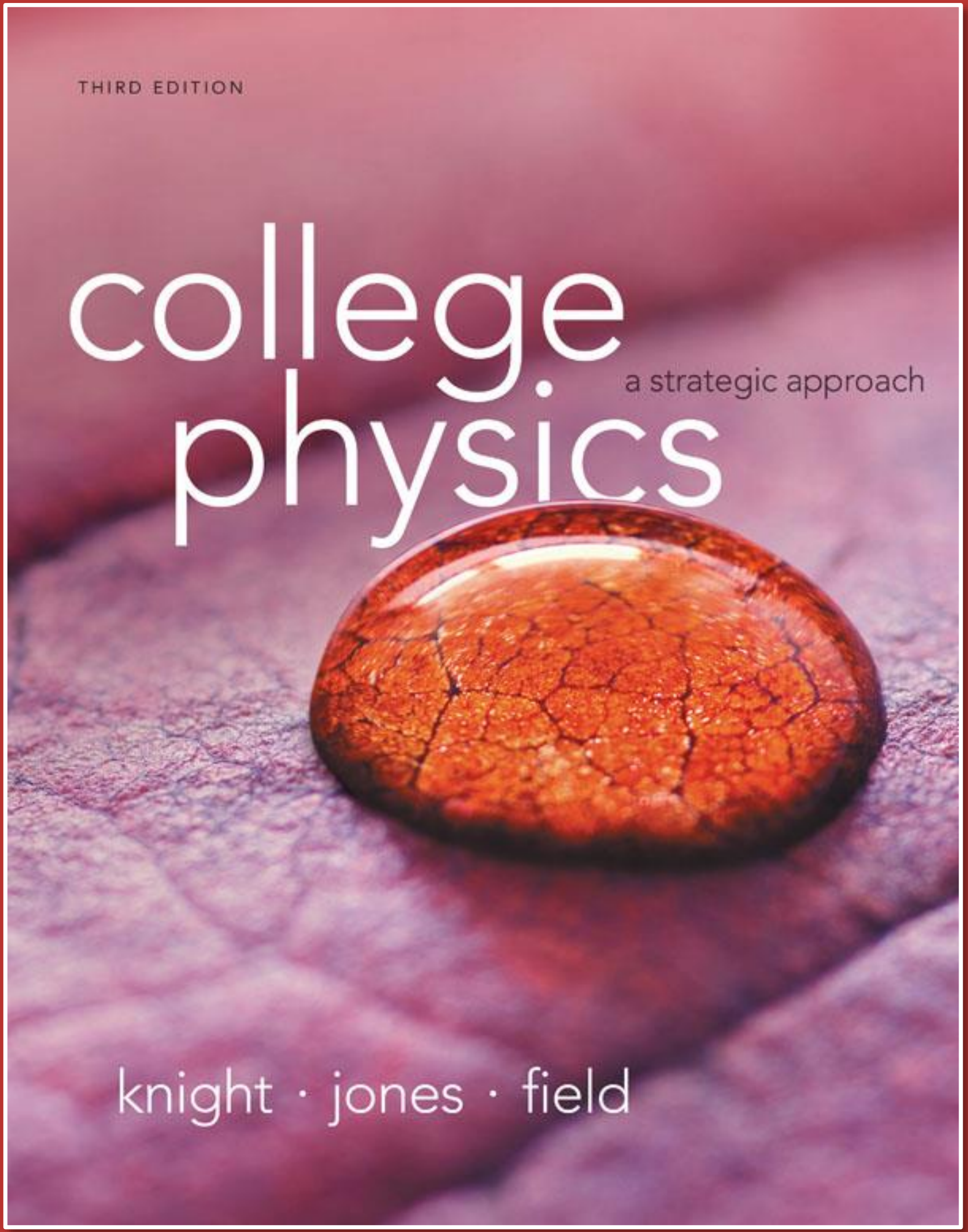


Lecture Presentation

Chapter 3

Vectors and Motion in Two Dimensions

Suggested Videos for Chapter 3

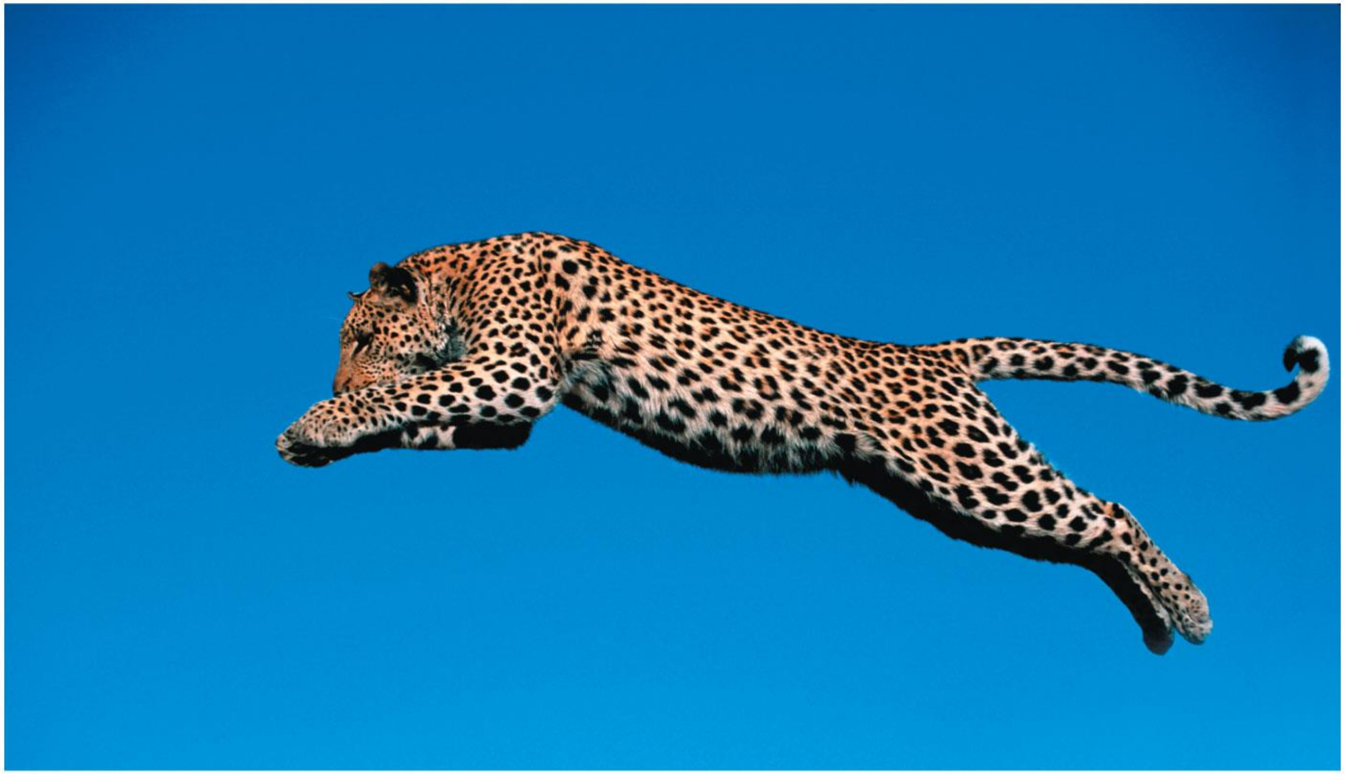
- **Prelecture Videos**
  - *Vectors and Motion*
  - *Projectile Motion*
  - *Circular Motion*
- **Class Videos**
  - *Motion on a Ramp*
  - *Acceleration Due to Changing Direction*
- **Video Tutor Solutions**
  - *Vectors and Motion in Two Dimensions*
- **Video Tutor Demos**
  - *Balls Take High and Low Tracks*
  - *Dropped and Thrown Balls*
  - *Ball Fired Upward from Moving Cart*
  - *Ball Fired Upward from Accelerating Cart*
  - *Ball Fired from Cart on Incline*
  - *Range of a Gun at Two Firing Angles*
  - *Ball Leaves Circular Track*

Suggested Simulations for Chapter 3

- **ActivPhysics**
  - *3.1–3.7*
  - *4.1*

- **PhETs**
  - *Vector Addition*
  - *Ladybug Motion 2D*
  - *Maze Game*
  - *Motion in 2D*
  - *Projectile Motion*
  - *Ladybug Revolution*

Chapter 3 Vectors and Motion in Two Dimensions



Chapter Goal: To learn more about vectors and to use vectors as a tool to analyze motion in two dimensions.

Chapter 3 Preview
Looking Ahead: Vectors and Components

- The dark green vector is the ball's initial velocity. The light green component vectors show initial horizontal and vertical velocity.

- You'll learn to describe motion in terms of quantities such as distance and velocity, an important first step in analyzing motion.

Chapter 3 Preview
Looking Ahead: Projectile Motion

- A leaping fish's parabolic arc is an example of projectile motion. The details are the same for a fish or a basketball.

- You'll see how to solve projectile motion problems, determining how long an object is in the air and how far it travels.

Chapter 3 Preview
Looking Ahead: Circular Motion

- The riders move in a circle at a constant speed, but they have an acceleration because the direction is constantly changing.

- You'll learn how to determine the magnitude and the direction of the acceleration for an object in circular motion.

Chapter 3 Preview
Looking Ahead

Vectors and Components

The dark green vector is the ball's initial velocity. The light green **component vectors** show initial horizontal and vertical velocity.

You'll learn how to find components of vectors and how to use these components to solve problems.

Projectile Motion

A leaping fish's parabolic arc is an example of **projectile motion.** The details are the same for a fish or a basketball.

You'll see how to solve projectile motion problems, determining how long an object is in the air and how far it travels.

Circular Motion

The riders move in a circle at a constant speed, but they have an acceleration because the direction is constantly changing.

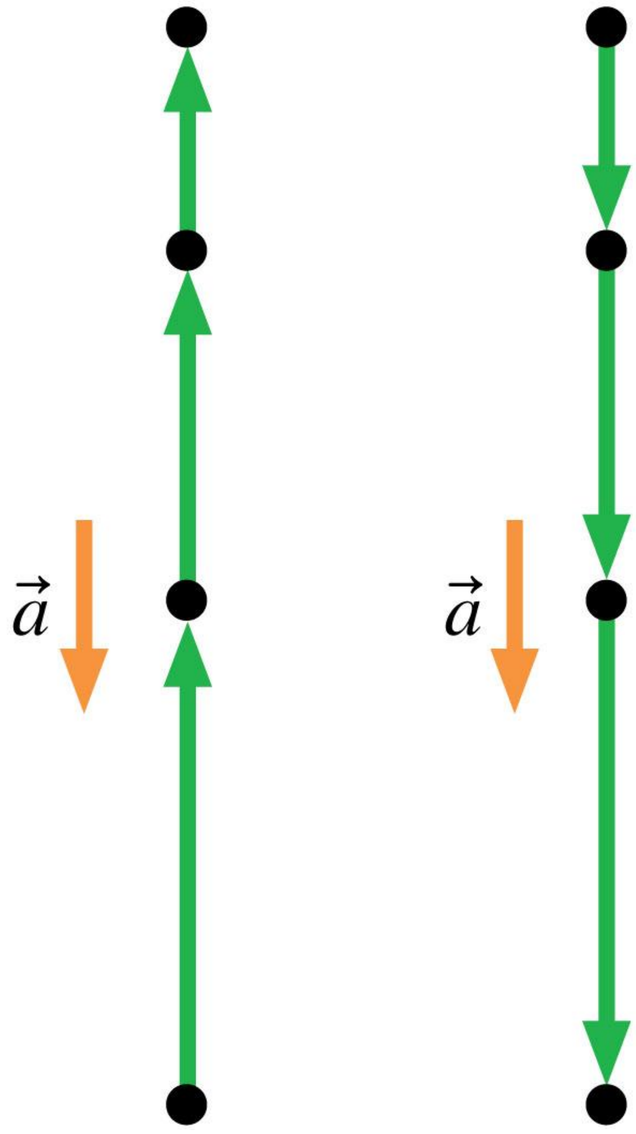
You'll learn how to determine the magnitude and the direction of the acceleration for an object in circular motion.

Text: p. 64

Chapter 3 Preview
Looking Back: Free Fall

- You learned in Section 2.7 that an object tossed straight up is in free fall. The acceleration is the same whether the object is going up or coming back down.

- For an object in projectile motion, the vertical component of the motion is also free fall. You'll use your knowledge of free fall to solve projectile motion problems.



Chapter 3 Preview
Stop to Think

A player kicks a football straight up into the air. The ball takes 2.0 s to reach its highest point. Approximately how fast was the ball moving when it left the player's foot?

A. 5 m/s
B. 10 m/s
C. 15 m/s
D. 20 m/s

Reading Question 3.1

The _______ of a vector is always a positive quantity.

A. *x*-component
B. *y*-component
C. Magnitude
D. Direction

Reading Question 3.1

The _______ of a vector is always a positive quantity.

A. *x*-component
B. *y*-component
✔ C. Magnitude
D. Direction

Reading Question 3.2

A_x is positive if $\vec{A}_x$ is directed ________; A_y is positive if $\vec{A}_y$ is directed ________.

A. Right, up
B. Left, up
C. Right, down
D. Left, down

Reading Question 3.2

A_x is positive if $\vec{A}_x$ is directed ________; A_y is positive if $\vec{A}_y$ is directed ________.

✔ A. Right, up

B. Left, up

C. Right, down

D. Left, down

Reading Question 3.3

The acceleration of a cart rolling down a ramp depends on

A. The angle of the ramp.
B. The length of the ramp.
C. Both the angle of the ramp and the length of the ramp.
D. Neither the angle of the ramp or the length of the ramp.

Reading Question 3.3

The acceleration of a cart rolling down a ramp depends on

✔ A. The angle of the ramp.
B. The length of the ramp.
C. Both the angle of the ramp and the length of the ramp.
D. Neither the angle of the ramp or the length of the ramp.

Reading Question 3.4

The acceleration vector of a particle in projectile motion

A. Points along the path of the particle.
B. Is directed horizontally.
C. Vanishes at the particle's highest point.
D. Is directed down at all times.
E. Is zero.

Reading Question 3.4

The acceleration vector of a particle in projectile motion

A. Points along the path of the particle.
B. Is directed horizontally.
C. Vanishes at the particle's highest point.
✔ D. Is directed down at all times.
E. Is zero.

Reading Question 3.5

The acceleration vector of a particle in uniform circular motion

A. Points tangent to the circle, in the direction of motion.
B. Points tangent to the circle, opposite the direction of motion.
C. Is zero.
D. Points toward the center of the circle.
E. Points outward from the center of the circle.

Reading Question 3.5

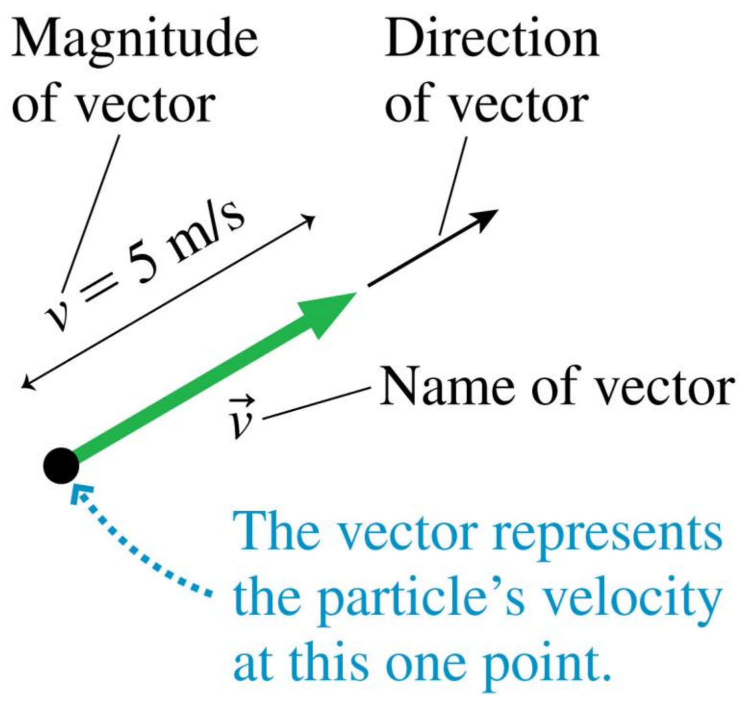
The acceleration vector of a particle in uniform circular motion

A. Points tangent to the circle, in the direction of motion.
B. Points tangent to the circle, opposite the direction of motion.
C. Is zero.
✔ D. Points toward the center of the circle.
E. Points outward from the center of the circle.

Section 3.1 Using Vectors

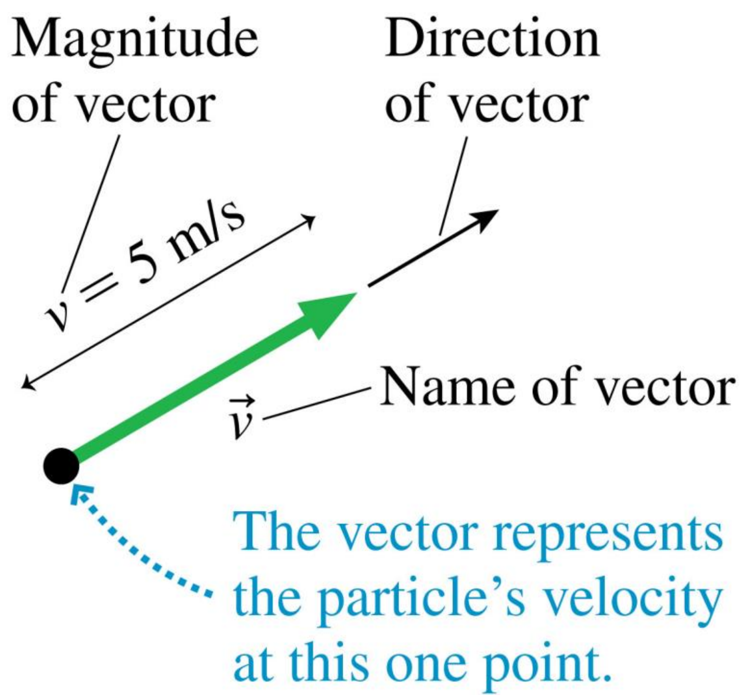
Using Vectors

- A vector is a quantity with both a size (magnitude) and a direction.
- Figure 3.1 shows how to represent a particle's velocity as a vector $\vec{v}$.
- The particle's speed at this point is 5 m/s and it is moving in the direction indicated by the arrow.



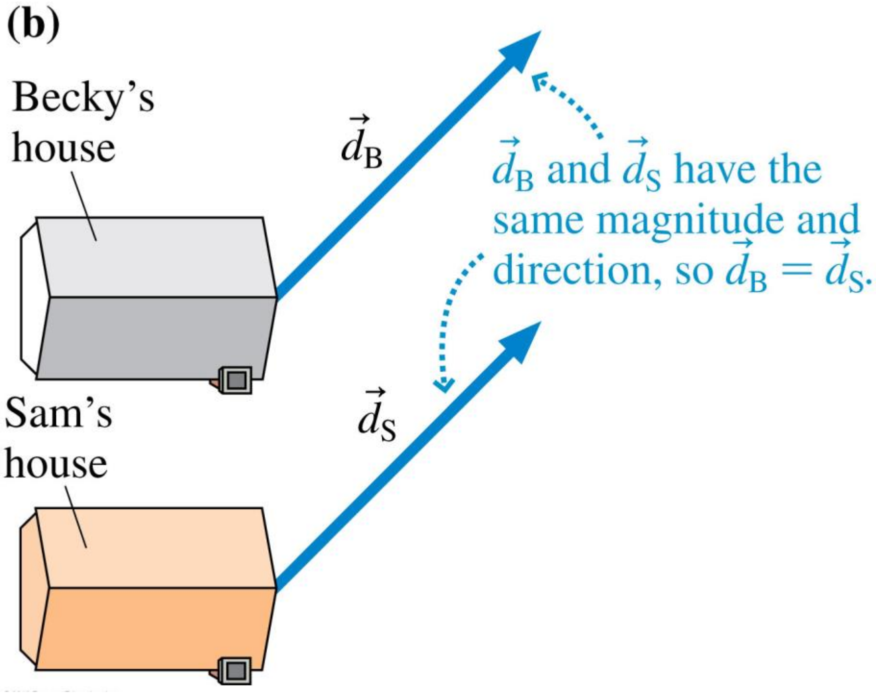
Using Vectors

- The magnitude of a vector is represented by the letter without an arrow.
- In this case, the particle's speed—the magnitude of the velocity vector $\vec{v}$—is $v = 5$ m/s.
- The magnitude of a vector, a *scalar* quantity, cannot be a negative number.



Using Vectors

- The displacement vector is a straight-line connection from the initial position to the final position, regardless of the actual path.
- **Two vectors are equal if they have the same magnitude and direction.** This is regardless of the individual starting points of the vectors.



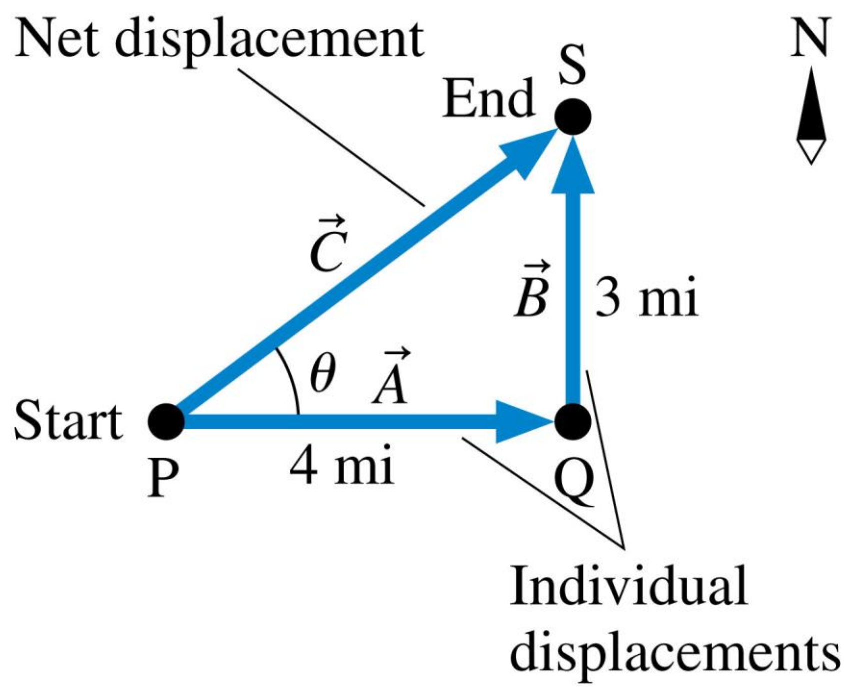
Vector Addition

- $\vec{C}$ is the *net displacement* because it describes the net result of the hiker's having first displacement $\vec{A}$, then displacement $\vec{B}$.
- The net displacement $\vec{C}$ is an initial displacement $\vec{A}$ *plus* a second displacement $\vec{B}$:

$$\vec{C} = \vec{A} + \vec{B}$$

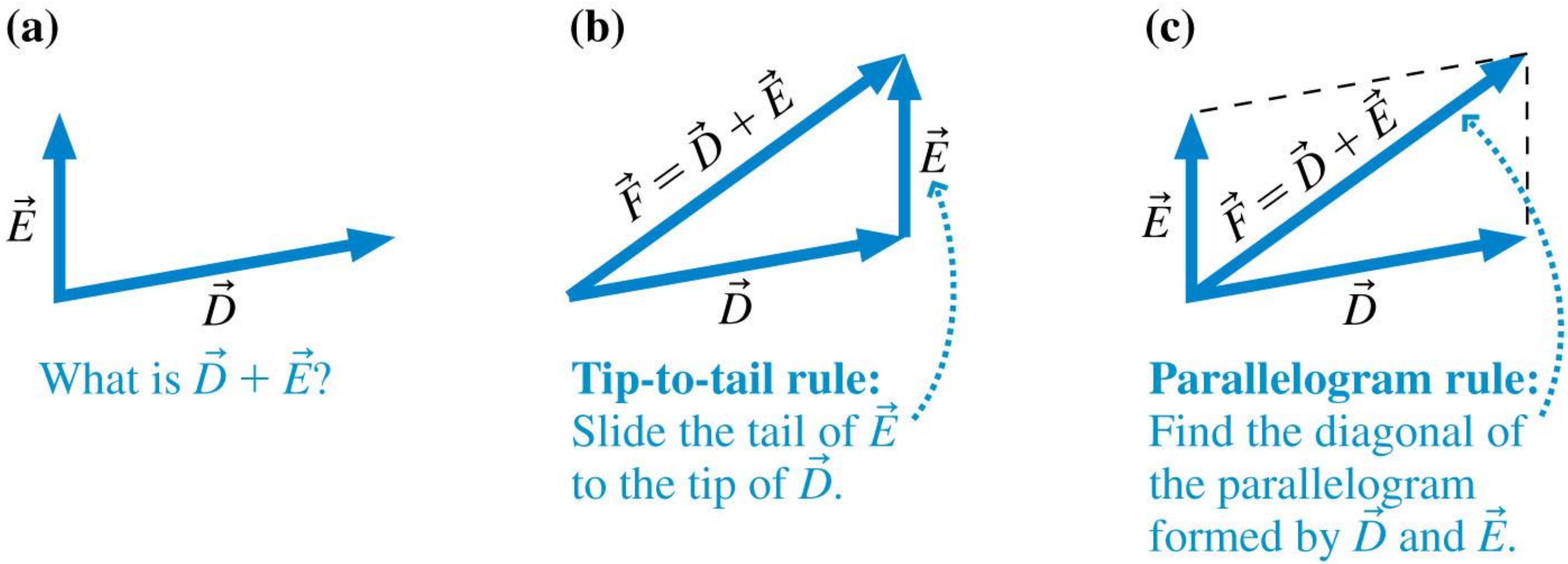
- The sum of the two vectors is called the resultant vector. Vector addition is commutative:

$$\vec{A} + \vec{B} = \vec{B} + \vec{A}$$



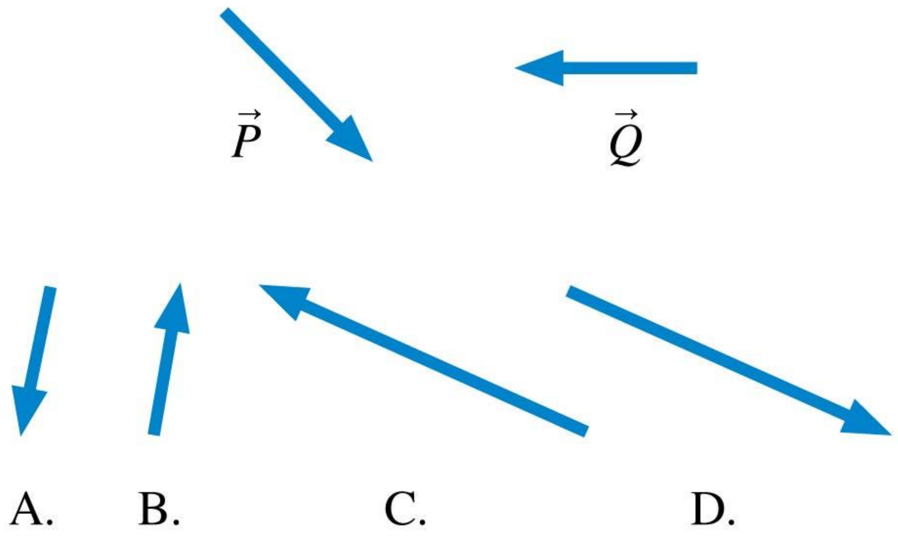
Vector Addition

- The figure shows the *tip-to-tail rule* of vector addition and the *parallelogram rule* of vector addition.



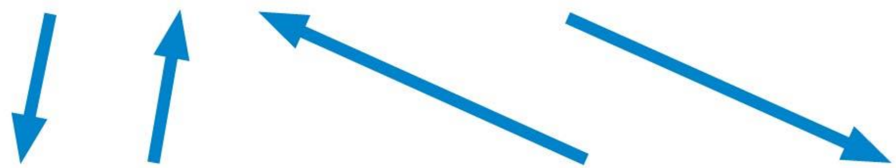
QuickCheck 3.1

Given vectors $\vec{P}$ and $\vec{Q}$, what is $\vec{P}+\vec{Q}$?

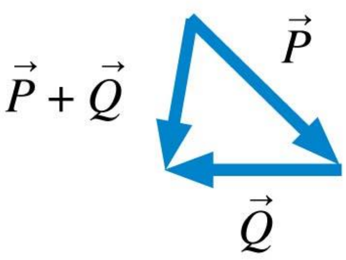


QuickCheck 3.1

Given vectors $\vec{P}$ and $\vec{Q}$, what is $\vec{P}+\vec{Q}$?

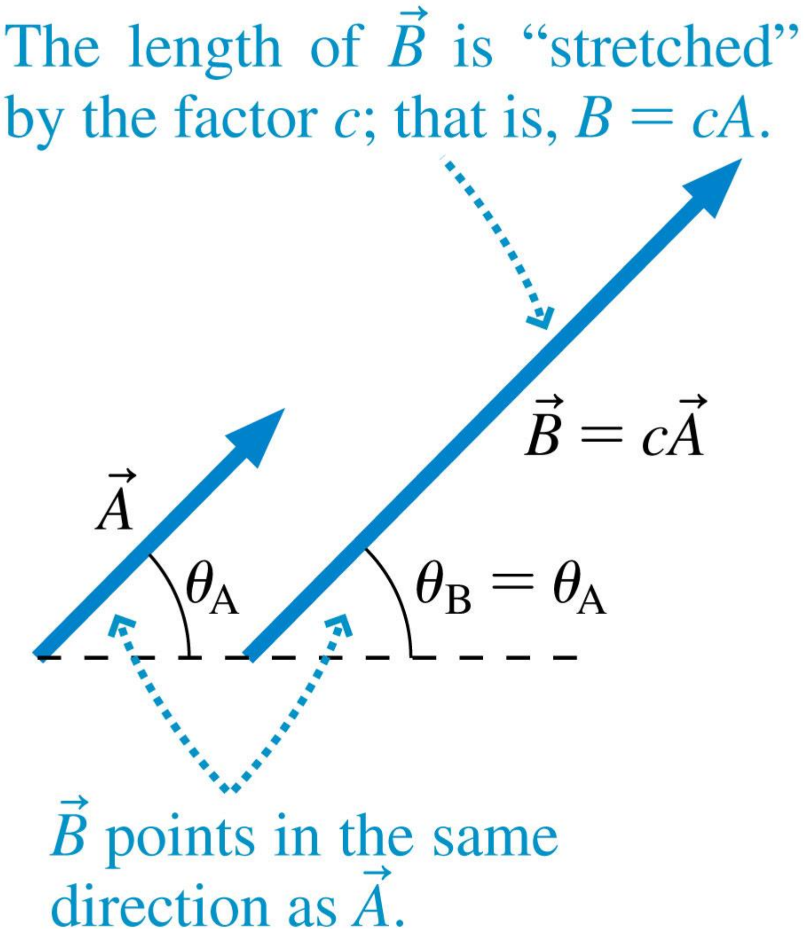


A. B. C. D.



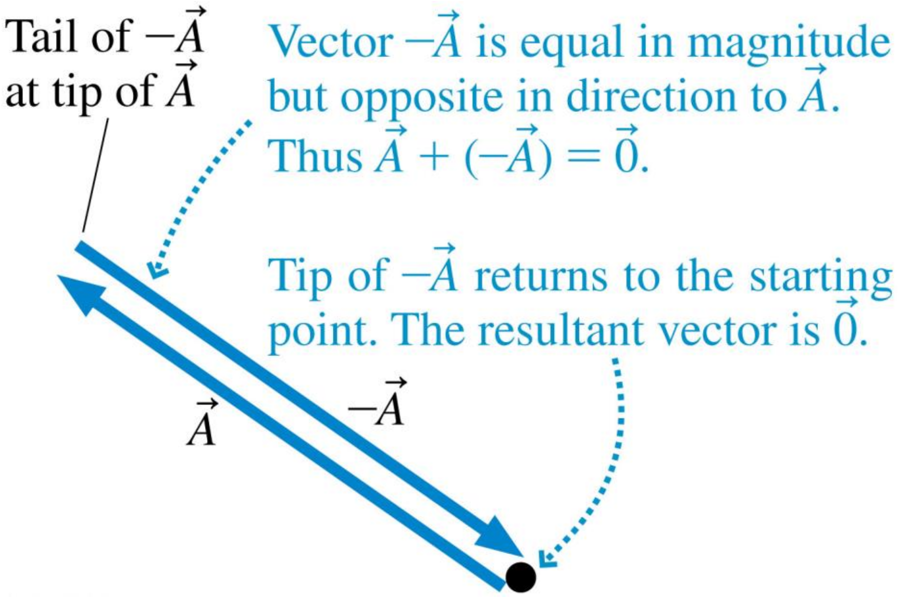
Multiplication by a Scalar

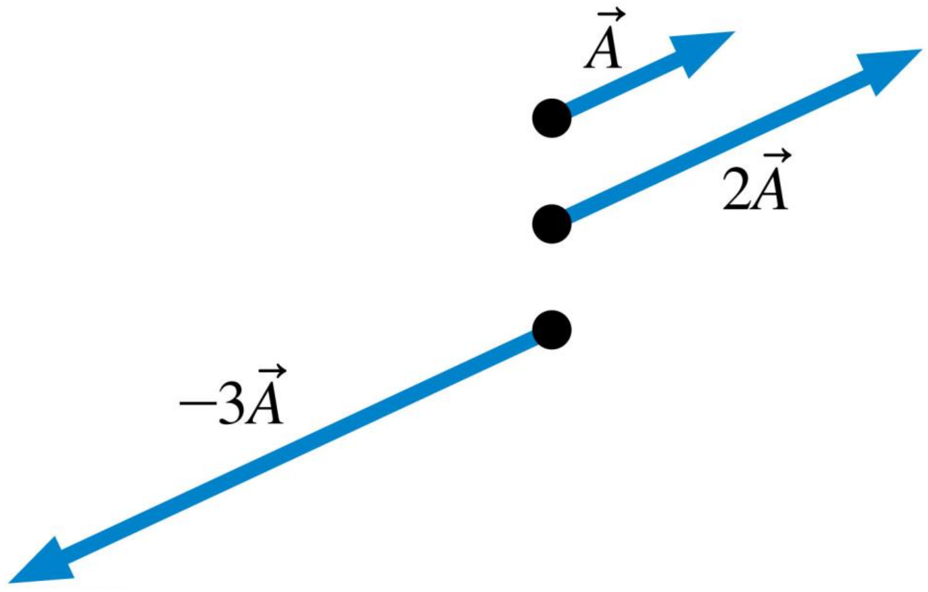
- **Multiplying a vector by a positive scalar gives another vector of *different magnitude* but pointing in the *same direction*.**
- If we multiply a vector by zero the product is a vector having zero length. The vector is known as the **zero vector**.



Multiplication by a Scalar

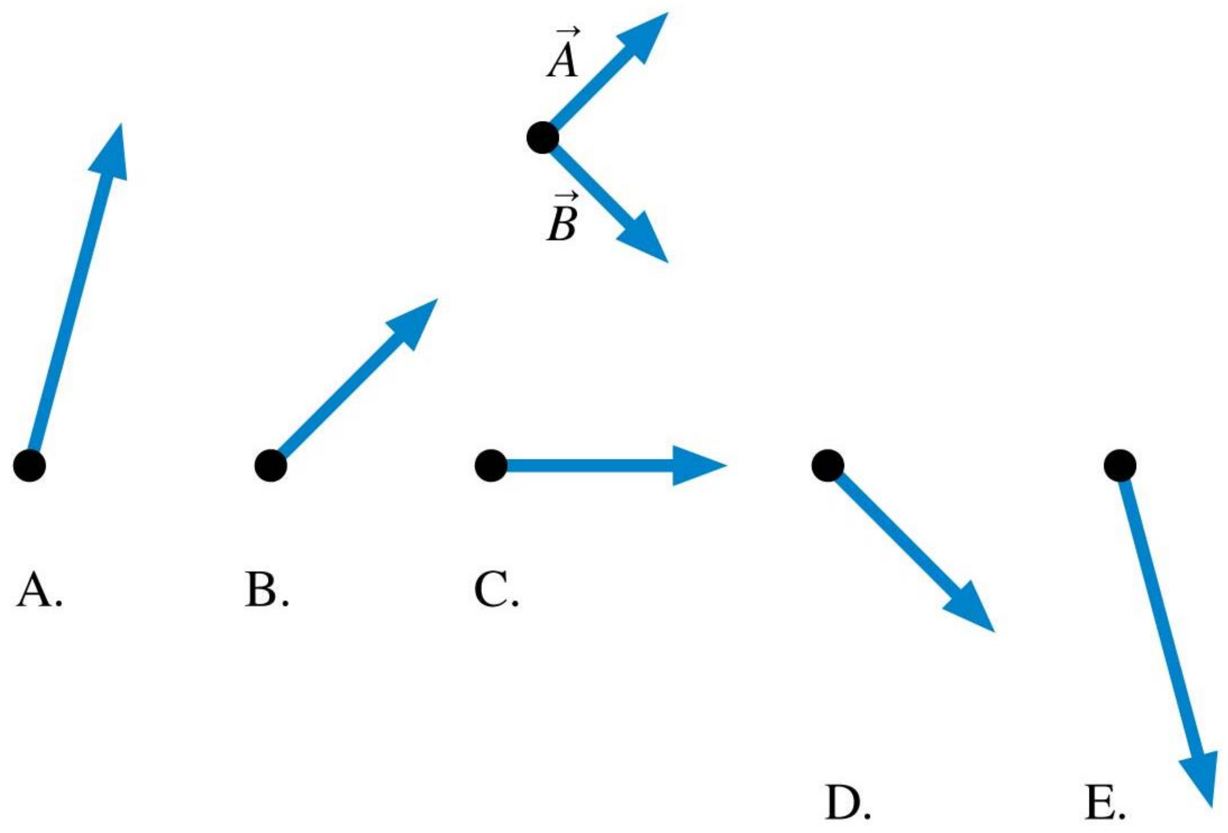
- **A vector cannot have a negative magnitude.**
- If we multiply a vector by a negative number we reverse its direction.
- Multiplying a vector by –1 reverses its direction without changing its length (magnitude).





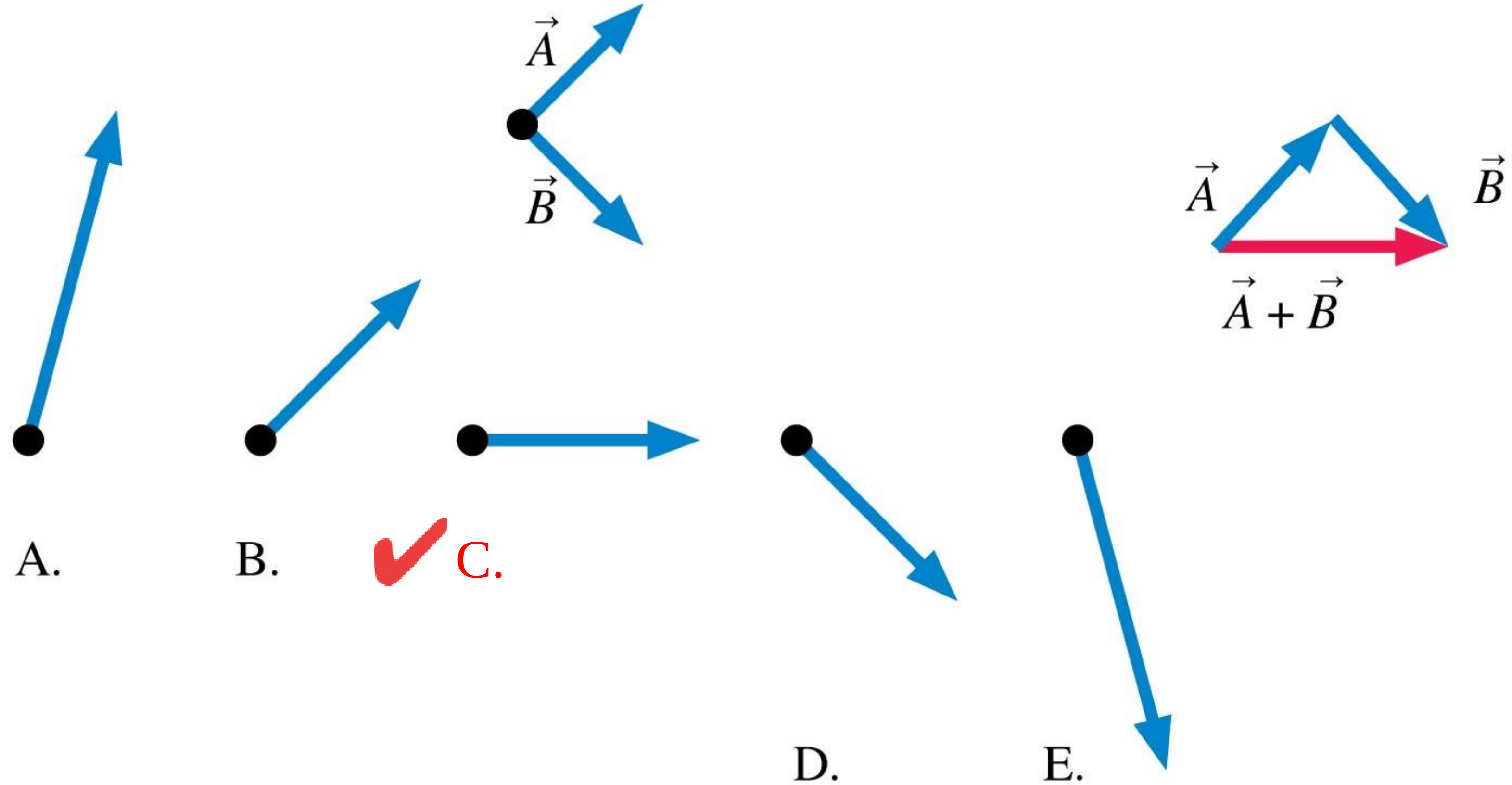
QuickCheck 3.2

Which of the vectors in the second row shows $\vec{A} + \vec{B}$?



QuickCheck 3.2

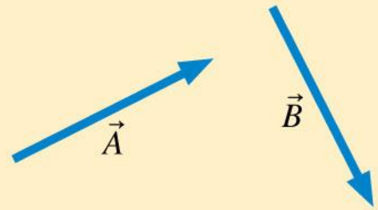
Which of the vectors in the second row shows $\vec{A} + \vec{B}$?



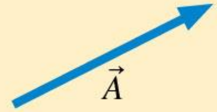
Vector Subtraction

TACTICS BOX 3.1 **Subtracting vectors**

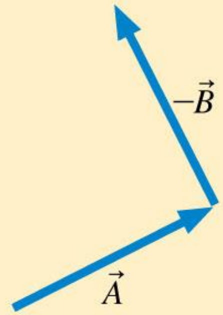
To subtract $\vec{B}$ from $\vec{A}$:



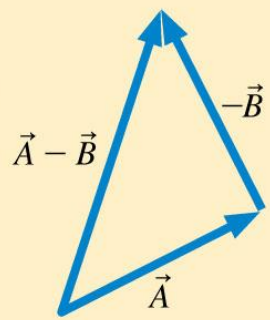
❶ Draw $\vec{A}$.



❷ Place the tail of $-\vec{B}$ at the tip of $\vec{A}$.



❸ Draw an arrow from the tail of $\vec{A}$ to the tip of $-\vec{B}$. This is vector $\vec{A} - \vec{B}$.

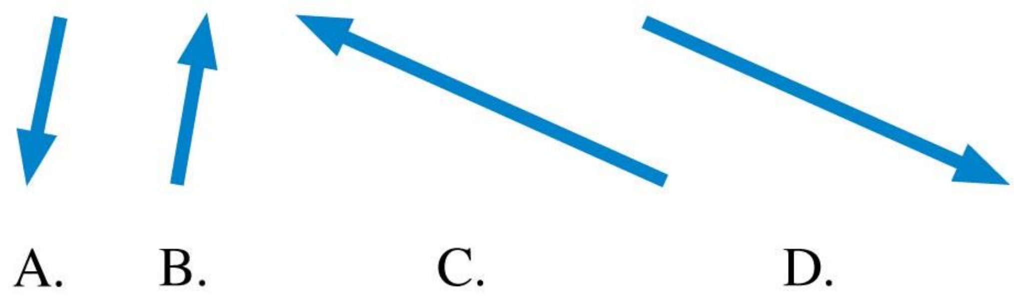


Exercises 5–8

Text: p. 67

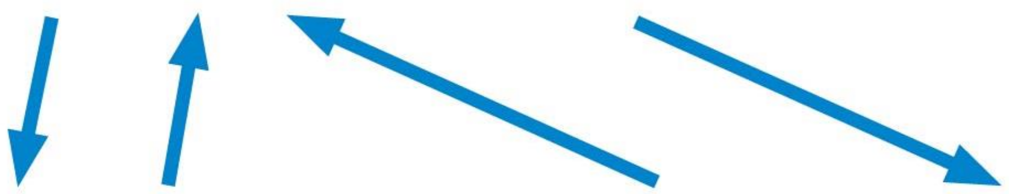
QuickCheck 3.3

Given vectors $\vec{P}$ and $\vec{Q}$, what is $\vec{P} - \vec{Q}$?

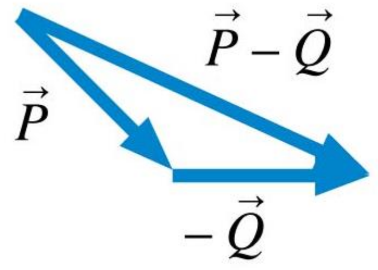


QuickCheck 3.3

Given vectors $\vec{P}$ and $\vec{Q}$, what is $\vec{P} - \vec{Q}$?

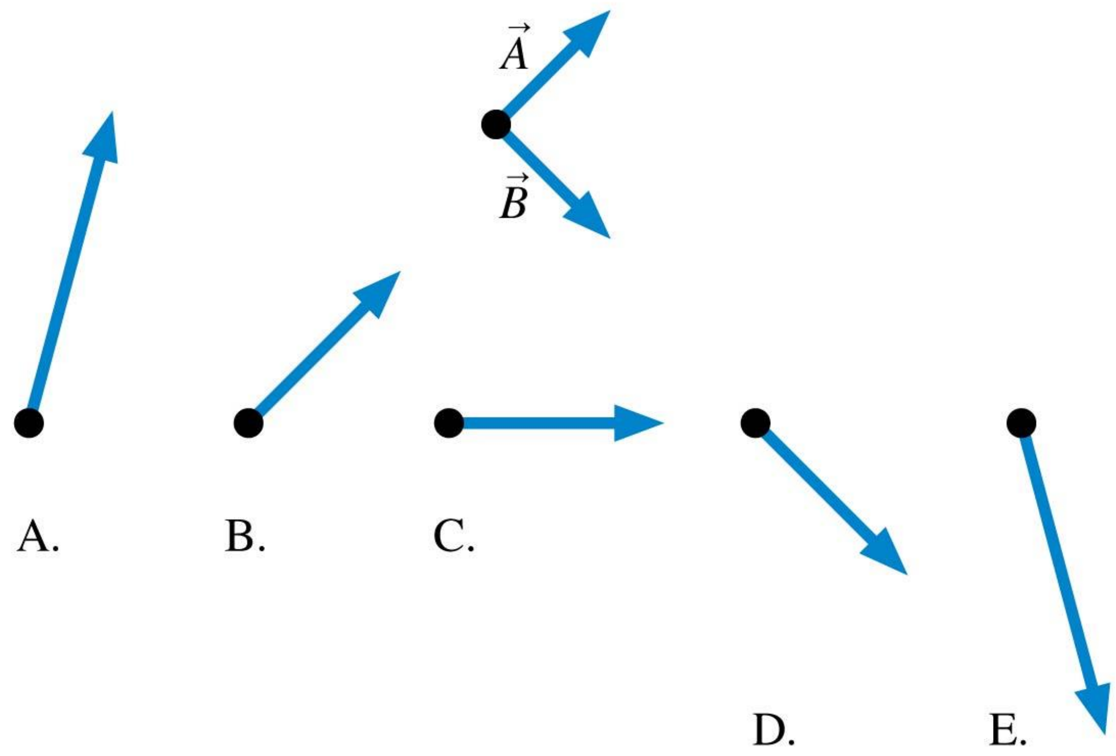


A. B. C. ✔D.



QuickCheck 3.4

Which of the vectors in the second row shows $2\vec{A} - \vec{B}$?



QuickCheck 3.4

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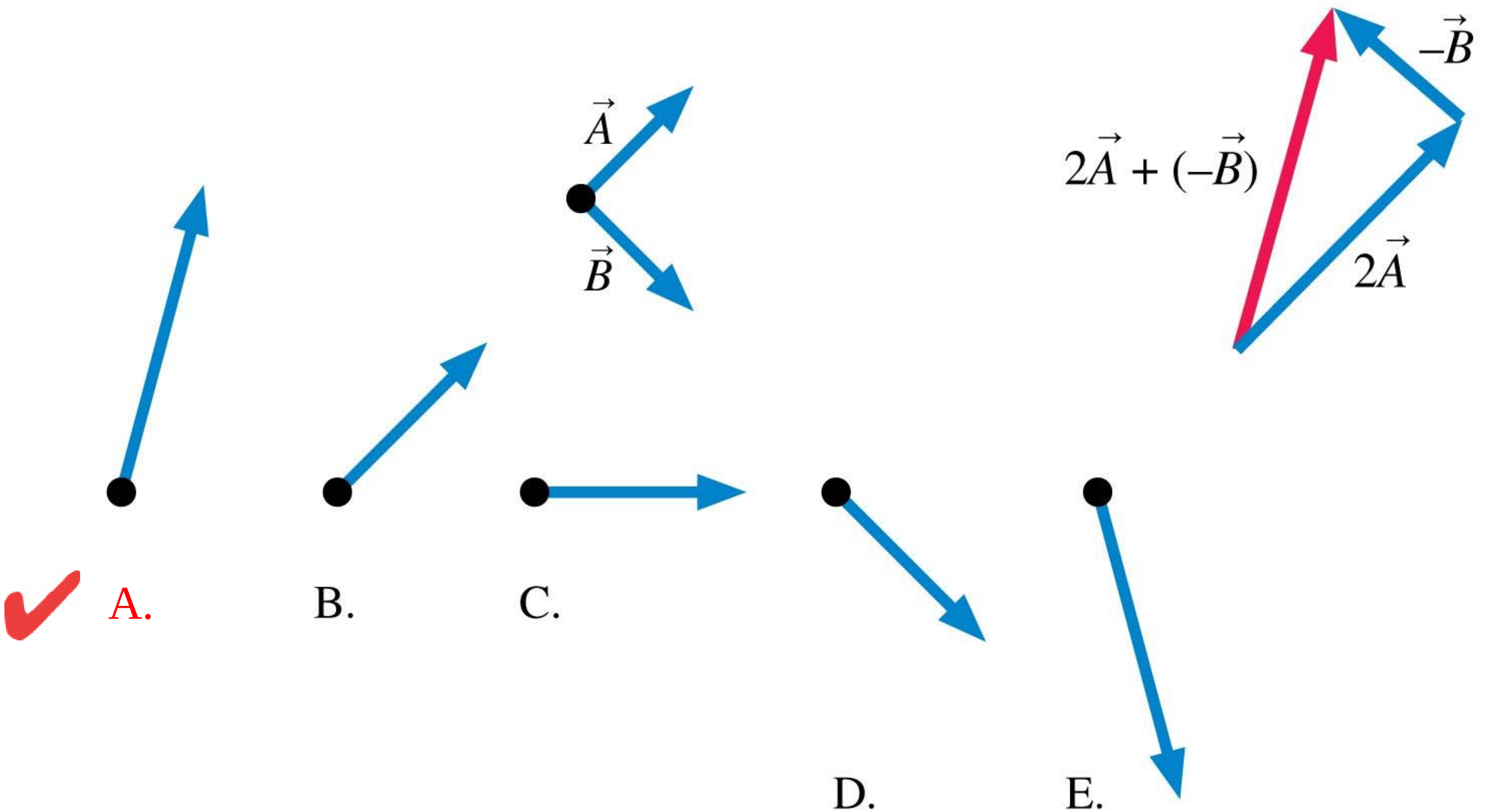


Section 3.2 Using Vectors on Motion Diagrams

Using Vectors on Motion Diagrams

- In two dimensions, an object's displacement is a vector:

$$\vec{v} = \frac{\vec{d}}{\Delta t} = \left(\frac{d}{\Delta t}, \text{same direction as } \vec{d}\right)$$

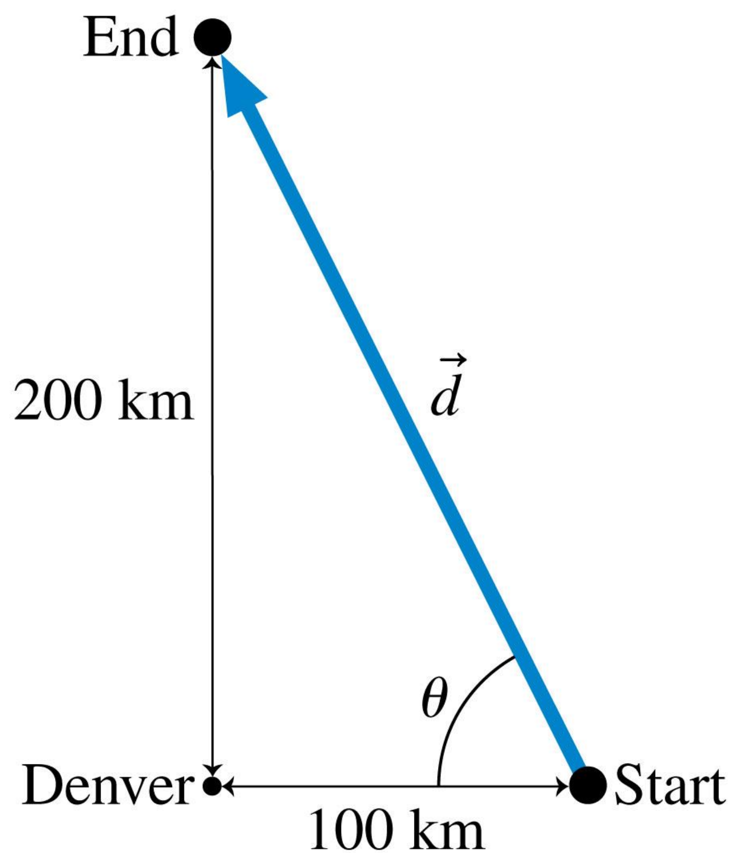
Definition of velocity in two or more dimensions

- The velocity vector is simply the displacement vector multiplied by the scalar $1/\Delta t$.
- Consequently **the velocity vector points in the direction of the displacement.**

Example 3.1 Finding the velocity of an airplane

A small plane is 100 km due east of Denver. After 1 hour of flying at a constant speed in the same direction, it is 200 km due north of Denver. What is the plane's velocity?

PREPARE The initial and final positions of the plane are shown in FIGURE 3.8; the displacement $\vec{d}$ is the vector that points from the initial to the final position.



Example 3.1 Finding the velocity of an airplane (cont.)

SOLVE The length of the displacement vector is the hypotenuse of a right triangle:

$$d = \sqrt{(100\text{ km})^2 + (200\text{ km})^2} = 224\text{ km}$$

The direction of the displacement vector is described by the angle θ in Figure 3.8. From trigonometry, this angle is

$$\theta = \tan^{-1}\left(\frac{200\text{ km}}{100\text{ km}}\right) = \tan^{-1}(2.00) = 63.4°$$

Example 3.1 Finding the velocity of an airplane (cont.)

Thus the plane's displacement vector is

$$\vec{d} = (224 \text{ km}, 63.4^\circ \text{ north of west})$$

Because the plane undergoes this displacement during 1 hour, its velocity is

$$\vec{v} = \left(\frac{d}{\Delta t}, \text{ same direction as } \vec{d}\right) = \left(\frac{224 \text{ km}}{1 \text{ h}}, 63.4^\circ \text{ north of west}\right)$$
$$= (224 \text{ km/h}, 63.4^\circ \text{ north of west})$$

ASSESS The plane's *speed* is the magnitude of the velocity, v = 224 km/h. This is approximately 140 mph, which is a reasonable speed for a small plane.

Using Vectors on Motion Diagrams

- The vector definition of acceleration is a straightforward extension of the one-dimensional version:

$$\vec{a} = \frac{\vec{v}_\mathrm{f} - \vec{v}_\mathrm{i}}{t_\mathrm{f} - t_\mathrm{i}} = \frac{\Delta \vec{v}}{\Delta t}$$

Definition of acceleration in two or more dimensions

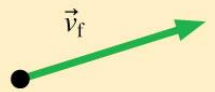
- There is an acceleration whenever there is a change in velocity. Velocity can change in either or both of two possible ways:
    1. The magnitude can change, indicating a change in speed.
    2. The direction of motion can change.

Finding the Acceleration Vector

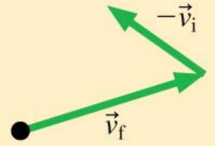
TACTICS BOX 3.2 **Finding the acceleration vector**

To find the acceleration between velocity $\vec{v}_i$ and velocity $\vec{v}_f$:

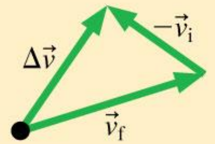
1. Draw the velocity vector $\vec{v}_f$.



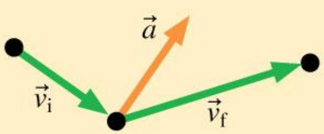
2. Draw $-\vec{v}_i$ at the tip of $\vec{v}_f$.



3. Draw $\Delta\vec{v} = \vec{v}_f - \vec{v}_i$
$= \vec{v}_f + (-\vec{v}_i)$
This is the direction of $\vec{a}$.



4. Return to the original motion diagram. Draw a vector at the middle point in the direction of $\Delta\vec{v}$; label it $\vec{a}$. This is the average acceleration at the midpoint between $\vec{v}_i$ and $\vec{v}_f$.

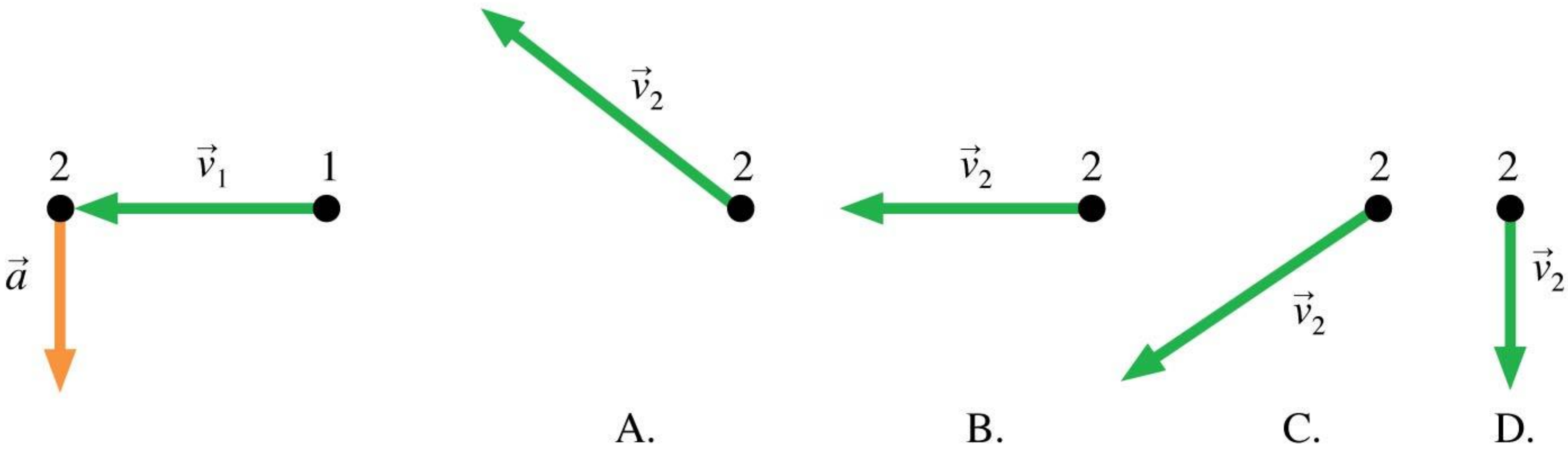


Exercises 11,12

Text: p. 69

QuickCheck 3.5

A particle undergoes acceleration $\vec{a}$ while moving from point 1 to point 2. Which of the choices shows the velocity vector $\vec{v}_2$ as the object moves away from point 2?



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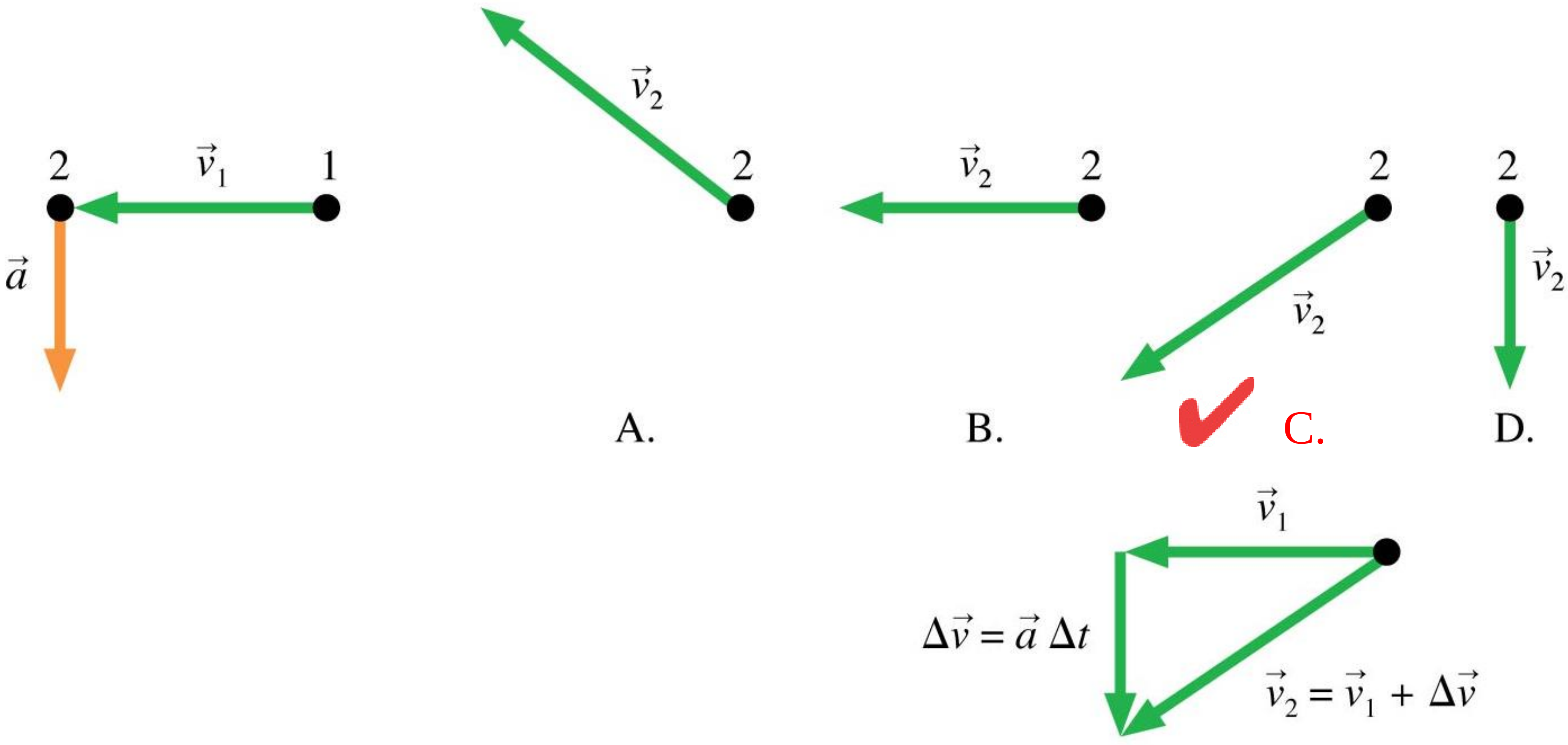
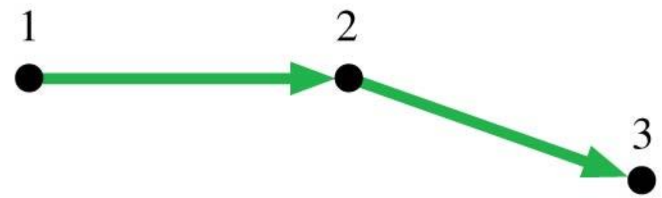


QuickCheck 3.6

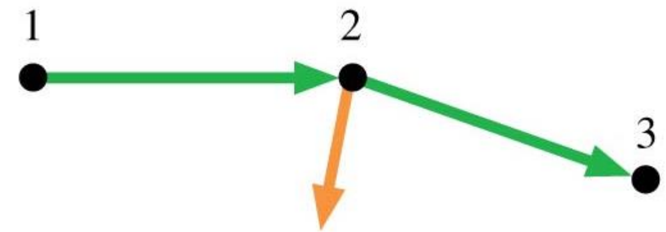
The diagram shows three points of a motion diagram. The particle changes direction with no change of speed. What is the acceleration at point 2?



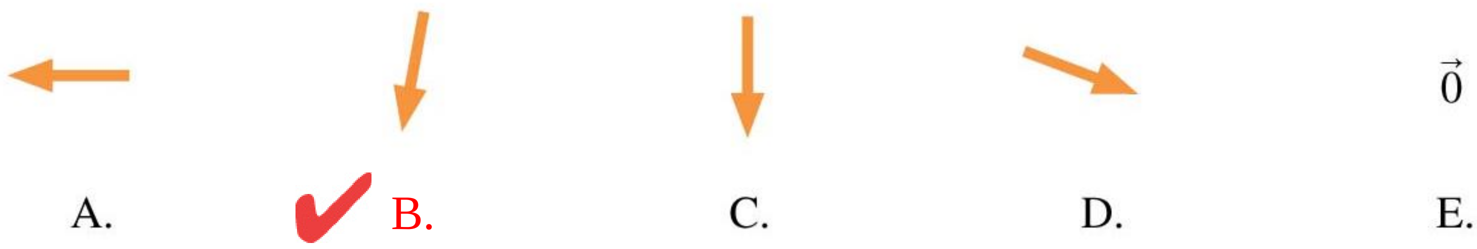
A. B. C. D. E. $\vec{0}$

QuickCheck 3.6

The diagram shows three points of a motion diagram. The particle changes direction with no change of speed. What is the acceleration at point 2?

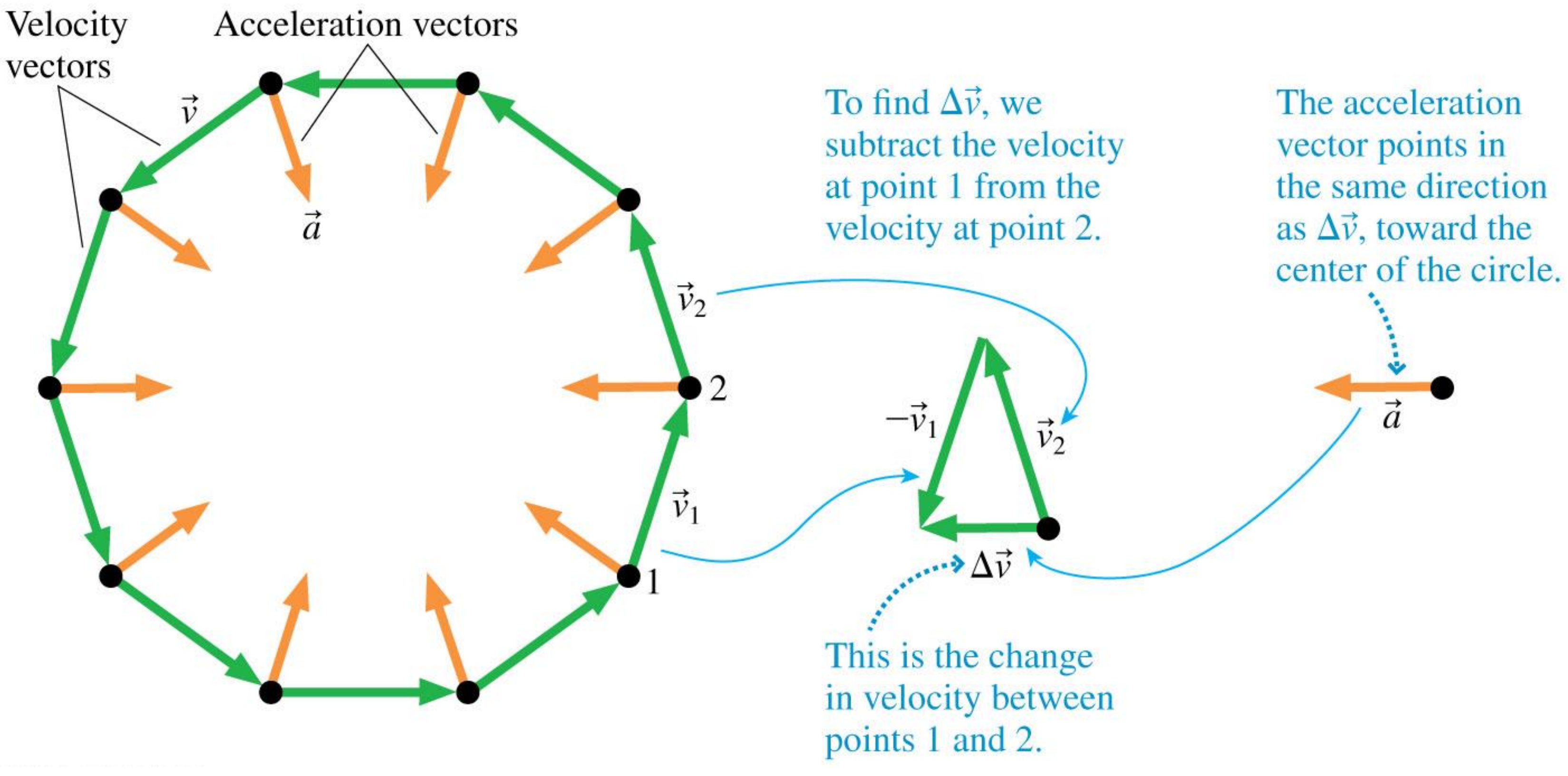


Acceleration of changing direction



Vectors and Circular Motion

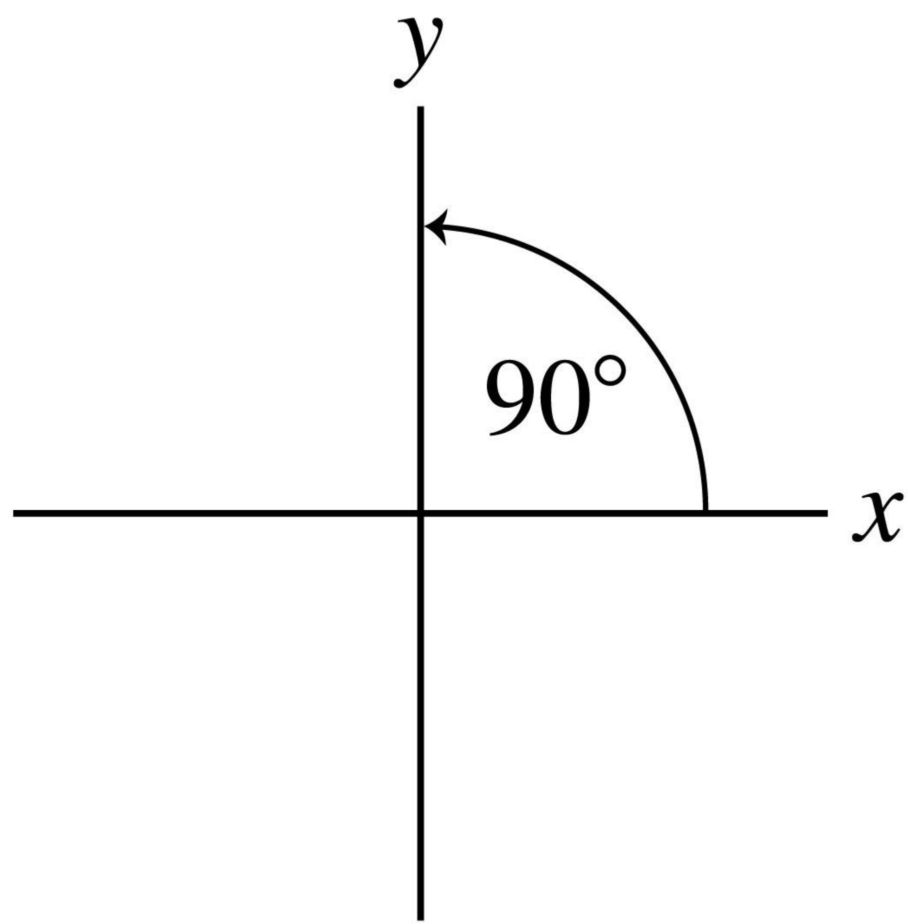
- Cars on a Ferris wheel move at a constant speed but in a continuously changing direction. They are in **uniform circular motion**.



Section 3.3 Coordinate Systems and Vector Components

Coordinate Systems

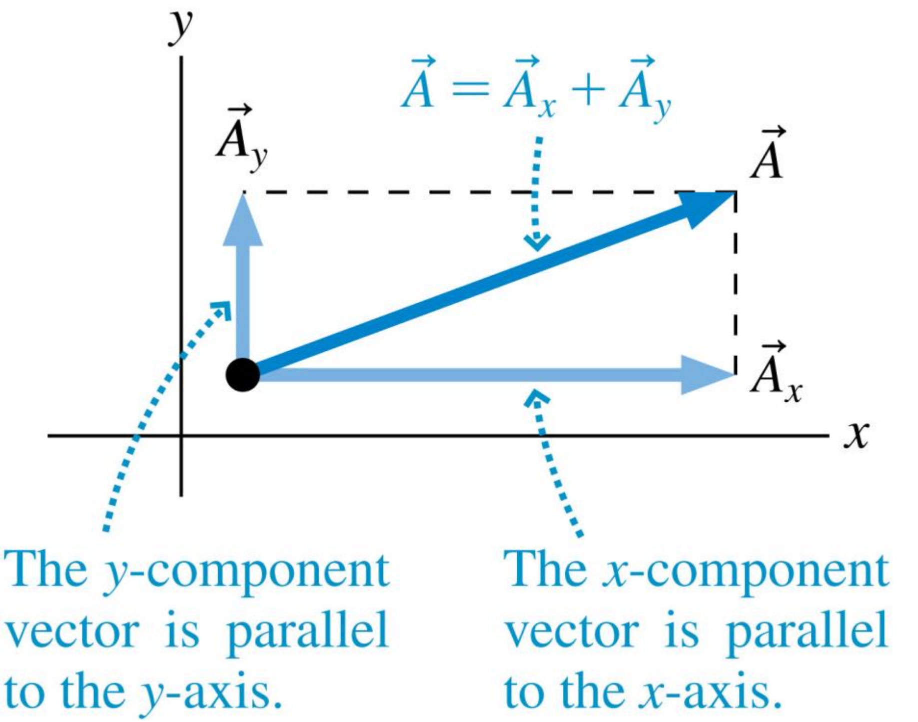
- A coordinate system is an artificially imposed grid that you place on a problem in order to make quantitative measurements.
- We will generally use **Cartesian coordinates**.
- Coordinate axes have a positive end and a negative end, separated by a zero at the origin where the two axes cross.



Component Vectors

- For a vector A and an xy-coordinate system we can define two new vectors parallel to the axes that we call the component vectors of $\vec{A}$.
- You can see, using the parallelogram rule, that $\vec{A}$ is the vector sum of the two component vectors:

$$\vec{A} = \vec{A}_x + \vec{A}_y$$



Components

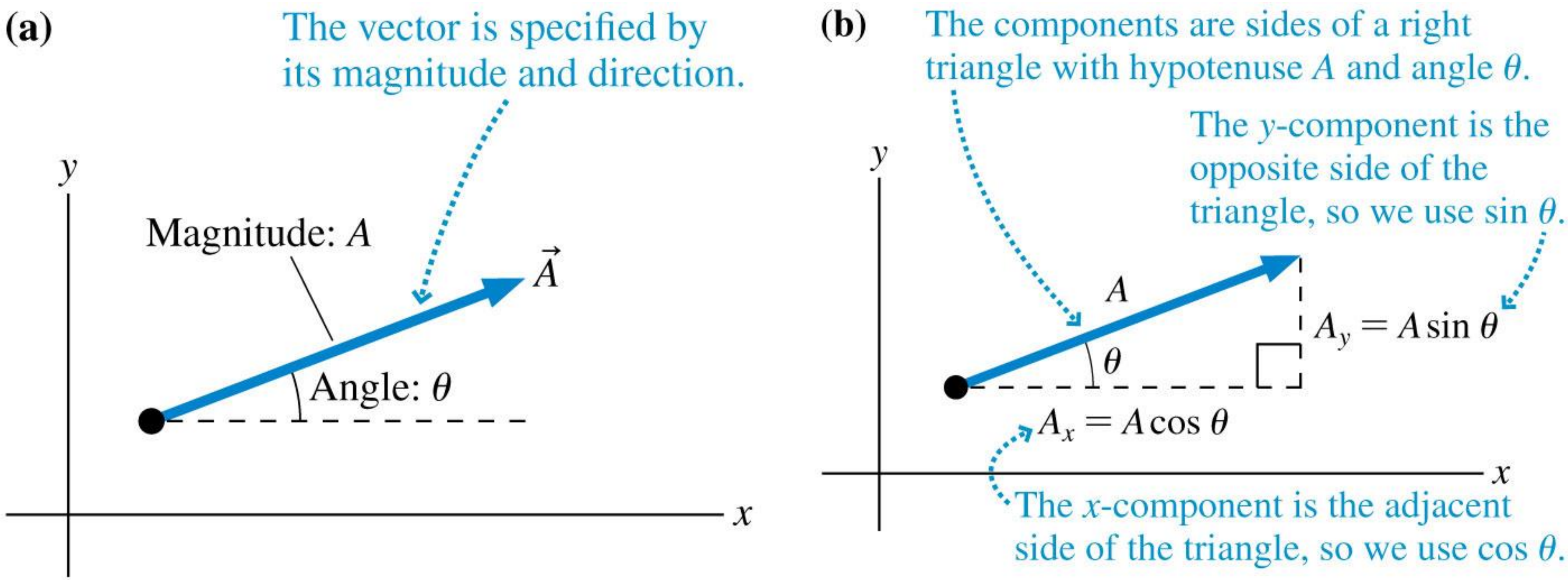
TACTICS BOX 3.3 **Determining the components of a vector**

1. The absolute value $|A_x|$ of the x-component A_x is the magnitude of the component vector $\vec{A}_x$.
2. The *sign* of A_x is positive if $\vec{A}_x$ points in the positive x-direction, negative if $\vec{A}_x$ points in the negative x-direction.
3. The y-component A_y is determined similarly.

Exercises 16–18

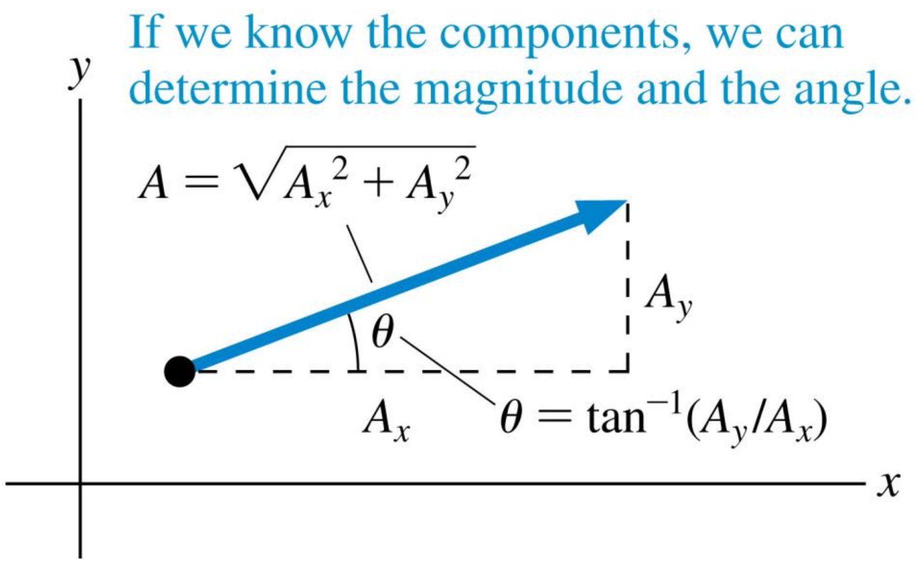
Text: p. 71

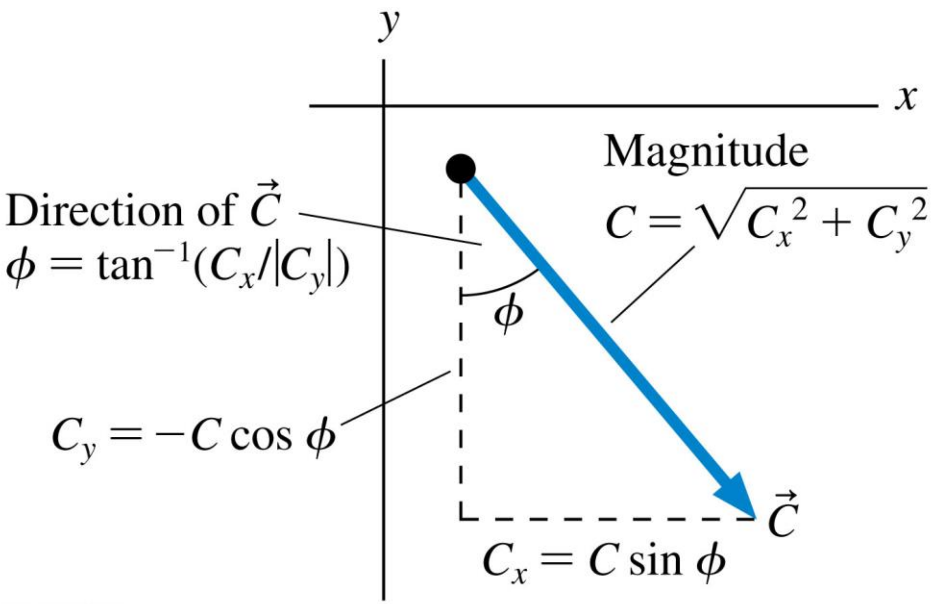
Components



(a)
The vector is specified by its magnitude and direction.
y
Magnitude: A
$\vec{A}$
Angle: θ
x
(b)
The components are sides of a right triangle with hypotenuse A and angle θ.
The y-component is the opposite side of the triangle, so we use sin θ.
y
A
θ
$A_y = A \sin \theta$
$A_x = A \cos \theta$
x
The x-component is the adjacent side of the triangle, so we use cos θ.

Components

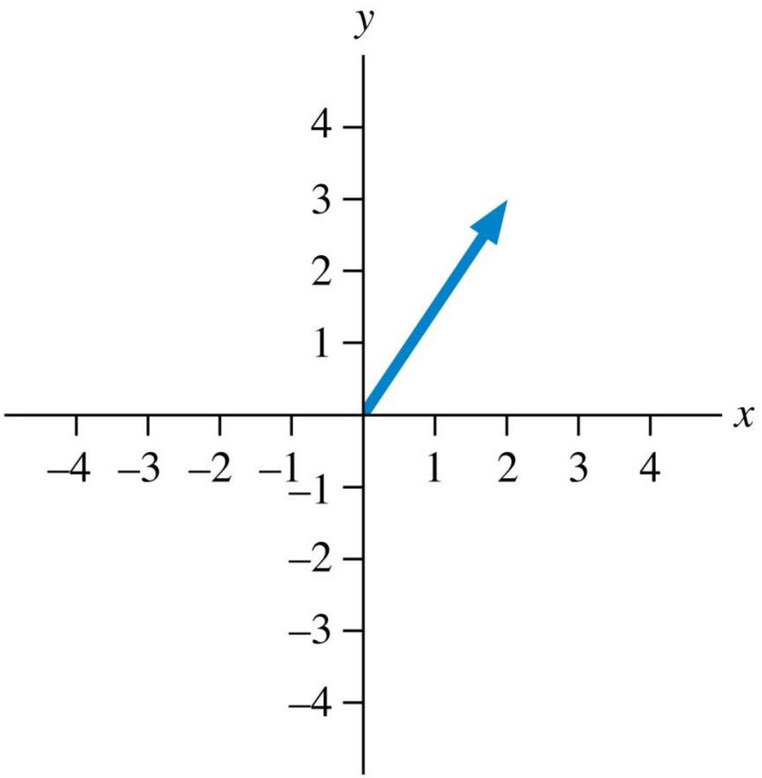

If we know the components, we can determine the magnitude and the angle.
y
x
$A = \sqrt{A_x^{\,2} + A_y^{\,2}}$
A_y
θ
A_x
$\theta = \tan^{-1}(A_y/A_x)$


y
x
Magnitude
$C = \sqrt{C_x^{\,2} + C_y^{\,2}}$
Direction of $\vec{C}$
$\phi = \tan^{-1}(C_x/|C_y|)$
ϕ
$C_y = -C\cos\phi$
$\vec{C}$
$C_x = C\sin\phi$

QuickCheck 3.7

What are the *x*- and *y*-components of this vector?

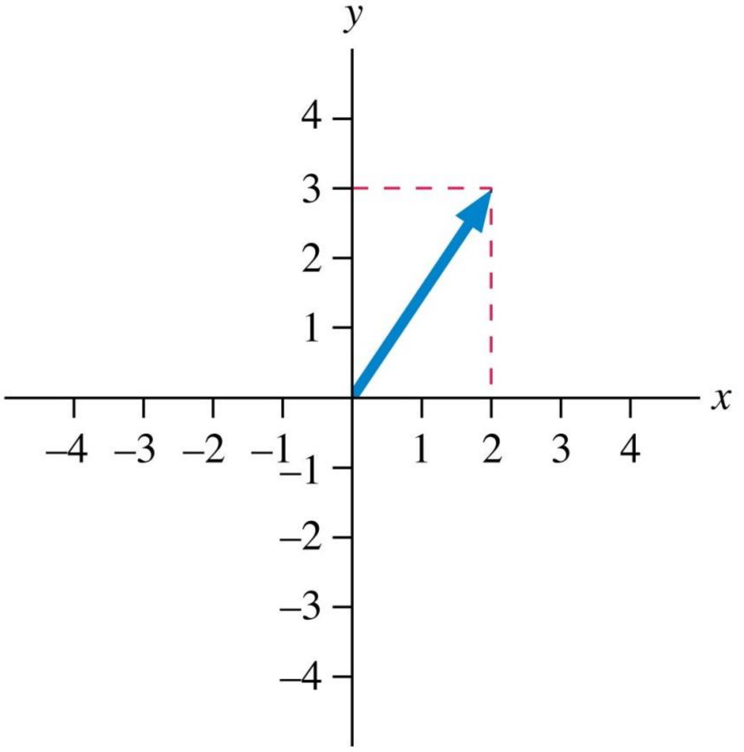
A. 3, 2
B. 2, 3
C. –3, 2
D. 2, –3
E. –3, –2



QuickCheck 3.7

What are the *x*- and *y*-components of this vector?

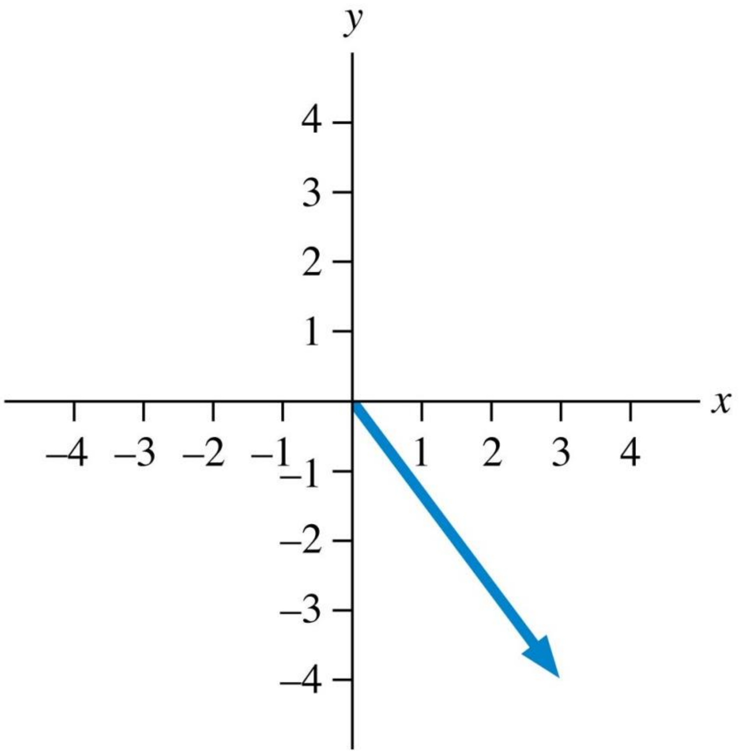
A. 3, 2
B. 2, 3
C. –3, 2
D. 2, –3
E. –3, –2



QuickCheck 3.8

What are the *x*- and *y*-components of this vector?

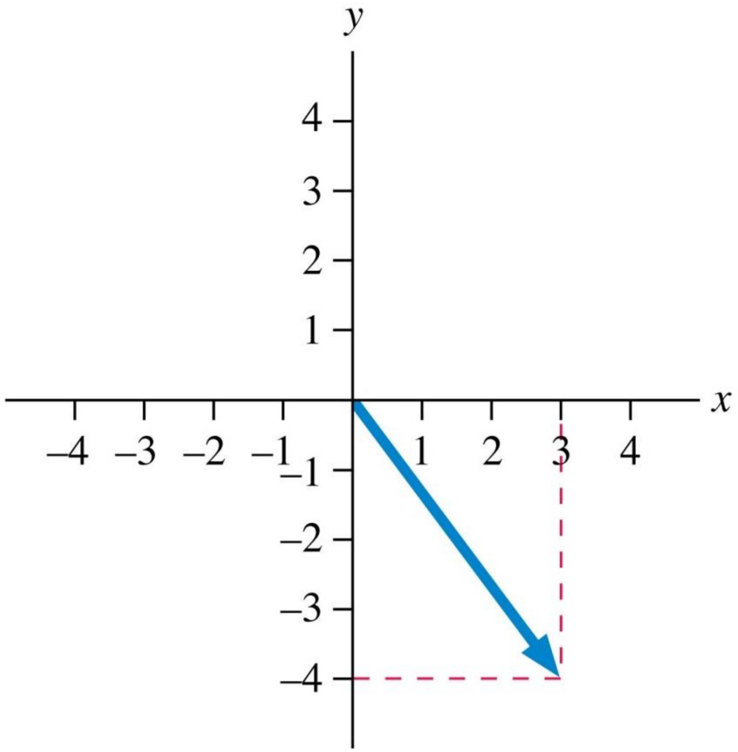
A. 3, 4
B. 4, 3
C. –3, 4
D. 4, –3
E. 3, –4



QuickCheck 3.8

What are the *x*- and *y*-components of this vector?

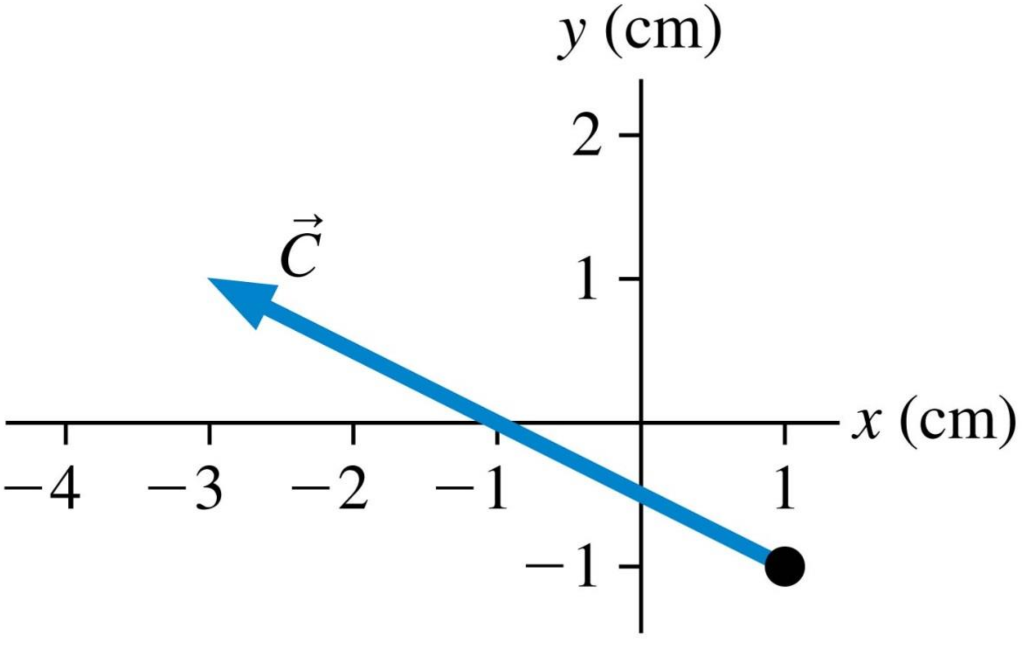
A. 3, 4
B. 4, 3
C. –3, 4
D. 4, –3
E. 3, –4



QuickCheck 3.9

What are the *x*- and *y*-components of vector $\vec{C}$?

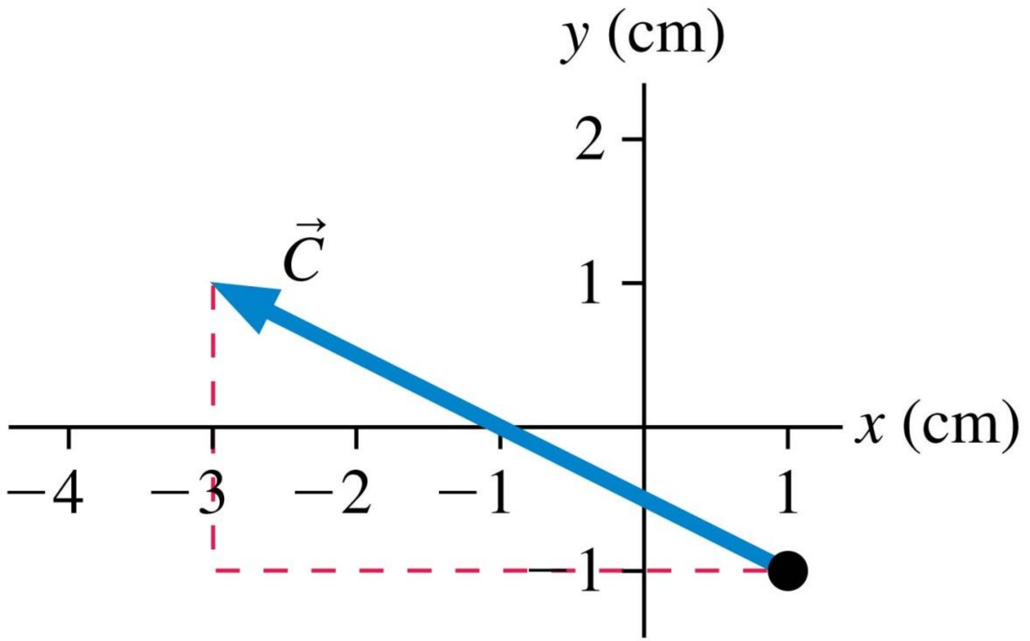
A. 1, –3
B. –3, 1
C. 1, –1
D. –4, 2
E. 2, –4



QuickCheck 3.9

What are the x- and y-components of vector $\vec{C}$?

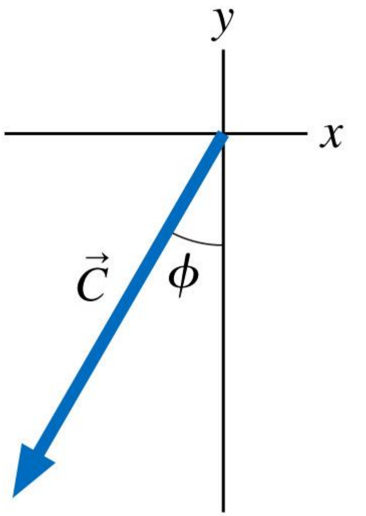
A. 1, –3
B. –3, 1
C. 1, –1
D. –4, 2
E. 2, –4



QuickCheck 3.10

The angle Φ that specifies the direction of vector $\vec{C}$ is

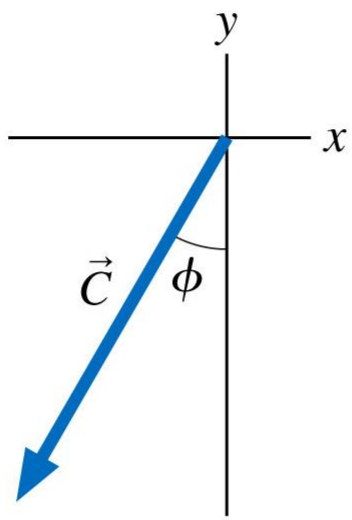
A. $\tan^{-1}(C_x/C_y)$
B. $\tan^{-1}(C_y/C_x)$
C. $\tan^{-1}(|C_x|/C_y)$
D. $\tan^{-1}(|C_x|/|C_y|)$
E. $\tan^{-1}(|C_y|/|C_x|)$



QuickCheck 3.10

The angle Φ that specifies the direction of vector $\vec{C}$ is

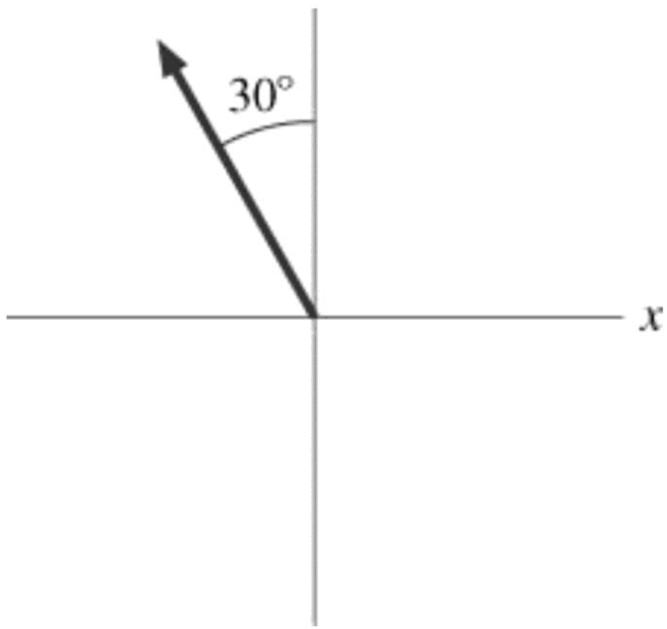
A. $\tan^{-1}(C_x/C_y)$
B. $\tan^{-1}(C_y/C_x)$
C. $\tan^{-1}(|C_x|/C_y)$
D. $\tan^{-1}(|C_x|/|C_y|)$
E. $\tan^{-1}(|C_y|/|C_x|)$



QuickCheck 3.11

The following vector has length 4.0 units. What are the *x*- and *y*-components?

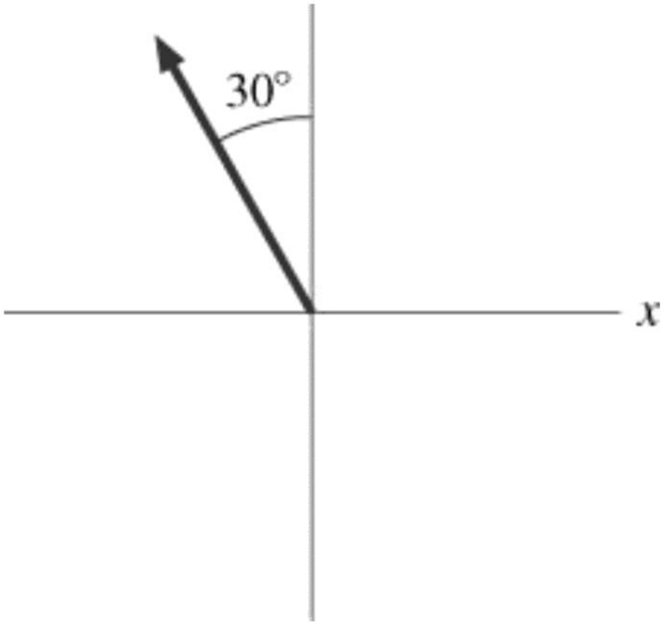
A. 3.5, 2.0
B. –2.0, 3.5
C. –3.5, 2.0
D. 2.0, –3.5
E. –3.5, –2.0



QuickCheck 3.11

The following vector has length 4.0 units. What are the *x*- and *y*-components?

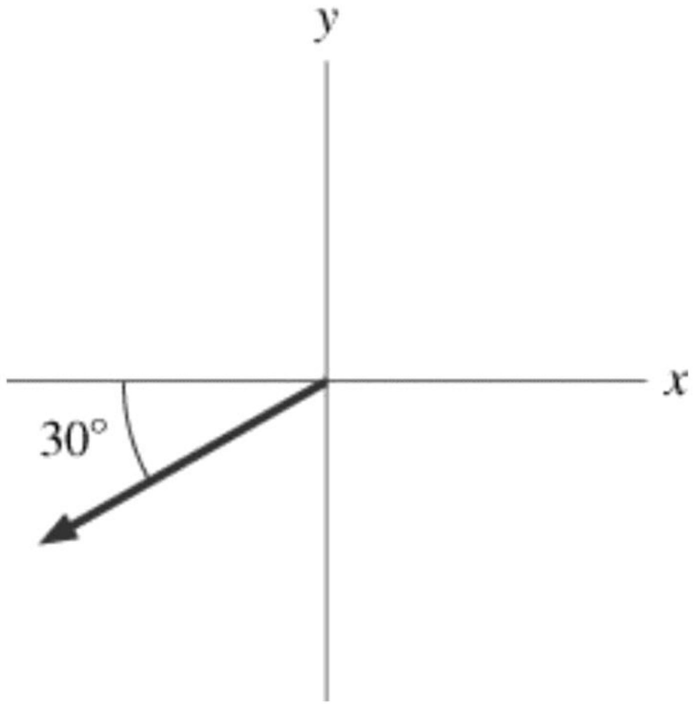
A. 3.5, 2.0
✔ B. –2.0, 3.5
C. –3.5, 2.0
D. 2.0, –3.5
E. –3.5, –2.0



QuickCheck 3.12

The following vector has length 4.0 units. What are the *x*- and *y*-components?

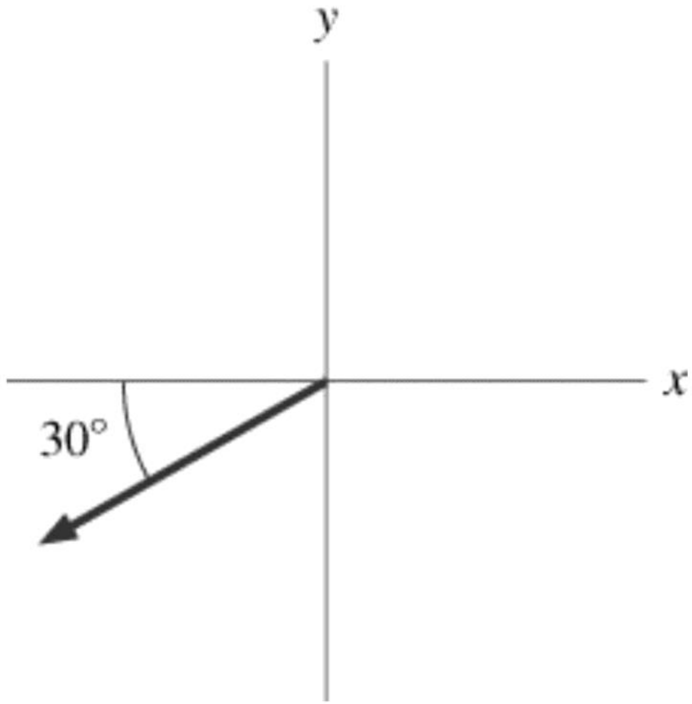
A. 3.5, 2.0
B. 2.0, 3.5
C. –3.5, 2.0
D. 2.0, –3.5
E. –3.5, –2.0



QuickCheck 3.12

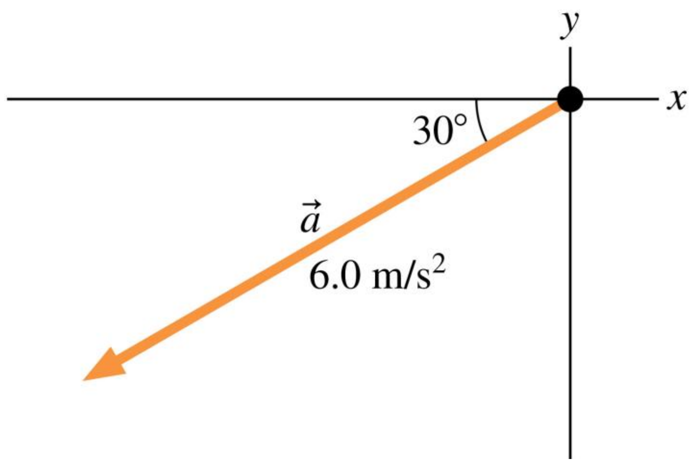
The following vector has length 4.0 units. What are the *x*- and *y*-components?

A. 3.5, 2.0
B. 2.0, 3.5
C. –3.5, 2.0
D. 2.0, –3.5
✔ E. –3.5, –2.0



Example 3.3 Finding the components of an acceleration vector

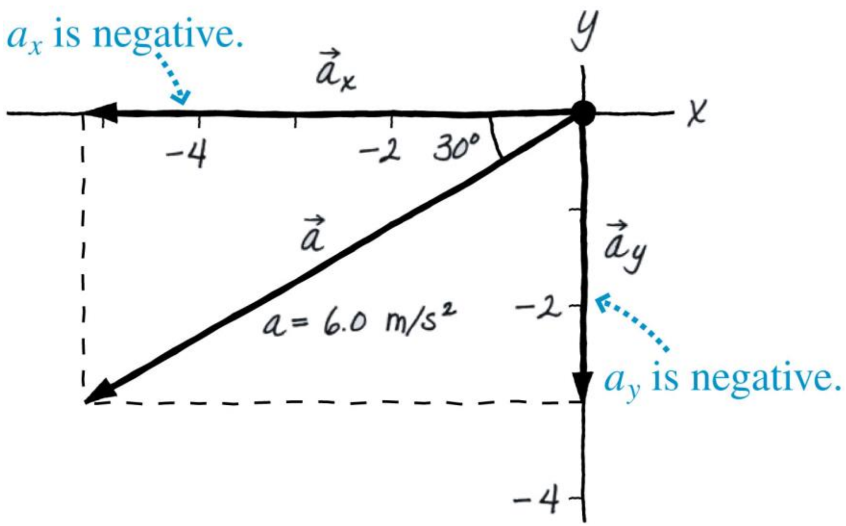
Find the x- and y-components of the acceleration vector $\vec{a}$ shown in FIGURE 3.17.



PREPARE It's important to *draw* the vectors. Making a sketch is crucial to setting up this problem. FIGURE 3.18 shows the original vector $\vec{a}$ decomposed into component vectors parallel to the axes.

Example 3.3 Finding the components of an acceleration vector (cont.)

SOLVE The acceleration vector $\vec{a}$ = (6.0 m/s^2, 30° below the negative x-axis) points to the left (negative x-direction) and down (negative y-direction), so the components a_x and a_y are both negative:



$$a_x = -a \cos 30° = -(6.0 \text{ m/s}^2) \cos 30° = -5.2 \text{ m/s}^2$$

$$a_y = -a \sin 30° = -(6.0 \text{ m/s}^2) \sin 30° = -3.0 \text{ m/s}^2$$

Example 3.3 Finding the components of an acceleration vector (cont.)

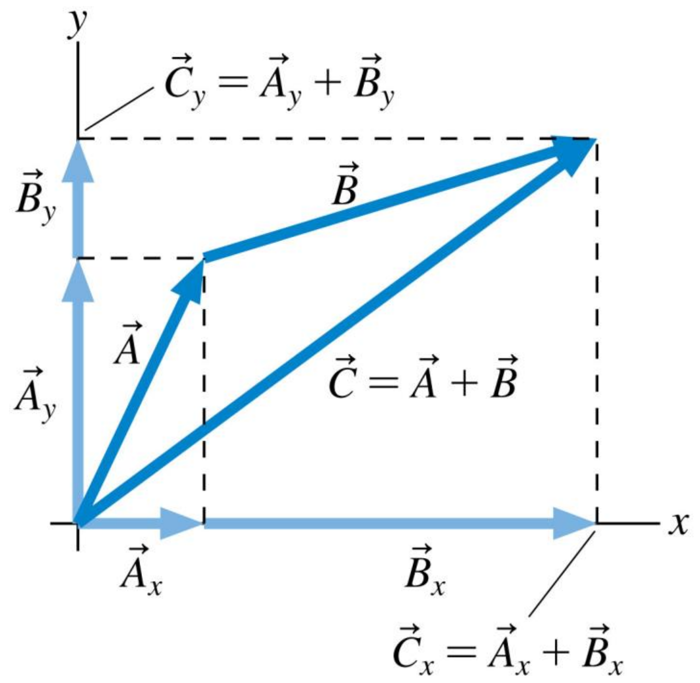
ASSESS The magnitude of the *y*-component is less than that of the *x*-component, as seems to be the case in Figure 3.18, a good check on our work. The units of a_x and a_y are the same as the units of vector $\vec{a}$. Notice that we had to insert the minus signs manually by observing that the vector points down and to the left.

Working with Components

- We can add vectors using components.
- Let's look at the vector sum $\vec{C} = \vec{A} + \vec{B}$ for the vectors shown in FIGURE 3.19. You can see that the component vectors of $\vec{C}$ are the sums of the component vectors of $\vec{A}$ and $\vec{B}$. The same is true of the components: $C_x = A_x + B_x$ and $C_y = A_y + B_y$.

$$D_x = A_x + B_x + C_x + \cdots$$

$$D_y = A_y + B_y + C_y + \cdots$$



Working with Components

$$\vec{F} = \vec{n} + \vec{w} + \vec{f}$$

- Equation 3.18 is really just a shorthand way of writing the two simultaneous equations:

$$F_x = n_x + w_x + f_x$$

$$F_y = n_y + w_y + f_y$$

- In other words, a vector equation is interpreted as meaning: Equate the *x*-components on both sides of the equals sign, then equate the *y*-components. Vector notation allows us to write these two equations in a more compact form.

QuickCheck 3.13

A_x is the ____________ of the vector $\vec{A}_x$.

A. Magnitude
B. *x*-component
C. Direction
D. Size
E. Displacement

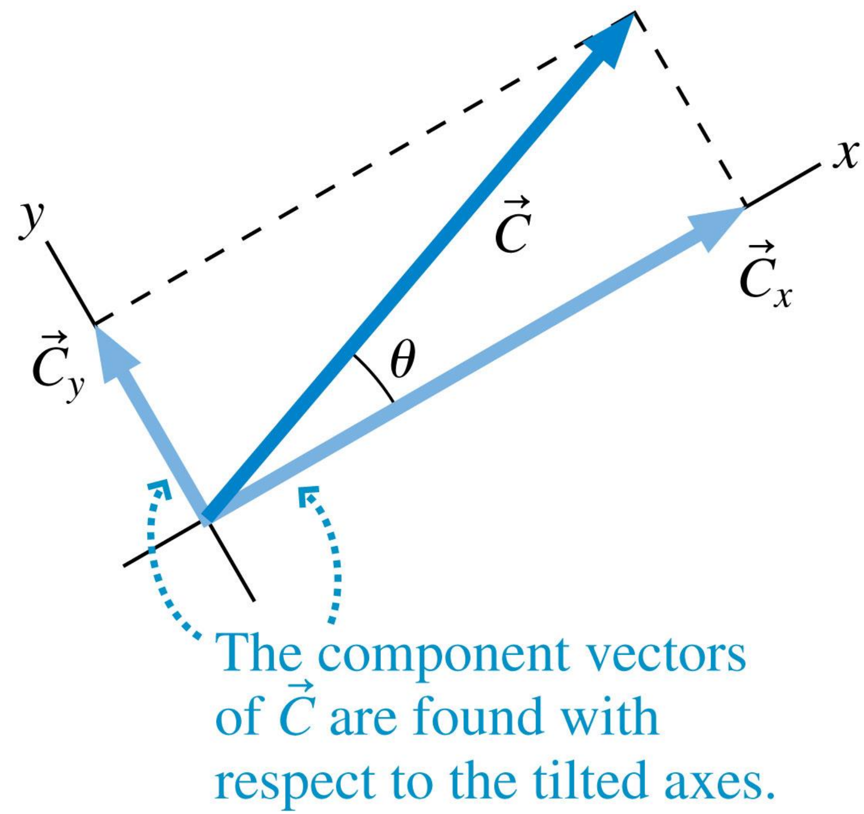
QuickCheck 3.13

A_x is the ____________ of the vector $\vec{A}_x$.

✔ A. Magnitude
B. x-component
C. Direction
D. Size
E. Displacement

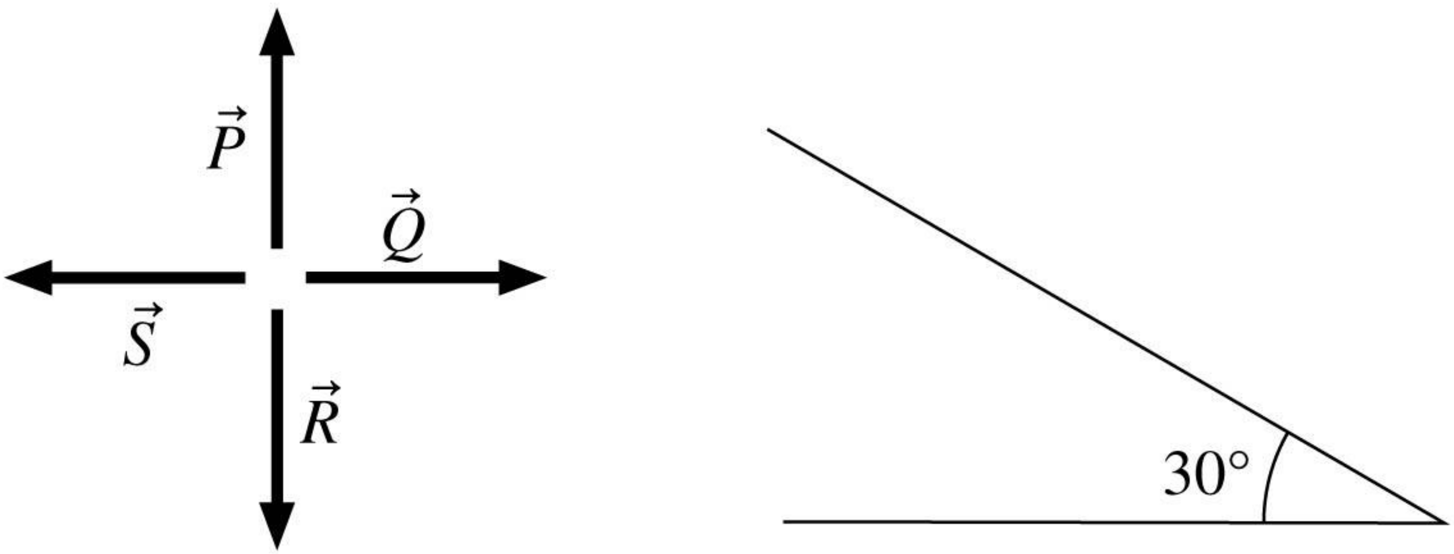
Tilted Axes

- For motion on a slope, it is often most convenient to put the *x*-axis along the slope.
- When we add the *y*-axis, this gives us a tilted coordinate system.
- Finding components with tilted axes is done the same way as with horizontal and vertical axes. The components are parallel to the tilted axes and the angles are measured from the tilted axes.



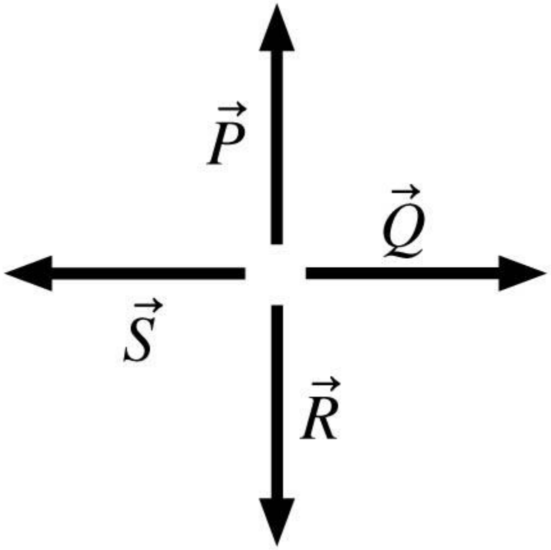
Example Problem

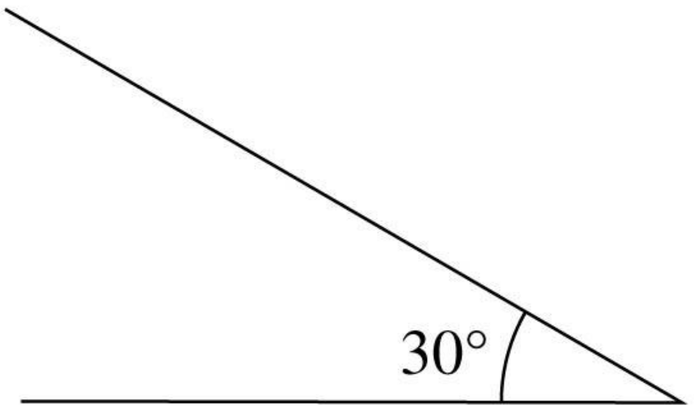
The following vectors have length 4.0 units. For each vector, what is the component parallel to the ramp?



Example Problem

The following vectors have length 4.0 units. For each vector, what is the component perpendicular to the ramp?

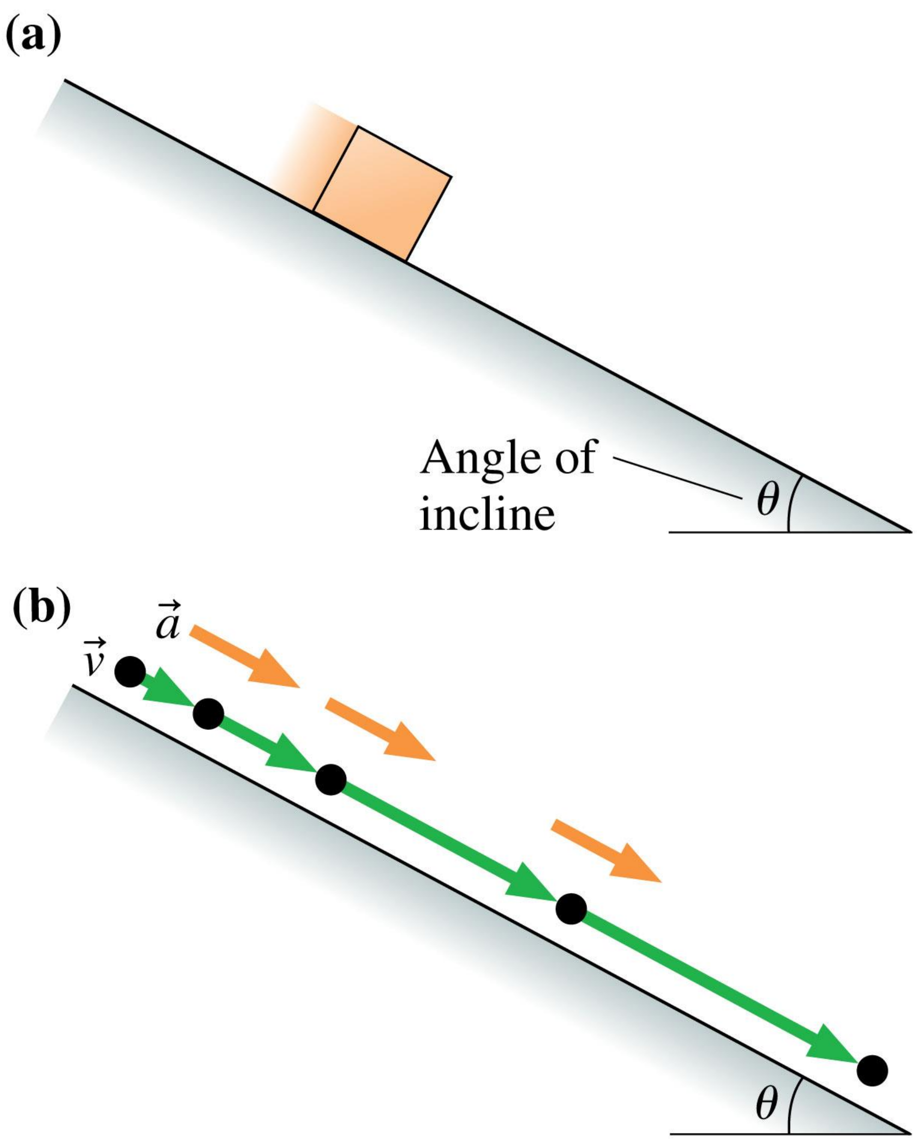




Section 3.4 Motion on a Ramp

Accelerated Motion on a Ramp

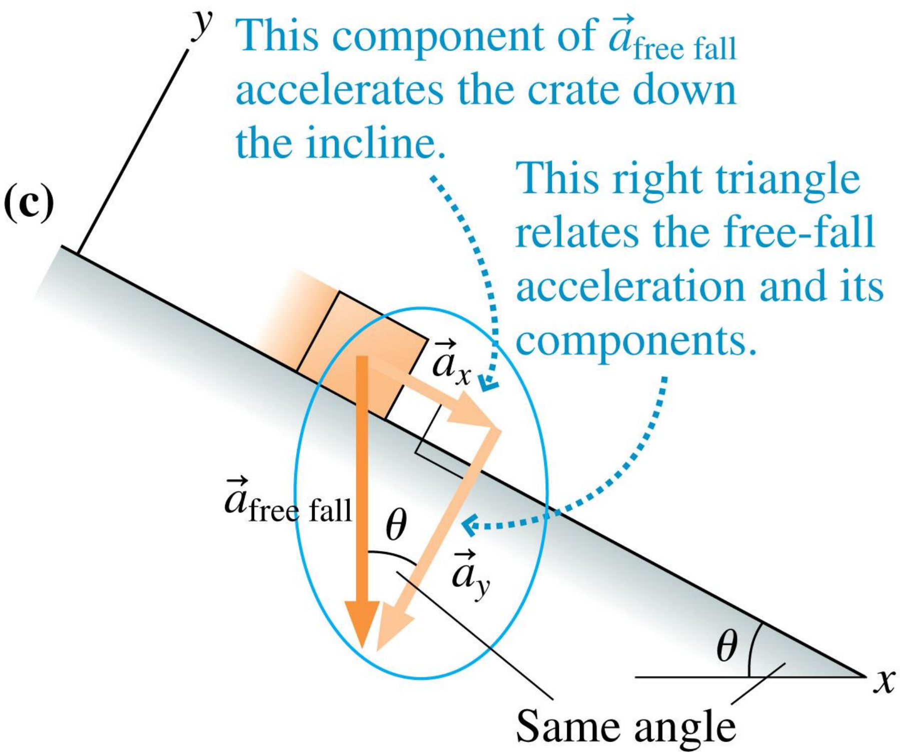
- A crate slides down a frictionless (i.e., smooth) ramp tilted at angle θ.
- The crate is constrained to accelerate parallel to the surface.
- Both the acceleration and velocity vectors are parallel to the ramp.



Accelerated Motion on a Ramp

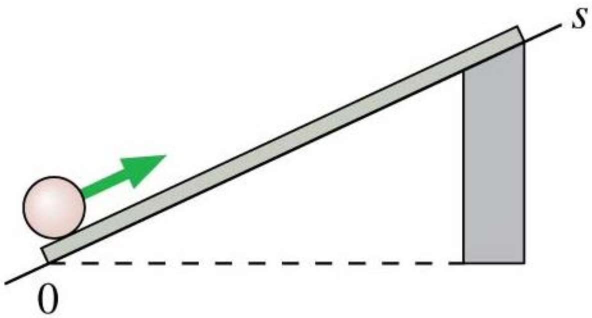
- We choose the coordinate system to have the *x*-axis along the ramp and the *y*-axis perpendicular. All motion will be along the *x*-axis.
- The acceleration parallel to the ramp is a component of the free-fall acceleration the object would have if the ramp vanished:

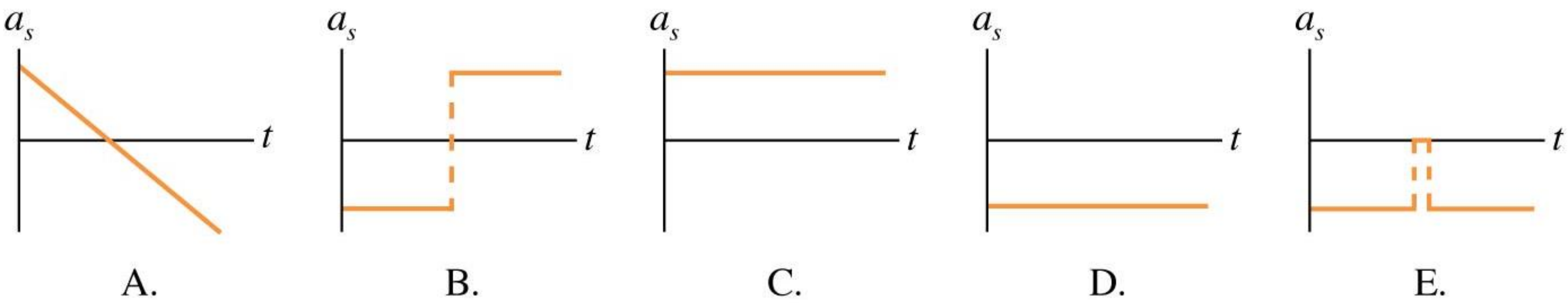
$$a_x = \pm g \sin\theta$$



QuickCheck 3.14

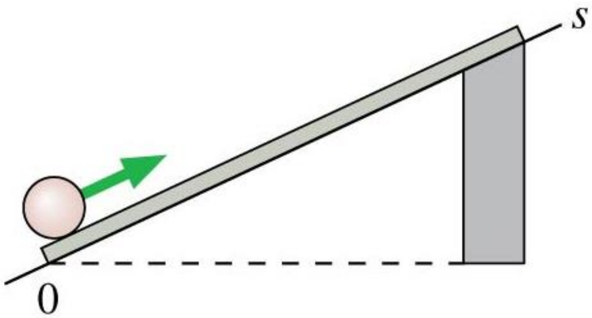
A ball rolls up the ramp, then back down. Which is the correct acceleration graph?

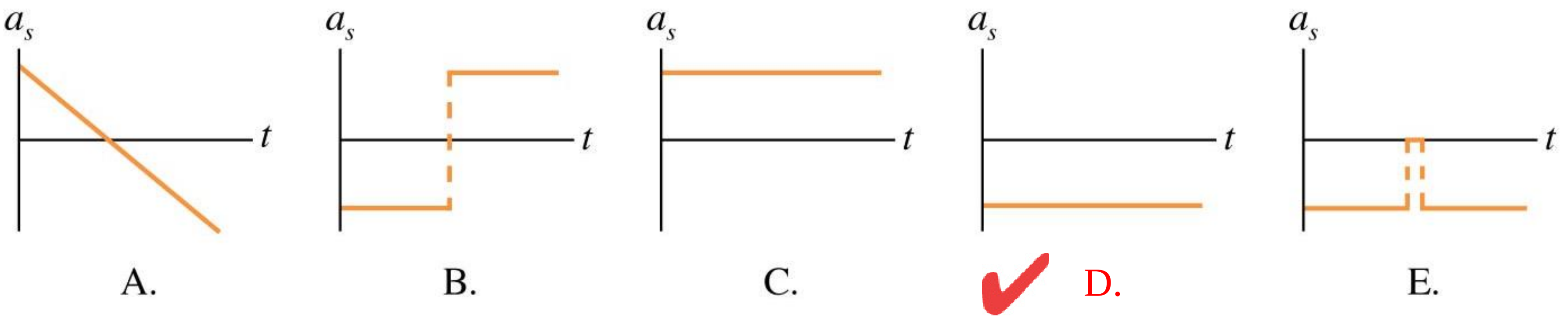




QuickCheck 3.14

A ball rolls up the ramp, then back down. Which is the correct acceleration graph?



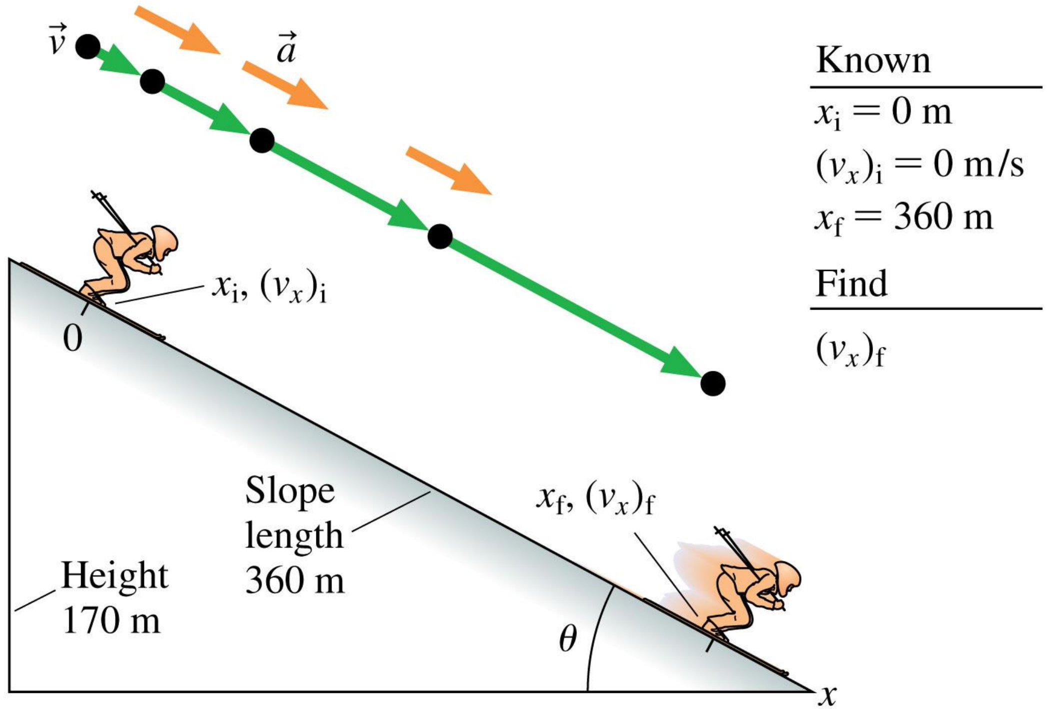


Example 3.6 Maximum possible speed for a skier

The Willamette Pass ski area in Oregon was the site of the 1993 U.S. National Speed Skiing Competition. The skiers started from rest and then accelerated down a stretch of the mountain with a reasonably constant slope, aiming for the highest possible speed at the end of this run. During this acceleration phase, the skiers traveled 360 m while dropping a vertical distance of 170 m. What is the fastest speed a skier could achieve at the end of this run?

Example 3.6 Maximum possible speed for a skier (cont.)

PREPARE We begin with the visual overview in FIGURE 3.24. The motion diagram shows the acceleration of the skier and the pictorial representation gives an overview of the problem including the dimensions of the slope. As before, we put the *x*-axis along the slope.



Example 3.6 Maximum possible speed for a skier (cont.)

SOLVE The fastest possible run would be one without any friction or air resistance, meaning the acceleration down the slope is given by Equation 3.20. The acceleration is in the positive *x*-direction, so we use the positive sign. What is the angle in Equation 3.20? Figure 3.24 shows that the 360-m-long slope is the hypotenuse of a triangle of height 170 m, so we use trigonometry to find

$$\sin\theta = \frac{170\text{ m}}{360\text{ m}}$$

which gives $\theta = \sin^{-1}(170/360) = 28°$. Equation 3.20 then gives

$$a_x = +g\sin\theta = (9.8\text{ m/s}^2)(\sin 28°) = 4.6\text{ m/s}^2$$

Example 3.6 Maximum possible speed for a skier (cont.)

For linear motion with constant acceleration, we can use the third of the kinematic equations in Synthesis 2.1: $(v_x)_f^2 = (v_x)_i^2 + 2a_x\,\Delta x$. The initial velocity $(v_x)_i$ is zero; thus

This is the distance along the slope, the length of the run.

$$(v_x)_f = \sqrt{2a_x\Delta x} = \sqrt{2(4.6\ \text{m/s}^2)(360\ \text{m})} = 58\ \text{m/s}$$

This is the fastest that any skier could hope to be moving at the end of the run. Any friction or air resistance would decrease this speed.

Example 3.6 Maximum possible speed for a skier (cont.)

ASSESS The final speed we calculated is 58 m/s, which is about 130 mph, reasonable because we expect a high speed for this sport. In the competition noted, the actual winning speed was 111 mph, not much slower than the result we calculated. Obviously, efforts to minimize friction and air resistance are working!

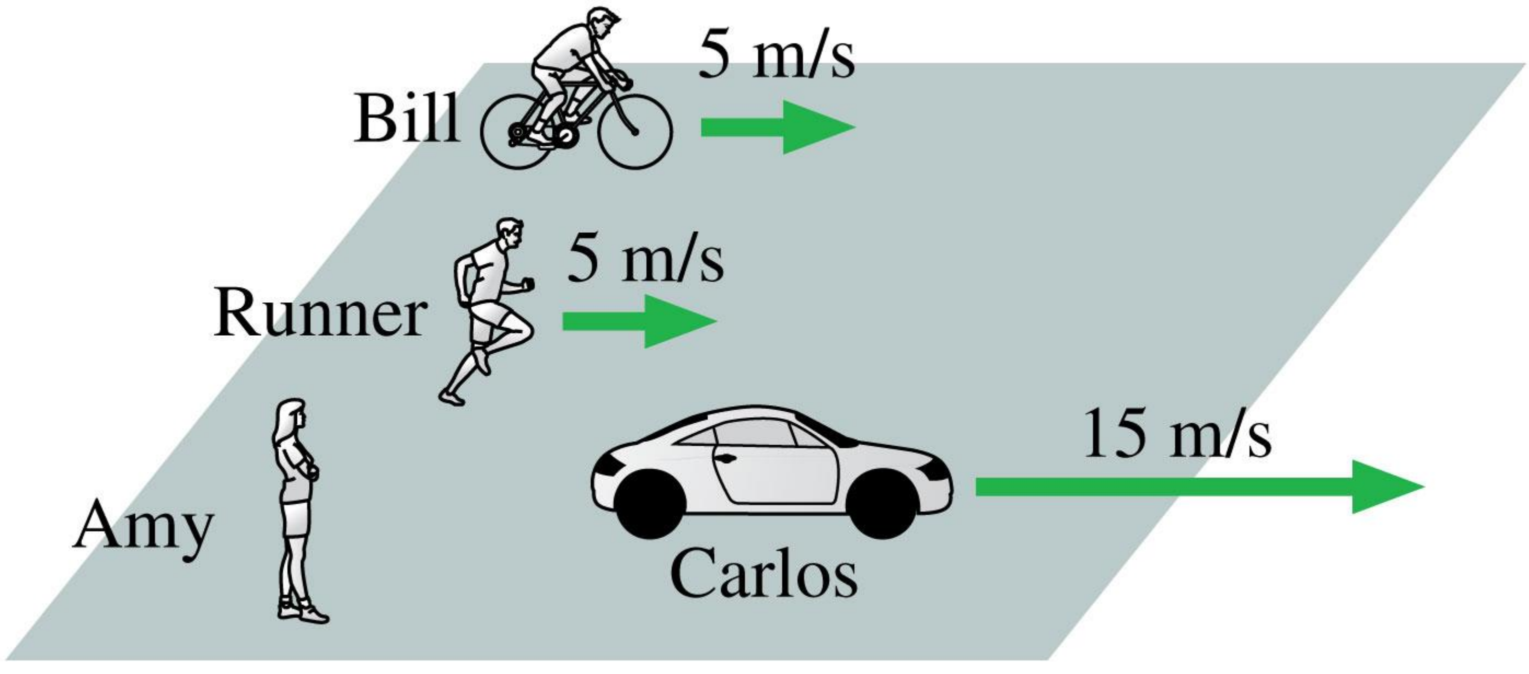
Example Problem

A new ski area has opened that emphasizes the extreme nature of the skiing possible on its slopes. Suppose an ad intones “Free-fall skydiving is the greatest rush you can experience . . . but we’ll take you as close as you can get on land. When you tip your skis down the slope of our steepest runs, you can accelerate at up to 75% of the acceleration you’d experience in free fall.” What angle slope could give such an acceleration?

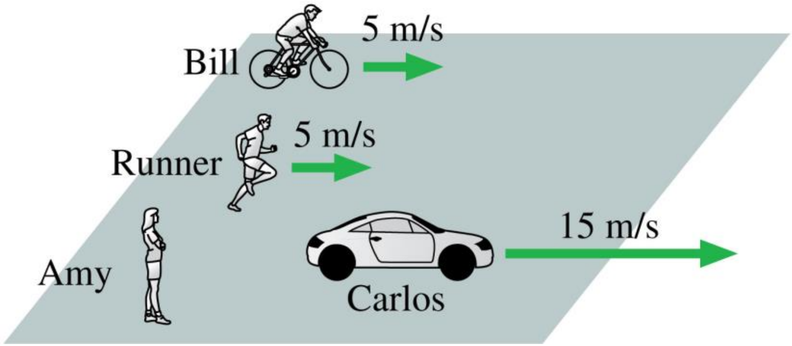
Section 3.5 Relative Motion

Relative Motion

- Amy, Bill, and Carlos are watching a runner.
- The runner moves at a different velocity relative to each of them.



Relative Velocity



$$(v_x)_{RC} = (v_x)_{RA} + (v_x)_{AC}$$

The “A” appears on the right of the first expression and on the left of the second; when we combine these velocities, we “cancel” the A to get $(v_x)_{RC}$.

- The runner’s velocity relative to Amy is

$$(v_x)_{RA} = 5 \text{ m/s}$$

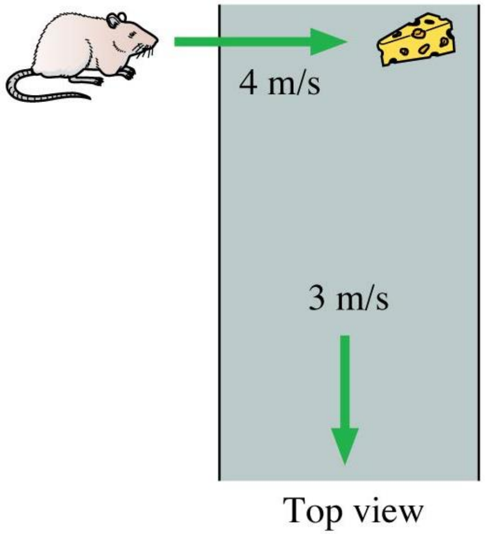
- The subscript “RA” means “**R**unner relative to **A**my.”
- The velocity of Carlos relative to Amy is

$$(v_x)_{CA} = 15 \text{ m/s}$$

- The subscript “CA” means “**C**arlos relative to **A**my.”

QuickCheck 3.15

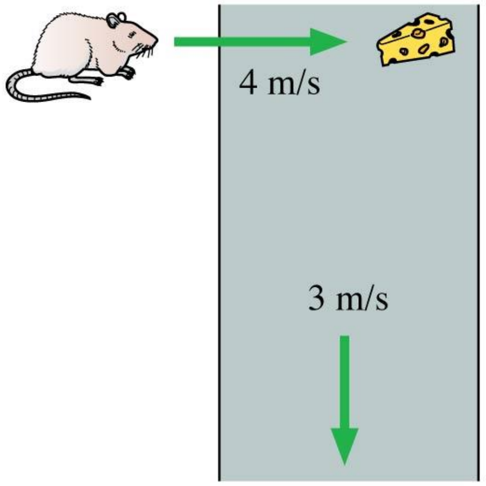
A factory conveyor belt rolls at 3 m/s. A mouse sees a piece of cheese directly across the belt and heads straight for the cheese at 4 m/s. What is the mouse's speed relative to the factory floor?



A. 1 m/s
B. 2 m/s
C. 3 m/s
D. 4 m/s
E. 5 m/s

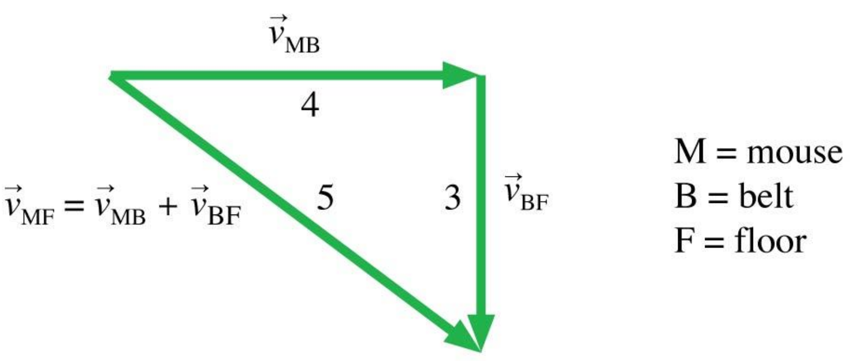
QuickCheck 3.15

A factory conveyor belt rolls at 3 m/s. A mouse sees a piece of cheese directly across the belt and heads straight for the cheese at 4 m/s. What is the mouse's speed relative to the factory floor?



Top view

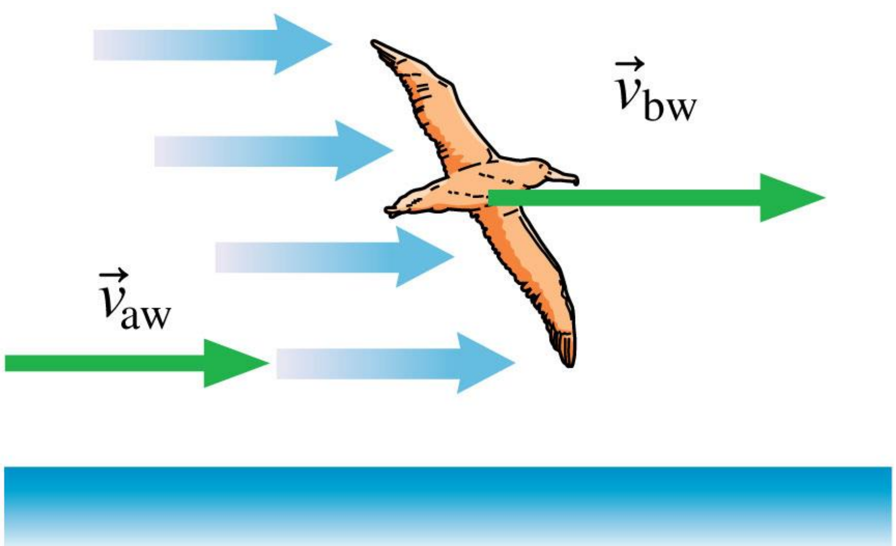
A. 1 m/s
B. 2 m/s
C. 3 m/s
D. 4 m/s
✔ E. 5 m/s



3-4-5 right triangle

Example 3.8 Speed of a seabird

Researchers doing satellite tracking of albatrosses in the Southern Ocean observed a bird maintaining sustained flight speeds of 35 m/s—nearly 80 mph! This seems surprisingly fast until you realize that this particular bird was flying with the wind, which was moving at 23 m/s. What was the bird's airspeed—its speed relative to the air? This is a truer measure of its flight speed.



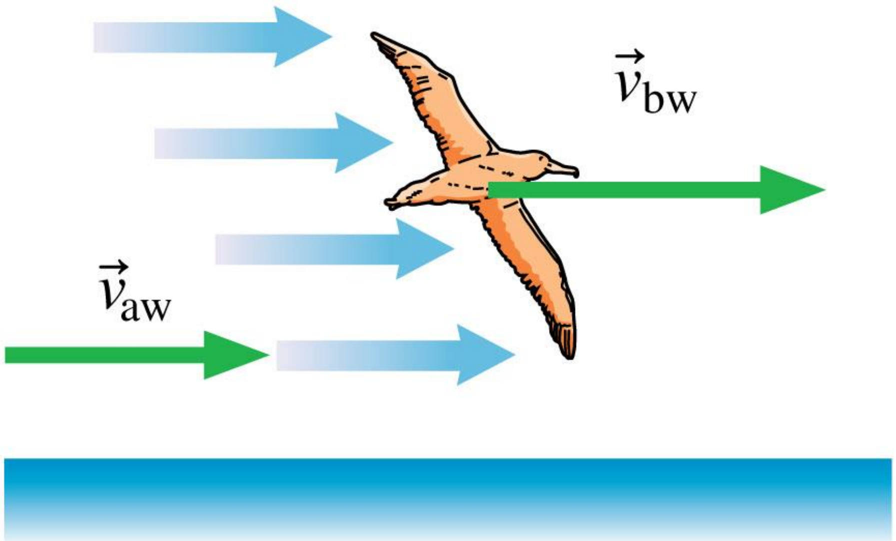
Known

$(v_x)_{bw} = 35$ m/s
$(v_x)_{aw} = 23$ m/s

Find

$(v_x)_{ba}$

Example 3.8 Speed of a seabird (cont.)



Known

$(v_x)_{bw} = 35$ m/s
$(v_x)_{aw} = 23$ m/s

Find

$(v_x)_{ba}$

PREPARE FIGURE 3.27 shows the wind and the albatross moving to the right, so all velocities will be positive. We've shown the velocity $(v_x)_{bw}$ of the bird with respect to the water, which is the measured flight speed, and the velocity $(v_x)_{aw}$ of the air with respect to the water, which is the known wind speed. We want to find the bird's airspeed—the speed of the bird with respect to the air.

Example 3.8 Speed of a seabird (cont.)

SOLVE We've noted three different velocities that are important in the problem: $(v_x)_{bw}$, $(v_x)_{aw}$, and $(v_x)_{ba}$. We can combine these in the usual way:

$$(v_x)_{bw} = (v_x)_{ba} + (v_x)_{aw}$$

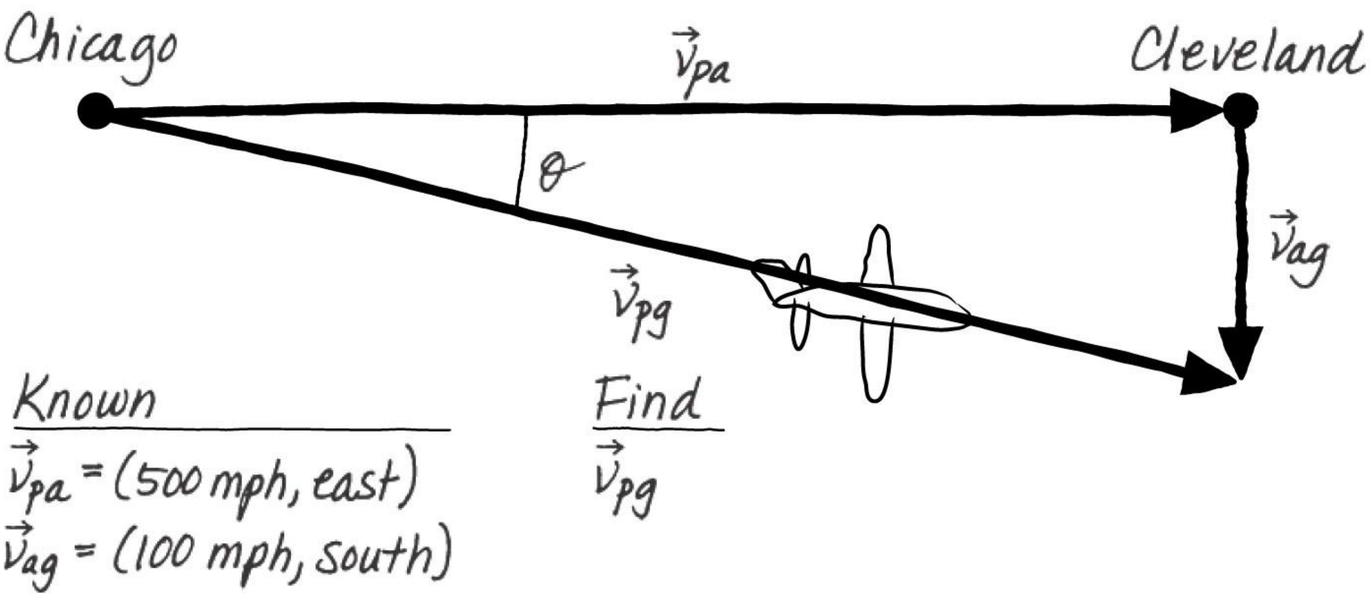
Then, to solve for $(v_x)_{ba}$, we can rearrange the terms:

$$(v_x)_{ba} = (v_x)_{bw} - (v_x)_{aw} = 35 \text{ m/s} - 23 \text{ m/s} = 12 \text{ m/s}$$

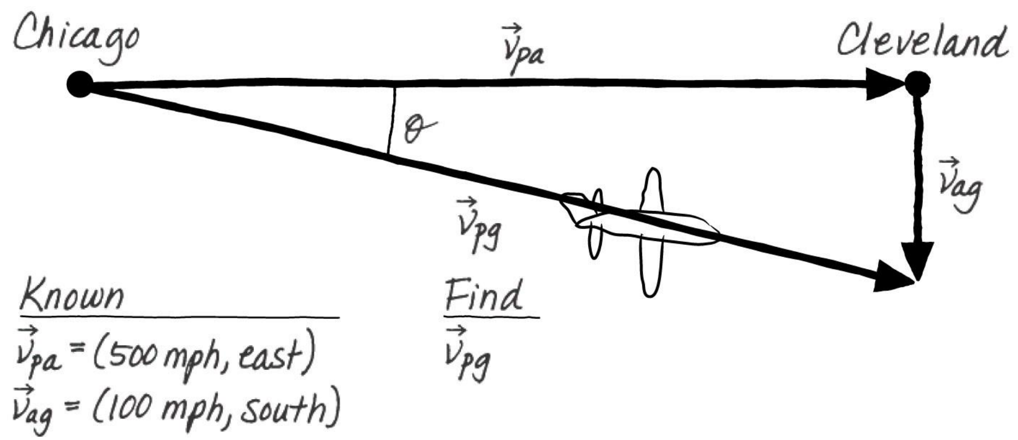
ASSESS 12 m/s—about 25 mph—is a reasonable airspeed for a bird. And it's slower than the observed flight speed, which makes sense because the bird is flying with the wind.

Example 3.9 Finding the ground speed of an airplane

Cleveland is approximately 300 miles east of Chicago. A plane leaves Chicago flying due east at 500 mph. The pilot forgot to check the weather and doesn't know that the wind is blowing to the south at 100 mph. What is the plane's velocity relative to the ground?



Example 3.9 Finding the ground speed of an airplane (cont.)



PREPARE FIGURE 3.28 is a visual overview of the situation. We are given the speed of the plane relative to the air ($\vec{v}_{\text{pa}}$) and the speed of the air relative to the ground ($\vec{v}_{\text{ag}}$); the speed of the plane relative to the ground will be the vector sum of these velocities:

$$\vec{v}_{\text{pg}} = \vec{v}_{\text{pa}} + \vec{v}_{\text{ag}}$$

This vector sum is shown in Figure 3.28.

Example 3.9 Finding the ground speed of an airplane (cont.)

SOLVE The plane's speed relative to the ground is the hypotenuse of the right triangle in Figure 3.28; thus:

$$v_{\text{pg}} = \sqrt{{v_{\text{pa}}}^2 + {v_{\text{ag}}}^2} = \sqrt{(500 \text{ mph})^2 + (100 \text{ mph})^2} = 510 \text{ mph}$$

The plane's direction can be specified by the angle θ measured from due east:

$$\theta = \tan^{-1}\left(\frac{100 \text{ mph}}{500 \text{ mph}}\right) = \tan^{-1}(0.20) = 11°$$

The velocity of the plane relative to the ground is thus

$$\vec{v}_{\text{pg}} = (510 \text{ mph, } 11° \text{ south of east})$$

ASSESS The good news is that the wind is making the plane move a bit faster relative to the ground. The bad news is that the wind is making the plane move in the wrong direction!

Example Problem

A skydiver jumps out of an airplane 1000 m directly above his desired landing spot. He quickly reaches a steady speed, falling through the air at 35 m/s. There is a breeze blowing at 7 m/s to the west.

A. At what angle with respect to vertical does he fall?
B. When he lands, what will be his displacement from his desired landing spot?

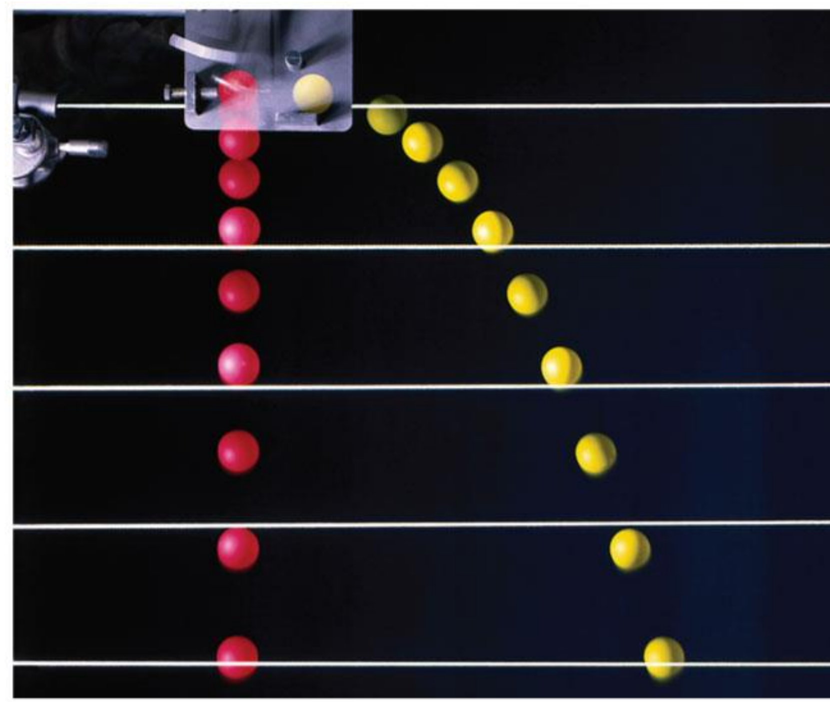
Section 3.6 Motion in Two Dimensions: Projectile Motion

Motion in Two Dimensions: Projectile Motion

- **Projectile motion** is an extension to two dimensions of free-fall motion.
- **A projectile is an object that moves in two dimensions under the influence of gravity and nothing else.**
- As long as we can neglect air resistance, any projectile will follow the same type of path.

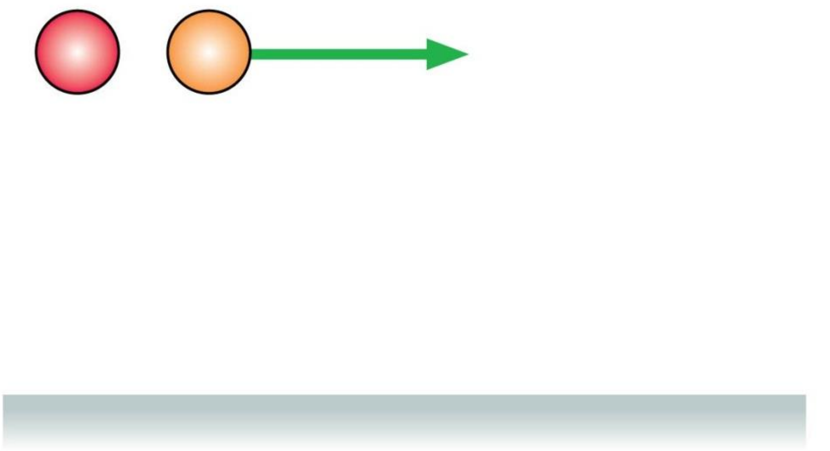
Motion in Two Dimensions: Projectile Motion

- The vertical motions of the two balls are identical.
- The vertical motion of the yellow ball is not affected by the fact that the ball is moving horizontally.
- **The horizontal and vertical components of an object undergoing projectile motion are independent of each other.**



QuickCheck 3.16

A heavy red ball is released from rest 2.0 m above a flat, horizontal surface. At exactly the same instant, a yellow ball with the same mass is fired horizontally at 3.0 m/s. Which ball hits the ground first?



A. The red ball hits first.
B. The yellow ball hits first.
C. They hit at the same time.

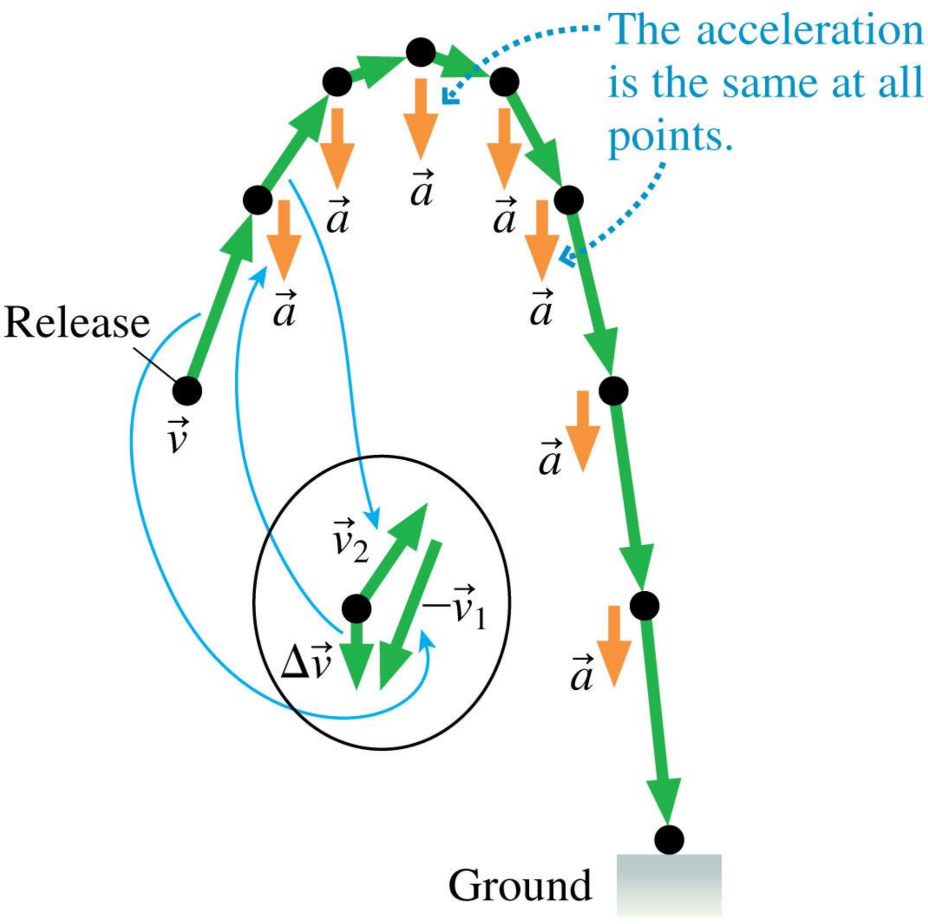
QuickCheck 3.16

A heavy red ball is released from rest 2.0 m above a flat, horizontal surface. At exactly the same instant, a yellow ball with the same mass is fired horizontally at 3.0 m/s. Which ball hits the ground first?

A. The red ball hits first.
B. The yellow ball hits first.
C. They hit at the same time.

Motion in Two Dimensions: Projectile Motion

- **The vertical component of acceleration a_y for all projectile motion is just the familiar $-g$ of free fall, while the horizontal component a_x is zero.**



QuickCheck 3.17

A 100-g ball rolls off a table and lands 2.0 m from the base of the table. A 200-g ball rolls off the same table with the same speed. It lands at distance

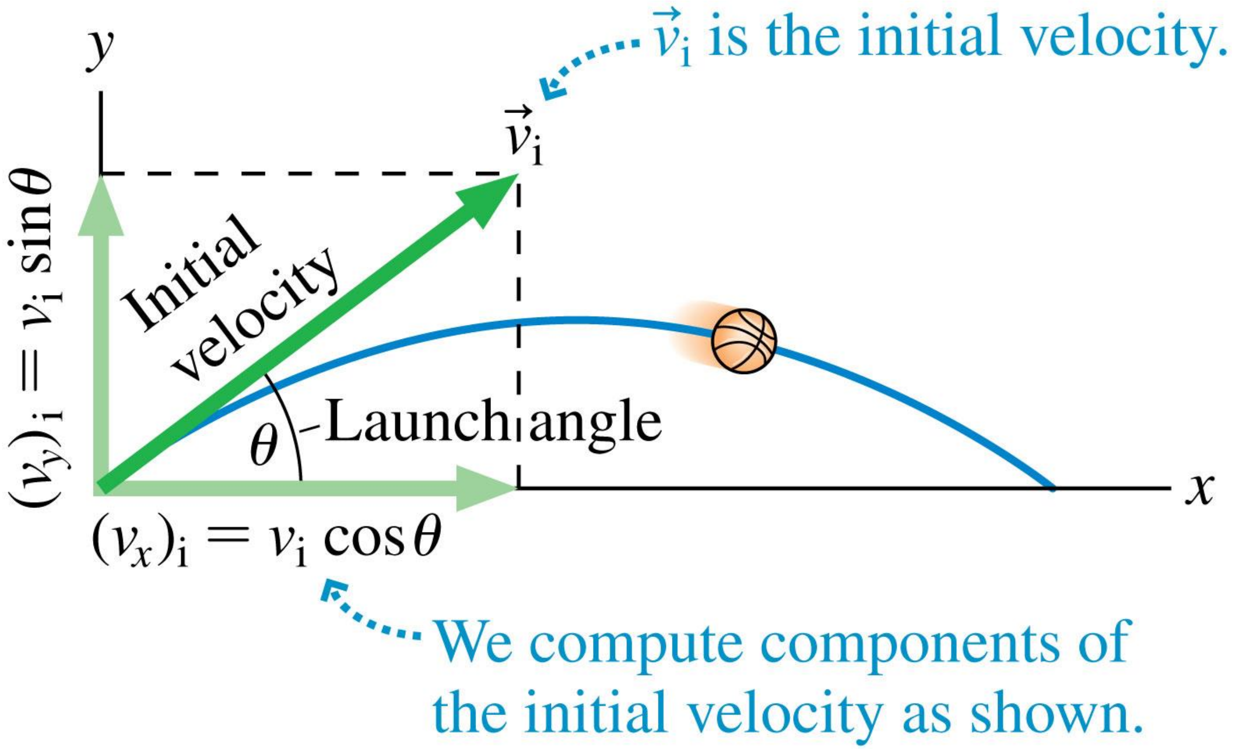
A. 1.0 m
B. Between 1 m and 2 m
C. 2.0 m
D. Between 2 m and 4 m
E. 4.0 m

QuickCheck 3.17

A 100-g ball rolls off a table and lands 2.0 m from the base of the table. A 200-g ball rolls off the same table with the same speed. It lands at distance

A. 1.0 m
B. Between 1 m and 2 m
C. 2.0 m
D. Between 2 m and 4 m
E. 4.0 m

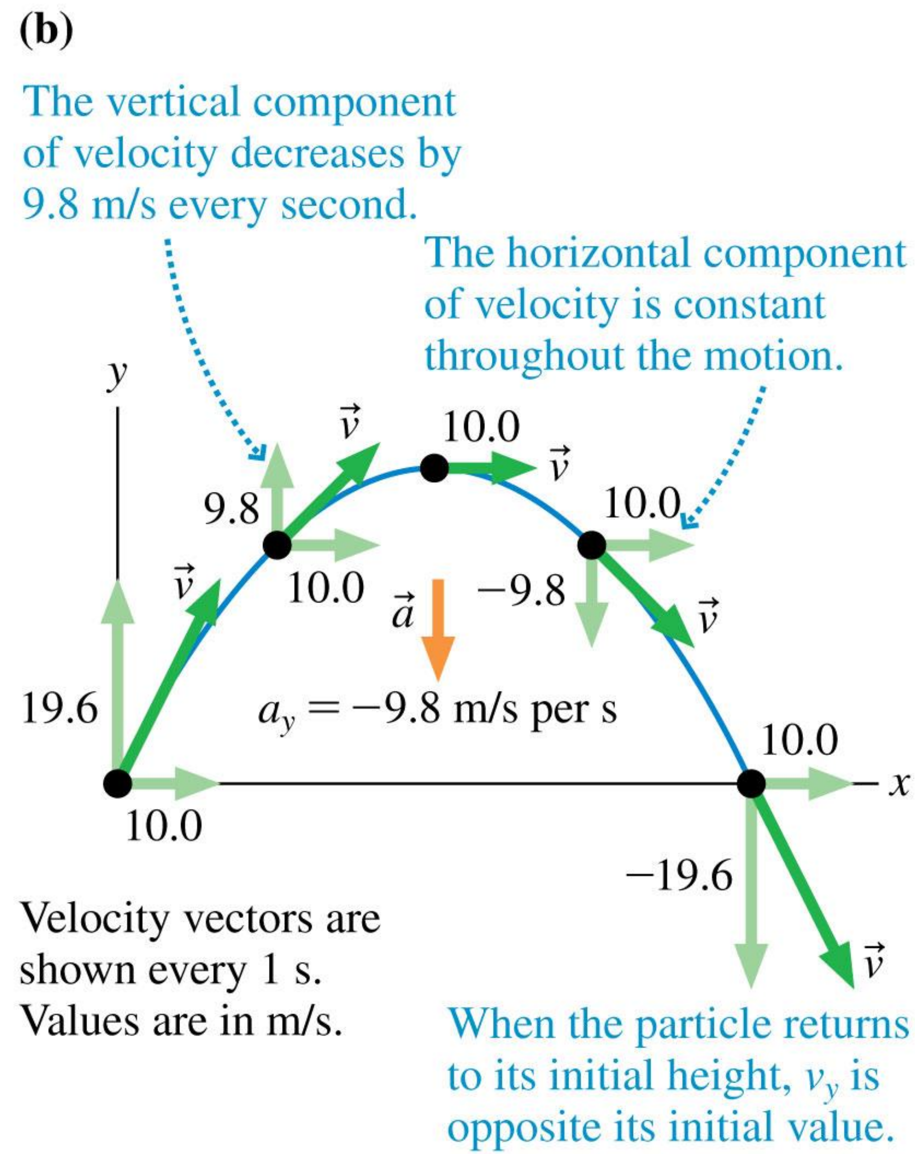
Analyzing Projectile Motion



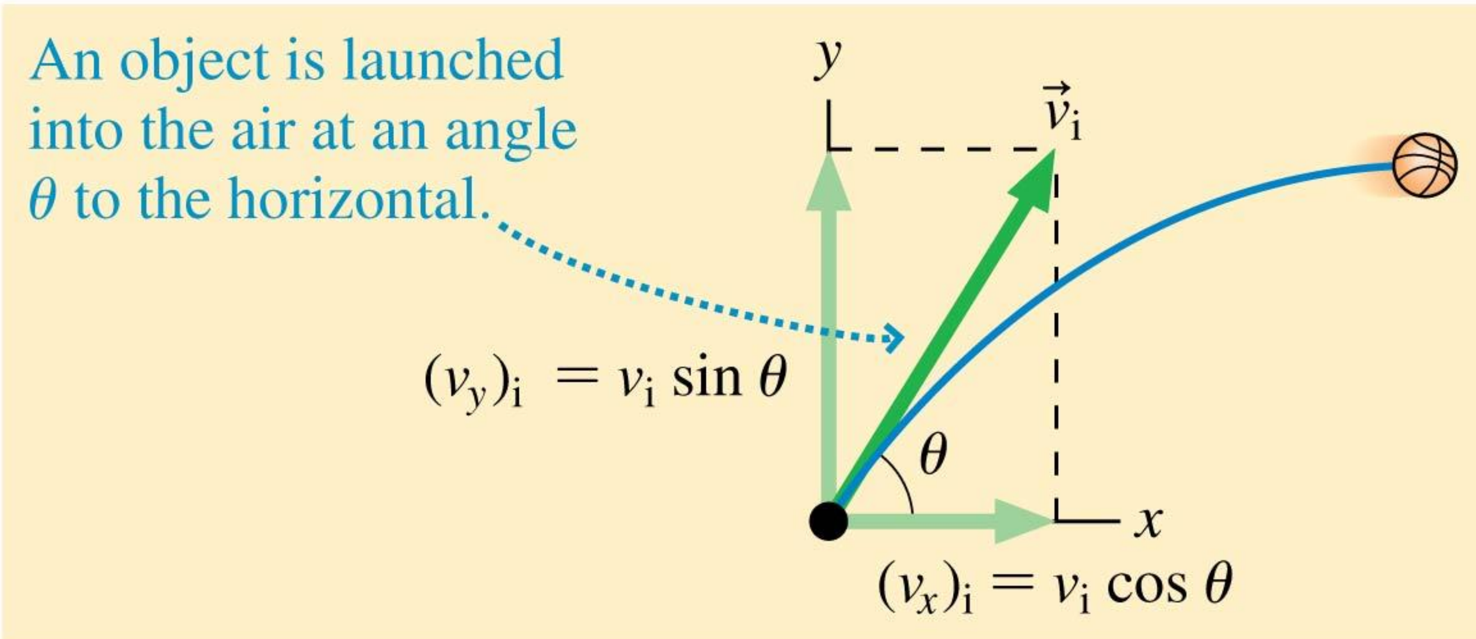
- The angle of the initial velocity above the horizontal (i.e., above the *x*-axis) is the **launch angle**.

Analyzing Projectile Motion

- The ball finishes its motion moving downward at the same speed as it started moving upward.
- Projectile motion is made up of two independent motions: uniform motion at constant velocity in the horizontal direction and free-fall motion in the vertical direction.

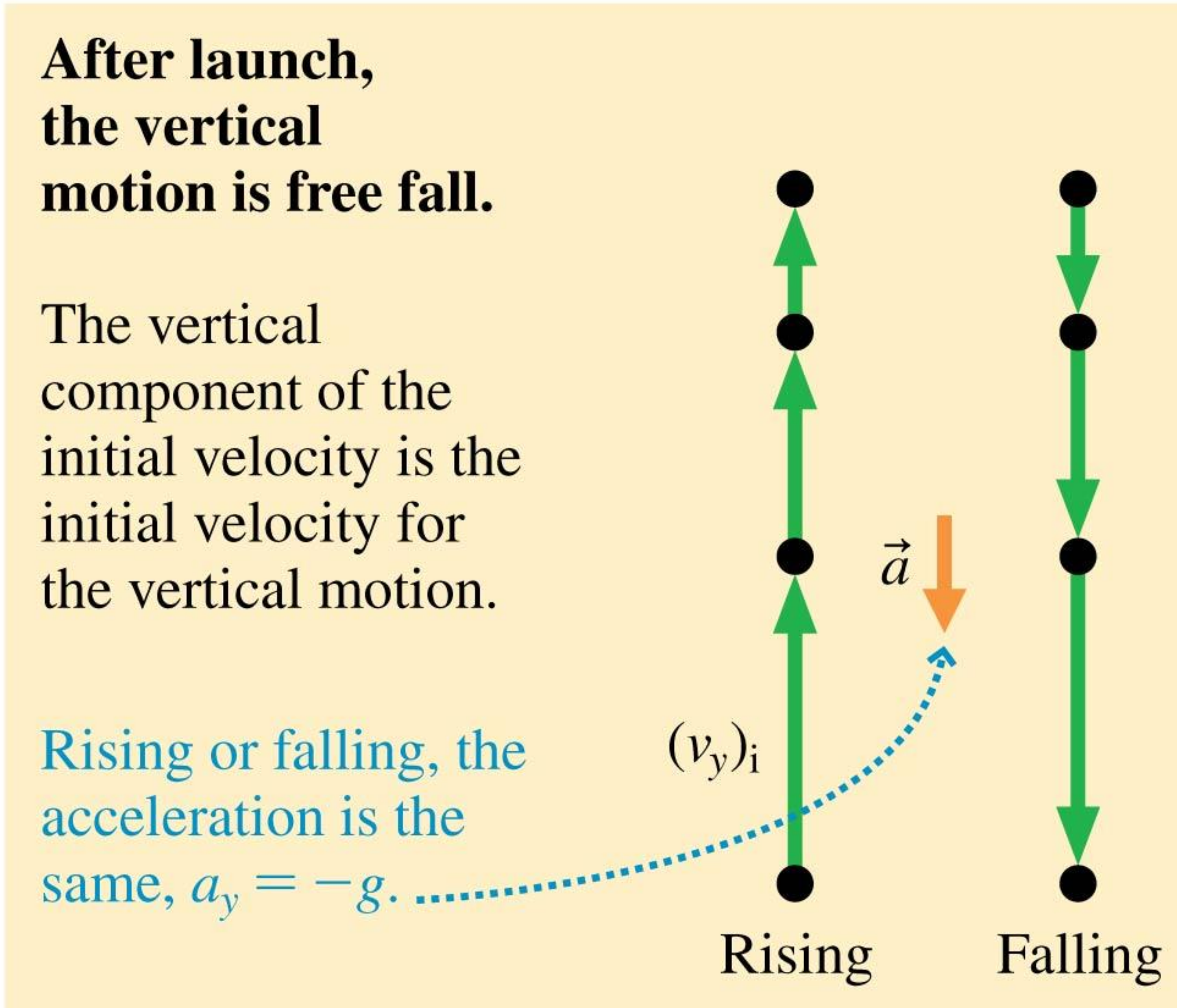


Synthesis 3.1 Projectile Motion



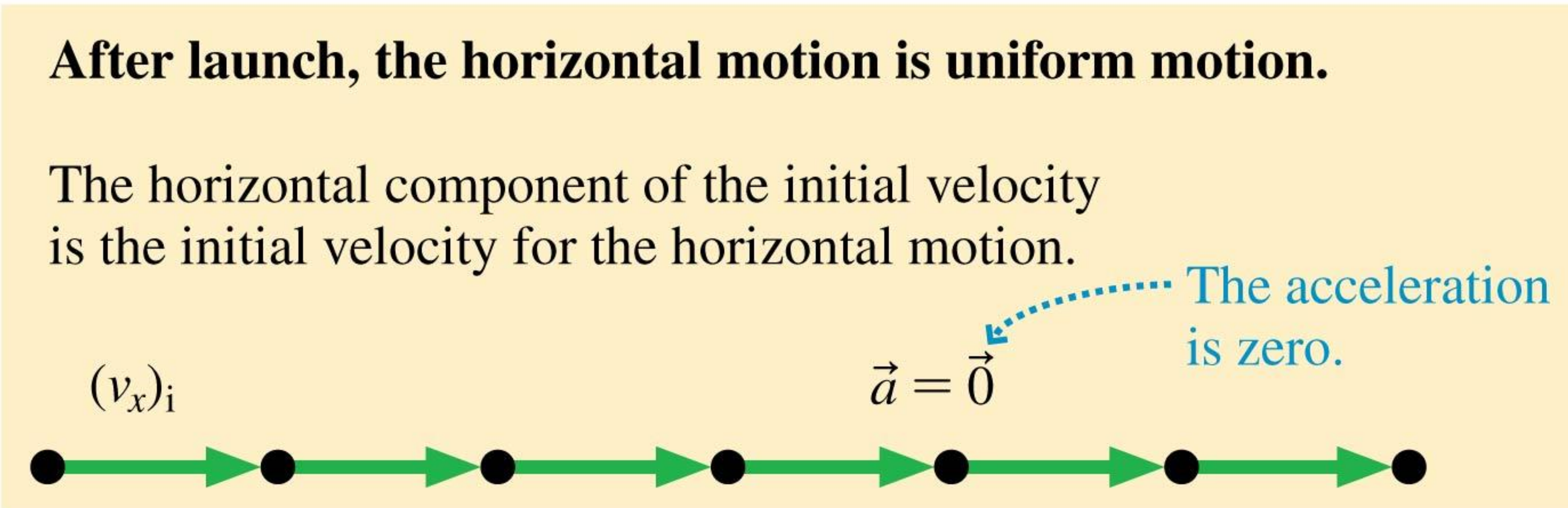
Text: p. 82

Synthesis 3.1 Projectile Motion



Text: p. 82

Synthesis 3.1 Projectile Motion



Text: p. 82

Synthesis 3.1 Projectile Motion

The kinematic equations for projectile motion are those for constant-acceleration motion vertically and constant-velocity horizontally:

The vertical motion is free fall.

The free fall acceleration, $g = 9.8\ \text{m/s}^2$.

The horizontal motion is uniform motion.

$$(v_y)_f = (v_y)_i - g\,\Delta t$$

$$y_f = y_i + (v_y)_i\,\Delta t - \tfrac{1}{2}g(\Delta t)^2$$

$$(v_x)_f = (v_x)_i = \text{constant}$$

$$x_f = x_i + (v_x)_i\,\Delta t$$

The two equations are linked by the time interval Δt, which is the same for the horizontal and vertical motion.

Text: p. 82

Section 3.7 Projectile Motion: Solving Problems

Projectile Motion Problems

We can solve projectile motion problems by considering the horizontal and vertical motions as separate but related problems.

PREPARE There are a number of steps that you should go through in setting up the solution to a projectile motion problem:

- Make simplifying assumptions. Whether the projectile is a car or a basketball, the motion will be the same.
- Draw a visual overview including a pictorial representation showing the beginning and ending points of the motion.
- Establish a coordinate system with the x-axis horizontal and the y-axis vertical. In this case, you know that the horizontal acceleration will be zero and the vertical acceleration will be free fall: $a_x = 0$ and $a_y = -g$.
- Define symbols and write down a list of known values. Identify what the problem is trying to find.

Text: p. 83

Projectile Motion Problems

SOLVE There are two sets of kinematic equations for projectile motion, one for the horizontal component and one for the vertical:

Horizontal	Vertical
$x_f = x_i + (v_x)_i \, \Delta t$	$y_f = y_i + (v_y)_i \, \Delta t - \frac{1}{2} g (\Delta t)^2$
$(v_x)_f = (v_x)_i = \text{constant}$	$(v_y)_f = (v_y)_i - g \, \Delta t$

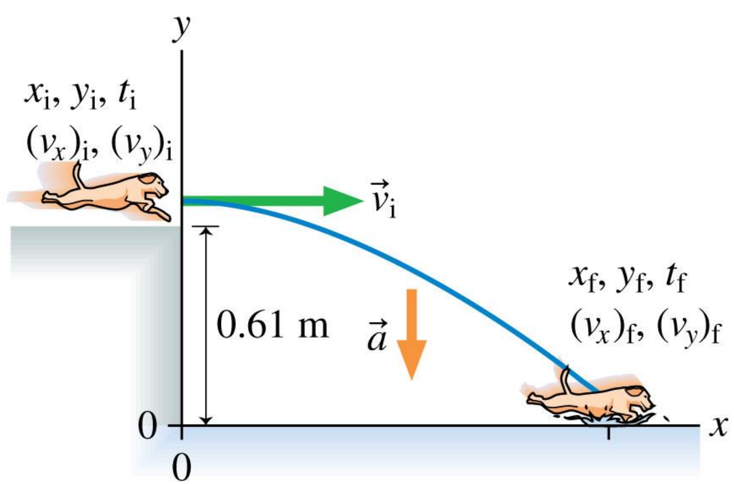
Δt is the same for the horizontal and vertical components of the motion. Find Δt by solving for the vertical or the horizontal component of the motion; then use that value to complete the solution for the other component.

ASSESS Check that your result has the correct units, is reasonable, and answers the question.

Text: p. 83

Example 3.11 Dock jumping

In the sport of dock jumping, dogs run at full speed off the end of a dock that sits a few feet above a pool of water. The winning dog is the one that lands farthest from the end of the dock. If a dog runs at 8.5 m/s (a pretty typical speed for this event) straight off the end of a dock that is 0.61 m (2 ft, a standard height) above the water, how far will the dog go before splashing down?



Known

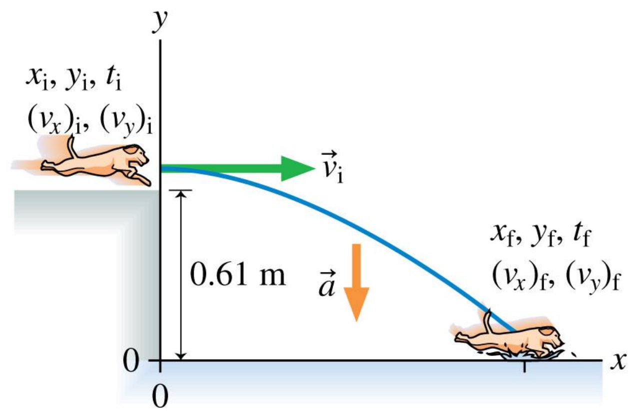
$x_i = 0$ m
$(v_y)_i = 0$ m/s
$t_i = 0$ s
$y_i = 0.61$ m, $y_f = 0$ m
$(v_x)_i = v_i = 8.5$ m/s
$a_x = 0$ m/s^2
$a_y = -g$

Find

x_f

Example 3.11 Dock jumping (cont.)

PREPARE We start with a visual overview of the situation in FIGURE 3.33. We have chosen to put the origin of the coordinate system at the base of the dock.



Known
$x_i = 0$ m
$(v_y)_i = 0$ m/s
$t_i = 0$ s
$y_i = 0.61$ m, $y_f = 0$ m
$(v_x)_i = v_i = 8.5$ m/s
$a_x = 0$ m/s^2
$a_y = -g$

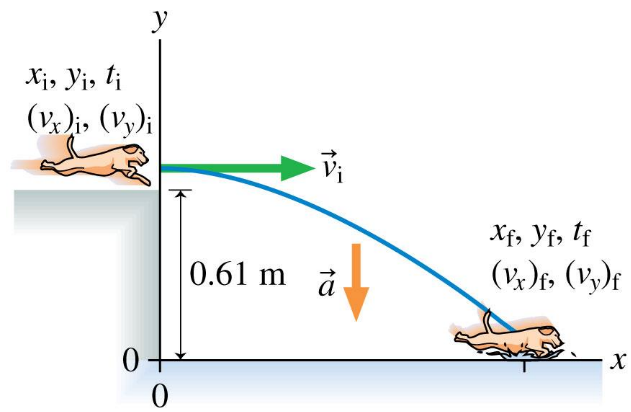
Find
x_f

The dog runs horizontally off the end of the dock, so the initial components of the velocity are $(v_x)_i = 8.5$ m/s and $(v_y)_i = 0$ m/s. We can treat this as a projectile motion problem, so we can use the details and equations presented in Synthesis 3.1 above.

We know that the horizontal and vertical motions are independent. The fact that the dog is falling toward the water doesn't affect its horizontal motion.

Example 3.11 Dock jumping (cont.)

When the dog leaves the end of the dock, it will continue to move horizontally at 8.5 m/s. The vertical motion is free fall. The jump ends when the dog hits the water—that is, when it has dropped by 0.61 m. We are ultimately interested in how far the dog goes, but to make this determination we'll need to find the time interval Δt that the dog is in the air.



Known

$x_i = 0$ m
$(v_y)_i = 0$ m/s
$t_i = 0$ s
$y_i = 0.61$ m, $y_f = 0$ m
$(v_x)_i = v_i = 8.5$ m/s
$a_x = 0$ m/s^2
$a_y = -g$

Find

x_f

Example 3.11 Dock jumping (cont.)

SOLVE We'll start by solving for the time interval Δt, the time the dog is in the air. This time is determined by the vertical motion, which is free fall with an initial velocity $(v_y)_i = 0$ m/s. We use the vertical position equation from Synthesis 3.1 to find the time interval:

$$y_f = y_i + (v_y)_i\,\Delta t - \frac{1}{2}g(\Delta t)^2$$

$$0\text{ m} = 0.61\text{ m} + (0\text{ m/s})\Delta t - \frac{1}{2}(9.8\text{ m/s}^2)(\Delta t)^2$$

Rearranging terms to solve for Δt, we find that

$$\Delta t = 0.35\text{ s}$$

Example 3.11 Dock jumping (cont.)

This is how long it takes the dog's vertical motion to reach the water. During this time interval, the dog's horizontal motion is uniform motion at the initial velocity. We can use the horizontal-position equation with the initial speed and Δt = 0.35 s to find how far the dog travels. This is the distance we are looking for:

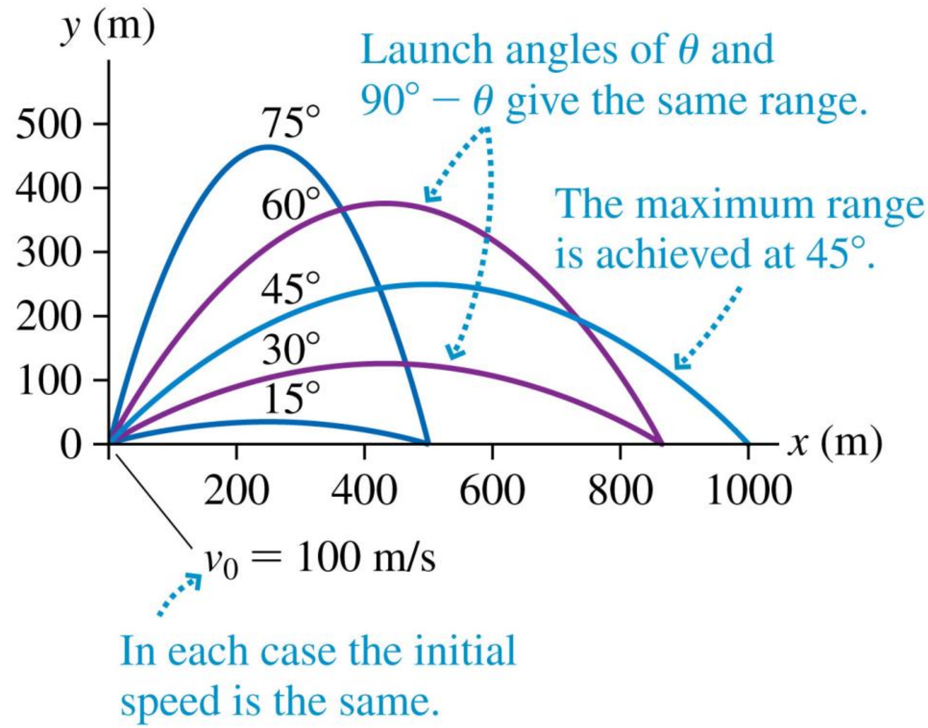
$$x_f = x_i + (v_x)_i \,\Delta t$$
$$= 0 \text{ m} + (8.5 \text{ m/s})(0.35 \text{ s}) = 3.0 \text{ m}$$

The dog hits the water 3.0 m from the edge of the dock.

ASSESS 3.0 m is about 10 feet. This seems like a reasonable distance for a dog running at a very fast clip off the end of a 2-foot-high dock. Indeed, this is a typical distance for dogs in such competitions.

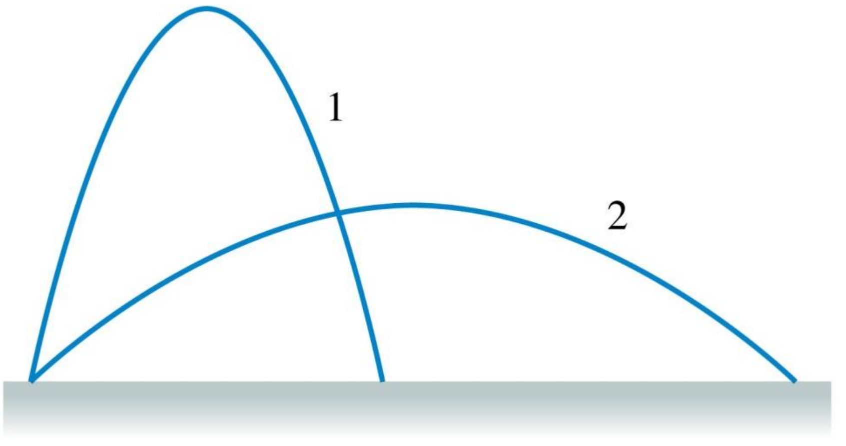
The Range of a Projectile

- The **range** of a projectile is the horizontal distance traveled.
- For smaller objects air resistance is critical, and the maximum range comes at an angle less than 45°.



QuickCheck 3.18

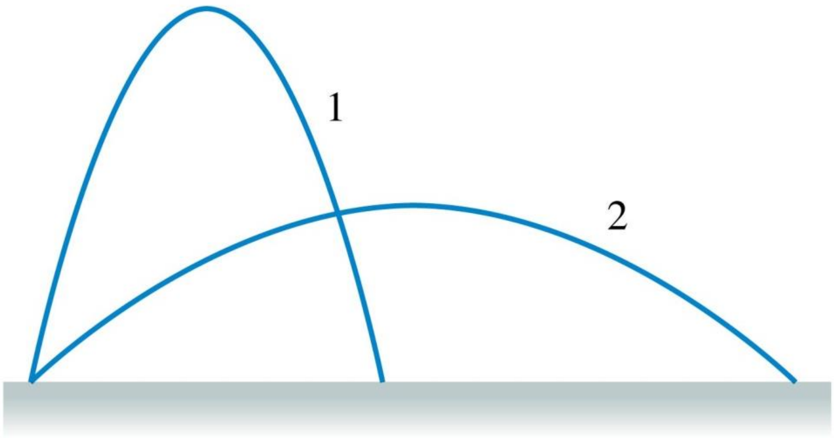
Projectiles 1 and 2 are launched over level ground with the same speed but at different angles. Which hits the ground first? Ignore air resistance.



A. Projectile 1 hits first.
B. Projectile 2 hits first.
C. They hit at the same time.
D. There's not enough information to tell.

QuickCheck 3.18

Projectiles 1 and 2 are launched over level ground with the same speed but at different angles. Which hits the ground first? Ignore air resistance.



A. Projectile 1 hits first.
B. Projectile 2 hits first.
C. They hit at the same time.
D. There's not enough information to tell.

Example Problem

A grasshopper can jump a distance of 30 in (0.76 m) from a standing start.

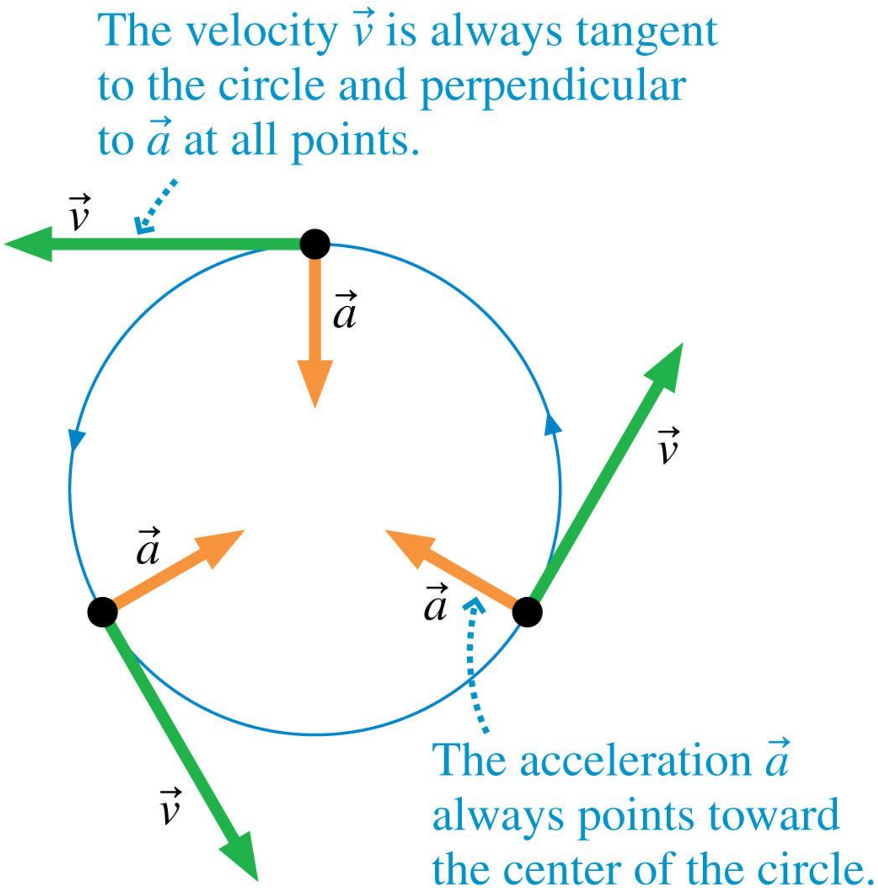
A. If the grasshopper takes off at the optimal angle for maximum distance of the jump, what is the initial speed of the jump?

B. Most animals jump at a lower angle than 45°. Suppose the grasshopper takes off at 30° from the horizontal. What jump speed is necessary to reach the noted distance?

Section 3.8 Motion in Two Dimensions: Circular Motion

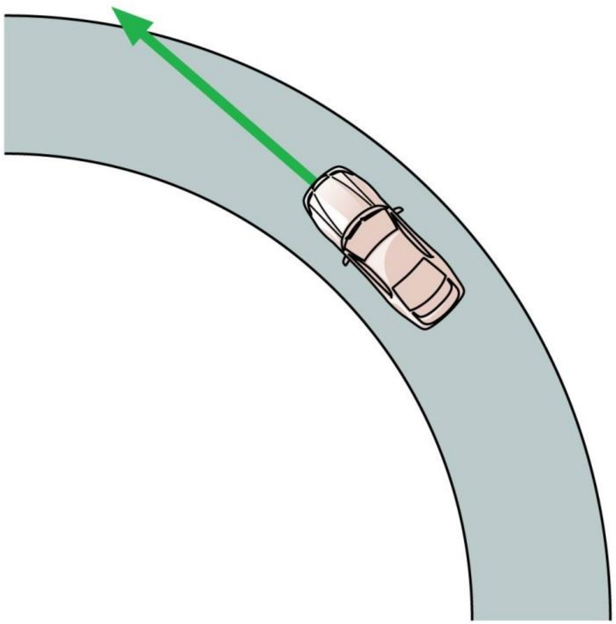
Motion in Two Dimensions: Circular Motion

- For circular motion at a constant speed, the acceleration vector a points toward the center of the circle.
- An acceleration that always points directly toward the center of a circle is called a **centripetal acceleration**.
- Centripetal acceleration is just the **name** for a particular type of motion. It is not a new type of acceleration.



QuickCheck 3.19

A car is traveling around a curve at a steady 45 mph. Is the car accelerating?

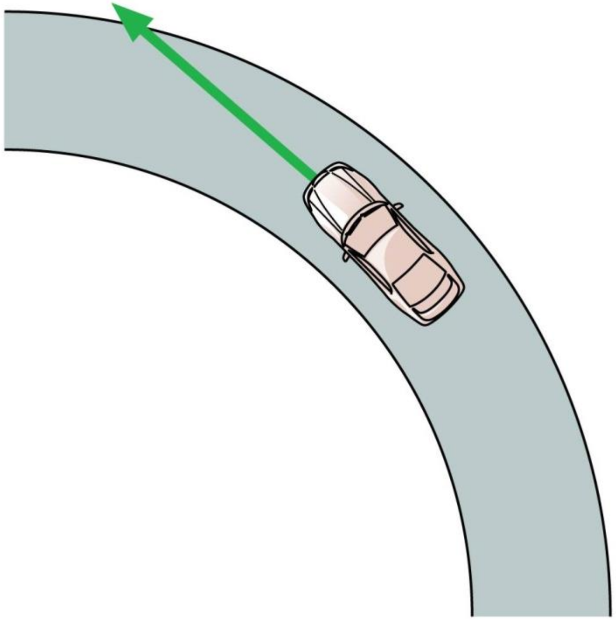


A. Yes

B. No

QuickCheck 3.19

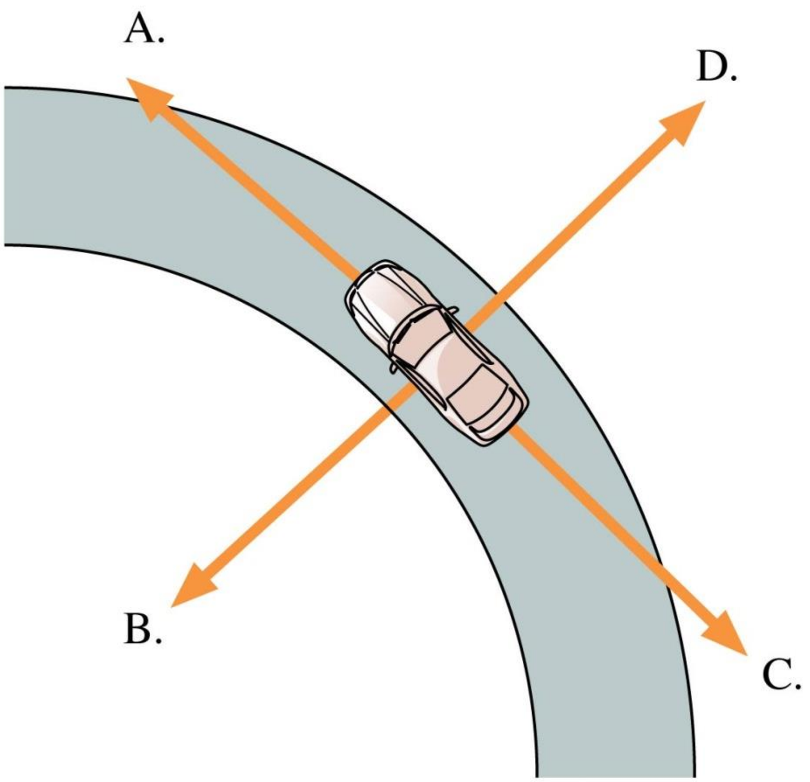
A car is traveling around a curve at a steady 45 mph. Is the car accelerating?



A. Yes
B. No

QuickCheck 3.20

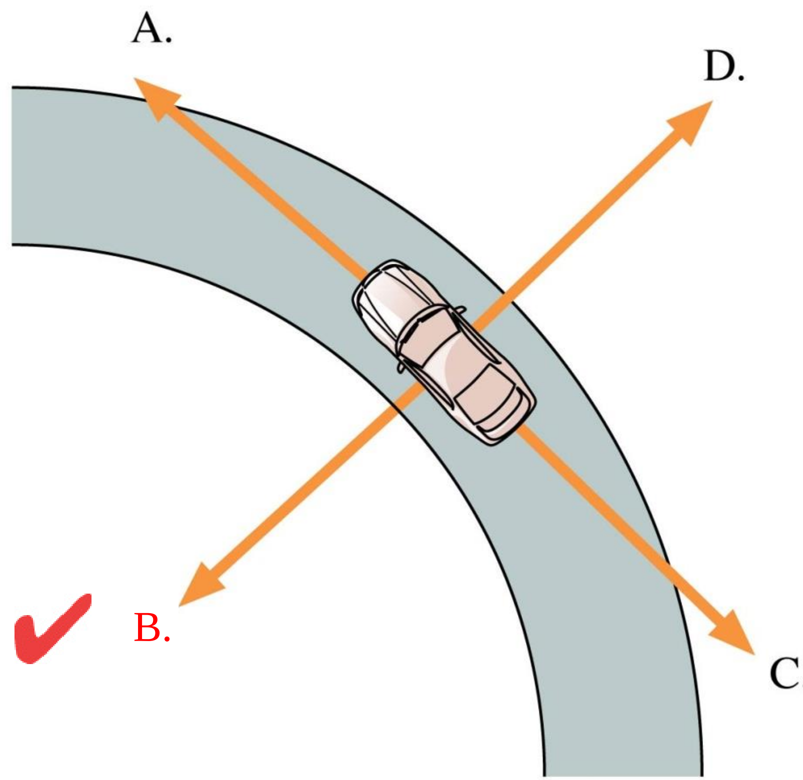
A car is traveling around a curve at a steady 45 mph. Which vector shows the direction of the car's acceleration?



E. The acceleration is zero.

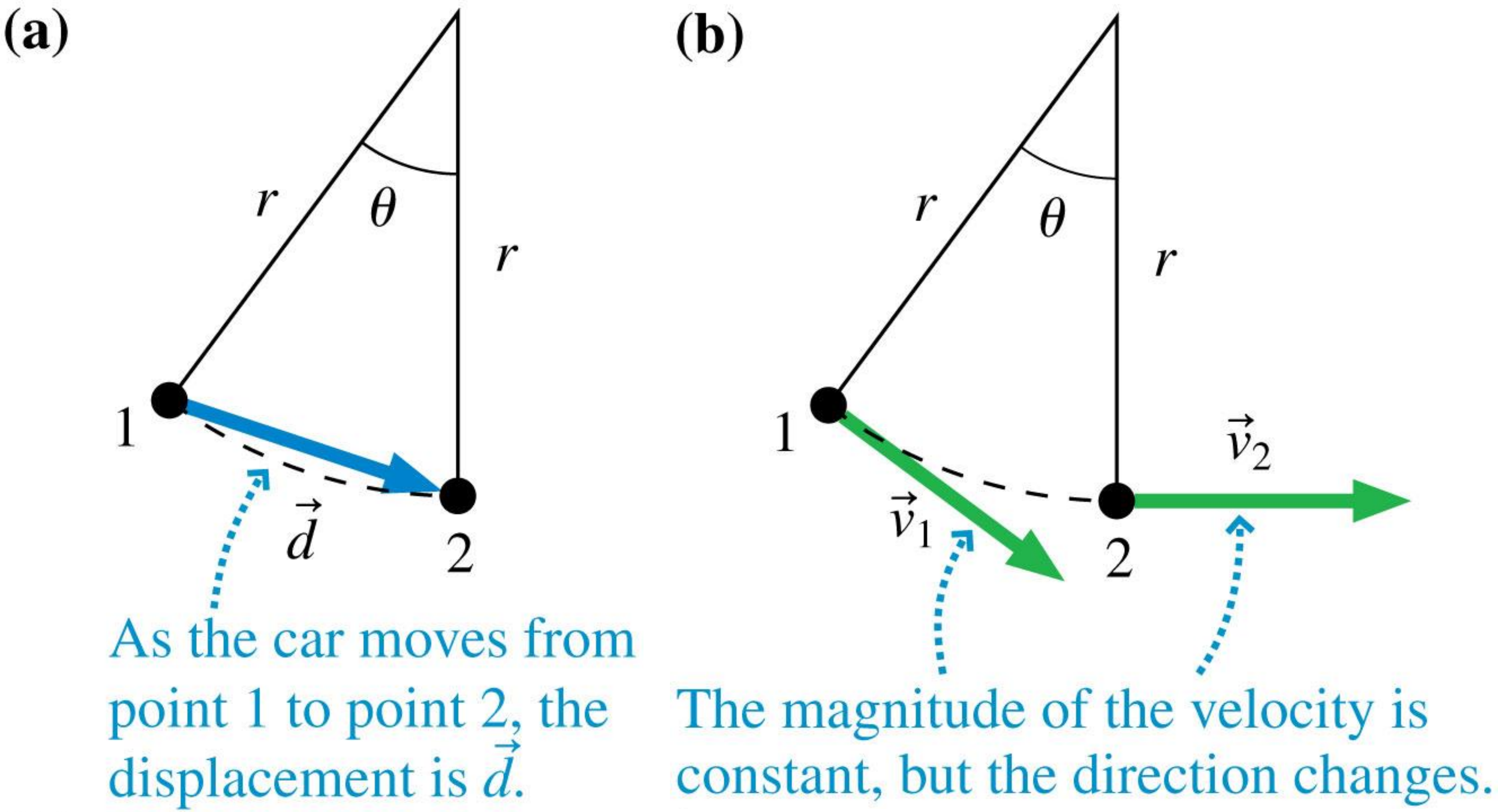
QuickCheck 3.20

A car is traveling around a curve at a steady 45 mph. Which vector shows the direction of the car's acceleration?

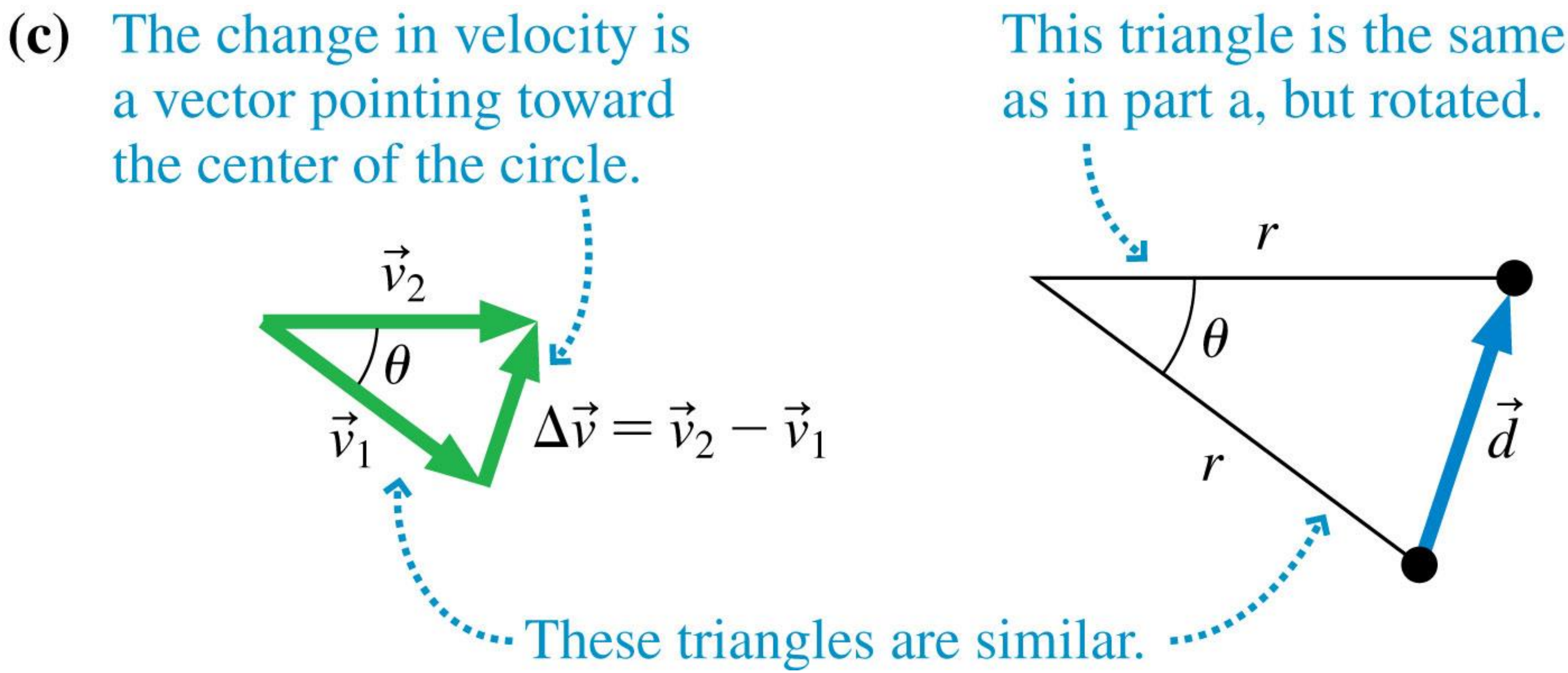


E. The acceleration is zero.

Motion in Two Dimensions: Circular Motion



Motion in Two Dimensions: Circular Motion



$$\frac{\Delta v}{v} = \frac{d}{r}$$

Motion in Two Dimensions: Circular Motion

$$d = v\Delta t$$

$$\frac{\Delta v}{v} = \frac{v\Delta t}{r}$$

$$\frac{\Delta v}{\Delta t} = \frac{v^2}{r}$$

$$a = \frac{v^2}{r}$$

$$\vec{a} = \left(\frac{v^2}{r}, \text{ toward center of circle}\right)$$

Centripetal acceleration of object moving in a circle of radius r at speed v

QuickCheck 3.21

A toy car moves around a circular track at constant speed. It suddenly doubles its speed — a change of a factor of 2. As a result, the centripetal acceleration changes by a factor of

A. 1/4
B. 1/2
C. No change since the radius doesn't change.
D. 2
E. 4

QuickCheck 3.21

A toy car moves around a circular track at constant speed. It suddenly doubles its speed — a change of a factor of 2. As a result, the centripetal acceleration changes by a factor of

A. 1/4
B. 1/2
C. No change since the radius doesn't change.
D. 2
✔ E. 4

Example 3.14 Acceleration in the turn

World-class female short-track speed skaters can cover the 500 m of a race in 45 s. The most challenging elements of the race are the turns, which are very tight, with a radius of approximately 11 m. Estimate the magnitude of the skater's centripetal acceleration in a turn.

Example 3.14 Acceleration in the turn (cont.)

PREPARE The centripetal acceleration depends on two quantities: the radius of the turn (given as approximately 11 m) and the speed. The speed varies during the race, but we can make a good estimate of the speed by using the total distance and time:

$$v \simeq \frac{500\ \text{m}}{45\ \text{s}} = 11\ \text{m/s}$$

SOLVE We can use these values to estimate the magnitude of the acceleration:

$$a = \frac{v^2}{r} \simeq \frac{(11\text{m/s})^2}{11\ \text{m}} = 11\ \text{m/s}^2$$

ASSESS This is a large acceleration—a bit more than g—but the photo shows the skaters leaning quite hard into the turn, so such a large acceleration seems quite reasonable.

Example Problem

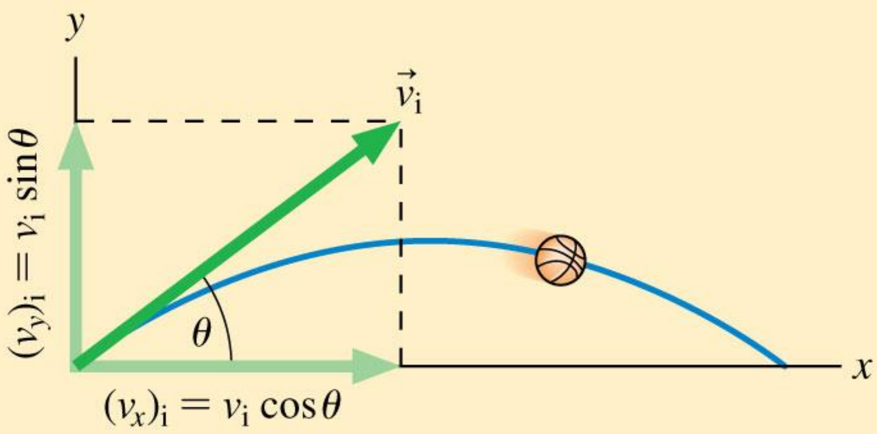
Turning a corner at a typical large intersection in a city means driving your car through a circular arc with a radius of about 25 m. If the maximum advisable acceleration of your vehicle through a turn on wet pavement is 0.40 times the free-fall acceleration, what is the maximum speed at which you should drive through this turn?

Summary: General Principles

Projectile Motion

A projectile is an object that moves through the air under the influence of gravity and nothing else.

The path of the motion is a parabola.



The motion consists of two pieces:

1. Vertical motion with free-fall acceleration, $a_y = -g$
2. Horizontal motion with constant velocity

Kinematic equations:

$$x_f = x_i + (v_x)_i\,\Delta t$$

$$(v_x)_f = (v_x)_i = \text{constant}$$

$$y_f = y_i + (v_y)_i\,\Delta t - \tfrac{1}{2}g(\Delta t)^2$$

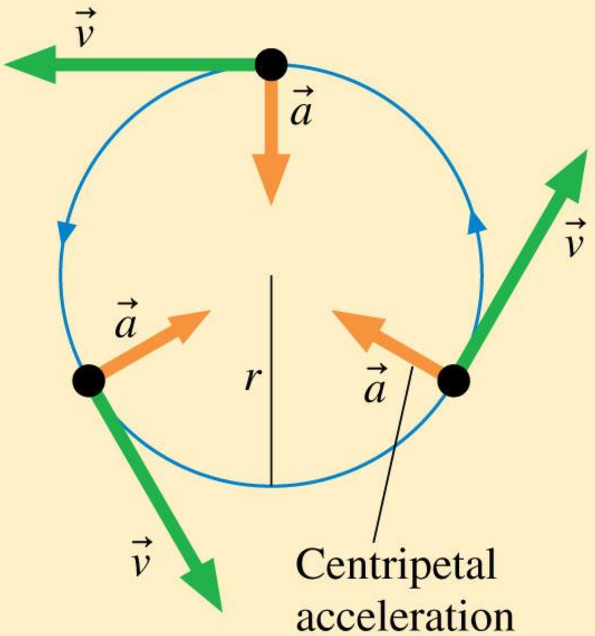
$$(v_y)_f = (v_y)_i - g\,\Delta t$$

Text: p. 89

Summary: General Principles

Circular Motion

An object moving in a circle at a constant speed has a velocity that is constantly changing direction, and so experiences an acceleration:



- The velocity is tangent to the circular path.
- The acceleration points toward the center of the circle and has magnitude

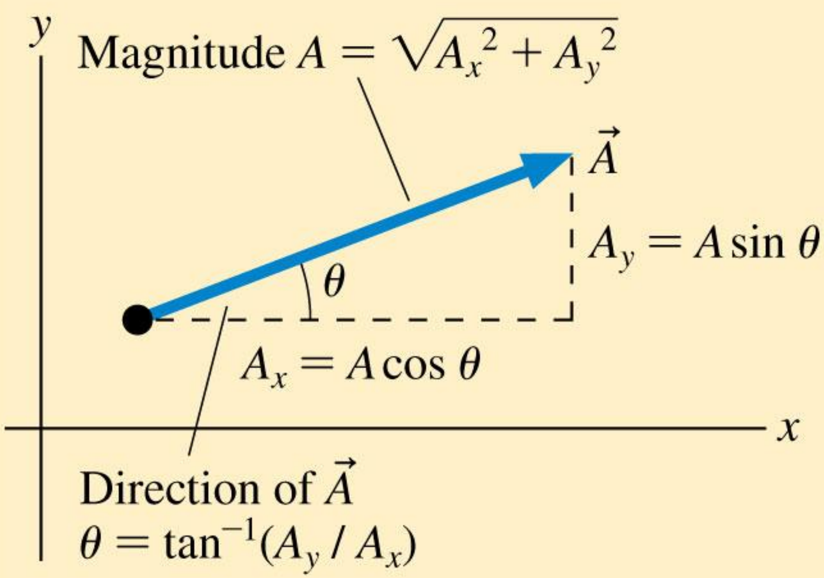
$$a = \frac{v^2}{r}$$

Text: p. 89

Summary: Important Concepts

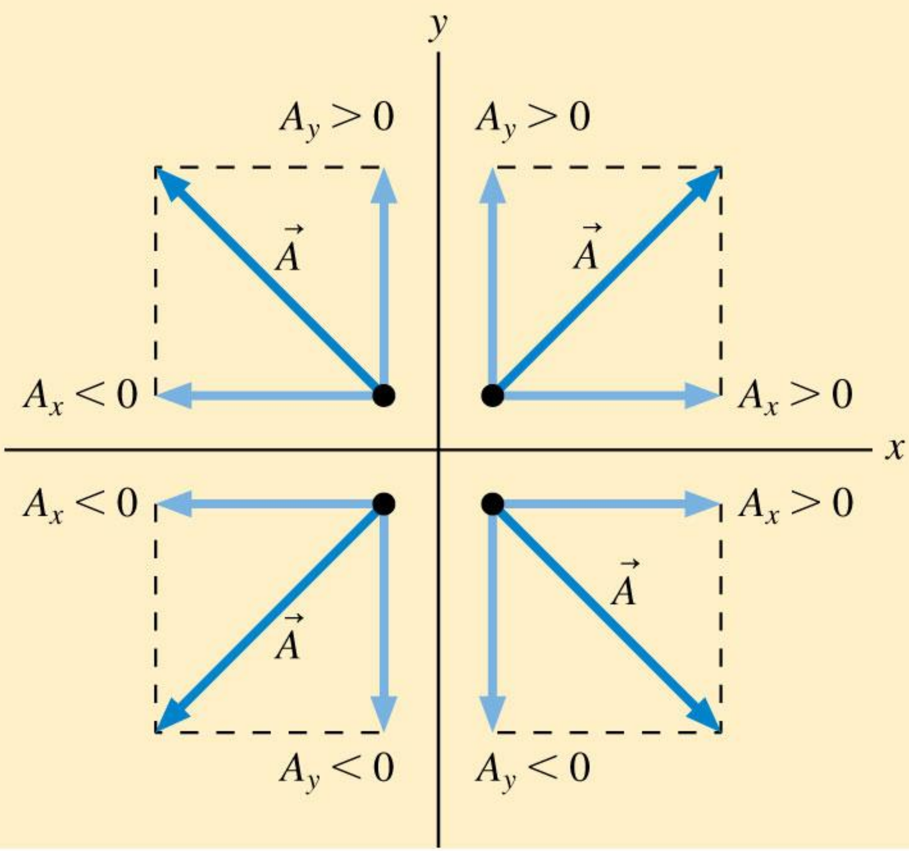
Vectors and Components

A vector can be decomposed into x- and y-**components**.



The magnitude and direction of a vector can be expressed in terms of its components.

The sign of the components depends on the direction of the vector:



Text: p. 89

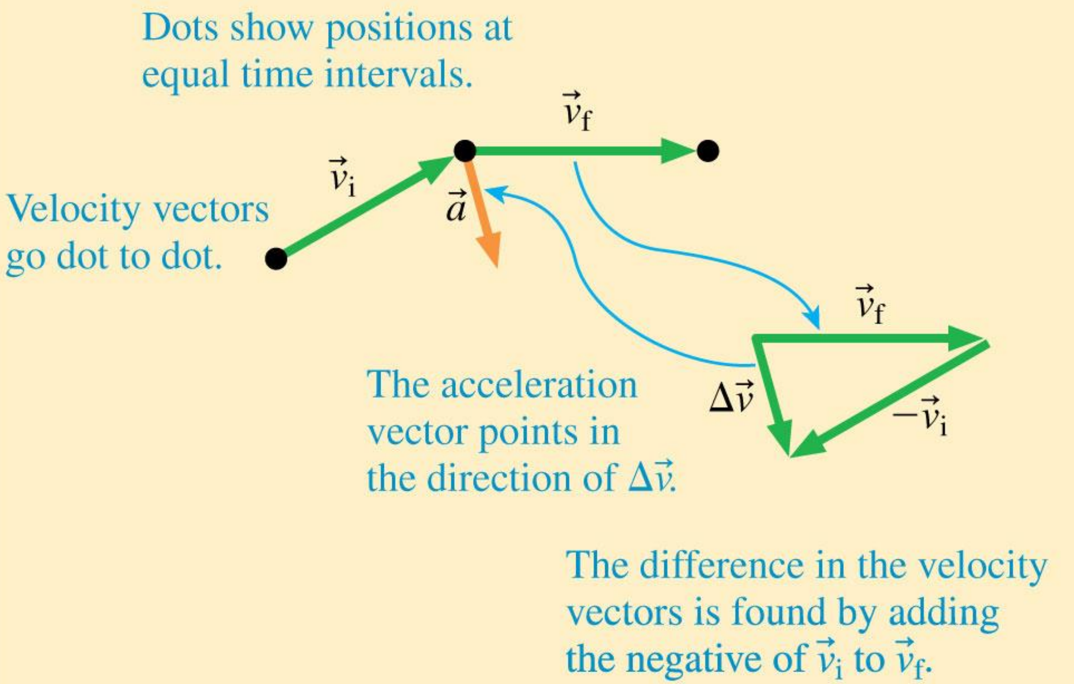
Summary: Important Concepts

The Acceleration Vector

We define the acceleration vector as

$$\vec{a} = \frac{\vec{v}_f - \vec{v}_i}{t_f - t_i} = \frac{\Delta\vec{v}}{\Delta t}$$

We find the acceleration vector on a motion diagram as follows:



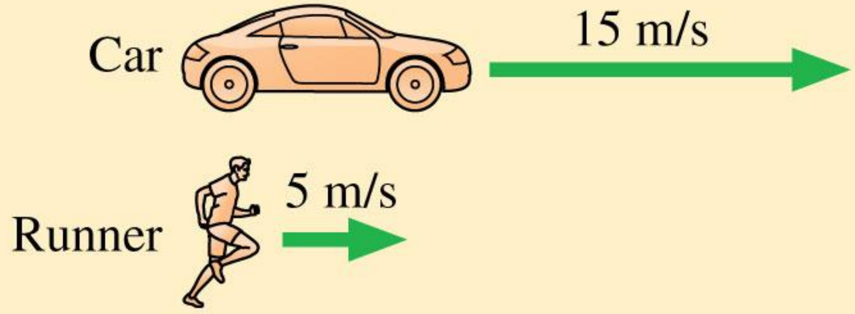
Text: p. 89

Summary: Applications

Relative motion

Velocities can be expressed relative to an observer. We can add relative velocities to convert to another observer's point of view.

c = car, r = runner, g = ground



The speed of the car with respect to the runner is

$$(v_x)_{cr} = (v_x)_{cg} + (v_x)_{gr}$$

Text: p. 89

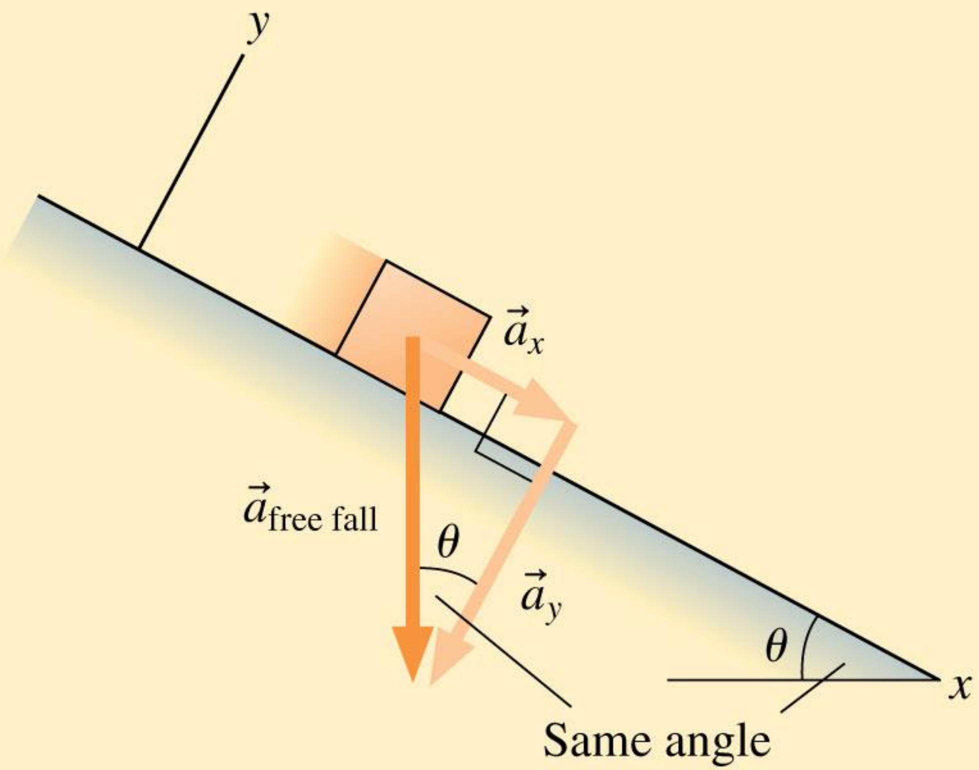
Summary: Applications

Motion on a ramp

An object sliding down a ramp will accelerate parallel to the ramp:

$$a_x = \pm g \sin\theta$$

The correct sign depends on the direction in which the ramp is tilted.



Text: p. 89

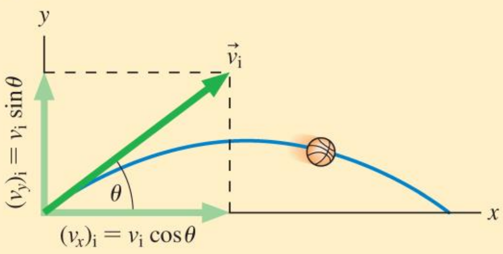
Summary

GENERAL PRINCIPLES

Projectile Motion

A projectile is an object that moves through the air under the influence of gravity and nothing else.

The path of the motion is a parabola.



The motion consists of two pieces:

1. Vertical motion with free-fall acceleration, $a_y = -g$
2. Horizontal motion with constant velocity

Kinematic equations:

$$x_f = x_i + (v_x)_i\,\Delta t$$

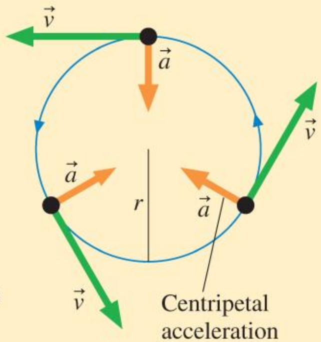
$$(v_x)_f = (v_x)_i = \text{constant}$$

$$y_f = y_i + (v_y)_i\,\Delta t - \tfrac{1}{2}g(\Delta t)^2$$

$$(v_y)_f = (v_y)_i - g\,\Delta t$$

Circular Motion

An object moving in a circle at a constant speed has a velocity that is constantly changing direction, and so experiences an acceleration:



- The velocity is tangent to the circular path.
- The acceleration points toward the center of the circle and has magnitude

$$a = \frac{v^2}{r}$$

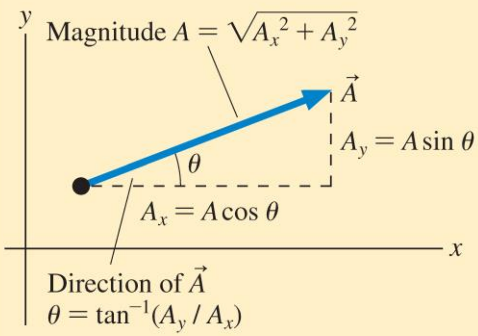
Text: p. 89

Summary

IMPORTANT CONCEPTS

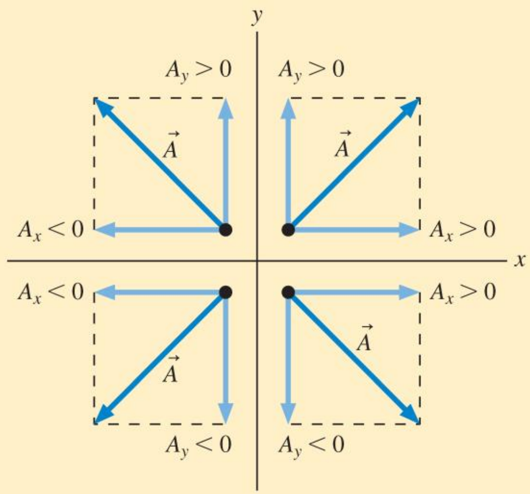
Vectors and Components

A vector can be decomposed into x- and y-**components**.



The magnitude and direction of a vector can be expressed in terms of its components.

The sign of the components depends on the direction of the vector:

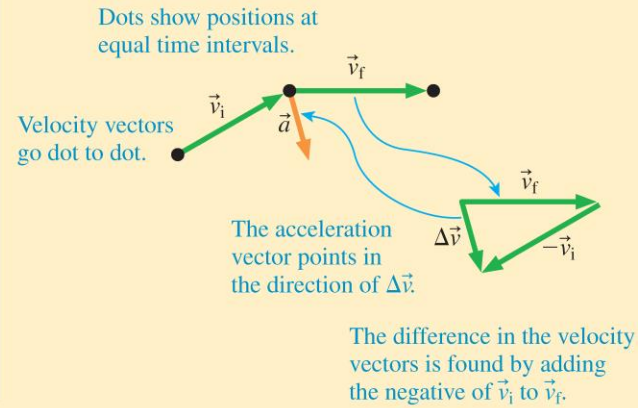


The Acceleration Vector

We define the acceleration vector as

$$\vec{a} = \frac{\vec{v}_f - \vec{v}_i}{t_f - t_i} = \frac{\Delta \vec{v}}{\Delta t}$$

We find the acceleration vector on a motion diagram as follows:



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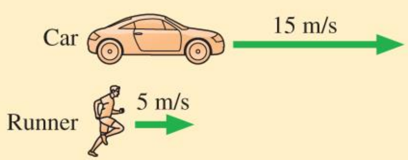
Summary

APPLICATIONS

Relative motion

Velocities can be expressed relative to an observer. We can add relative velocities to convert to another observer's point of view.

c = car, r = runner, g = ground



The speed of the car with respect to the runner is

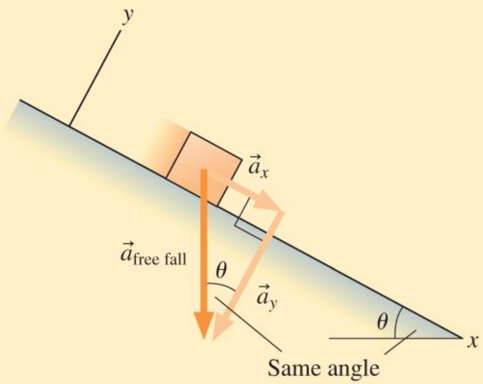
$$(v_x)_{cr} = (v_x)_{cg} + (v_x)_{gr}$$

Motion on a ramp

An object sliding down a ramp will accelerate parallel to the ramp:

$$a_x = \pm g \sin\theta$$

The correct sign depends on the direction in which the ramp is tilted.



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