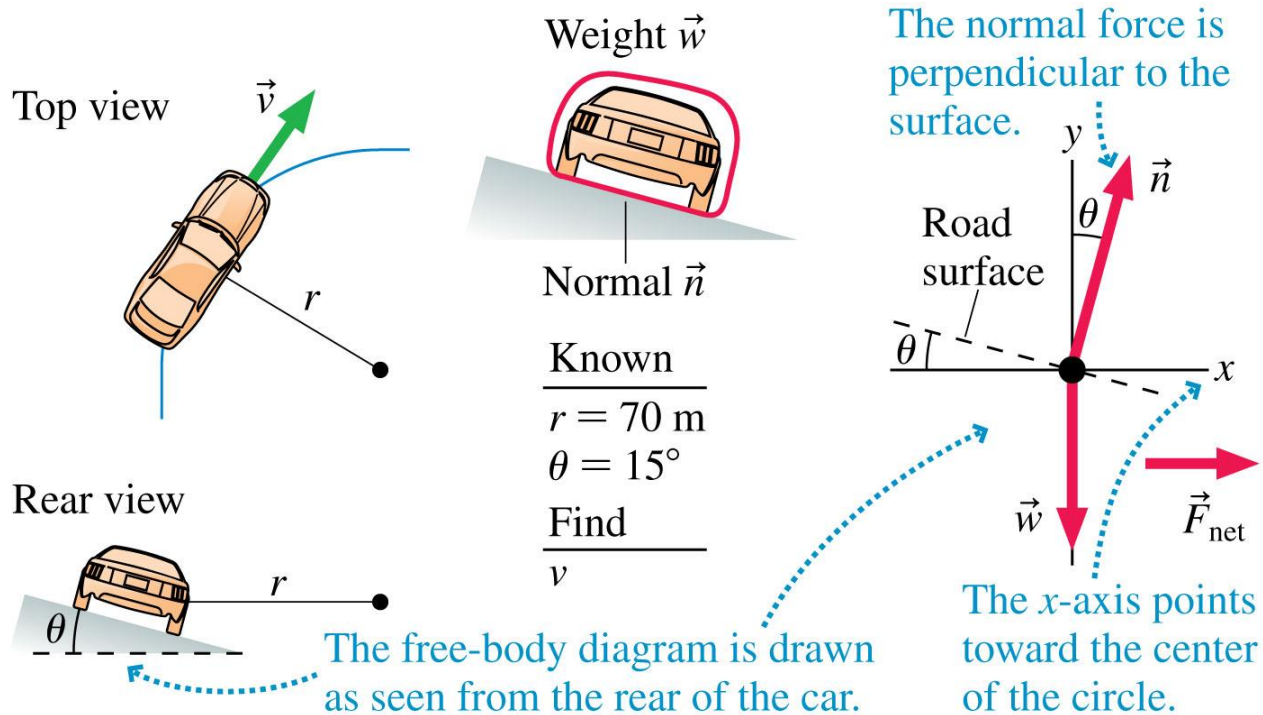
Example 6.7 Finding the maximum speed to turn a corner (cont.)

ASSESS $14 \text{ m/s} \approx 30 \text{ mph}$, which seems like a reasonable upper limit for the speed at which a car can go around a curve without sliding. There are two other things to note about the solution:

- The car's mass canceled out. The maximum speed *does not* depend on the mass of the vehicle, though this may seem surprising.
- The final expression for v_{max} *does* depend on the coefficient of friction and the radius of the turn. Both of these factors make sense. You know, from experience, that the speed at which you can take a turn decreases if μ_s is less (the road is wet or icy) or if r is smaller (the turn is tighter).

Example 6.8 Finding speed on a banked turn

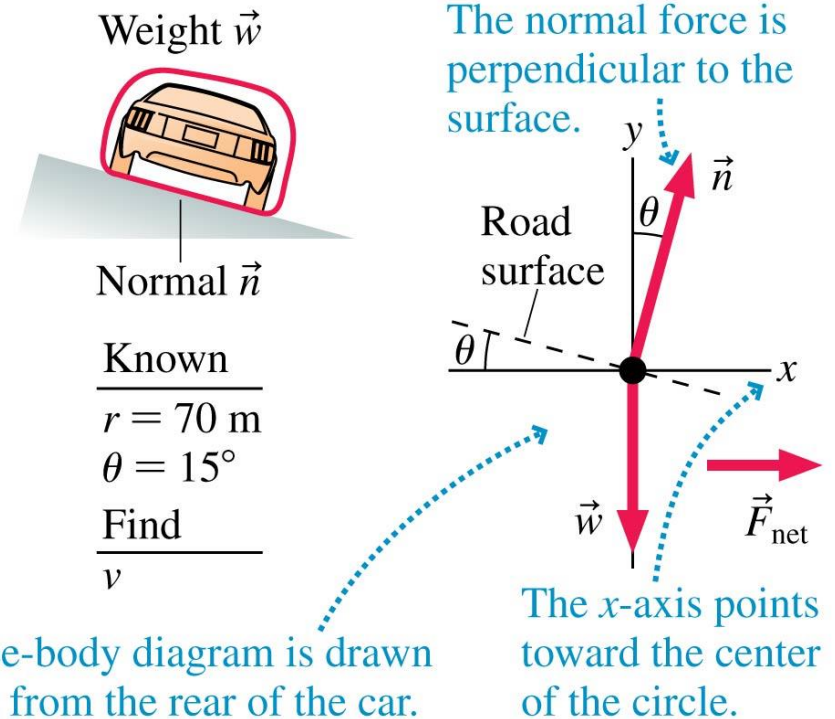
A curve on a racetrack of radius 70 m is banked at a 15° angle. At what speed can a car take this curve without assistance from friction?



Example 6.8 Finding speed on a banked turn (cont.)

PREPARE After drawing the pictorial representation in FIGURE 6.11, we use the force identification diagram to find that, given that there is no friction acting, the only two forces are the normal force and the car's weight. We can then construct the free-body diagram, making sure that we draw the normal force perpendicular to the road's surface.

Even though the car is tilted, it is still moving in a *horizontal* circle. Thus, following Problem-Solving Strategy 6.1, we choose the x -axis to be horizontal and pointing toward the center of the circle.



Example 6.8 Finding speed on a banked turn (cont.)

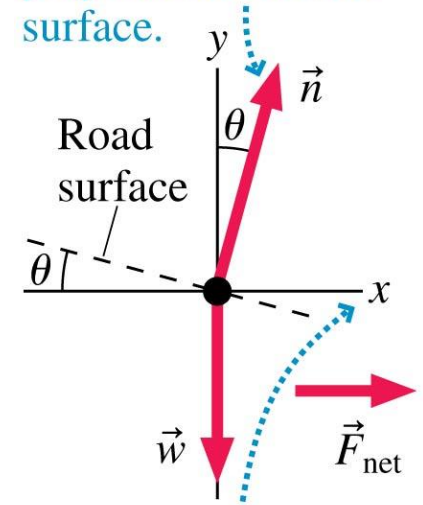
SOLVE Without friction, $n_x = n \sin \theta$ is the only component of force toward the center of the circle. It is this inward component of the normal force on the car that causes it to turn the corner. Newton's second law is

$$\sum F_x = n \sin \theta = \frac{mv^2}{r}$$

$$\sum F_y = n \cos \theta - w = 0$$

where θ is the angle at which the road is banked, and we've assumed that the car is traveling at the correct speed v .

The normal force is perpendicular to the surface.



The x-axis points toward the center of the circle.

Example 6.8 Finding speed on a banked turn (cont.)

From the y -equation,

$$n = \frac{w}{\cos \theta} = \frac{mg}{\cos \theta}$$

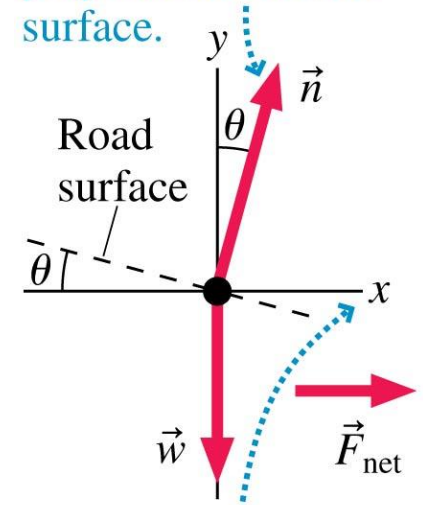
Substituting this into the x -equation and solving for v give

$$\left(\frac{mg}{\cos \theta} \right) \sin \theta = mg \tan \theta = \frac{mv^2}{r}$$

$$v = \sqrt{rg \tan \theta} = 14 \text{ m/s}$$

ASSESS This is ≈ 30 mph, a reasonable speed. Only at this exact speed can the turn be negotiated without reliance on friction forces.

The normal force is perpendicular to the surface.



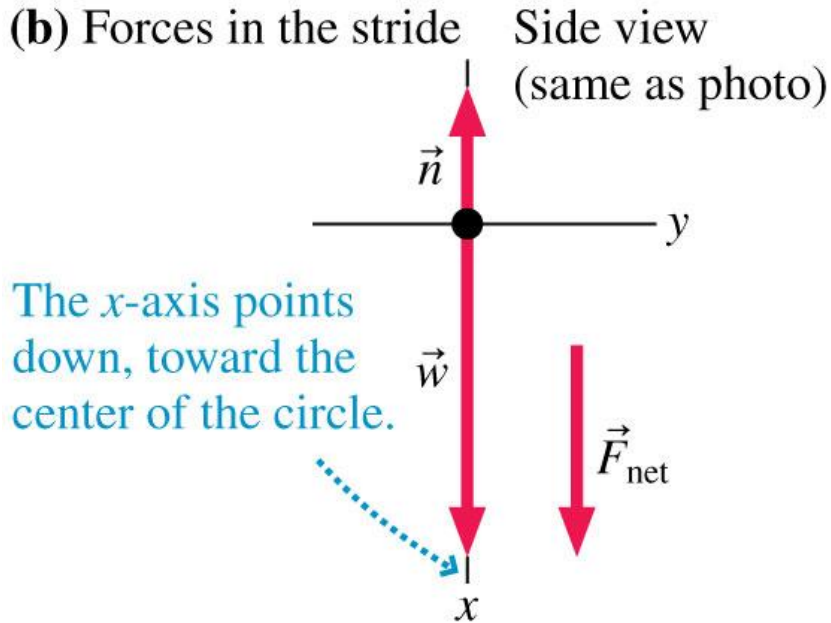
The x -axis points toward the center of the circle.

Example Problem

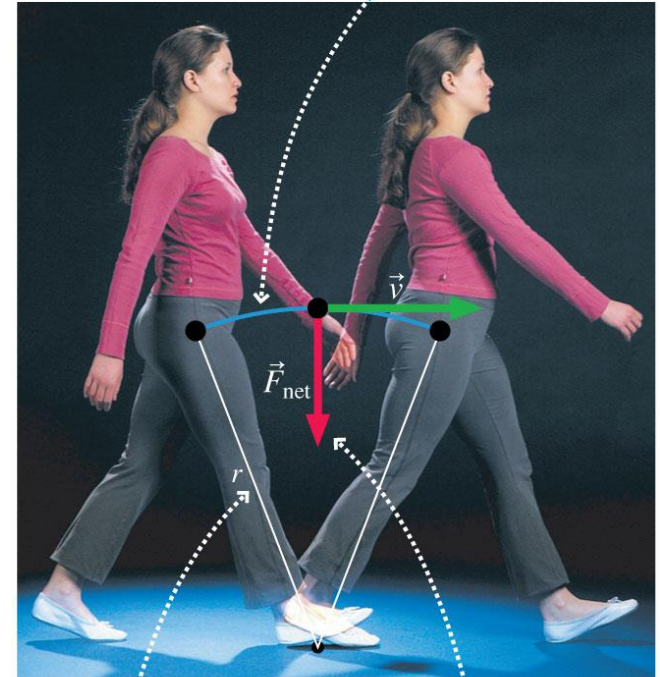
A level curve on a country road has a radius of 150 m. What is the maximum speed at which this curve can be safely negotiated on a rainy day when the coefficient of friction between the tires on a car and the road is 0.40?

Maximum Walking Speed

In a walking gait, your body is in circular motion as you pivot on your forward foot.



(a) Walking stride During each stride, her hip undergoes circular motion.



The radius of the circular motion is the length of the leg from the foot to the hip.

The circular motion requires a force directed toward the center of the circle.

Maximum Walking Speed

Newton's second law for the x -axis is

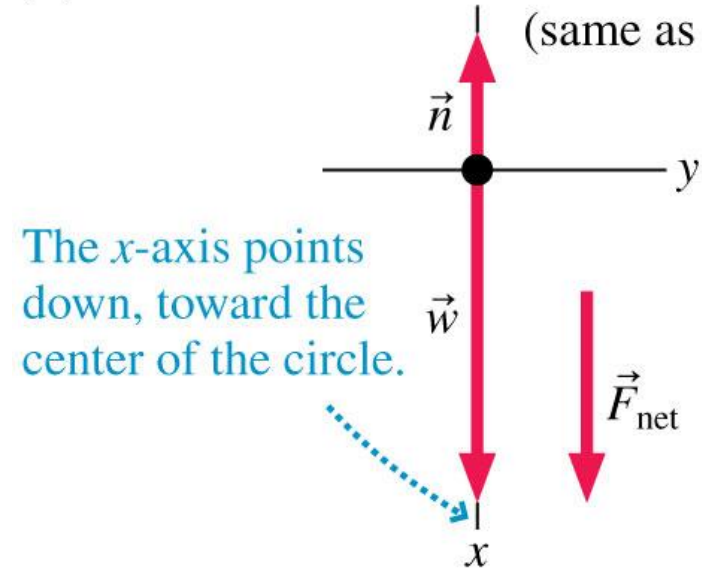
$$\sum F_x = w - n = \frac{mv^2}{r}$$

Setting $n = 0$ in Newton's second law gives

$$w = mg = \frac{mv_{\max}^2}{r}$$

$$v_{\max} = \sqrt{gr}$$

(b) Forces in the stride Side view
(same as photo)

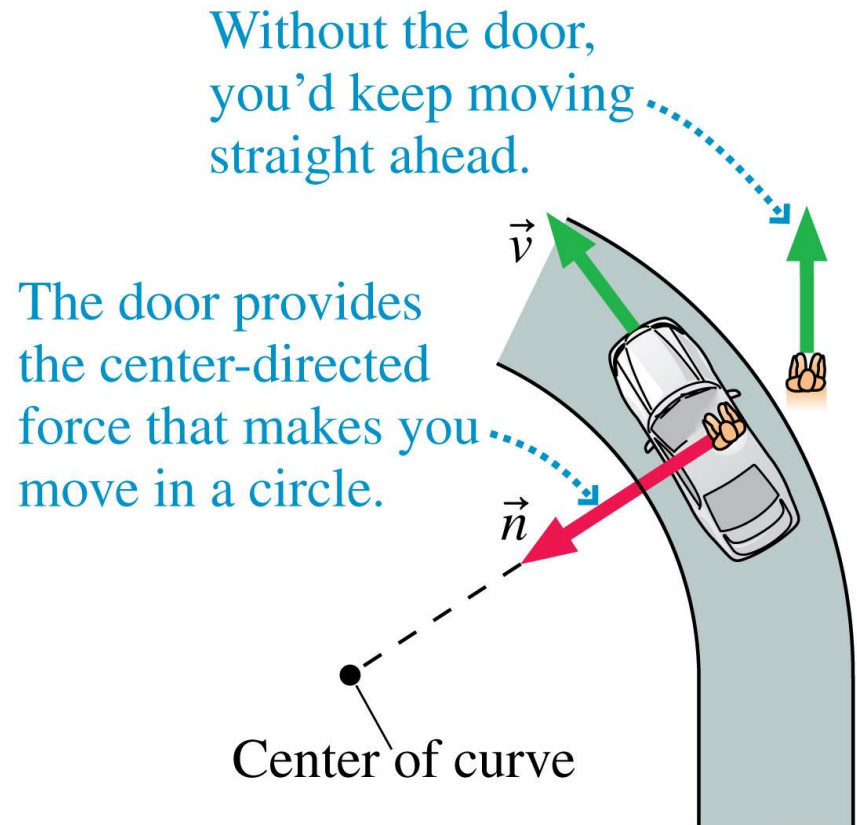


The x -axis points down, toward the center of the circle.

Section 6.3 Apparent Forces in Circular Motion

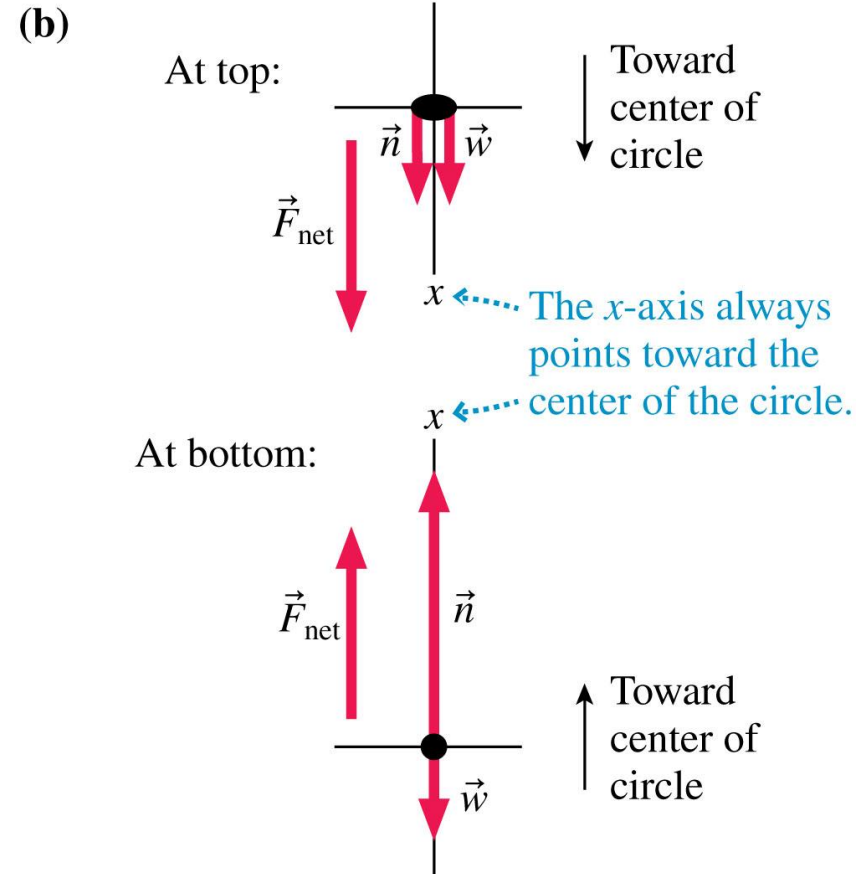
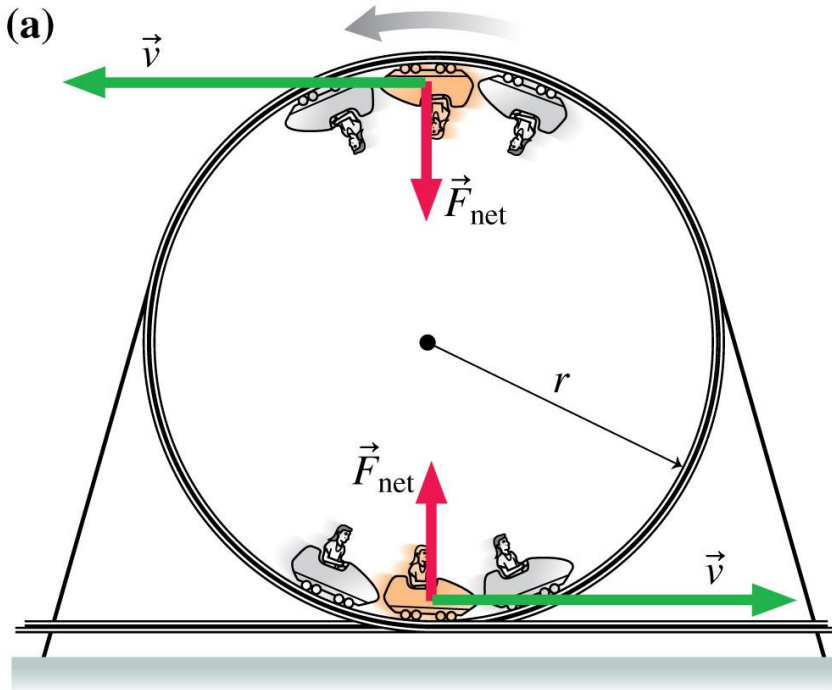
Centrifugal Force?

- If you are a passenger in a car that turns a corner quickly, it is the force of the car door, pushing inward toward the center of the curve, that causes you to turn the corner.
- What you feel is your body trying to move ahead in a straight line as outside forces (the door) act to turn you in a circle.



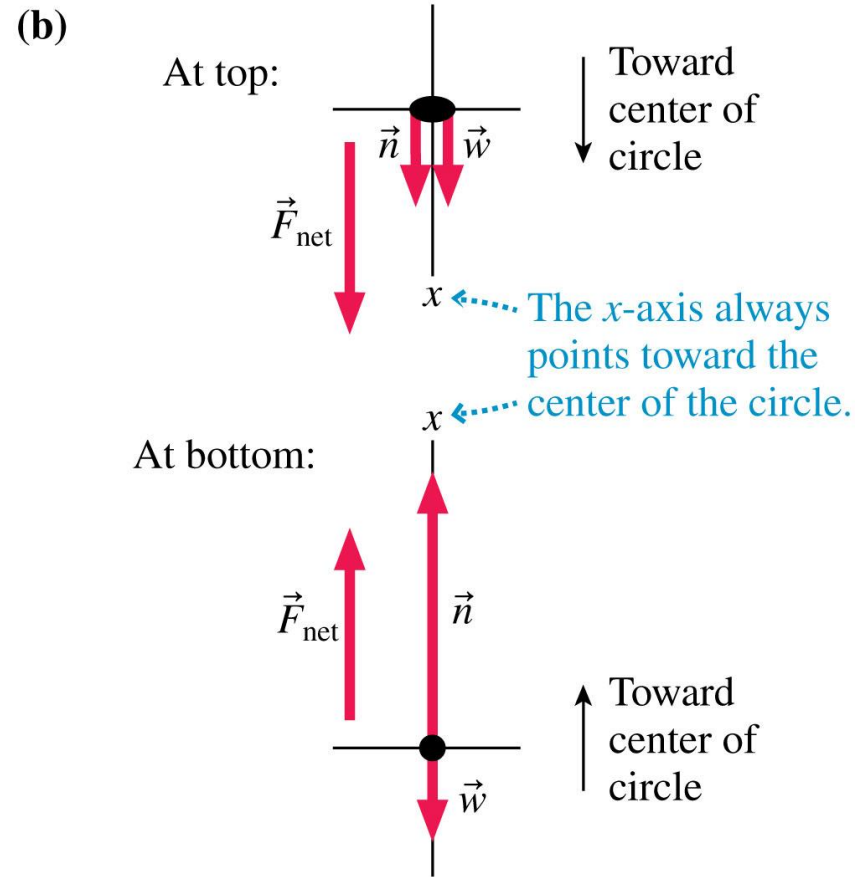
A centrifugal force will never appear on a free-body diagram and never be included in Newton's laws.

Apparent Weight in Circular Motion



Apparent Weight in Circular Motion

- The force you feel, your apparent weight, is the magnitude of the contact force that supports you.
- When the passenger on the roller coaster is at the bottom of the loop:
 - The net force points upward, so $n > w$.
 - Her apparent weight is $w_{\text{app}} = n$, so her apparent weight is greater than her true weight.



Apparent Weight in Circular Motion

- Newton's second law for the passenger at the *bottom* of the circle is

$$\sum F_x = n_x + w_x = n - w = \frac{mv^2}{r}$$

- From this equation, the passenger's apparent weight is

$$w_{\text{app}} = n = w + \frac{mv^2}{r}$$

- Her apparent weight at the bottom is *greater* than her true weight, w .

Apparent Weight in Circular Motion

- Newton's second law for the passenger at the *top* of the circle is

$$\sum F_x = n_x + w_x = n + w = \frac{mv^2}{r}$$

- Note that w_x is now *positive* because the x -axis is directed downward. We can solve for the passenger's apparent weight:

$$w_{\text{app}} = n = \frac{mv^2}{r} - w$$

- If v is sufficiently large, her apparent weight can exceed the true weight.

Apparent Weight in Circular Motion

- As the car goes slower there comes a point where n becomes zero:

$$w_{\text{app}} = n = \frac{mv^2}{r} - w$$

- The speed for which $n = 0$ is called the *critical speed* v_c . Because for n to be zero we must have $mv_c^2/r = w$, the critical speed is

$$v_c = \sqrt{\frac{rw}{m}} = \sqrt{\frac{rmg}{m}} = \sqrt{gr}$$

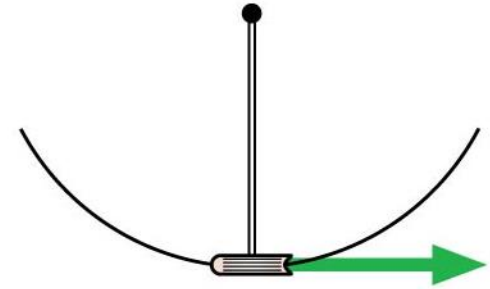
- **The critical speed is the slowest speed at which the car can complete the circle.**

Example Problem

A handful of professional skaters have taken a skateboard through an inverted loop in a full pipe. For a typical pipe with a diameter 14 feet, what is the minimum speed the skater must have at the very top of the loop?

QuickCheck 6.10

A physics textbook swings back and forth as a pendulum. Which is the correct free-body diagram when the book is at the bottom and moving to the right?



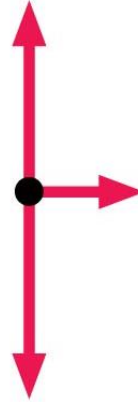
A.



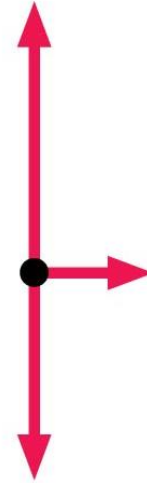
B.



C.



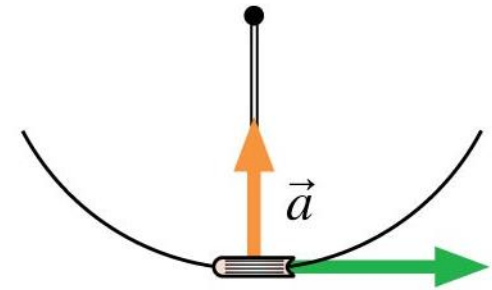
D.



E.

QuickCheck 6.10

A physics textbook swings back and forth as a pendulum. Which is the correct free-body diagram when the book is at the bottom and moving to the right?



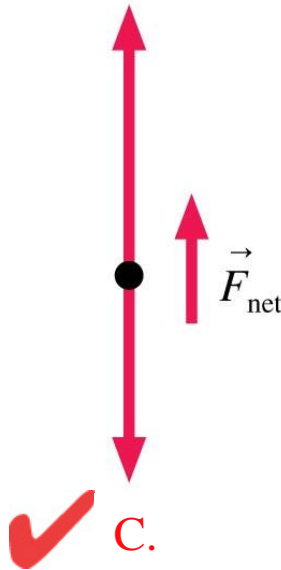
Centripetal acceleration requires an upward force.



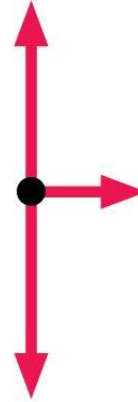
A.



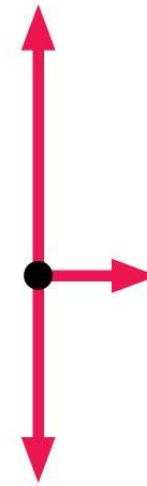
B.



C.



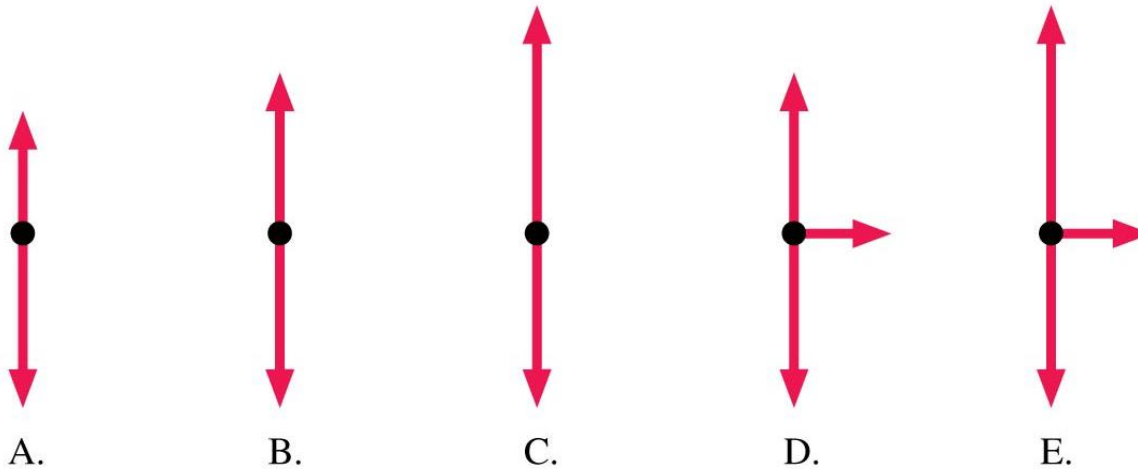
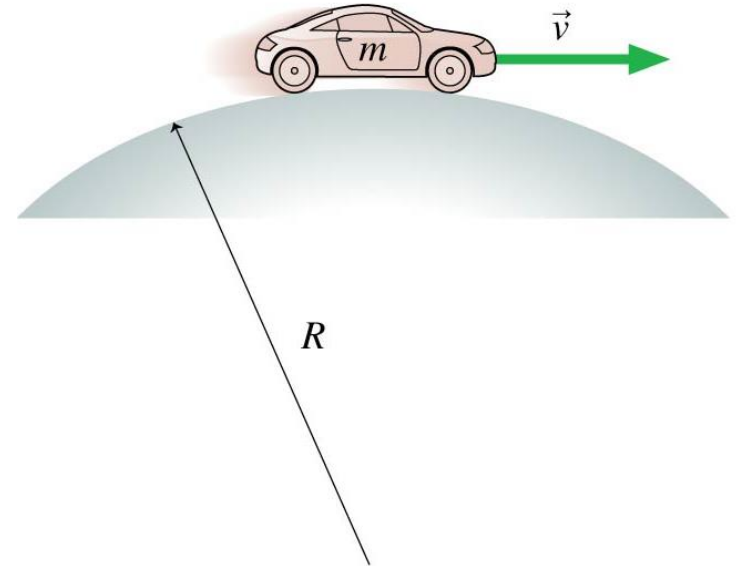
D.



E.

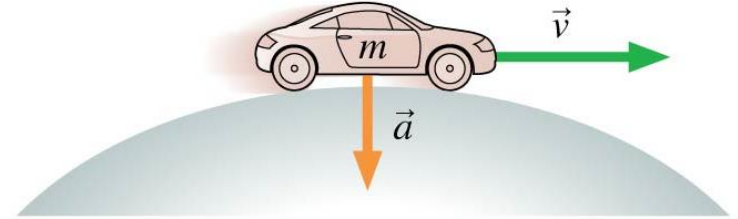
QuickCheck 6.11

A car that's out of gas coasts over the top of a hill at a steady 20 m/s. Assume air resistance is negligible. Which free-body diagram describes the car at this instant?



QuickCheck 6.11

A car that's out of gas coasts over the top of a hill at a steady 20 m/s. Assume air resistance is negligible. Which free-body diagram describes the car at this instant?

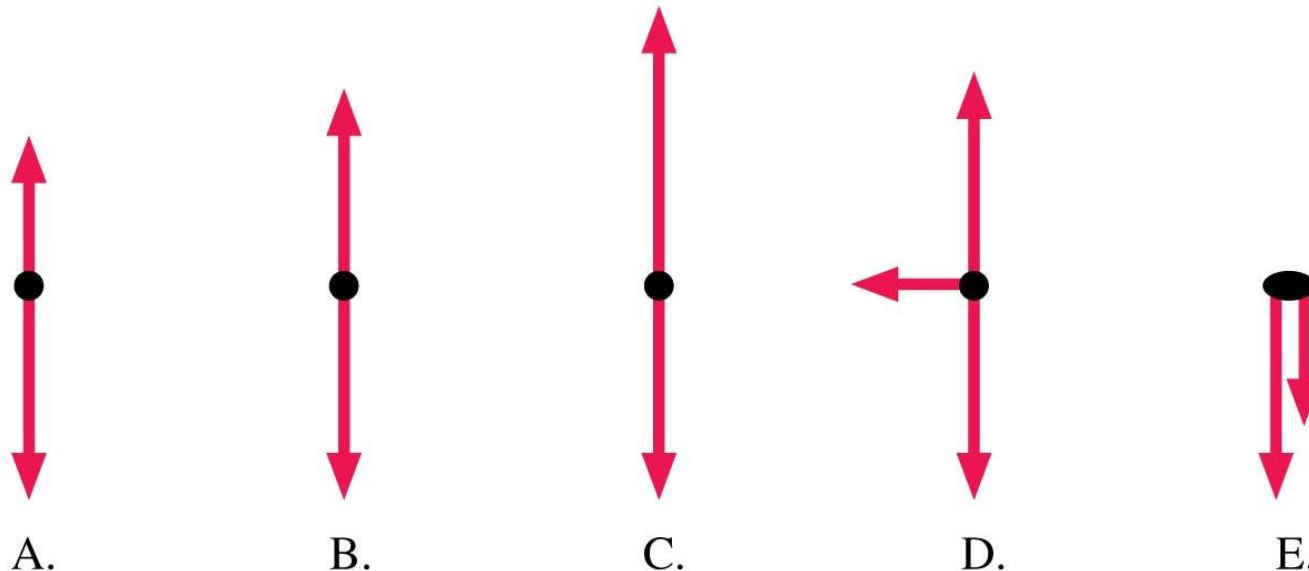


Now the centripetal acceleration points down.

- A. \vec{F}_{net}
- B.
- C.
- D.
- E.

QuickCheck 6.12

A roller coaster car does a loop-the-loop. Which of the free-body diagrams shows the forces on the car at the top of the loop? Rolling friction can be neglected.



A.

B.

C.

D.

E.

QuickCheck 6.12

A roller coaster car does a loop-the-loop. Which of the free-body diagrams shows the forces on the car at the top of the loop? Rolling friction can be neglected.



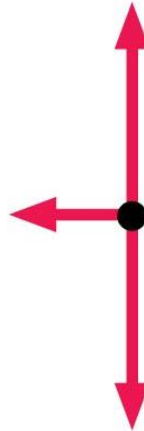
A.



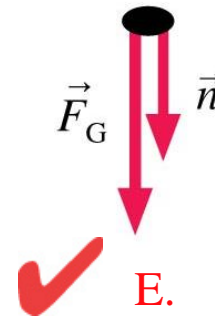
B.



C.



D.



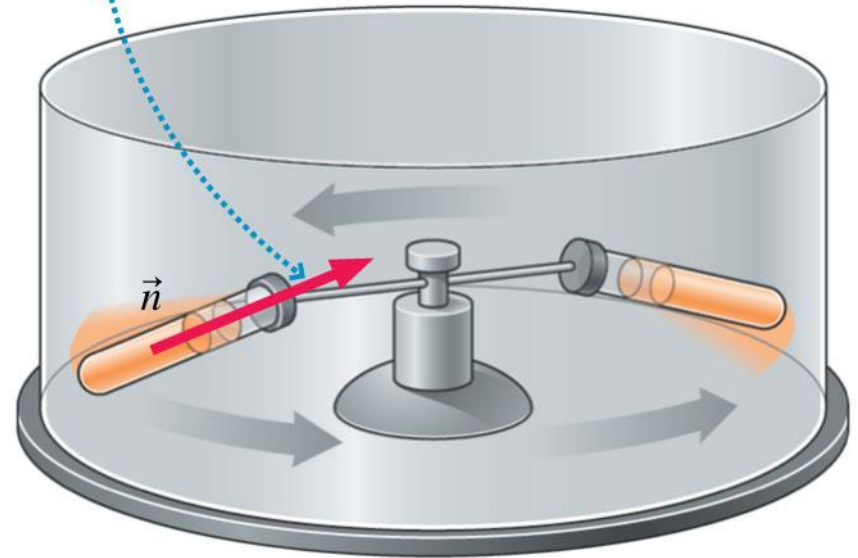
E.

The track is *above* the car, so the normal force of the track pushes down.

Centrifuges



The high centripetal acceleration requires a large normal force, which leads to a large apparent weight.



Try It Yourself: Human Centrifuge

If you spin your arm rapidly in a vertical circle, the motion will produce an effect like that in a centrifuge. The motion will assist outbound blood flow in your arteries and retard inbound blood flow in your veins. There will be a buildup of fluid in your hand that you will be able to see (and feel!) quite easily.



Example 6.10 Analyzing the ultracentrifuge

An 18-cm-diameter ultracentrifuge produces an extraordinarily large centripetal acceleration of $250,000g$, where g is the free-fall acceleration due to gravity. What is its frequency in rpm? What is the apparent weight of a sample with a mass of 0.0030 kg ?

PREPARE The acceleration in SI units is

$$a = 250,000(9.80 \text{ m/s}^2) = 2.45 \times 10^6 \text{ m/s}^2$$

The radius is half the diameter, or $r = 9.0 \text{ cm} = 0.090 \text{ m}$.

SOLVE We can rearrange Equation 6.5 to find the frequency given the centripetal acceleration:

$$f = \frac{1}{2\pi} \sqrt{\frac{a}{r}} = \frac{1}{2\pi} \sqrt{\frac{2.45 \times 10^6 \text{ m/s}^2}{0.090 \text{ m}}} = 830 \text{ rev/s}$$

Example 6.10 Analyzing the ultracentrifuge (cont.)

Converting to rpm, we find

$$830 \frac{\text{rev}}{\text{s}} \times \frac{60 \text{ s}}{1 \text{ min}} = 50,000 \text{ rpm}$$

The acceleration is so high that every force is negligible except for the force that provides the centripetal acceleration. The net force is simply equal to the inward force, which is also the sample's apparent weight:

$$w_{\text{app}} = F_{\text{net}} = ma = (3.0 \times 10^{-3} \text{ kg})(2.45 \times 10^6 \text{ m/s}^2) = 7.4 \times 10^3 \text{ N}$$

The 3 gram sample has an effective weight of about 1700 pounds!

Example 6.10 Analyzing the ultracentrifuge (cont.)

ASSESS Because the acceleration is $250,000g$, the apparent weight is 250,000 times the actual weight. This makes sense, as does the fact that we calculated a very high frequency, which is necessary to give the large acceleration.

QuickCheck 6.13

A coin sits on a turntable as the table steadily rotates counterclockwise. What force or forces act in the plane of the turntable?



Five diagrams (A-E) show a coin on a rotating turntable with different force vectors:

- A.** A single red arrow labeled "Static friction" points radially inward from the coin.
- B.** Two red arrows: one labeled "Static friction" pointing radially inward, and one labeled "Centripetal force" pointing tangentially backward relative to the rotation.
- C.** Two red arrows: one labeled "Static friction" pointing tangentially backward, and one labeled "Centripetal force" pointing radially inward.
- D.** Two red arrows: one labeled "Kinetic friction" pointing tangentially backward, and one labeled "Static friction" pointing radially inward.
- E.** No force vectors are shown, with the text "No forces in this plane" below the diagram.

QuickCheck 6.13

A coin sits on a turntable as the table steadily rotates counterclockwise. What force or forces act in the plane of the turntable?



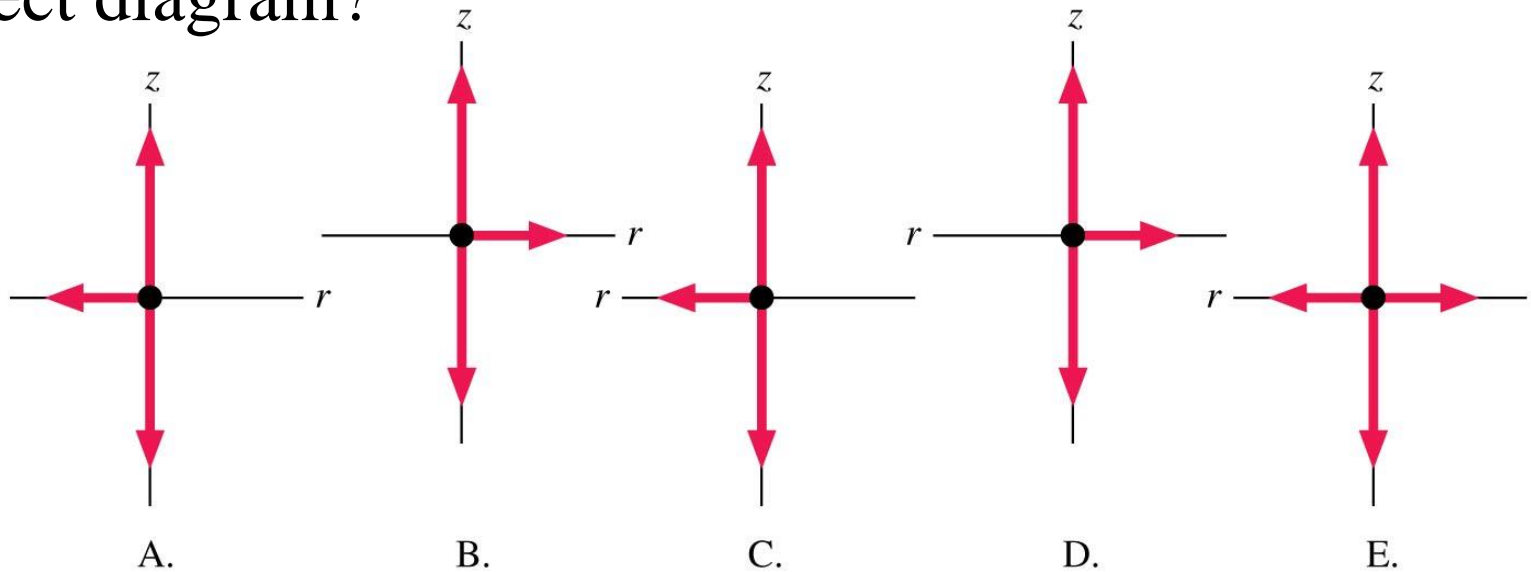
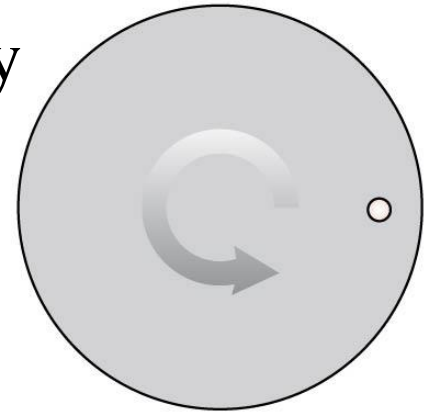
Five diagrams (A-E) show a coin on a rotating turntable with different force vectors:

- A.** A red arrow labeled "Static friction" points from the coin towards the center of the turntable.
- B.** A red arrow labeled "Static friction" points from the coin towards the center, and another red arrow labeled "Centripetal force" points from the center towards the coin.
- C.** A red arrow labeled "Static friction" points from the coin towards the center, and another red arrow labeled "Centripetal force" points from the coin towards the center.
- D.** A red arrow labeled "Kinetic friction" points from the coin towards the center, and another red arrow labeled "Static friction" points from the center towards the coin.
- E.** No force vectors are shown, with the text "No forces in this plane" below the diagram.

A large red checkmark is placed to the left of option A.

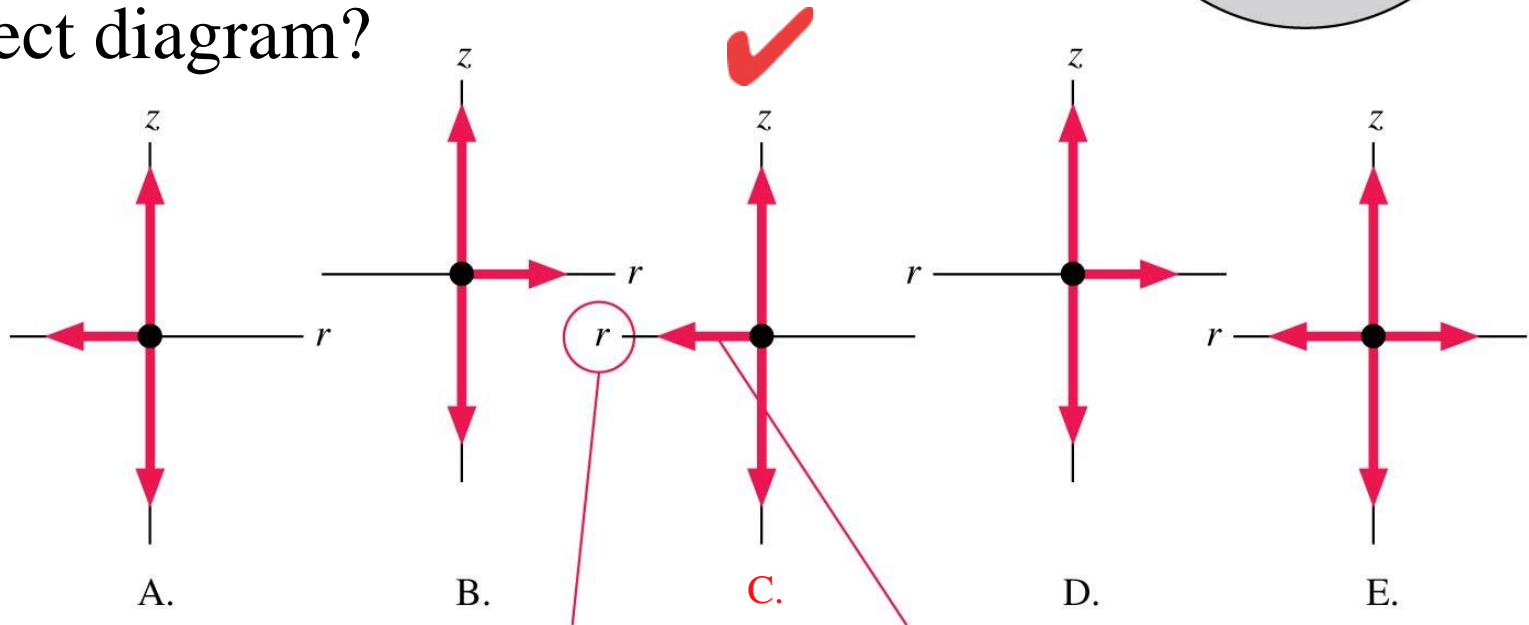
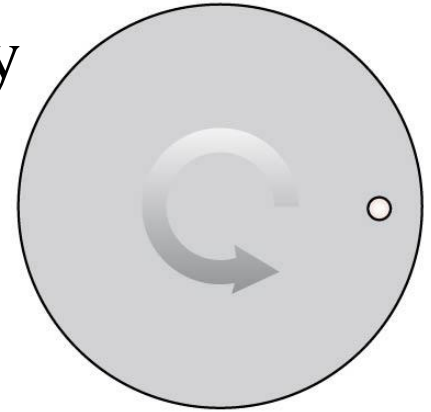
QuickCheck 6.14

A coin sits on a turntable as the table steadily rotates counterclockwise. The free-body diagrams below show the coin from behind, moving away from you. Which is the correct diagram?



QuickCheck 6.14

A coin sits on a turntable as the table steadily rotates counterclockwise. The free-body diagrams below show the coin from behind, moving away from you. Which is the correct diagram?

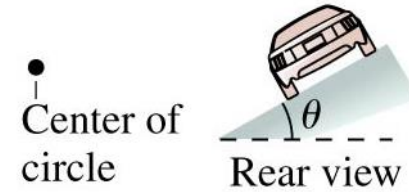


Center of circle is to the left.

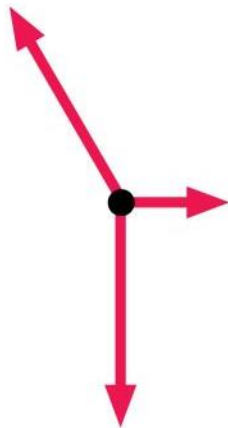
Net force must point to the center of the circle.

QuickCheck 6.15

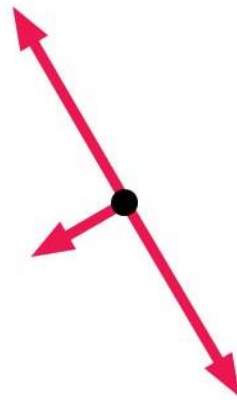
A car turns a corner on a banked road.
Which of the diagrams could be the car's free-body diagram?



A.



B.



C.



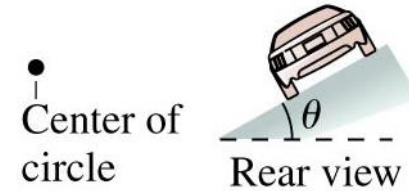
D.



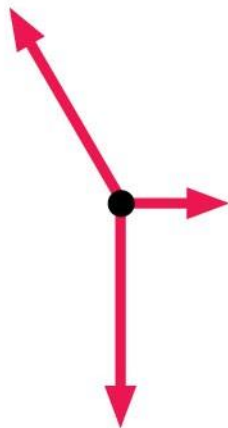
E.

QuickCheck 6.15

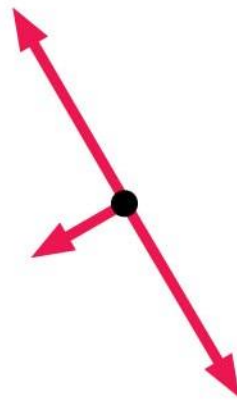
A car turns a corner on a banked road.
Which of the diagrams could be the car's free-body diagram?



A.



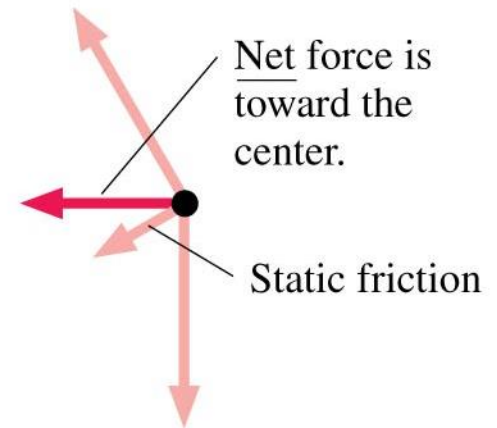
B.



C.



D.

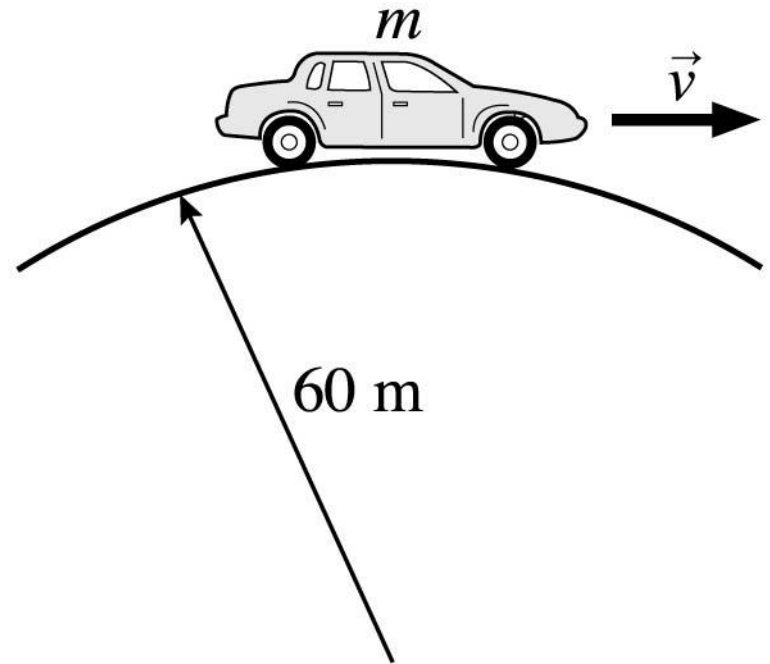


E.



Example Problem

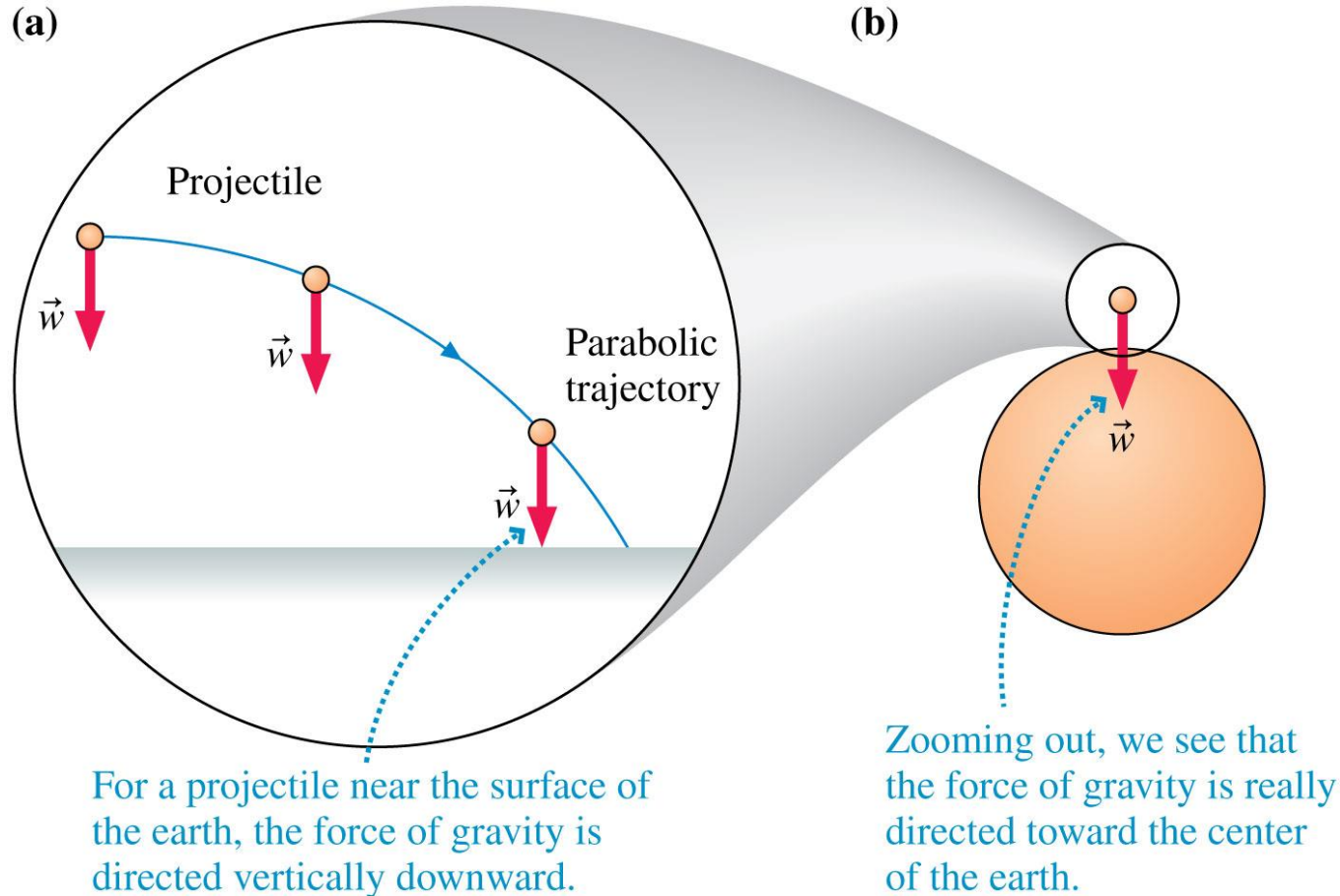
A car of mass 1500 kg goes over a hill at a speed of 20 m/s. The shape of the hill is approximately circular, with a radius of 60 m, as in the figure. When the car is at the highest point of the hill,



- A. What is the force of gravity on the car?
- B. What is the normal force of the road on the car at this point?

Section 6.4 Circular Orbits and Weightlessness

Orbital Motion

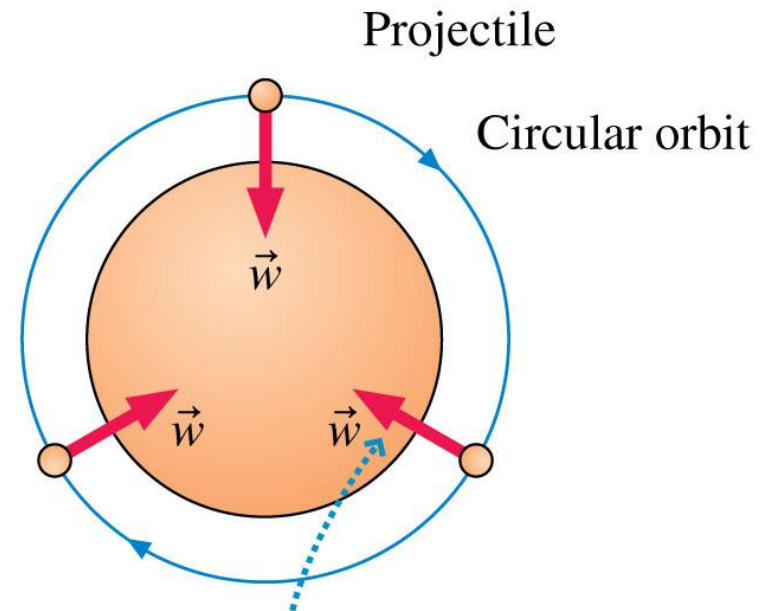


The force of gravity on a projectile is directed toward the center of the earth.

Orbital Motion

- If the launch speed of a projectile is sufficiently large, there comes a point at which the curve of the trajectory and the curve of the earth are parallel.
- Such a *closed trajectory* is called an **orbit**.
- **An orbiting projectile is in free fall.**

(c)



For a projectile in orbit, as the projectile moves around the earth, the direction of the force of gravity changes as well.

Orbital Motion

- The force of gravity is the force that causes the centripetal acceleration of an orbiting object:

$$a = \frac{F_{\text{net}}}{m} = \frac{w}{m} = \frac{mg}{m} = g$$

- An object moving in a circle of radius r at speed v_{orbit} will have this centripetal acceleration if

$$a = \frac{(v_{\text{orbit}})^2}{r} = g$$

- That is, if an object moves parallel to the surface with the speed

$$v_{\text{orbit}} = \sqrt{gr}$$

Orbital Motion

- The orbital speed of a projectile just skimming the surface of a smooth, airless earth is

$$\begin{aligned}v_{\text{orbit}} &= \sqrt{gR_e} = \sqrt{(9.80 \text{ m/s}^2)(6.37 \times 10^6 \text{ m})} \\ &= 7900 \text{ m/s} \approx 18,000 \text{ mph} \quad | \end{aligned}$$

- We can use v_{orbit} to calculate the period of the satellite's orbit:

$$T = \frac{2\pi r}{v_{\text{orbit}}} = 2\pi \sqrt{\frac{r}{g}}$$

Weightlessness in Orbit

- Astronauts and their spacecraft are in free fall.




QuickCheck 6.19

Astronauts on the International Space Station are weightless because

- A. There's no gravity in outer space.
- B. The net force on them is zero.
- C. The centrifugal force balances the gravitational force.
- D. g is very small, although not zero.
- E. They are in free fall.

QuickCheck 6.19

Astronauts on the International Space Station are weightless because

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- D. g is very small, although not zero.
-  E. They are in free fall.

The Orbit of the Moon

- The moon, like all satellites, is simply “falling” around the earth.
- If we use the distance to the moon, $r = 3.84 \times 10^8$ m, in:

$$T = \frac{2\pi r}{v_{\text{orbit}}} = 2\pi \sqrt{\frac{r}{g}}$$

we get a period of approximately 11 hours instead of one month.

- This is because the magnitude of the force of gravity, and thus the size of g , decreases with increasing distance from the earth.

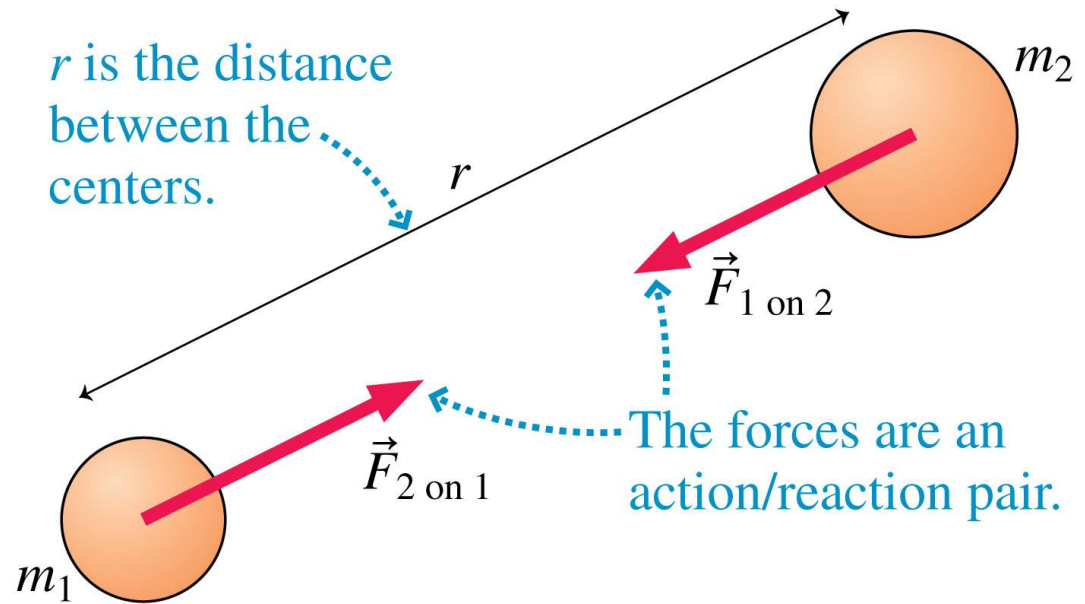
Example Problem

Phobos is one of two small moons that orbit Mars. Phobos is a very small moon, and has correspondingly small gravity—it varies, but a typical value is about 6 mm/s^2 . Phobos isn't quite round, but it has an average radius of about 11 km. What would be the orbital speed around Phobos, assuming it was round with gravity and radius as noted?

Section 6.5 Newton's Law of Gravity

Gravity Obeys an Inverse-Square Law

- Gravity is a universal force that affects all objects in the universe.
- Newton proposed that the force of gravity has the following properties:



1. The force is inversely proportional to the square of the distance between the objects.
2. The force is directly proportional to the product of the masses of the two objects.

Gravity Obeys an Inverse-Square Law

$$F_{1\text{ on }2} = F_{2\text{ on }1} = \frac{Gm_1m_2}{r^2}$$

- Newton's law of gravity is an inverse-square law.
- Doubling the distance between two masses causes the force between them to decrease by a factor of 4.

Conceptual Example 6.11 Varying gravitational force

The gravitational force between two giant lead spheres is 0.010 N when the centers of the spheres are 20 m apart. What is the distance between their centers when the gravitational force between them is 0.160 N?

REASON We can solve this problem without knowing the masses of the two spheres. The key is to consider the ratios of forces and distances. Gravity is an inverse-square relationship; the force is related to the inverse square of the distance. The force *increases* by a factor of $(0.160 \text{ N})/(0.010 \text{ N}) = 16$, so the distance must *decrease* by a factor of $\sqrt{16} = 4$. The distance is thus $(20 \text{ m})/4 = 5.0 \text{ m}$.

ASSESS This type of ratio reasoning is a very good way to get a quick handle on the solution to a problem.

Example 6.12 Gravitational force between two people

You are seated in your physics class next to another student 0.60 m away. Estimate the magnitude of the gravitational force between you. Assume that you each have a mass of 65 kg.

PREPARE We will model each of you as a sphere. This is not a particularly good model, but it will do for making an estimate. We will take the 0.60 m as the distance between your centers.

Example 6.12 Gravitational force between two people (cont.)

SOLVE The gravitational force is given by Equation 6.15:

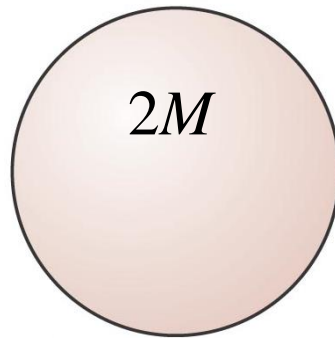
$$\begin{aligned} F_{(\text{you}) \text{ on } (\text{other student})} &= \frac{Gm_{\text{you}}m_{\text{other student}}}{r^2} \\ &= \frac{(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(65 \text{ kg})(65 \text{ kg})}{(0.60 \text{ m})^2} \\ &= 7.8 \times 10^{-7} \text{ N} \end{aligned}$$

ASSESS The force is quite small, roughly the weight of one hair on your head. This seems reasonable; you don't normally sense this attractive force!

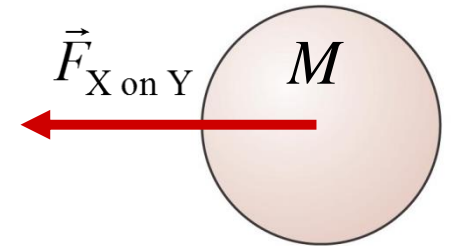
QuickCheck 6.16

The force of Planet Y on Planet X is ____ the magnitude of $\vec{F}_{X \text{ on } Y}$.

- A. One quarter
- B. One half
- C. The same as
- D. Twice
- E. Four times



Planet X

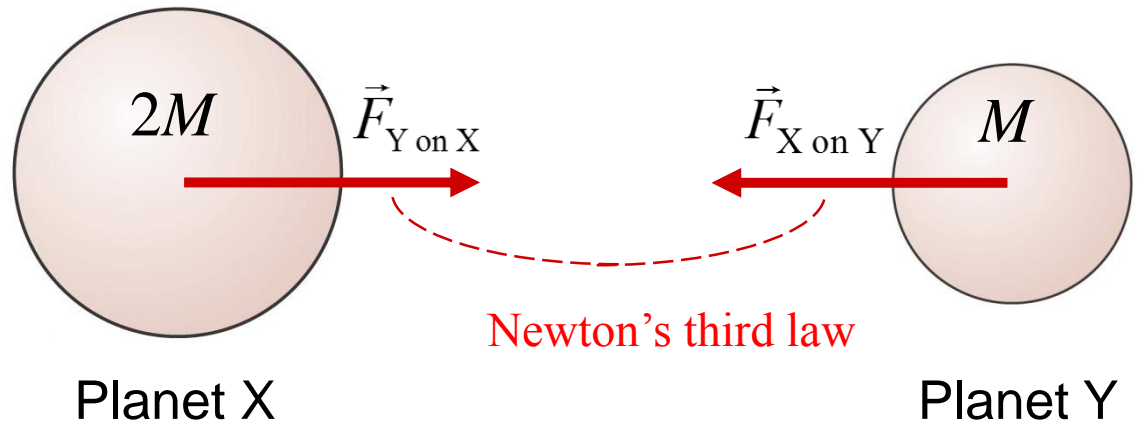


Planet Y

QuickCheck 6.16

The force of Planet Y on Planet X is ____ the magnitude of $\vec{F}_{X \text{ on } Y}$.

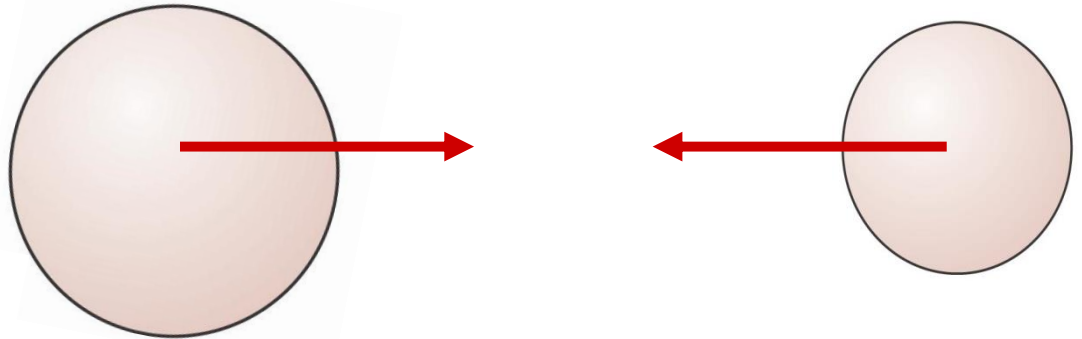
- A. One quarter
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QuickCheck 6.17

The gravitational force between two asteroids is 1,000,000 N. What will the force be if the distance between the asteroids is doubled?

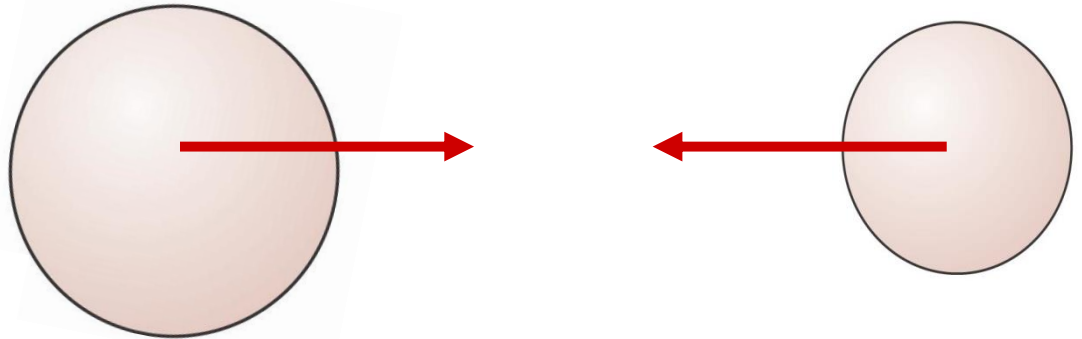
- A. 250,000 N
- B. 500,000 N
- C. 1,000,000 N
- D. 2,000,000 N
- E. 4,000,000 N



QuickCheck 6.17

The gravitational force between two asteroids is 1,000,000 N. What will the force be if the distance between the asteroids is doubled?

- ✓ A. 250,000 N
- B. 500,000 N
- C. 1,000,000 N
- D. 2,000,000 N
- E. 4,000,000 N



Gravity on Other Worlds

- If you traveled to another planet, your *mass* would be the same but your *weight* would vary. The weight of a mass m on the moon is given by

$$w = mg_{\text{moon}}$$

- Using Newton's law of gravity (Eq. (6.15)) the weight is given by

$$F_{\text{moon on } m} = \frac{GM_{\text{moon}}m}{R_{\text{moon}}^2}$$

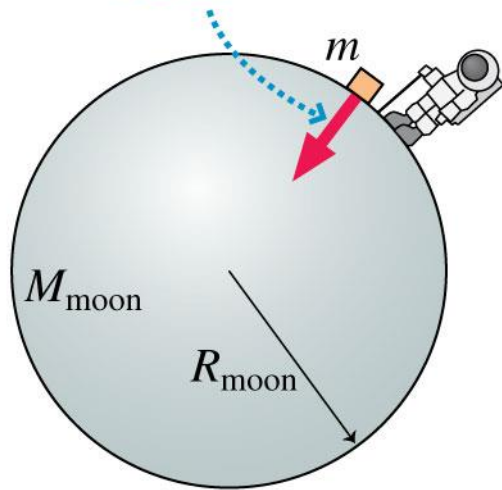
- Since these are two expressions for the same force, they are equal and

$$g_{\text{planet}} = \frac{GM_{\text{planet}}}{R_{\text{planet}}^2}$$

Gravity on Other Worlds

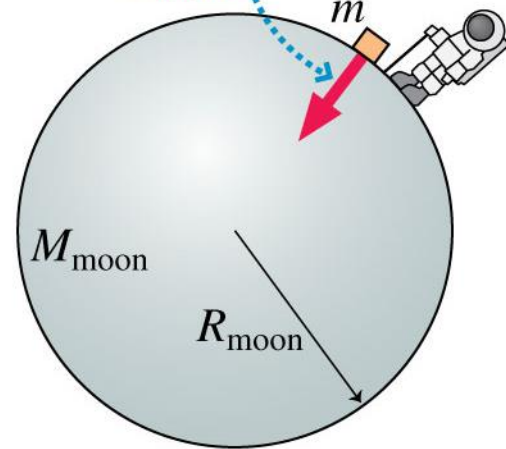
“Little g ” perspective:

$$F = mg_{\text{moon}}$$



“Big G ” perspective:

$$F = \frac{GM_{\text{moon}}m}{R_{\text{moon}}^2}$$



- If we use values for the mass and the radius of the moon, we compute $g_{\text{moon}} = 1.62 \text{ m/s}^2$.
- A 70-kg astronaut wearing an 80-kg spacesuit would weigh more than 330 lb on the earth but only 54 lb on the moon.

QuickCheck 6.18

Planet X has free-fall acceleration 8 m/s^2 at the surface. Planet Y has twice the mass and twice the radius of planet X. On Planet Y

- A. $g = 2 \text{ m/s}^2$
- B. $g = 4 \text{ m/s}^2$
- C. $g = 8 \text{ m/s}^2$
- D. $g = 16 \text{ m/s}^2$
- E. $g = 32 \text{ m/s}^2$

QuickCheck 6.18

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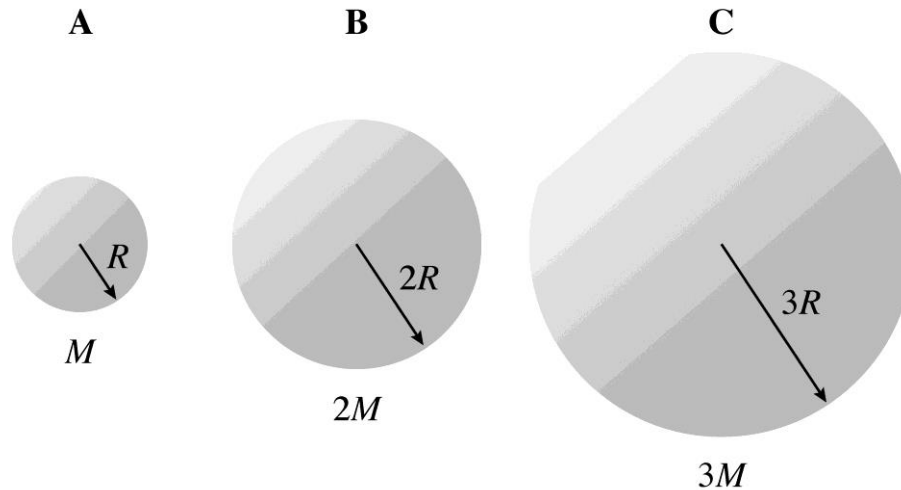
C. $g = 8 \text{ m/s}^2$

D. $g = 16 \text{ m/s}^2$

E. $g = 32 \text{ m/s}^2$

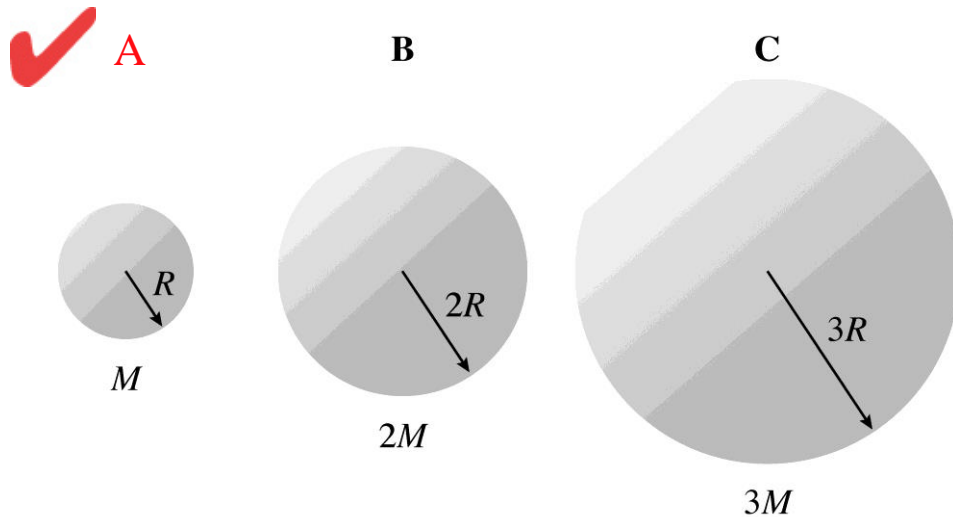
QuickCheck 6.22

A 60-kg person stands on each of the following planets. On which planet is his or her weight the greatest?



QuickCheck 6.22

A 60-kg person stands on each of the following planets.
On which planet is his or her weight the greatest?



Example 6.14 Finding the speed to orbit Deimos

Mars has two moons, each much smaller than the earth's moon. The smaller of these two bodies, Deimos, isn't quite spherical, but we can model it as a sphere of radius 6.3 km. Its mass is 1.8×10^{15} kg. At what speed would a projectile move in a very low orbit around Deimos?

Example 6.14 Finding the speed to orbit Deimos (cont.)

SOLVE The free-fall acceleration at the surface of Deimos is small:

$$\begin{aligned}g_{\text{Deimos}} &= \frac{GM_{\text{Deimos}}}{R_{\text{Deimos}}^2} \\ &= \frac{(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(1.8 \times 10^{15} \text{ kg})}{(6.3 \times 10^3 \text{ m})^2} \\ &= 0.0030 \text{ m/s}^2\end{aligned}$$

Example 6.14 Finding the speed to orbit Deimos (cont.)

Given this, we can use Equation 6.13 to calculate the orbital speed:

$$\begin{aligned}v_{\text{orbit}} &= \sqrt{gr} = \sqrt{(0.0030 \text{ m/s}^2)(6.3 \times 10^3 \text{ m})} \\ &= 4.3 \text{ m/s} \approx 10 \text{ mph}\end{aligned}$$

ASSESS This is quite slow. With a good jump, you could easily launch yourself into an orbit around Deimos!

Example Problem

A typical bowling ball is spherical, weighs 16 pounds, and has a diameter of 8.5 in. Suppose two bowling balls are right next to each other in the rack. What is the gravitational force between the two—magnitude and direction?

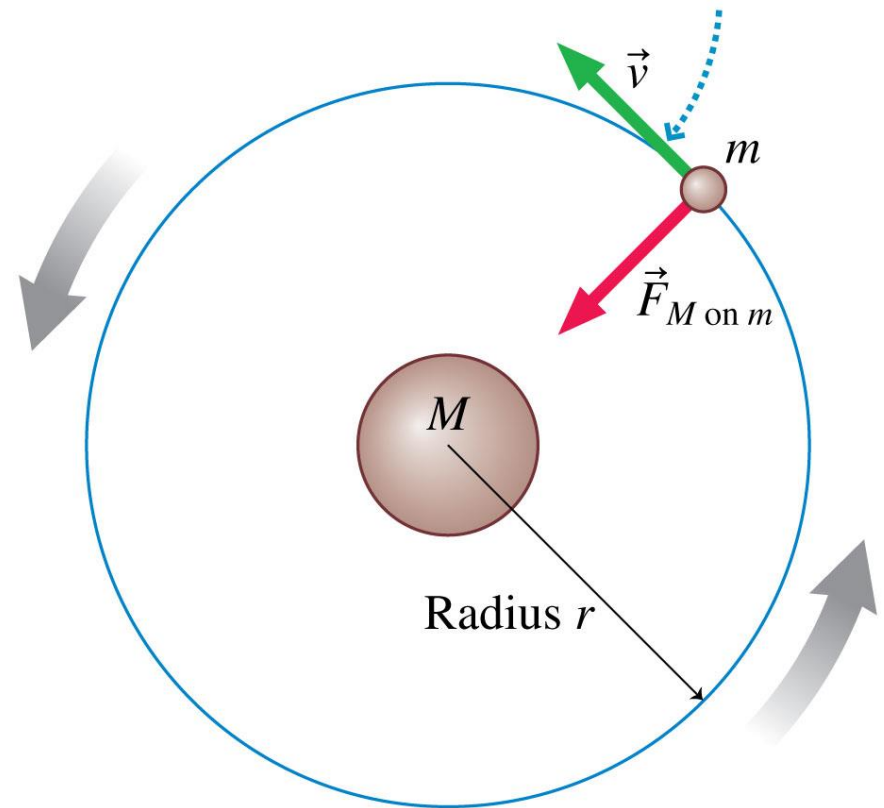
Section 6.6 Gravity and Orbits

Gravity and Orbits

- Newton's second law tells us that $F_{M \text{ on } m} = ma$, where $F_{M \text{ on } m}$ is the gravitational force of the large body on the satellite and a is the satellite's acceleration.
- Because it's moving in a circular orbit, Newton's second law gives

$$F_{M \text{ on } m} = \frac{GMm}{r^2} = ma = \frac{mv^2}{r}$$

The satellite must have speed $\sqrt{GM/r}$ to maintain a circular orbit of radius r .



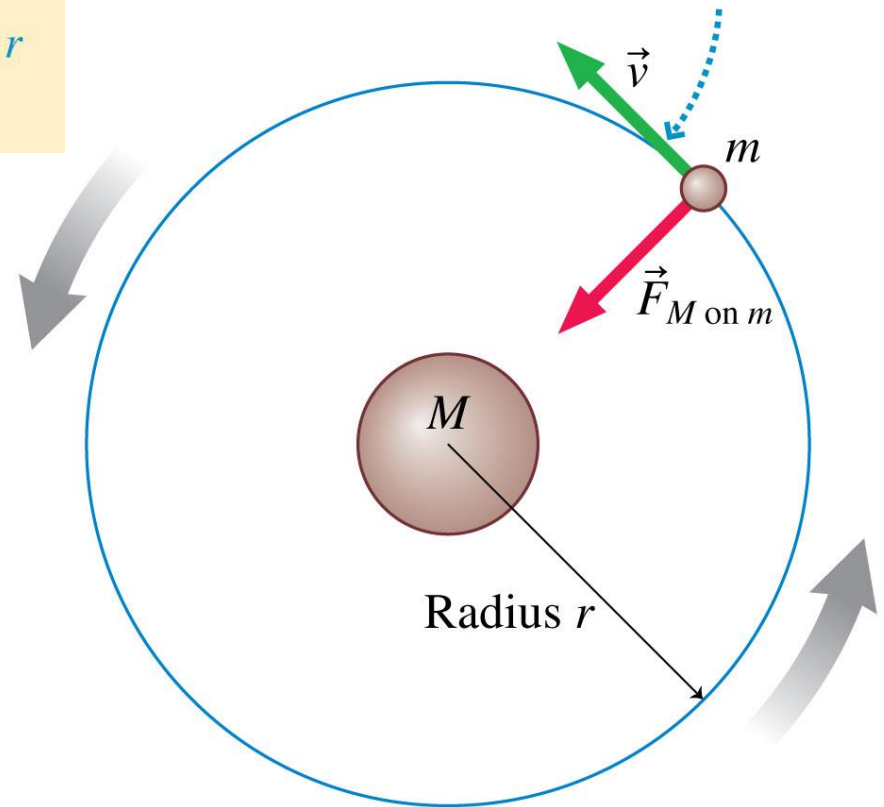
Gravity and Orbits

$$v = \sqrt{\frac{GM}{r}}$$

Speed of a satellite in a circular orbit of radius r about a star or planet of mass M

- A satellite must have this specific speed in order to maintain a circular orbit of radius r about the larger mass M .

The satellite must have speed $\sqrt{GM/r}$ to maintain a circular orbit of radius r .



Gravity and Orbits

- For a planet orbiting the sun, the period T is the time to complete one full orbit. The relationship among speed, radius, and period is the same as for any circular motion:

$$v = 2\pi r/T$$

- Combining this with the value of v for a circular orbit from Equation 6.21 gives

$$\sqrt{\frac{GM}{r}} = \frac{2\pi r}{T}$$

- If we square both sides and rearrange, we find the period of a satellite:

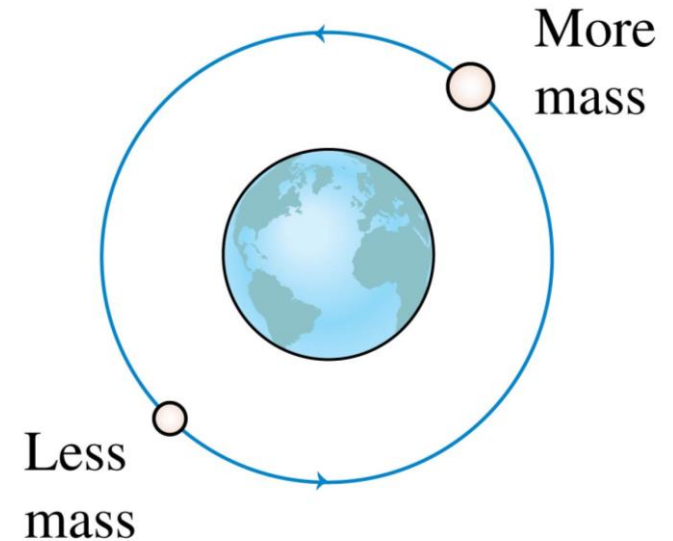
$$T^2 = \left(\frac{4\pi^2}{GM}\right)r^3$$

Relationship between the orbital period T and radius r for a satellite in a circular orbit around an object of mass M

QuickCheck 6.20

Two satellites have circular orbits with the same radius.
Which has a higher speed?

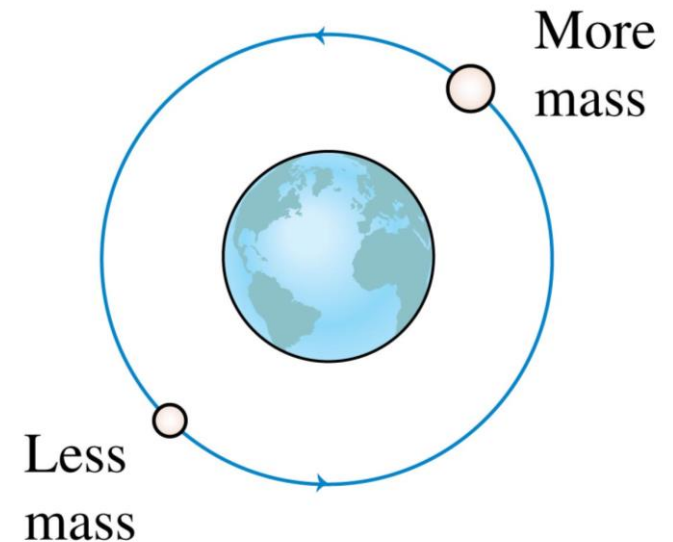
- A. The one with more mass.
- B. The one with less mass.
- C. They have the same speed.



QuickCheck 6.20

Two satellites have circular orbits with the same radius.
Which has a higher speed?

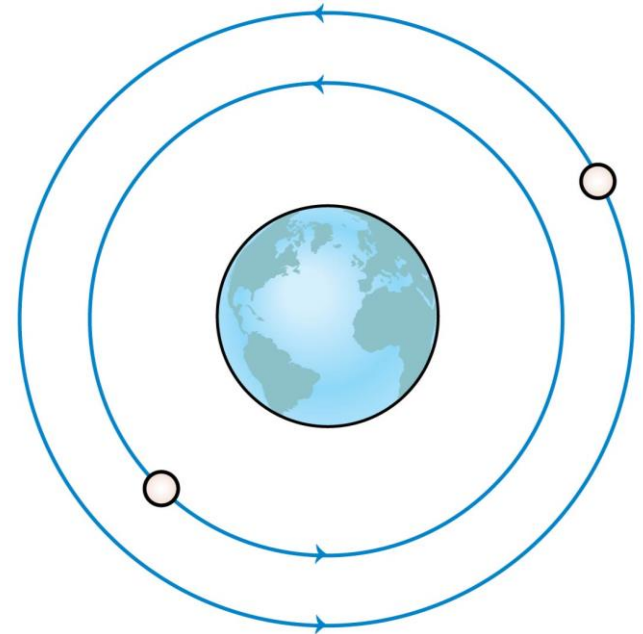
- A. The one with more mass.
- B. The one with less mass.
- ✓ C. They have the same speed.



QuickCheck 6.21

Two identical satellites have different circular orbits. Which has a higher speed?

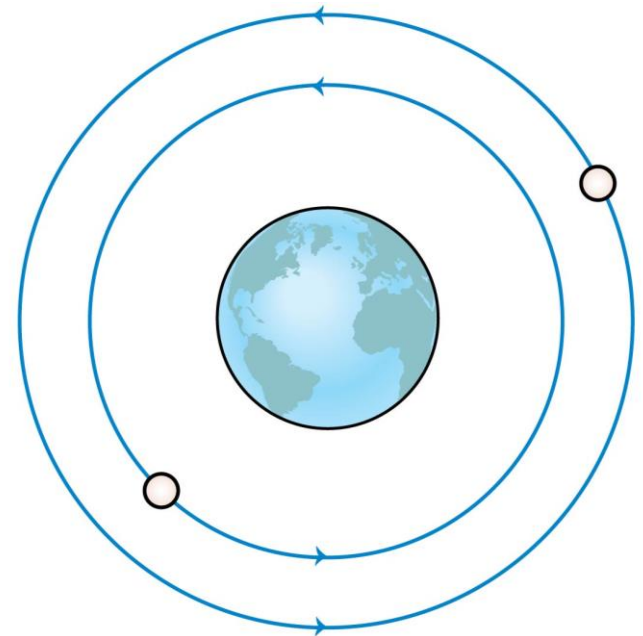
- A. The one in the larger orbit
- B. The one in the smaller orbit
- C. They have the same speed.



QuickCheck 6.21

Two identical satellites have different circular orbits. Which has a higher speed?

- A. The one in the larger orbit
- ✓ B. The one in the smaller orbit
- C. They have the same speed.




QuickCheck 6.23

A satellite orbits the earth. A Space Shuttle crew is sent to boost the satellite into a higher orbit. Which of these quantities increases?

- A. Speed
- B. Angular speed
- C. Period
- D. Centripetal acceleration
- E. Gravitational force of the earth

QuickCheck 6.23

A satellite orbits the earth. A Space Shuttle crew is sent to boost the satellite into a higher orbit. Which of these quantities increases?

- A. Speed
- B. Angular speed
-  C. Period
- D. Centripetal acceleration
- E. Gravitational force of the earth

Example 6.15 Locating a geostationary satellite

Communication satellites appear to “hover” over one point on the earth’s equator. A satellite that appears to remain stationary as the earth rotates is said to be in a *geostationary orbit*. What is the radius of the orbit of such a satellite?

PREPARE For the satellite to remain stationary with respect to the earth, the satellite’s orbital period must be 24 hours; in seconds this is $T = 8.64 \times 10^4$ s.

Example 6.15 Locating a geostationary satellite (cont.)

SOLVE We solve for the radius of the orbit by rearranging Equation 6.22. The mass at the center of the orbit is the earth:

$$\begin{aligned} r &= \left(\frac{GM_e T^2}{4\pi^2} \right)^{\frac{1}{3}} \\ &= \left(\frac{(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(5.98 \times 10^{24} \text{ kg})(8.64 \times 10^4 \text{ s})^2}{4\pi^2} \right)^{\frac{1}{3}} \\ &= 4.22 \times 10^7 \text{ m} \end{aligned}$$

Example 6.15 Locating a geostationary satellite (cont.)

ASSESS This is a high orbit, and the radius is about 7 times the radius of the earth. Recall that the radius of the International Space Station's orbit is only about 5% larger than that of the earth.

Gravity on a Grand Scale

- No matter how far apart two objects may be, there is a gravitational attraction between them.



- Galaxies are held together by gravity.
- All of the stars in a galaxy are different distances from the galaxy's center, and so orbit with different periods.

Example Problem

Phobos is the closer of Mars's two small moons, orbiting at 9400 km from the center of Mars, a planet of mass 6.4×10^{23} kg. What is Phobos's orbital period? How does this compare to the length of the Martian day, which is just shy of 25 hours?

Summary: General Principles

Uniform Circular Motion

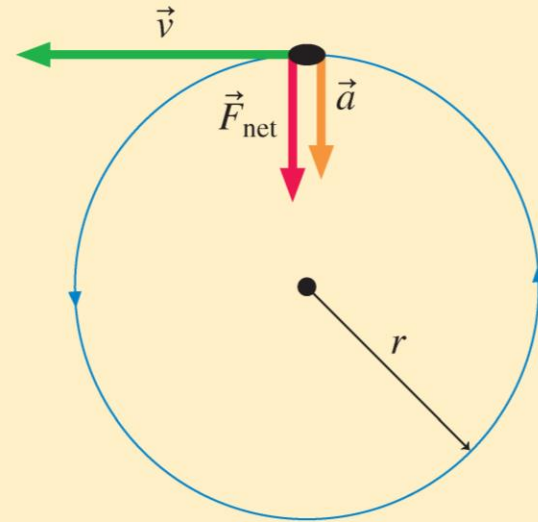
An object moving in a circular path is in uniform circular motion if v is constant.

- The speed is constant, but the direction of motion is constantly changing.
- The **centripetal acceleration** is directed toward the center of the circle and has magnitude

$$a = \frac{v^2}{r}$$

- This acceleration requires a net force directed toward the center of the circle. Newton's second law for circular motion is

$$\vec{F}_{\text{net}} = m\vec{a} = \left(\frac{mv^2}{r}, \text{ toward center of circle} \right)$$



Text: p. 182

Summary: General Principles

Universal Gravitation

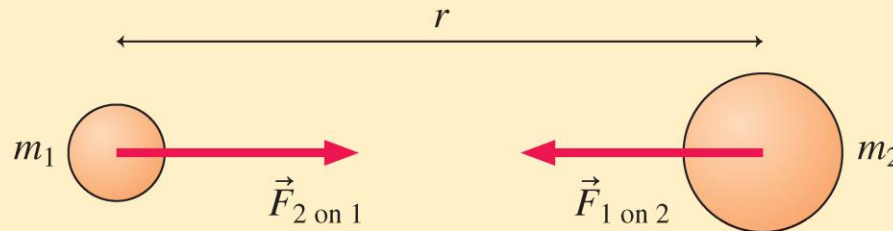
Two objects with masses m_1 and m_2 that are distance r apart exert attractive gravitational forces on each other of magnitude

$$F_{1\text{ on }2} = F_{2\text{ on }1} = \frac{Gm_1m_2}{r^2}$$

where the gravitational constant is

$$G = 6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2$$

This is **Newton's law of gravity**. Gravity is an inverse-square law.



Text: p. 182

Summary: Important Concepts

Describing circular motion

For an object moving in a circle of radius r at a constant speed v :

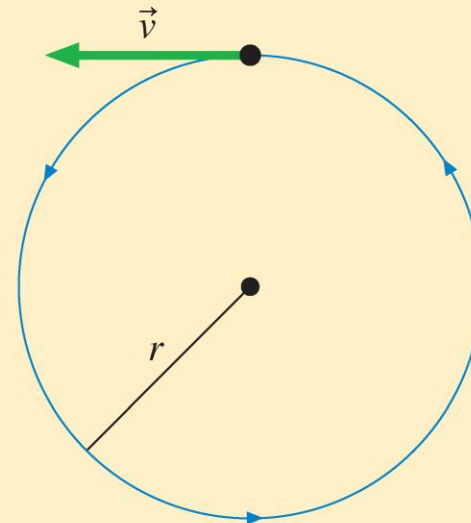
- The **period** T is the time to go once around the circle.

$T =$ time for one revolution

- The **frequency** f is defined as the number of revolutions per second. It is defined in terms of the period:

$$f = \frac{1}{T}$$

- The frequency and period are related to the speed and the radius: $v = 2\pi fr = \frac{2\pi r}{T}$



Text: p. 182

Summary: Important Concepts

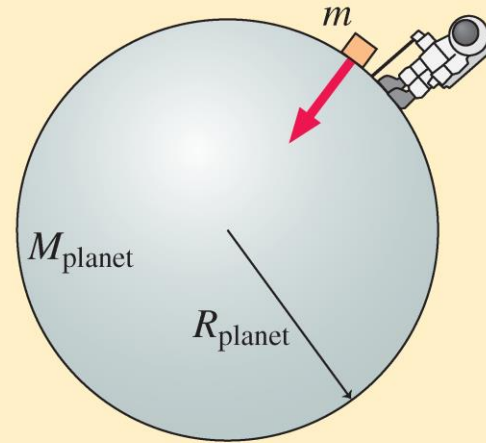
Planetary gravity

The gravitational attraction between a planet and a mass on the surface depends on the two masses and the distance to the center of the planet.

$$F_{\text{planet on } m} = \frac{GM_{\text{planet}}m}{R_{\text{planet}}^2}$$

We can use this to define a value of the free-fall acceleration at the surface of a planet:

$$g_{\text{planet}} = \frac{GM_{\text{planet}}}{R_{\text{planet}}^2}$$

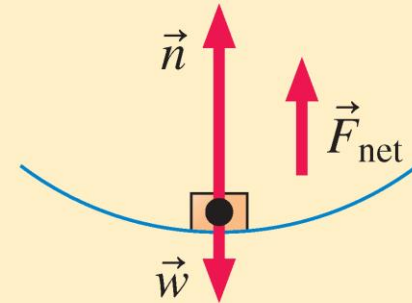


Text: p. 182

Summary: Applications

Apparent weight and weightlessness

Circular motion requires a net force pointing to the center. The apparent weight $w_{\text{app}} = n$ is usually not the same as the true weight w . n must be > 0 for the object to be in contact with a surface.



In orbital motion, the net force is provided by gravity. An astronaut and his spacecraft are both in free fall, so he feels weightless.

Summary: Applications

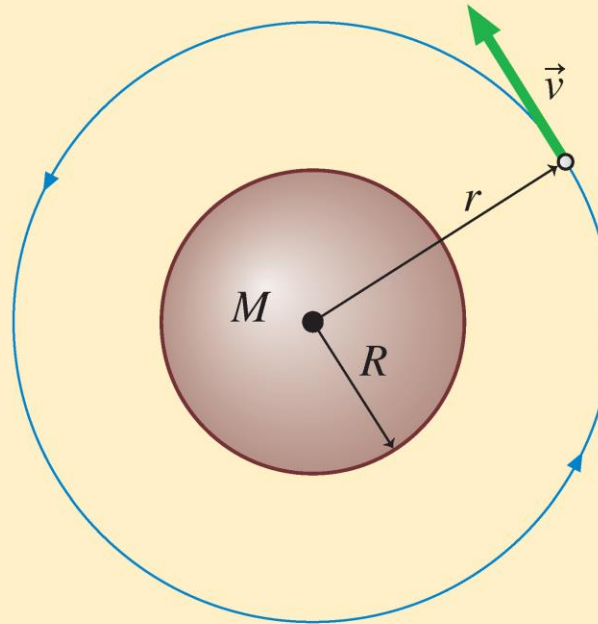
Orbital motion

A **satellite** in a circular orbit of radius r around an object of mass M moves at a speed v given by

$$v = \sqrt{\frac{GM}{r}}$$

The period and radius are related as follows:

$$T^2 = \left(\frac{4\pi^2}{GM}\right)r^3$$



The speed of a satellite in a low orbit is

$$v = \sqrt{gr}$$

The orbital period is

$$T = 2\pi\sqrt{\frac{r}{g}}$$

Text: p. 182

Summary

GENERAL PRINCIPLES

Uniform Circular Motion

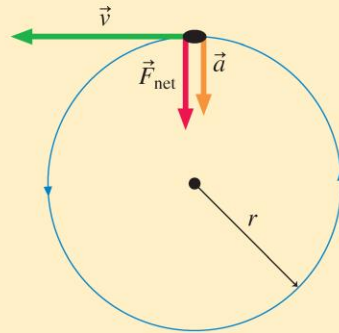
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Universal Gravitation

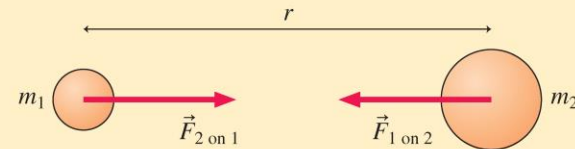
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This is **Newton's law of gravity**. Gravity is an inverse-square law.



Text: p. 182

Summary

IMPORTANT CONCEPTS

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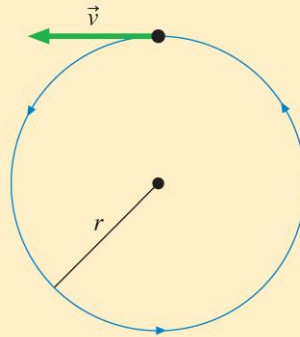
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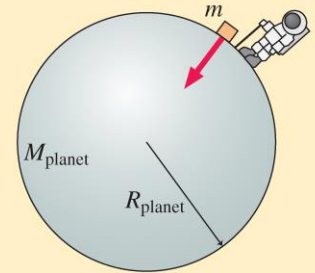
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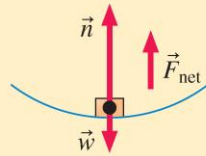
Text: p. 182

Summary

APPLICATIONS

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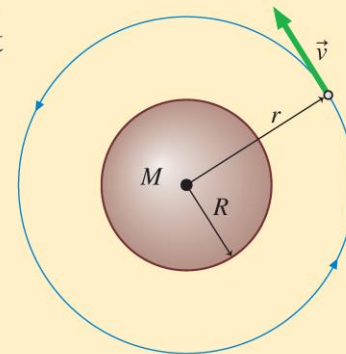
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Text: p. 182