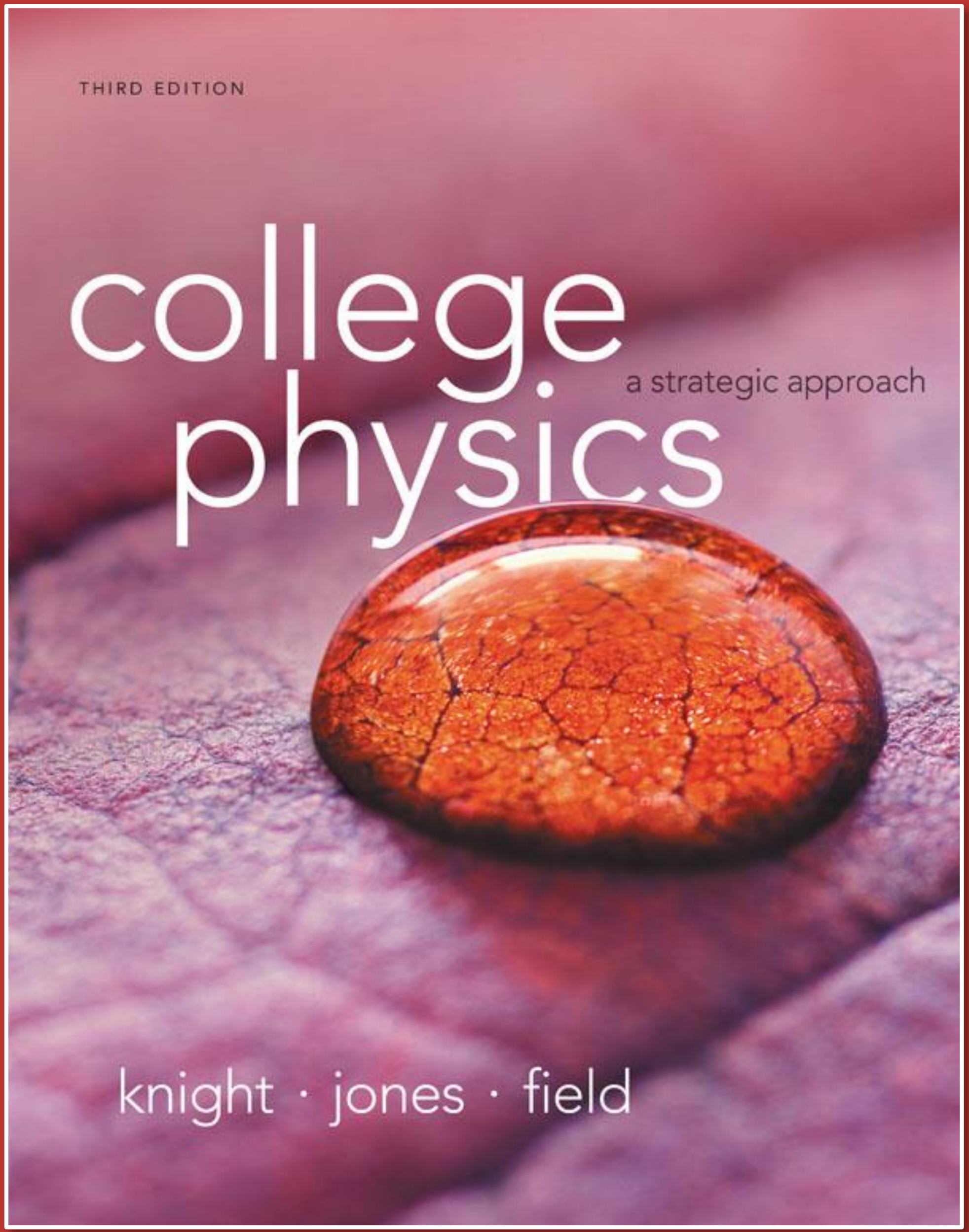


# Lecture Presentation

## Chapter 2

## *Motion in One Dimension*

# Suggested Videos for Chapter 2

- **Prelecture Videos**
  - *Motion Along a Line*
  - *Acceleration*
  - *Free Fall*
- **Class Videos**
  - *Motion Along a Straight Line*
  - *Motion with Constant Acceleration*
  - *Free Fall*
- **Video Tutor Solutions**
  - *Motion in One Dimension*

# Suggested Simulations for Chapter 2

- **ActivPhysics**
  - *1.1–1.14*

- **PhETs**
  - *The Moving Man*
  - *Equation Grapher*

# Chapter 2 Motion in One Dimension

**Chapter Goal:** To describe and analyze linear motion.

# Chapter 2 Preview
## Looking Ahead: Uniform Motion

- Successive images of the rider are the same distance apart, so the velocity is constant. This is **uniform motion**.

- You'll learn to describe motion in terms of quantities such as distance and velocity, an important first step in analyzing motion.

# Chapter 2 Preview
## Looking Ahead: Acceleration

- A cheetah is capable of very high speeds but, more importantly, it is capable of a rapid *change* in speed—a large **acceleration**.

- You'll use the concept of acceleration to solve problems of changing velocity, such as races, or predators chasing prey.

# Chapter 2 Preview
## Looking Ahead: Free Fall

- When you toss a coin, the motion—both going up and coming down—is determined by gravity alone. We call this **free fall**.

- How long does it take the coin to go up and come back down? This is the type of free-fall problem you'll learn to solve.

# Chapter 2 Preview
## Looking Ahead

### Uniform Motion

Successive images of the rider are the same distance apart, so the velocity is constant. This is **uniform motion.**

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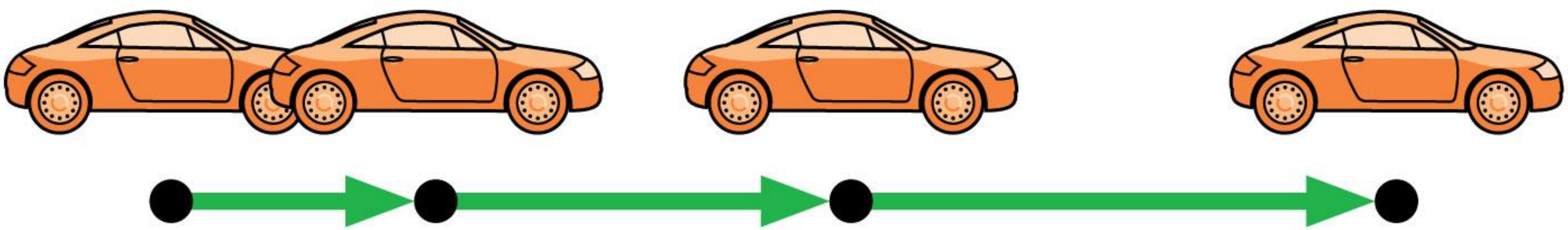
Text: p. 28

# Chapter 2 Preview
## Looking Back: Motion Diagrams

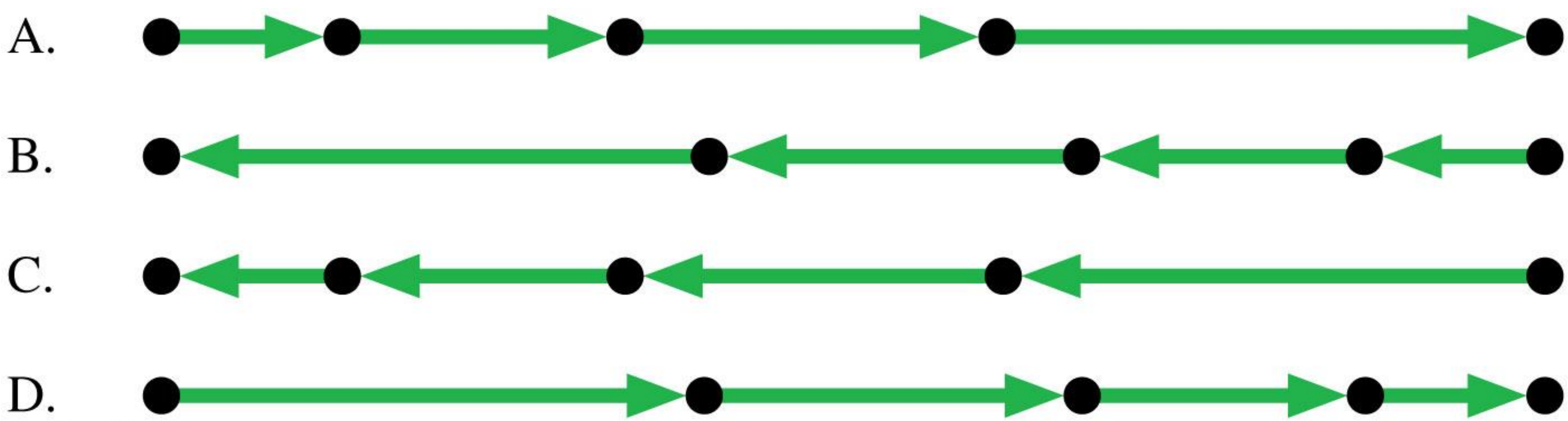
- As you saw in Section 1.5, a good first step in analyzing motion is to draw a motion diagram, marking the position of an object in subsequent times.



- In this chapter, you'll learn to create motion diagrams for different types of motion along a line. Drawing pictures like this is a good staring point for solving problems.

# Chapter 2 Preview
## Stop to Think

A bicycle is moving to the left with increasing speed. Which of the following motion diagrams illustrates this motion?



# Reading Question 2.1

The slope at a point on a position-versus-time graph of an object is the

A. Object's speed at that point.
B. Object's average velocity at that point.
C. Object's instantaneous velocity at that point.
D. Object's acceleration at that point.
E. Distance traveled by the object to that point.

# Reading Question 2.1

The slope at a point on a position-versus-time graph of an object is the

A. Object's speed at that point.
B. Object's average velocity at that point.
✔ C. Object's instantaneous velocity at that point.
D. Object's acceleration at that point.
E. Distance traveled by the object to that point.

# Reading Question 2.2

Which of the following is an example of uniform motion?

A. A car going around a circular track at a constant speed.
B. A person at rest starts running in a straight line in a fixed direction.
C. A ball dropped from the top of a building.
D. A hockey puck sliding in a straight line at a constant speed.

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Which of the following is an example of uniform motion?

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C. A ball dropped from the top of a building.
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# Reading Question 2.3

The area under a velocity-versus-time graph of an object is

A. The object's speed at that point.
B. The object's acceleration at that point.
C. The distance traveled by the object.
D. The displacement of the object.
E. This topic was not covered in this chapter.

# Reading Question 2.3

The area under a velocity-versus-time graph of an object is

A. The object's speed at that point.
B. The object's acceleration at that point.
C. The distance traveled by the object.
✔ D. The displacement of the object.
E. This topic was not covered in this chapter.

# Reading Question 2.4

If an object is speeding up,

A. Its acceleration is positive.
B. Its acceleration is negative.
C. Its acceleration can be positive or negative depending on the direction of motion.

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# Reading Question 2.5

A 1-pound ball and a 100-pound ball are dropped from a height of 10 feet at the same time. In the absence of air resistance

A. The 1-pound ball wins the race.
B. The 100-pound ball wins the race.
C. The two balls end in a tie.
D. There's not enough information to determine which ball wins the race.

# Reading Question 2.5

A 1-pound ball and a 100-pound ball are dropped from a height of 10 feet at the same time. In the absence of air resistance

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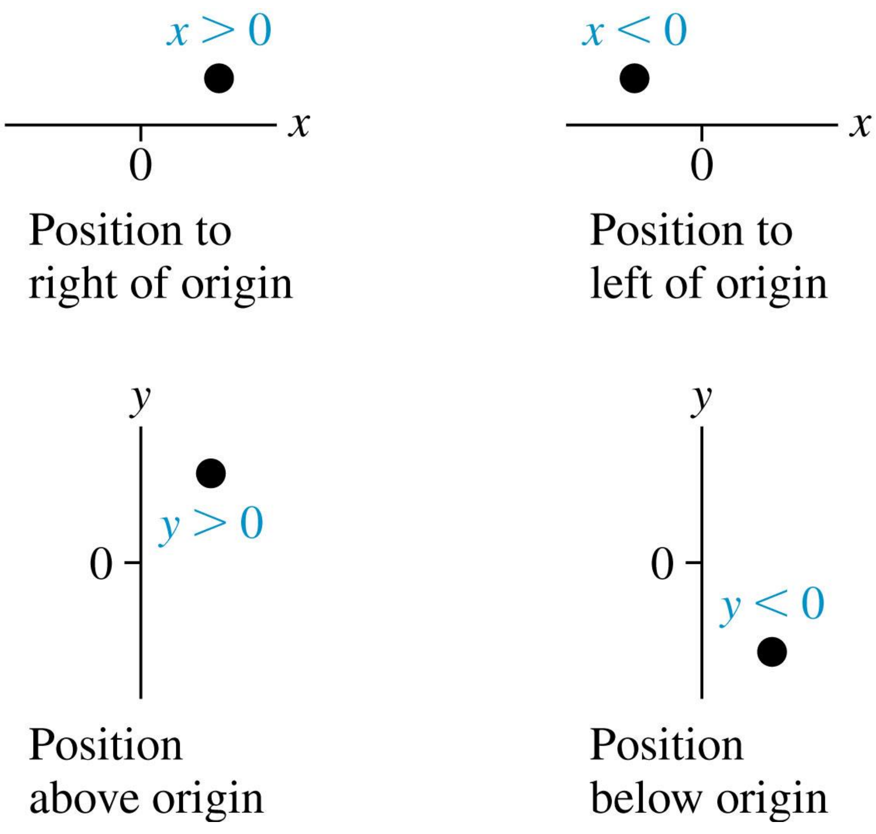
B. The 100-pound ball wins the race.
✔ C. The two balls end in a tie.
D. There's not enough information to determine which ball wins the race.

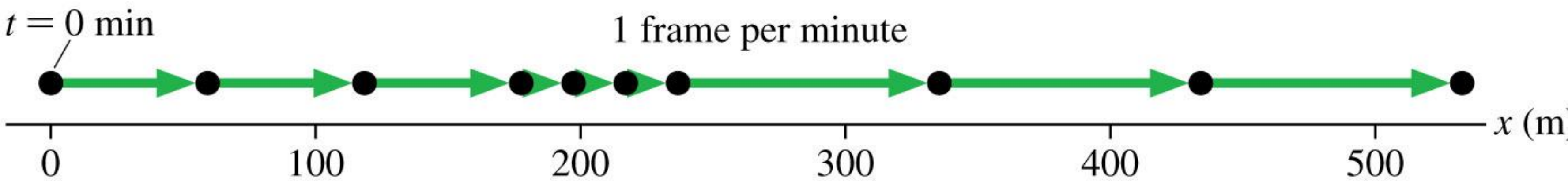
# Section 2.1 Describing Motion

# Representing Position

- We will use an ***x*-axis** to analyze horizontal motion and motion on a ramp, with the positive end to the right.
- We will use a ***y*-axis** to analyze vertical motion, with the positive end up.

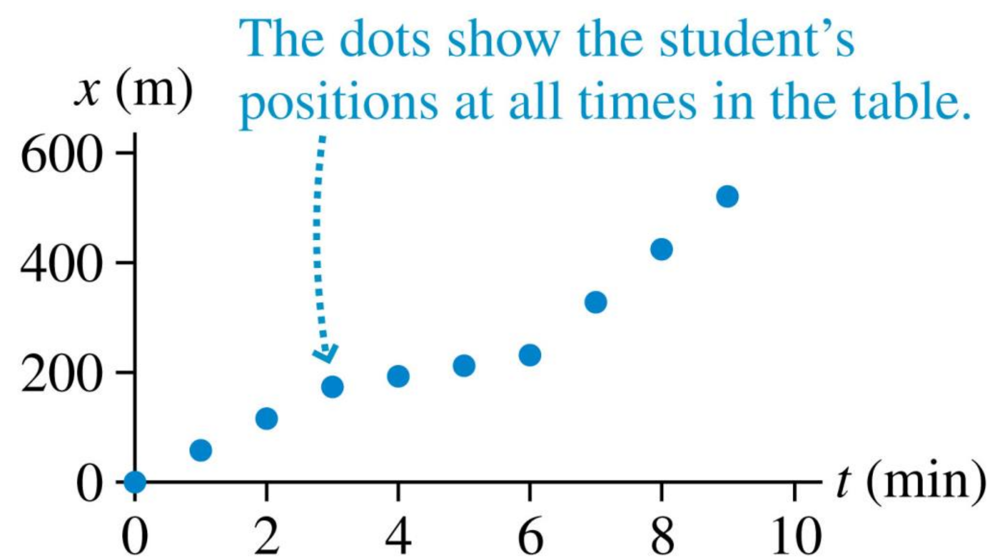


# Representing Position




The motion diagram of a student walking to school and a coordinate axis for making measurements

- Every dot in the motion diagram of Figure 2.2 represents the student's position at a particular time.
- Figure 2.3 shows the student's motion shows the student's position as a **graph** of *x* versus *t*.

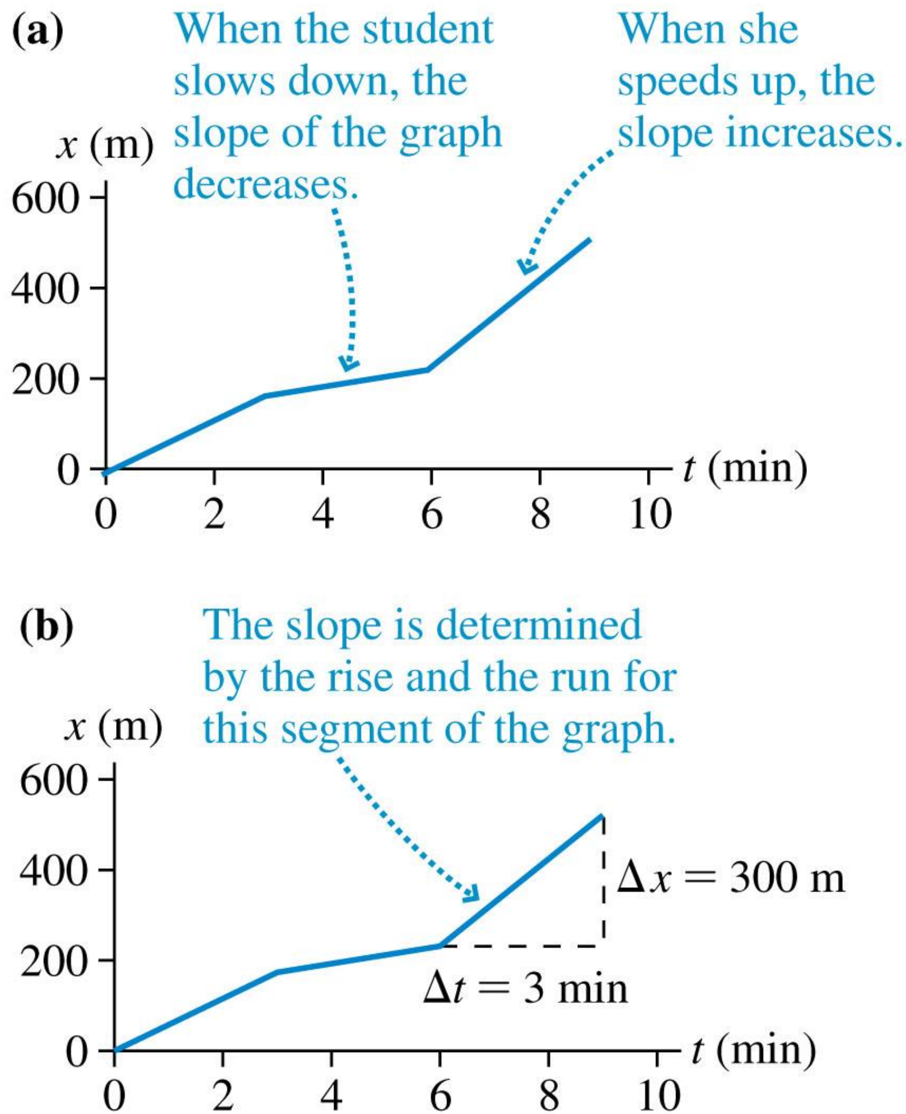


# From Position to Velocity

- On a position-versus-time graph, **a faster speed corresponds to a steeper slope**.

$$\text{slope of graph} = \frac{\text{rise}}{\text{run}} = \frac{\Delta x}{\Delta t}$$

- **The slope of an object's position-versus-time graph is the object's velocity at that point in the motion.**



# From Position to Velocity

**TACTICS BOX 2.1** **Interpreting position-versus-time graphs**

Information about motion can be obtained from position-versus-time graphs as follows:

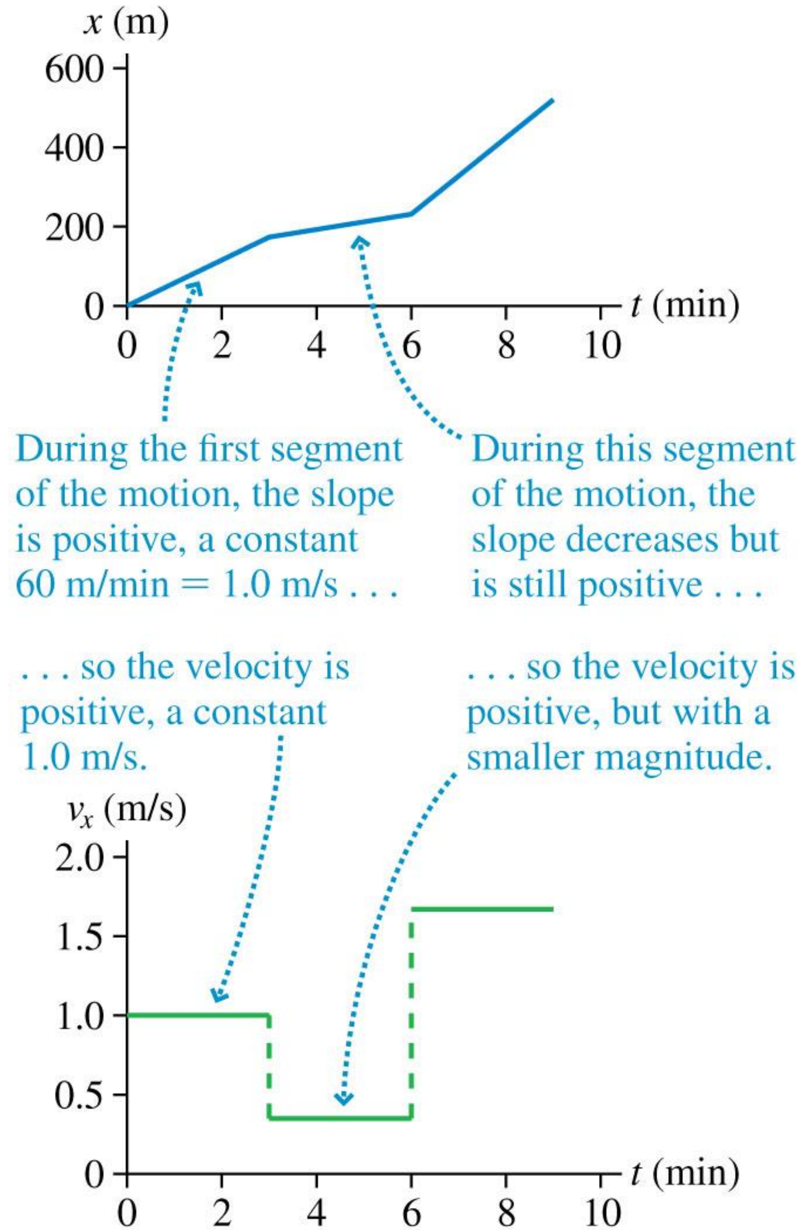
1. Determine an object's *position* at time *t* by reading the graph at that instant of time.
2. Determine the object's *velocity* at time *t* by finding the slope of the position graph at that point. Steeper slopes correspond to faster speeds.
3. Determine the *direction of motion* by noting the sign of the slope. Positive slopes correspond to positive velocities and, hence, to motion to the right (or up). Negative slopes correspond to negative velocities and, hence, to motion to the left (or down).

Exercises 2,3

Text: p. 31

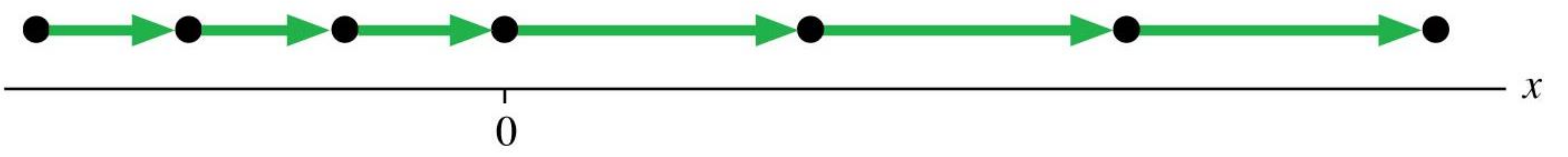
# From Position to Velocity

- We can deduce the **velocity-versus-time graph** from the position-versus-time graph.
- The velocity-versus-time graph is yet another way to represent an object's motion.

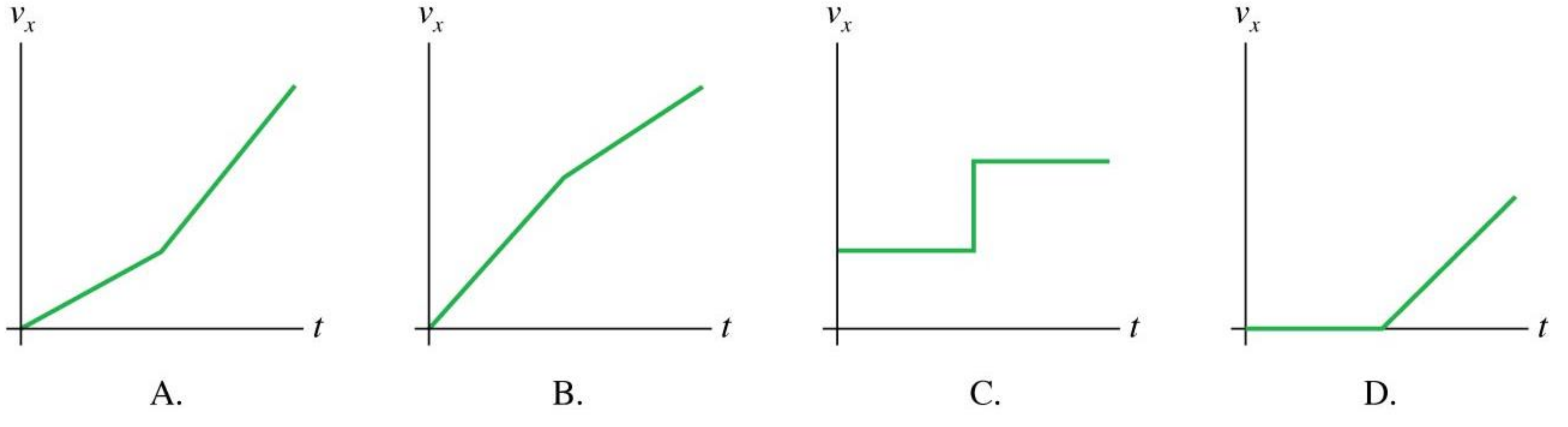


# QuickCheck 2.2

Here is a motion diagram of a car moving along a straight road:



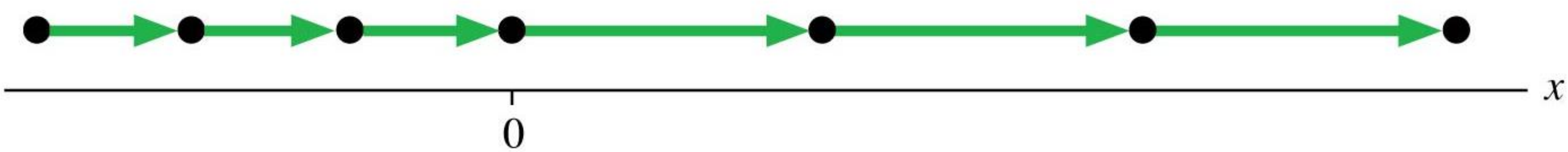

Which velocity-versus-time graph matches this motion diagram?



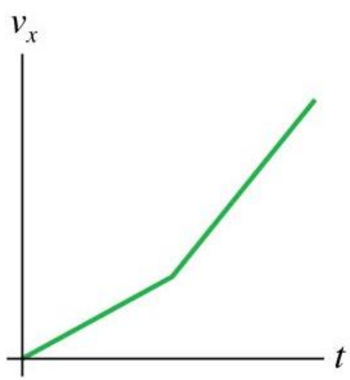

E. None of the above.

# QuickCheck 2.2

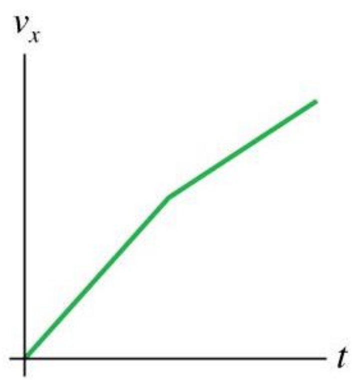
Here is a motion diagram of a car moving along a straight road:



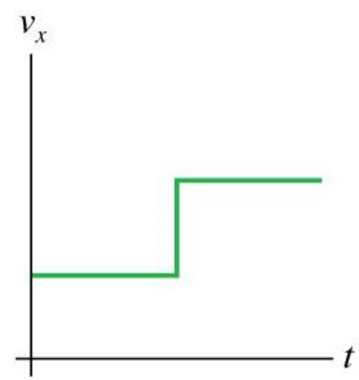

Which velocity-versus-time graph matches this motion diagram?



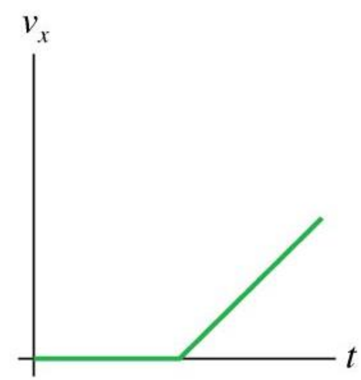

A.




B.




✔ C.

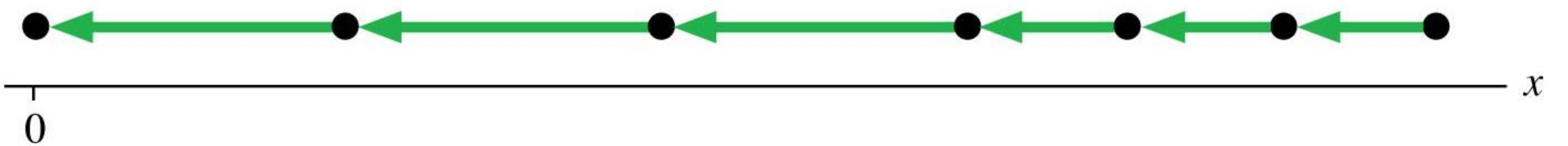



D.

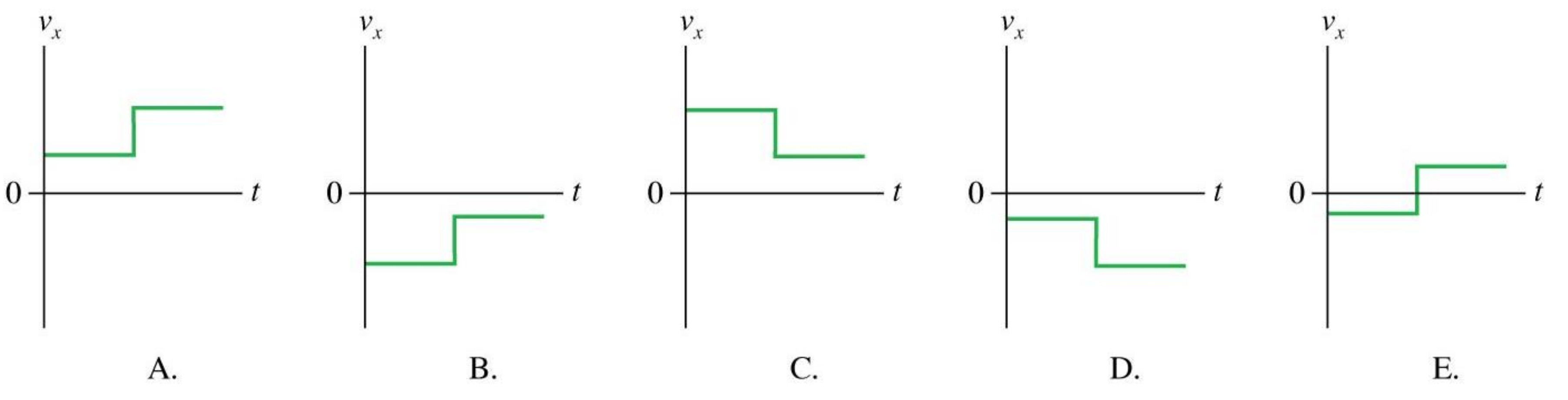
E. None of the above.

# QuickCheck 2.3

Here is a motion diagram of a car moving along a straight road:




Which velocity-versus-time graph matches this motion diagram?



# QuickCheck 2.3

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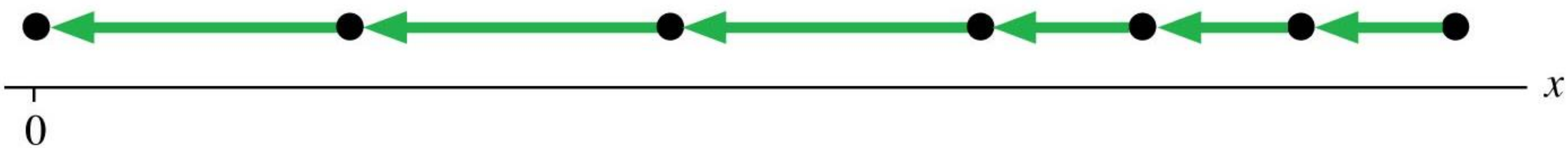
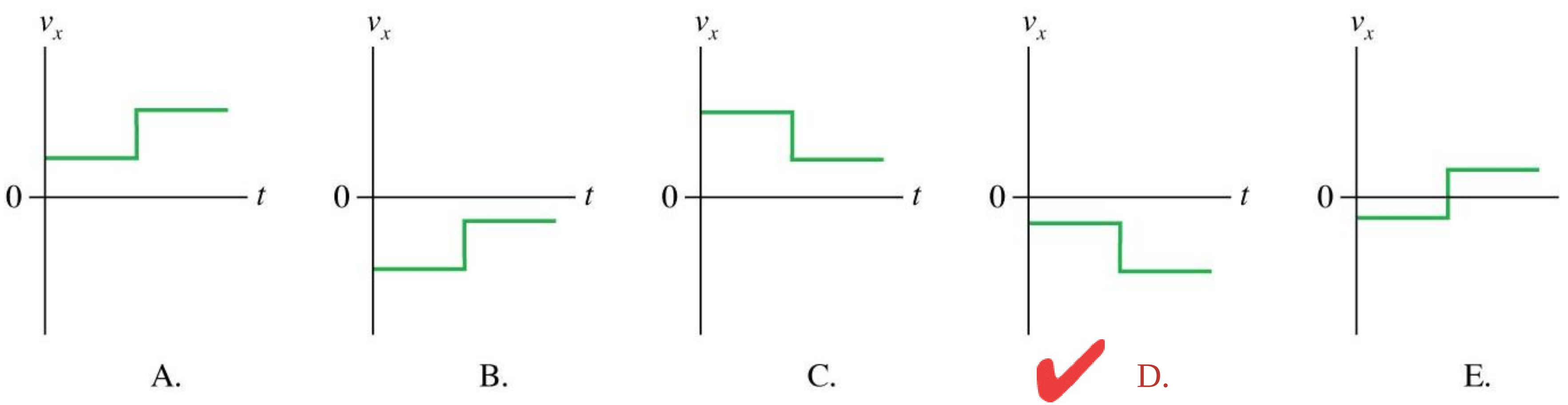
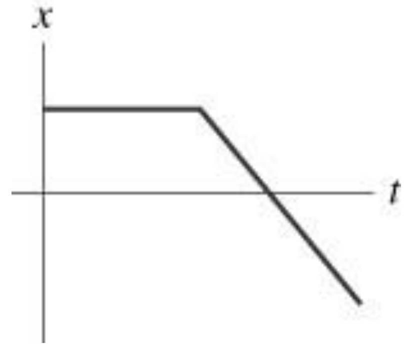



Which velocity-versus-time graph matches this motion diagram?

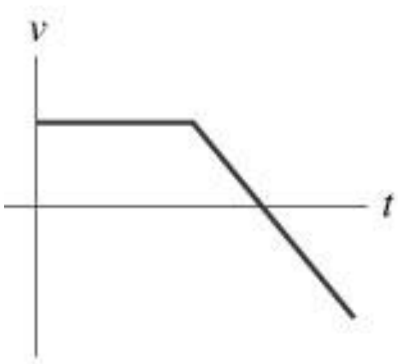


# QuickCheck 2.4

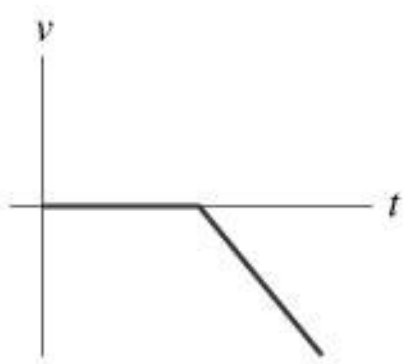
A graph of position versus time for a basketball player moving down the court appears as follows:



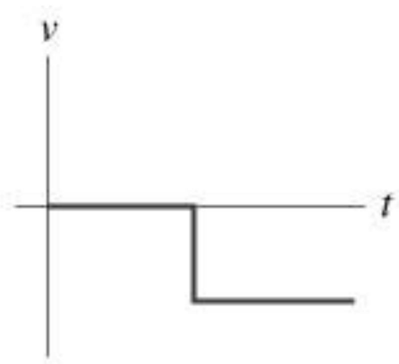

Which of the following velocity graphs matches the position graph?



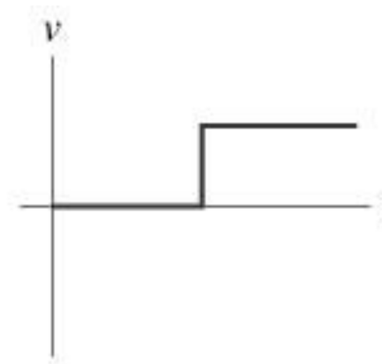

A.




B.




C.




D.

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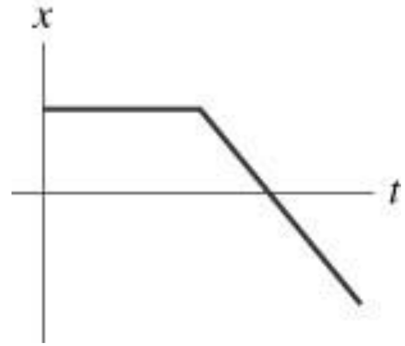


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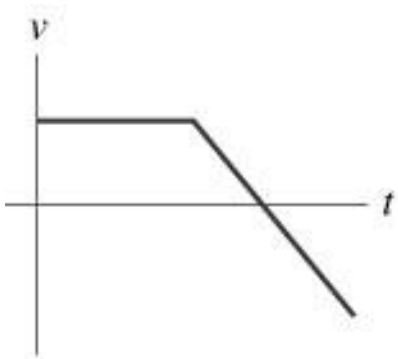
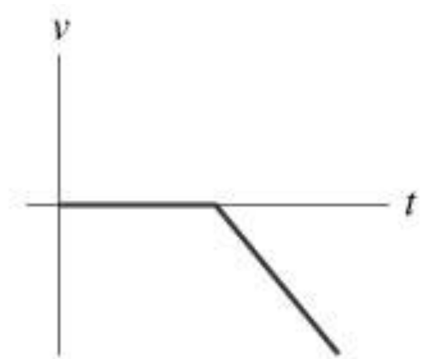
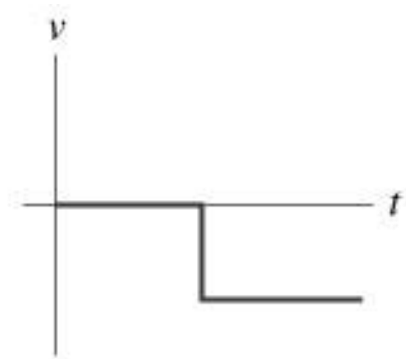


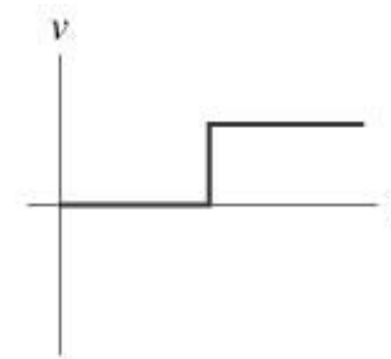

A.




B.




✔ C.



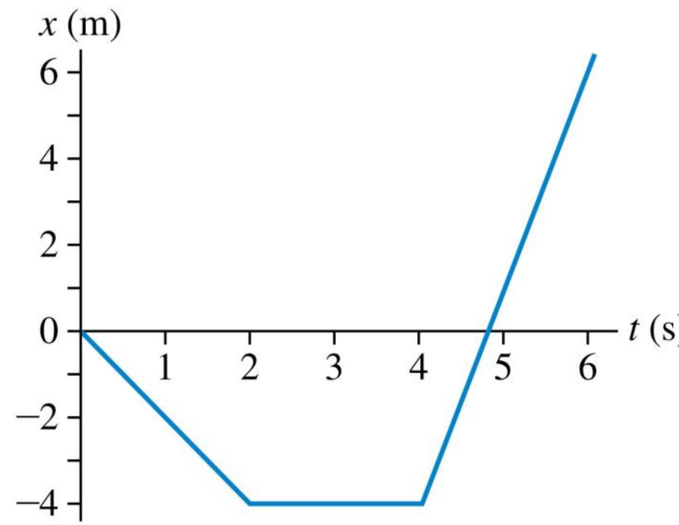

D.

# Example 2.2 Analyzing a car's position graph

FIGURE 2.11 gives the position-versus-time graph of a car.

a. Draw the car's velocity-versus-time graph.

b. Describe the car's motion in words.




**PREPARE** Figure 2.11 is a graphical representation of the motion. The car's position-versus-time graph is a sequence of three straight lines. Each of these straight lines represents uniform motion at a constant velocity. We can determine the car's velocity during each interval of time by measuring the slope of the line.

# Example 2.2 Analyzing a car's position graph (cont.)

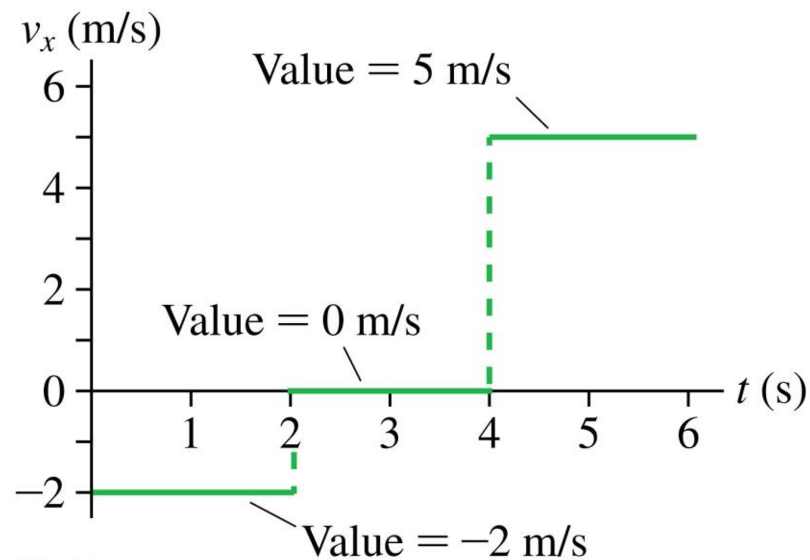
**SOLVE**

a. From $t$ = 0 s to $t$ = 2 s ($\Delta t$ = 2 s) the car's displacement is $\Delta x$ = –4 m – 0 m = –4 m. The velocity during this interval is

$$v_x = \frac{\Delta x}{\Delta t} = \frac{-4\ \text{m}}{2\ \text{s}} = -2\ \text{m/s}$$

The car's position does not change from $t$ = 2 s to $t$ = 4 s ($\Delta x$ = 0 m), so $v_x$ = 0 m/s. Finally, the displacement between $t$ = 4 s and $t$ = 6 s ($\Delta t$ = 2 s) is $\Delta x$ = 10 m. Thus the velocity during this interval is

$$v_x = \frac{10\ \text{m}}{2\ \text{s}} = 5\ \text{m/s}$$

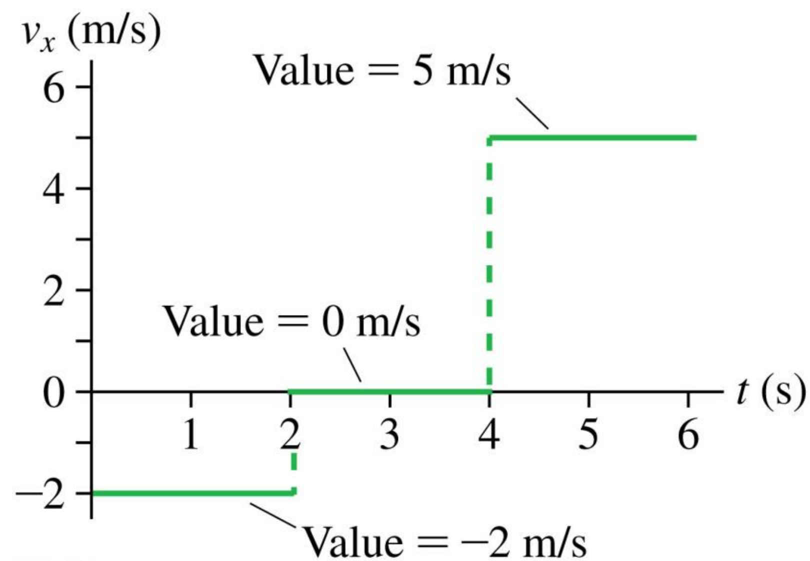



These velocities are represented graphically in FIGURE 2.12.

# Example 2.2 Analyzing a car's position graph (cont.)

**SOLVE**

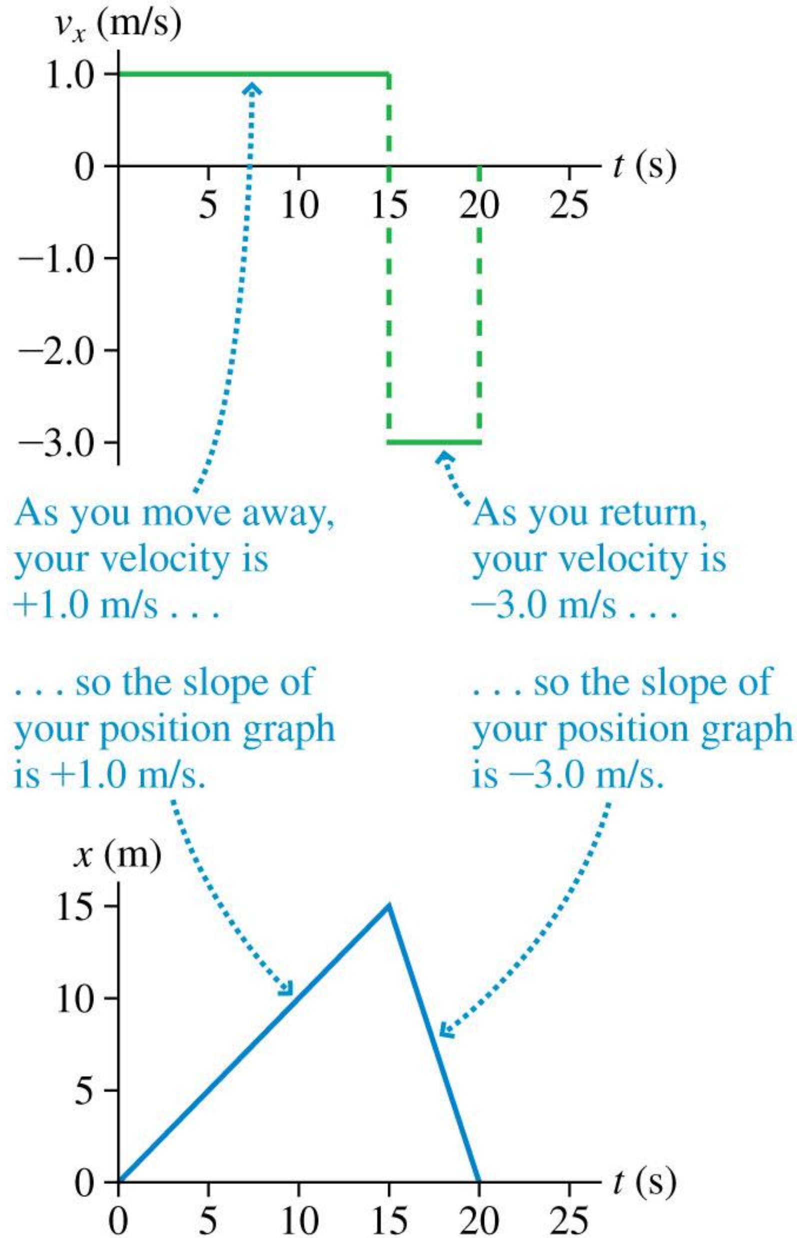
b. The velocity-versus-time graph of Figure 2.12 shows the motion in a way that we can describe in a straightforward manner: The car backs up for 2 s at 2 m/s, sits at rest for 2 s, then drives forward at 5 m/s for 2 s.




**ASSESS** Notice that the velocity graph and the position graph look completely different. They should! The value of the velocity graph at any instant of time equals the *slope* of the position graph. Since the position graph is made up of segments of constant slope, the velocity graph should be made up of segments of constant *value*, as it is. This gives us confidence that the graph we have drawn is correct.

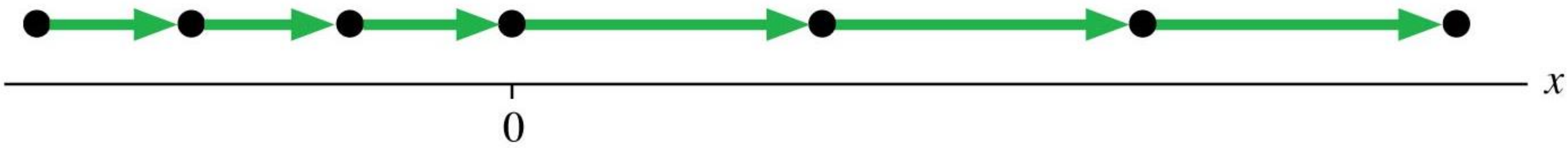
# From Velocity to Position

- We can deduce the position-versus-time graph from the velocity-versus-time graph.
- The sign of the velocity tells us whether the slope of the position graph is positive or negative.
- The magnitude of the velocity tells us how steep the slope is.

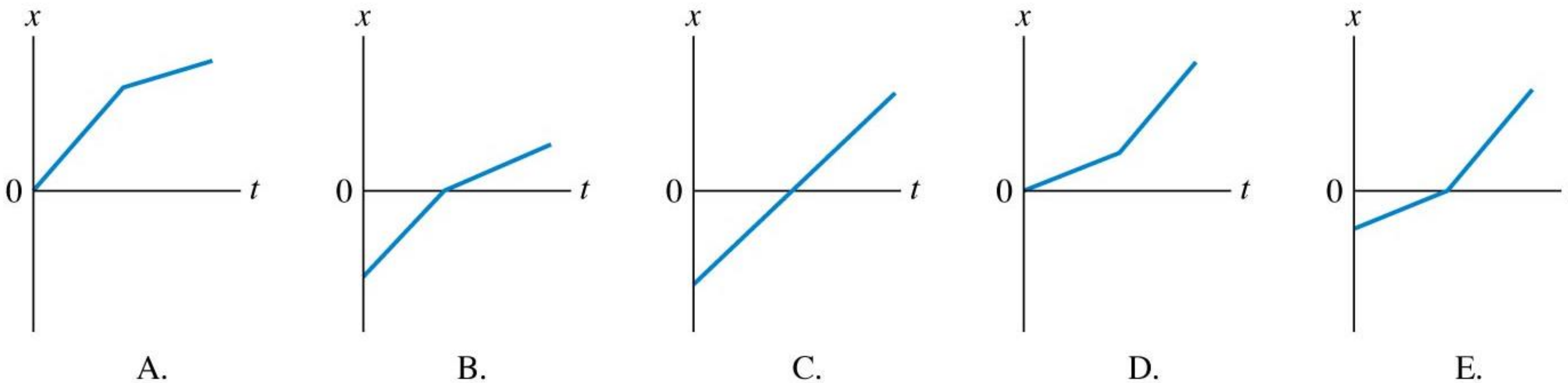


# QuickCheck 2.1

Here is a motion diagram of a car moving along a straight road:




Which position-versus-time graph matches this motion diagram?



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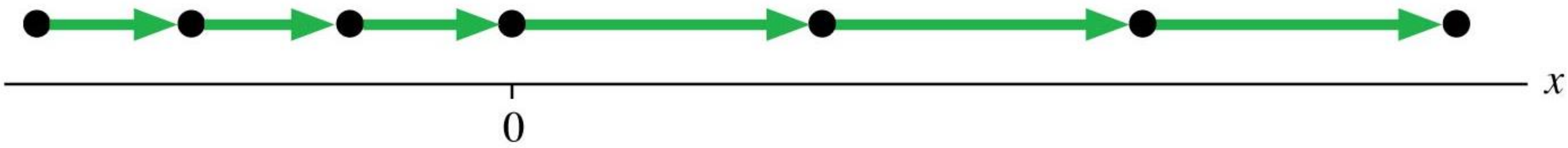


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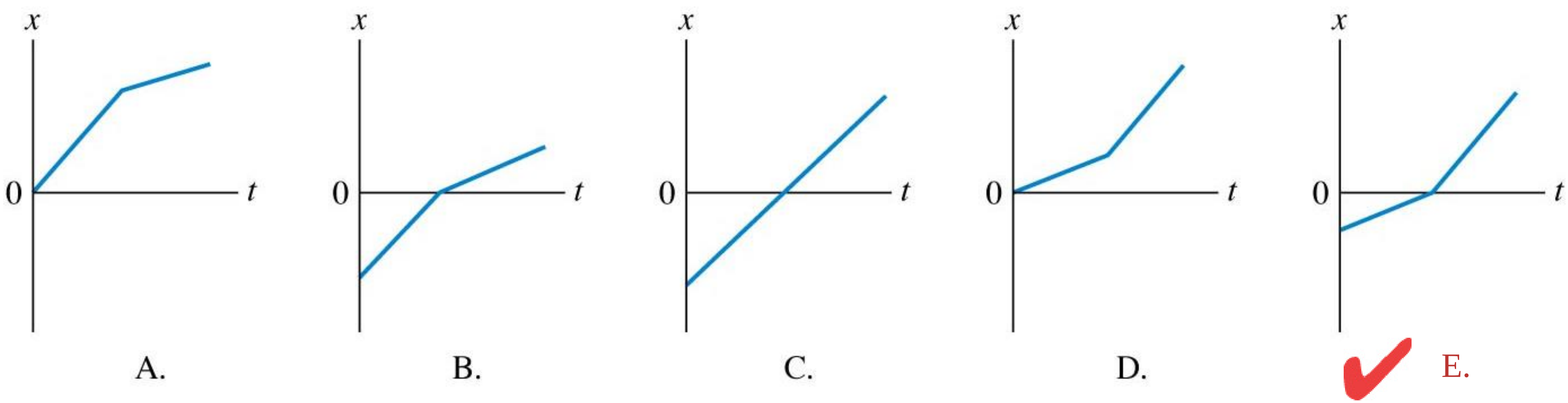
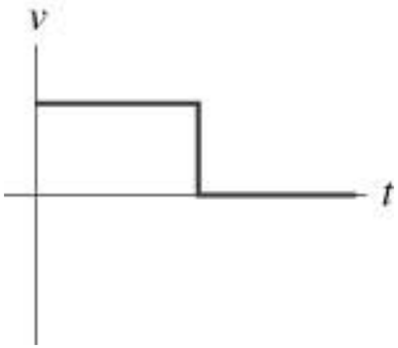
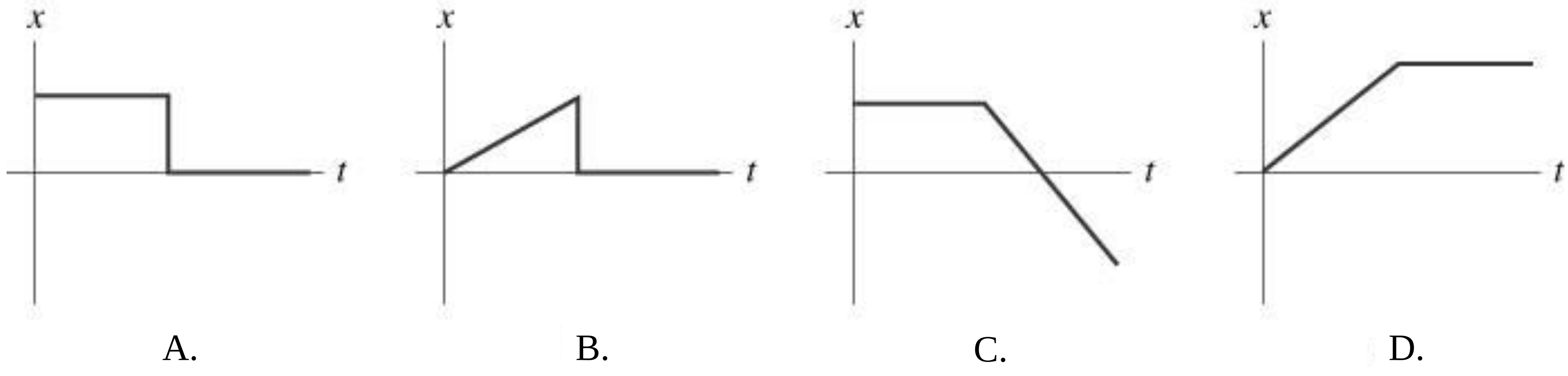


# QuickCheck 2.6

A graph of velocity versus time for a hockey puck shot into a goal appears as follows:




Which of the following position graphs matches the velocity graph?



# QuickCheck 2.6

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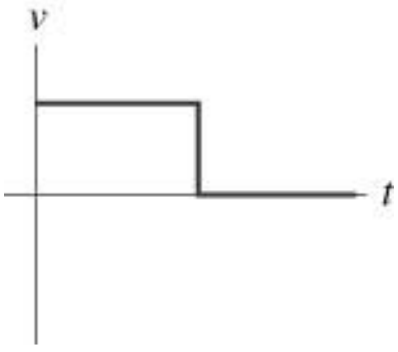
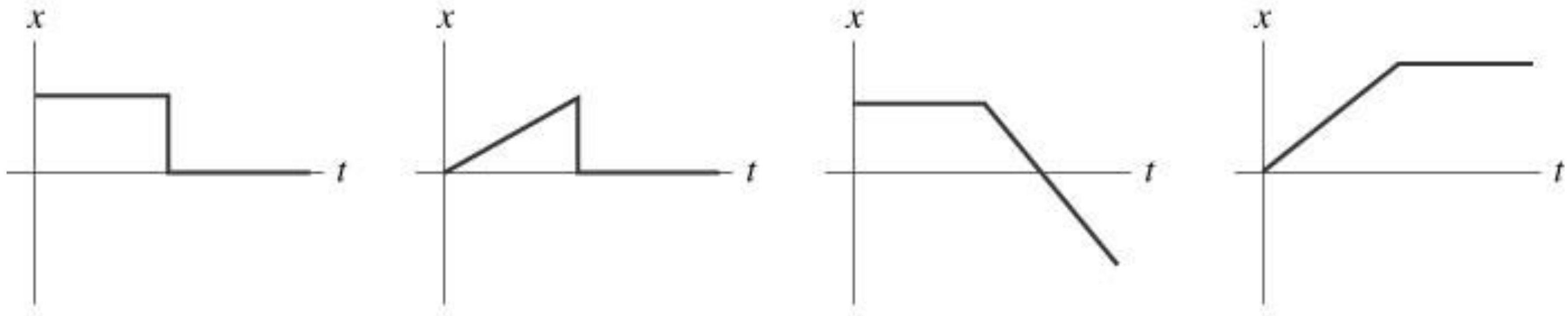


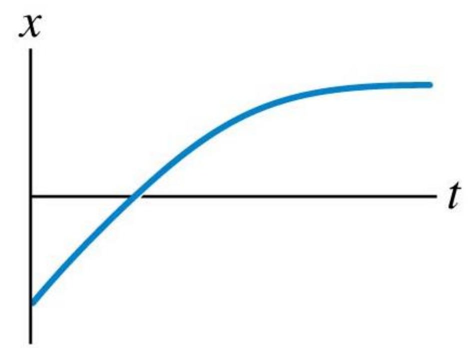

Which of the following position graphs matches the velocity graph?

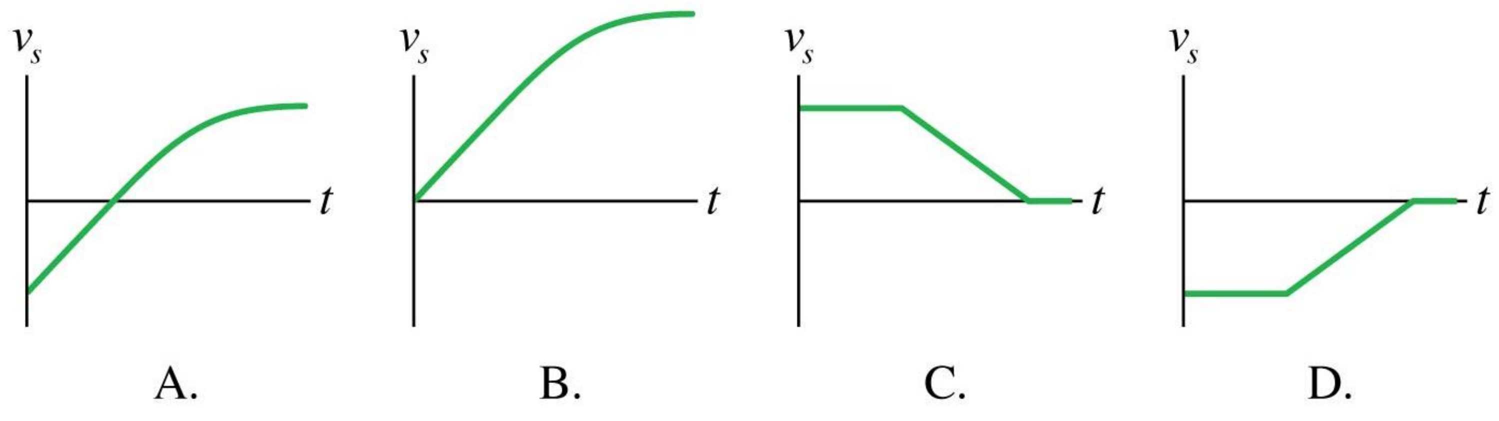



A. B. C. ✔ D.

# QuickCheck 2.7

Which velocity-versus-time graph goes with this position graph?



# QuickCheck 2.7

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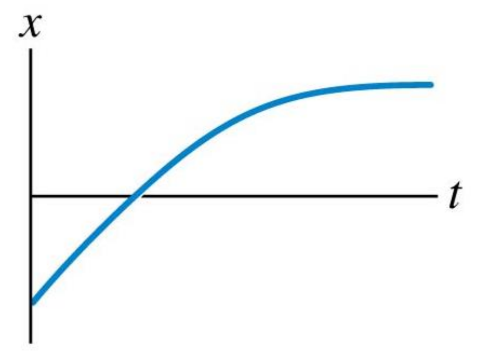
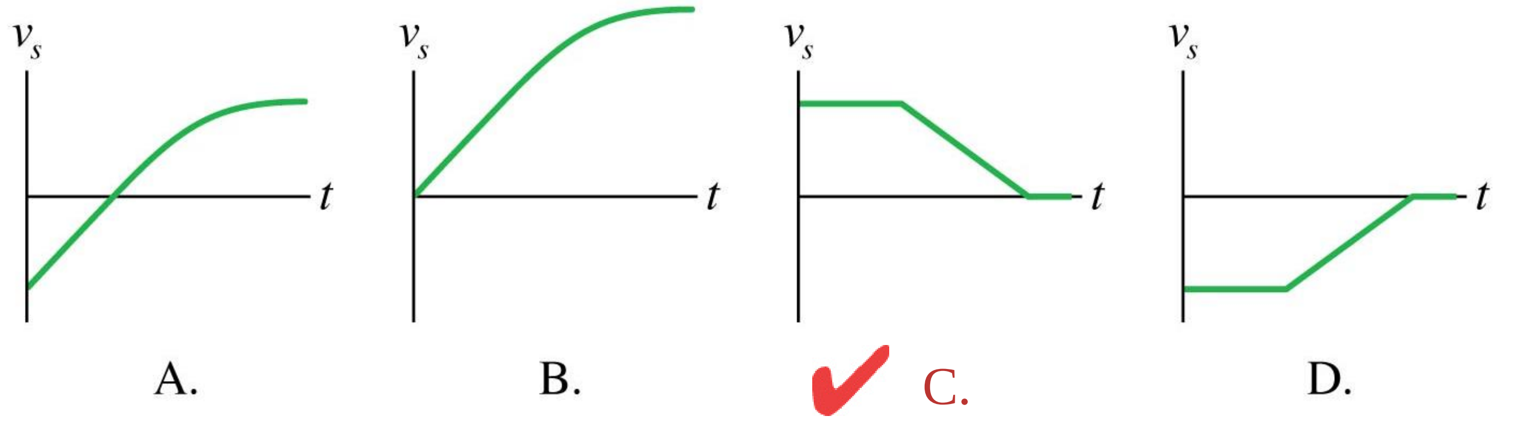
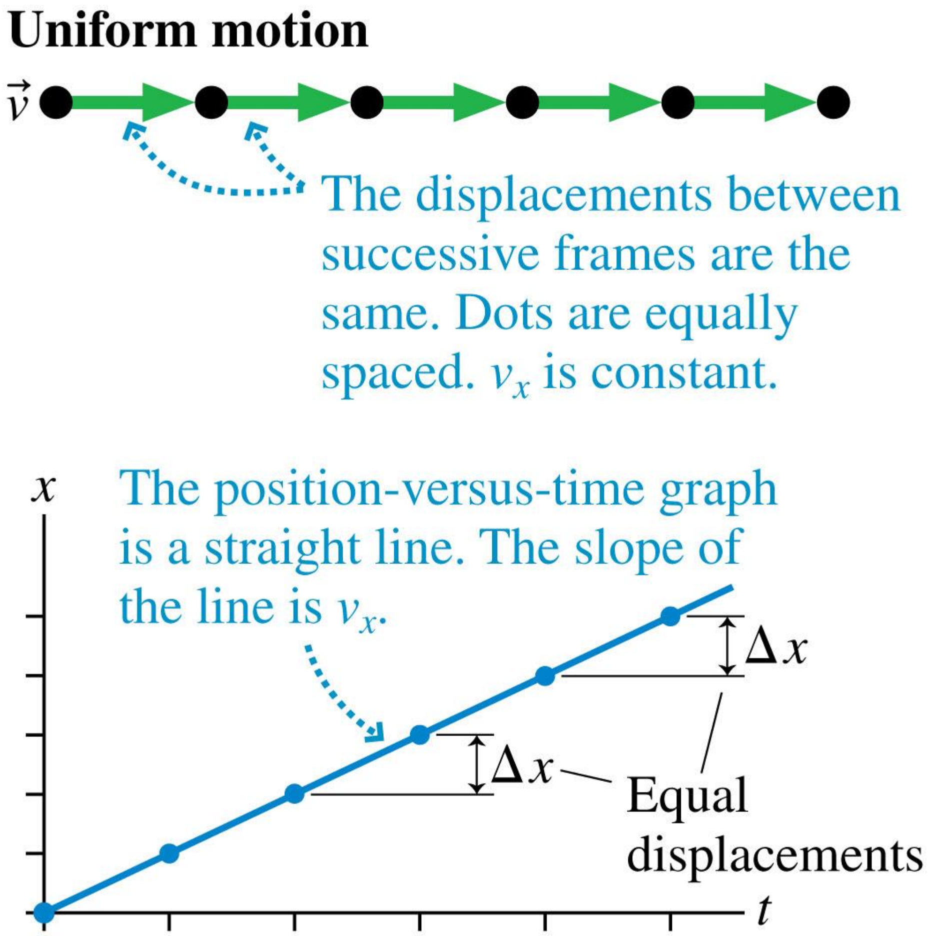


# Section 2.2 Uniform Motion

# Uniform Motion

- Straight-line motion in which equal displacements occur during any successive equal-time intervals is called **uniform motion** or **constant-velocity motion.**
- **An object's motion is uniform if and only if its position-versus-time graph is a straight line.**



# Equations of Uniform Motion

- The velocity of an object in uniform motion tells us the amount by which its position changes during each second.

$$v_x = \frac{\text{rise}}{\text{run}} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}$$

$$x_f = x_i + v_x \, \Delta t$$

Position equation for an object in uniform motion ($v_x$ is constant)

$$\Delta x = v_x \, \Delta t$$

- The displacement $\Delta x$ is proportional to the time interval $\Delta t$.

# Equations of Uniform Motion

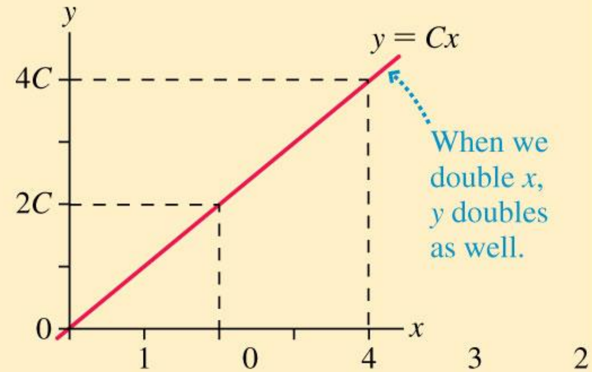
## Proportional relationships

We say that $y$ is **proportional** to $x$ if they are related by an equation of this form:

$$y = Cx$$

*y is proportional to x*

We call $C$ the **proportionality constant.** A graph of $y$ versus $x$ is a straight line that passes through the origin.




**SCALING** If $x$ has the initial value $x_1$, then $y$ has the initial value $y_1 = Cx_1$. Changing $x$ from $x_1$ to $x_2$ changes $y$ from $y_1$ to $y_2$ The ratio of $y_2$ to $y_1$ is

$$\frac{y_2}{y_1} = \frac{Cx_2}{Cx_1} = \frac{x_2}{x_1}$$

The ratio of $y_2$ to $y_1$ is exactly the same as the ratio of $x_2$ to $x_1$. If $y$ is proportional to $x$, which is often written $y \propto x$, then $x$ and $y$ change by the same factor:

- If you double $x$, you double $y$.
- If you decrease $x$ by a factor of 3, you decrease $y$ by a factor of 3.

If two variables have a proportional relationship, we can draw important conclusions from ratios without knowing the value of the proportionality constant $C$. We can often solve problems in a very straightforward manner by looking at such ratios. This is an important skill called *ratio reasoning*.

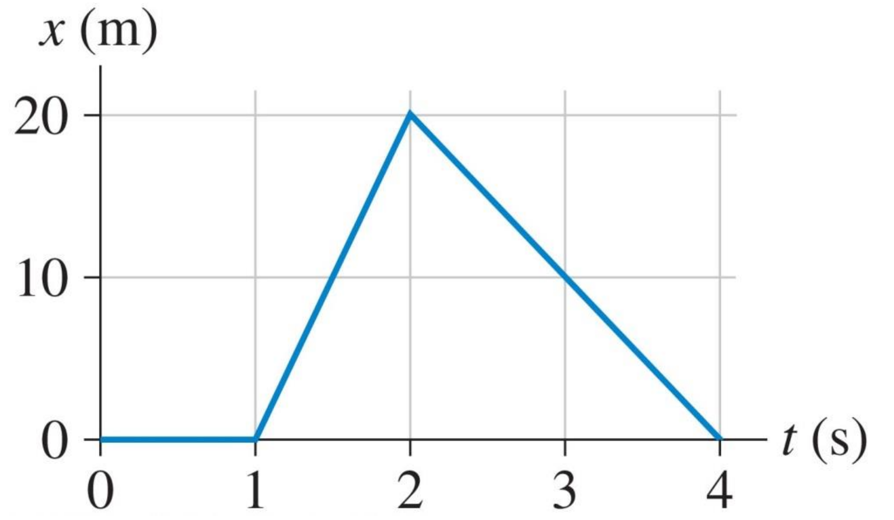
Exercise 11



Text: p. 34

# QuickCheck 2.8

Here is a position graph of an object:

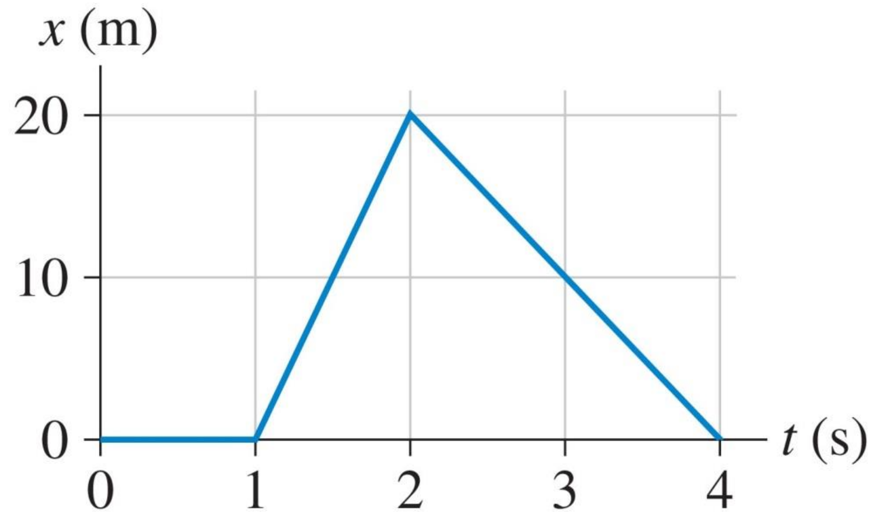



At $t$ = 1.5 s, the object's velocity is

A. 40 m/s
B. 20 m/s
C. 10 m/s
D. –10 m/s
E. None of the above

# QuickCheck 2.8

Here is a position graph of an object:




At $t = 1.5$ s, the object's velocity is

A. 40 m/s
✔ B. 20 m/s
C. 10 m/s
D. –10 m/s
E. None of the above

# Example 2.3 If a train leaves Cleveland at 2:00…

A train is moving due west at a constant speed. A passenger notes that it takes 10 minutes to travel 12 km. How long will it take the train to travel 60 km?

**PREPARE** For an object in uniform motion, Equation 2.5 shows that the distance traveled $\Delta x$ is proportional to the time interval $\Delta t$, so this is a good problem to solve using ratio reasoning.

# Example 2.3 If a train leaves Cleveland at 2:00…(cont.)

**SOLVE** We are comparing two cases: the time to travel 12 km and the time to travel 60 km. Because $\Delta x$ is proportional to $\Delta t$, the ratio of the times will be equal to the ratio of the distances. The ratio of the distances is

$$\frac{\Delta x_2}{\Delta x_1} = \frac{60\text{ km}}{12\text{ km}} = 5$$

This is equal to the ratio of the times:

$$\frac{\Delta t_2}{\Delta t_1} = 5$$

$$\begin{aligned}\Delta t_2 = \text{ time to travel 60 km} &= 5\Delta t_1 = 5 \times (10\text{ min}) \\ &= 50\text{ min}\end{aligned}$$

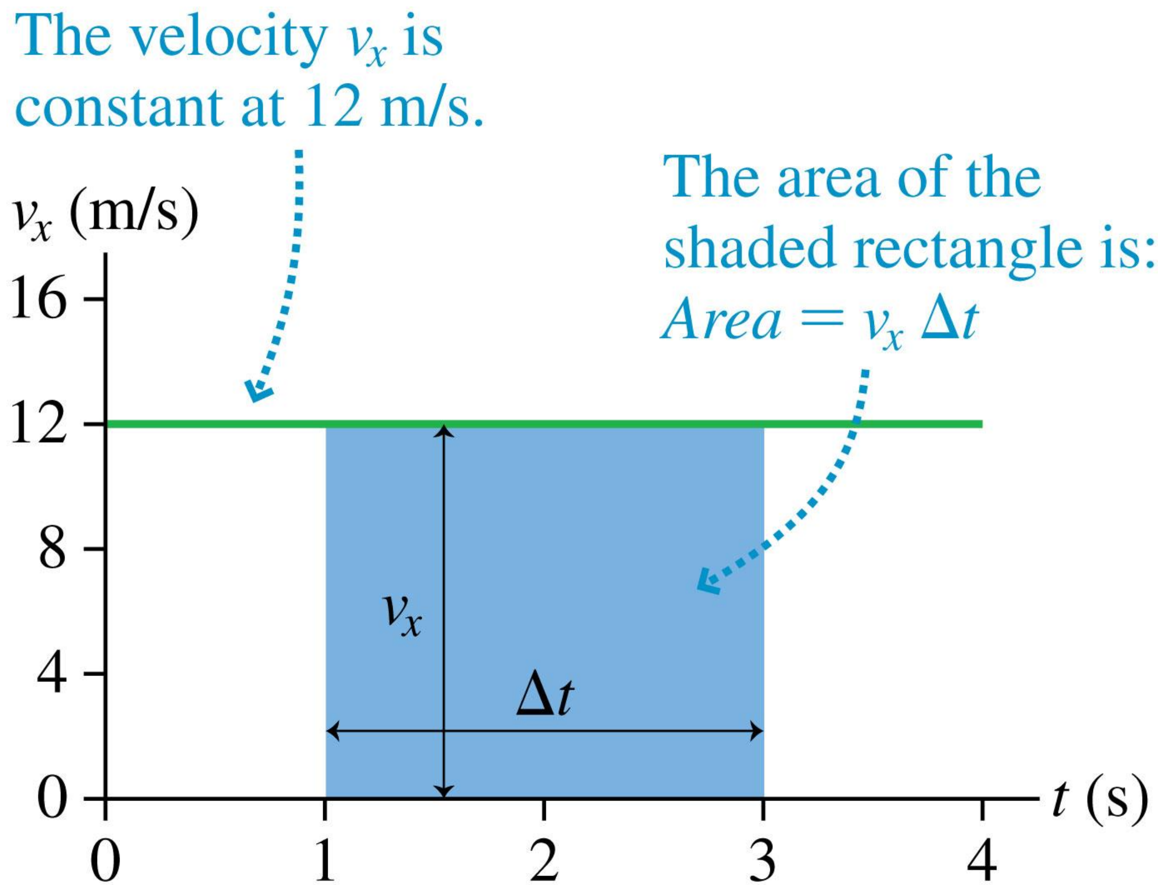
It takes 10 minutes to travel 12 km, so it will take 50 minutes—5 times as long—to travel 60 km.

# Example Problem

A soccer player is 15 m from her opponent's goal. She kicks the ball hard; after 0.50 s, it flies past a defender who stands 5 m away, and continues toward the goal. How much time does the goalie have to move into position to block the kick from the moment the ball leaves the kicker's foot?

# From Velocity to Position, One More Time

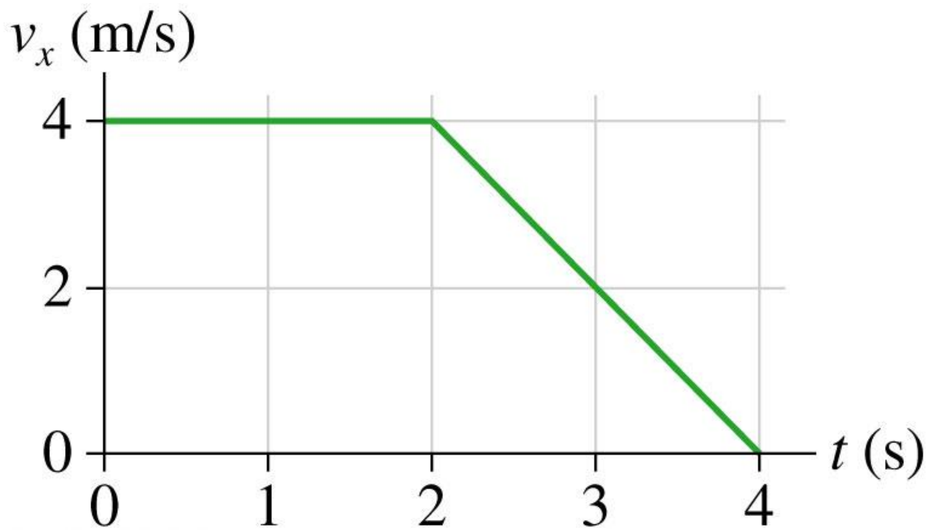
- **The displacement $\Delta x$ is equal to the area under the velocity graph during the time interval $\Delta t$.**



# QuickCheck 2.11

Here is the velocity graph of an object that is at the origin ($x = 0$ m) at $t = 0$ s.

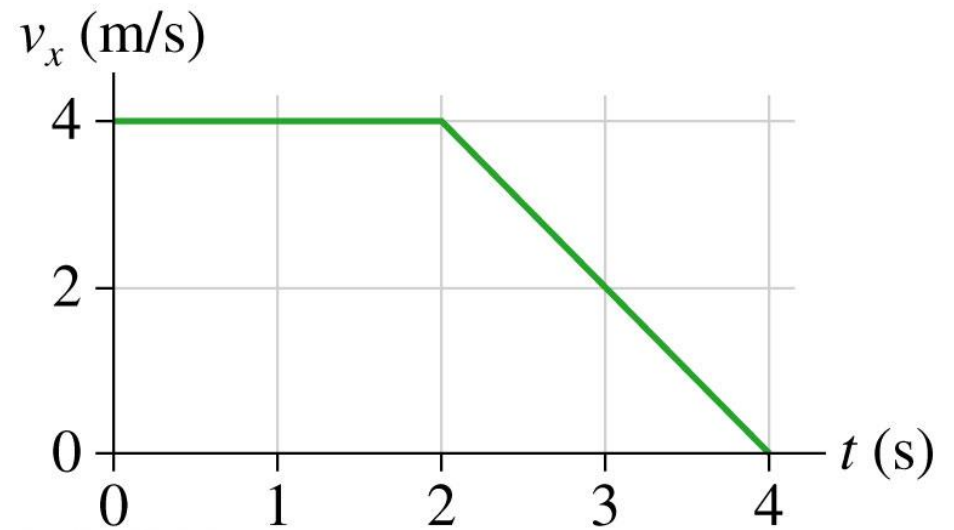
At $t = 4.0$ s, the object's position is




A. 20 m
B. 16 m
C. 12 m
D. 8 m
E. 4 m

# QuickCheck 2.11

Here is the velocity graph of an object that is at the origin ($x = 0$ m) at $t = 0$ s.



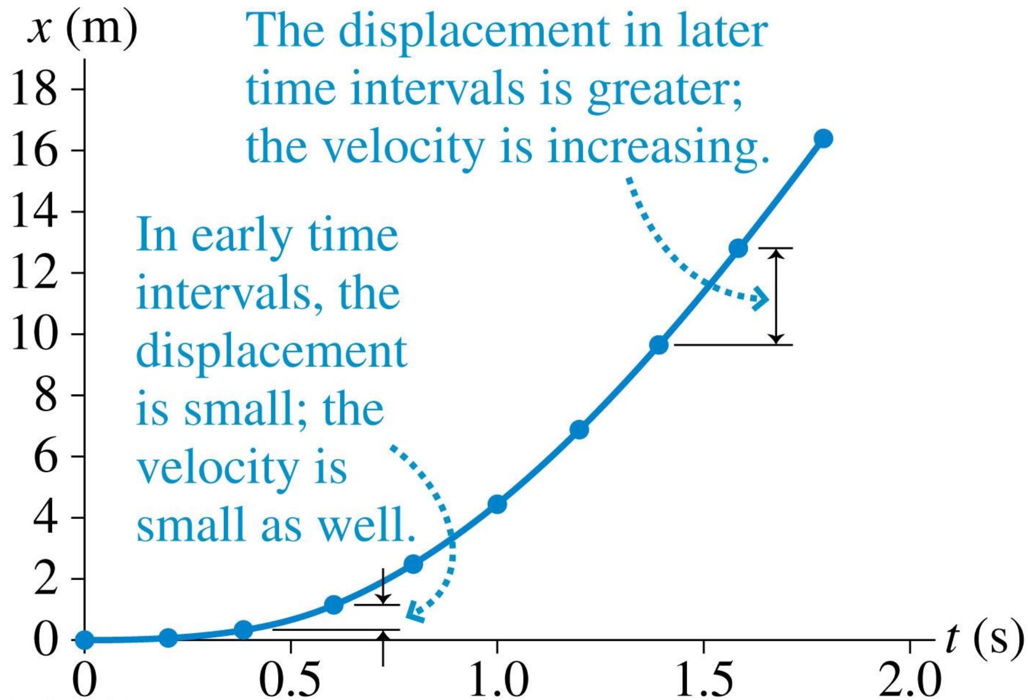

At $t = 4.0$ s, the object's position is

A. 20 m
B. 16 m
✔ C. 12 m Displacement = area under the curve
D. 8 m
E. 4 m

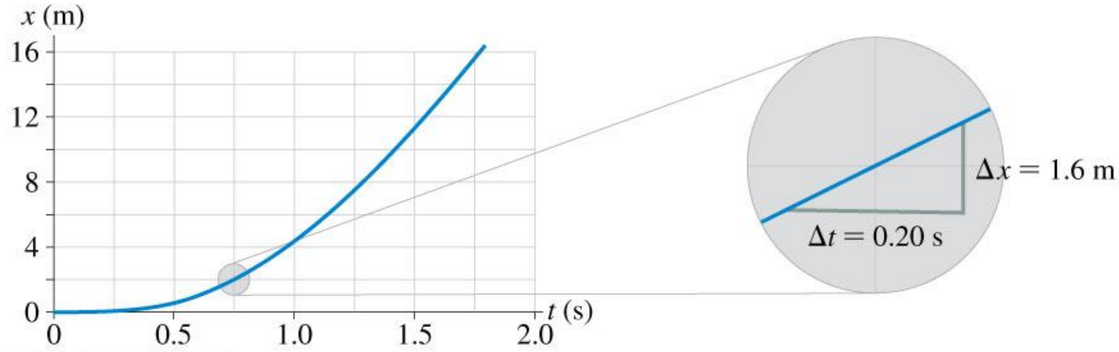
# Section 2.3 Instantaneous Velocity

# Instantaneous Velocity

- For one-dimensional motion, an object changing its velocity is either speeding up or slowing down.
- An object's velocity—a speed *and* a direction—at a specific *instant* of time *t* is called the object's **instantaneous velocity.**
- **From now on, the word "velocity" will always mean instantaneous velocity.**

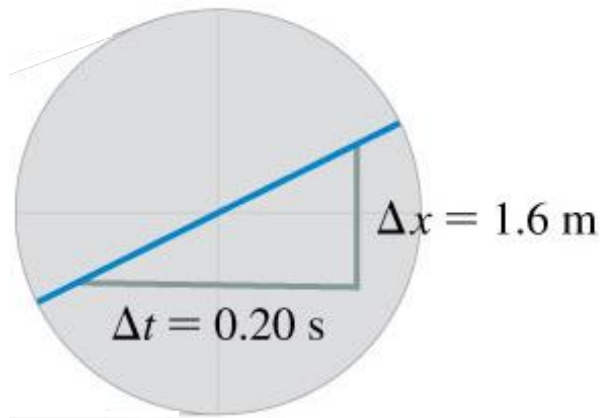


# Finding the Instantaneous Velocity




- If the velocity changes, the position graph is a curved line. But we can compute a slope at a point by considering a small segment of the graph. Let's look at the motion in a very small time interval right around $t$ = 0.75 s. This is highlighted with a circle, and we show a closeup in the next graph.

# Finding the Instantaneous Velocity



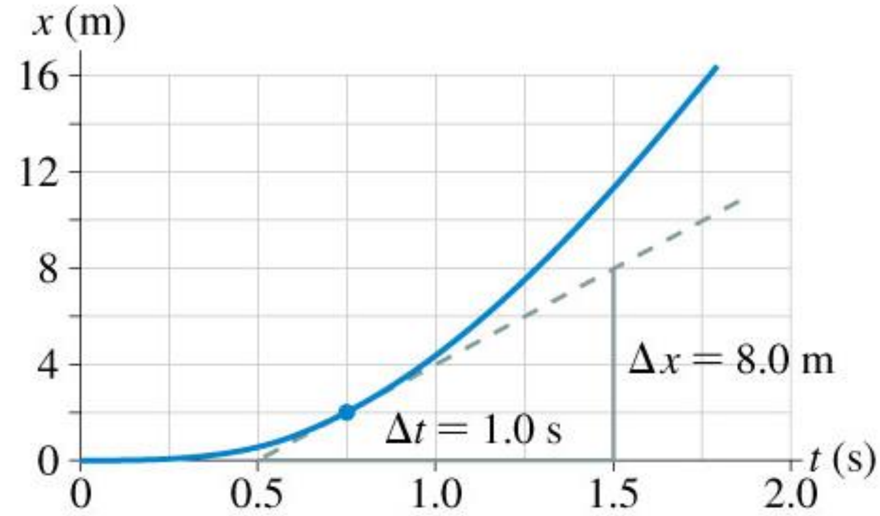

- In this magnified segment of the position graph, the curve isn't apparent. It appears to be a line segment. We can find the slope by calculating the rise over the run, just as before:

$$v_x = (1.6\text{ m})/(0.20\text{ s}) = 8.0\text{ m/s}$$

- This is the slope at $t = 0.75$ s and thus the velocity at this instant of time.

# Finding the Instantaneous Velocity

- Graphically, the slope of the curve at a point is the same as the slope of a straight line drawn *tangent* to the curve at that point. Calculating rise over run for the tangent line, we get

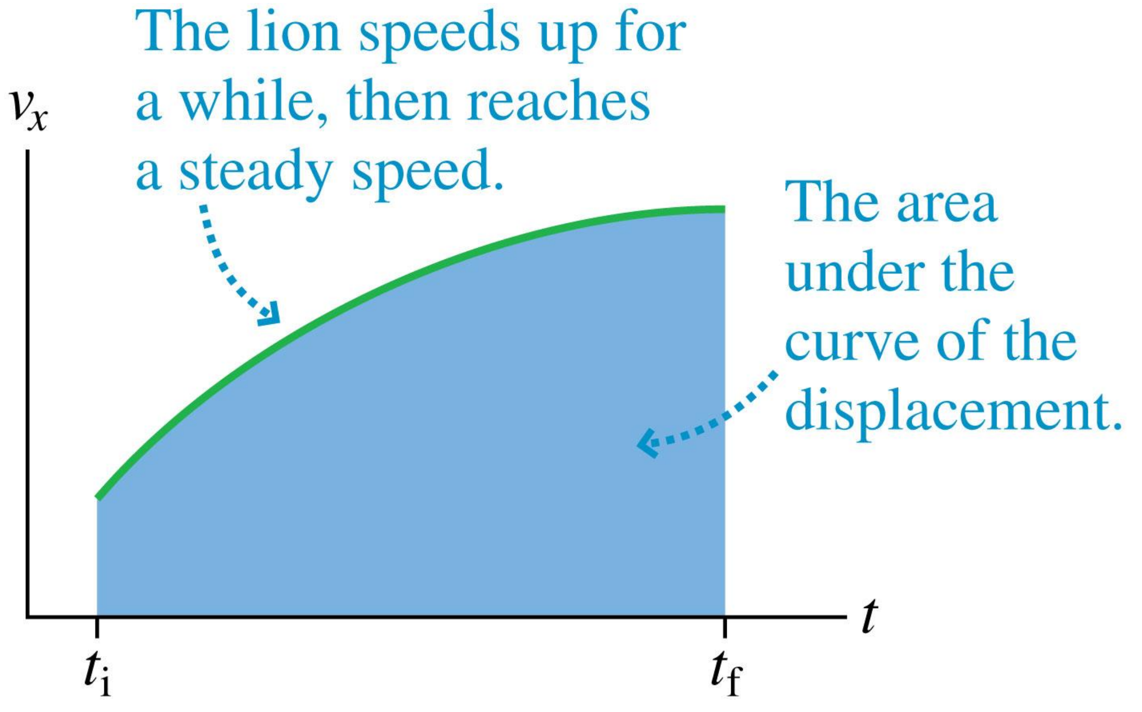



$$v_x = (8.0 \text{ m})/(1.0 \text{ s}) = 8.0 \text{ m/s}$$

- This is the same value we obtained from the closeup view. The slope of the tangent line is the instantaneous velocity at that instant of time.

# Instantaneous Velocity

- Even when the speed varies we can still use the velocity-versus-time graph to determine displacement.
- The area under the curve in a velocity-versus-time graph equals the displacement even for non-uniform motion.



# QuickCheck 2.5

The slope at a point on a position-versus-time graph of an object is

A. The object's speed at that point.
B. The object's velocity at that point.
C. The object's acceleration at that point.
D. The distance traveled by the object to that point.
E. I am not sure.

# QuickCheck 2.5

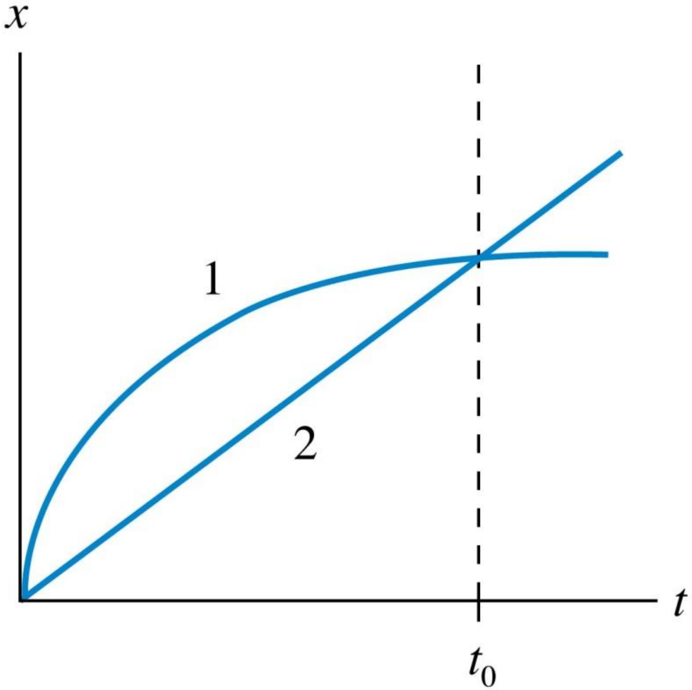
The slope at a point on a position-versus-time graph of an object is

A. The object's speed at that point.
✔ B. The object's velocity at that point.
C. The object's acceleration at that point.
D. The distance traveled by the object to that point.
E. I am not sure.

# QuickCheck 2.9

When do objects 1 and 2 have the same velocity?

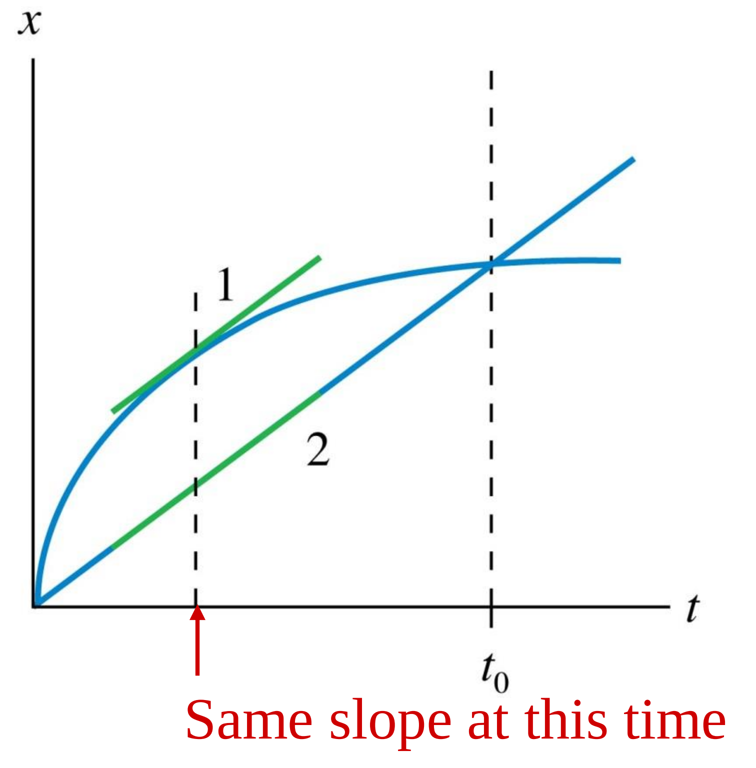
A. At some instant before time $t_0$
B. At time $t_0$
C. At some instant after time $t_0$
D. Both A and B
E. Never



# QuickCheck 2.9

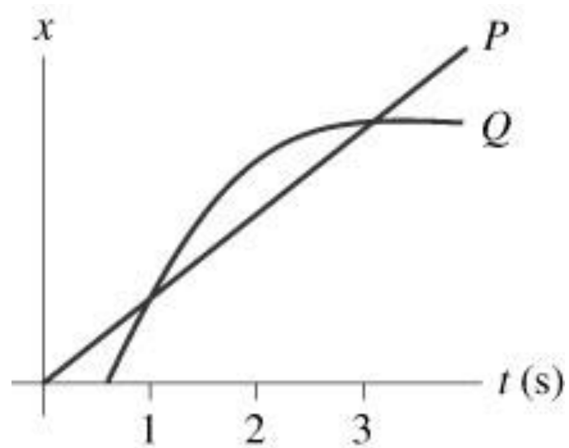
When do objects 1 and 2 have the same velocity?

- ✔ A. At some instant before time $t_0$
- B. At time $t_0$
- C. At some instant after time $t_0$
- D. Both A and B
- E. Never



# QuickCheck 2.10

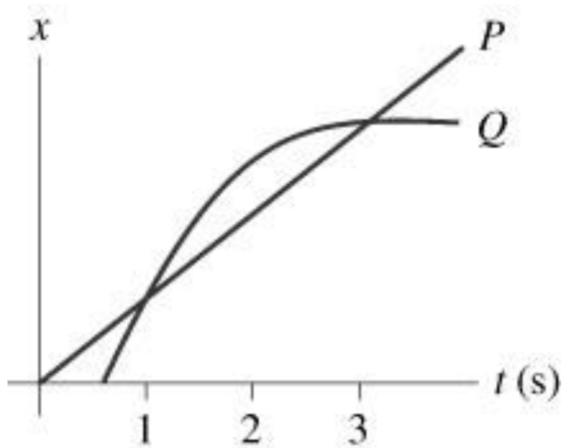
Masses *P* and *Q* move with the position graphs shown. Do *P* and *Q* ever have the same velocity? If so, at what time or times?




A. *P* and *Q* have the same velocity at 2 s.
B. *P* and *Q* have the same velocity at 1 s and 3 s.
C. *P* and *Q* have the same velocity at 1 s, 2 s, and 3 s.
D. *P* and *Q* never have the same velocity.

# QuickCheck 2.10

Masses *P* and *Q* move with the position graphs shown. Do *P* and *Q* ever have the same velocity? If so, at what time or times?




✔ A. *P* and *Q* have the same velocity at 2 s.

B. *P* and *Q* have the same velocity at 1 s and 3 s.

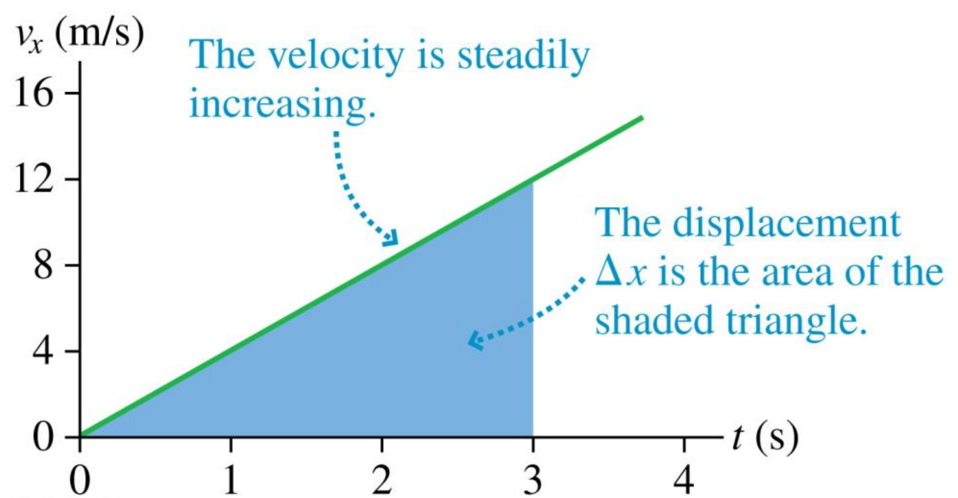
C. *P* and *Q* have the same velocity at 1 s, 2 s, and 3 s.

D. *P* and *Q* never have the same velocity.

# Example 2.5 The displacement during a rapid start

FIGURE 2.21 shows the velocity-versus-time graph of a car pulling away from a stop. How far does the car move during the first 3.0 s?

**PREPARE** Figure 2.21 is a graphical representation of the motion. The question How far? indicates that we need to find a displacement $\Delta x$ rather than a position $x$. According to Equation 2.7, the car's displacement $\Delta x = x_f - x_i$ between $t = 0$ s and $t = 3$ s is the area under the curve from $t = 0$ s to $t = 3$ s.



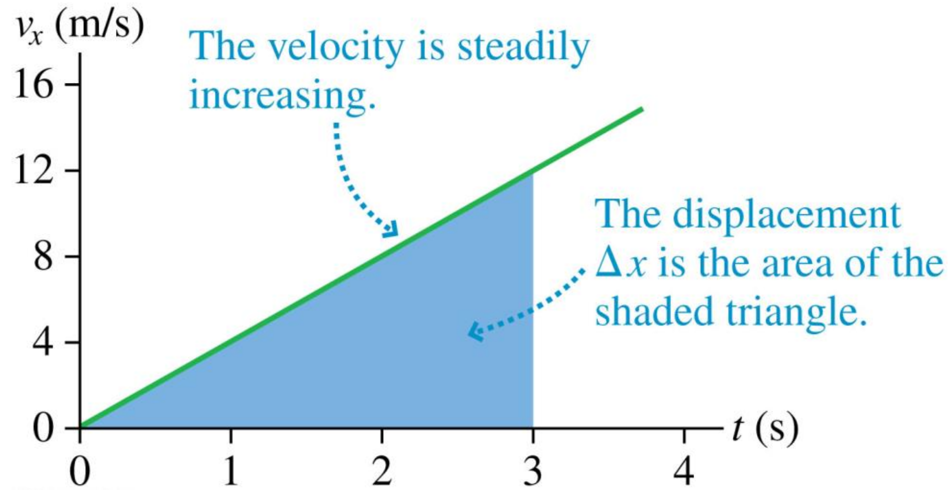
# Example 2.5 The displacement during a rapid start (cont.)

**SOLVE** The curve in this case is an angled line, so the area is that of a triangle:

$$\Delta x = \text{area of triangle between } t = 0 \text{ s and } t = 3 \text{ s}$$

$$= \tfrac{1}{2} \times \text{base} \times \text{height} = \tfrac{1}{2} \times 3 \text{ s} \times 12 \text{ m/s} = 18 \text{ m}$$

The car moves 18 m during the first 3 seconds as its velocity changes from 0 to 12 m/s.

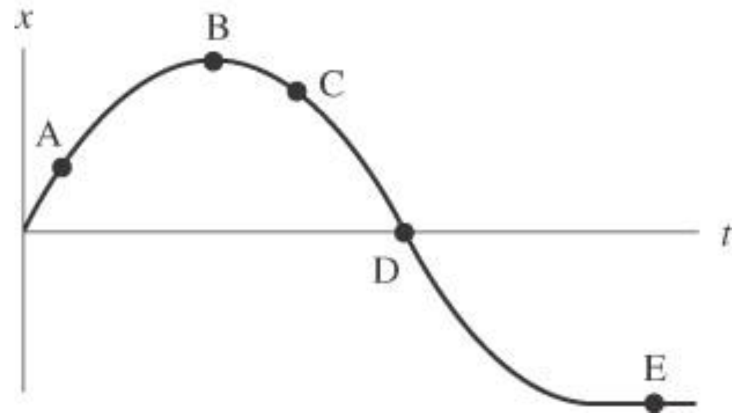


# Example 2.5 The displacement during a rapid start (cont.)

**ASSESS** The physically meaningful area is a product of s and m/s, so $\Delta x$ has the proper units of m. Let's check the numbers to see if they make physical sense. The final velocity, 12 m/s, is about 25 mph. Pulling away from a stop, you might expect to reach this speed in about 3 s—at least if you have a reasonably sporty vehicle! If the car had moved at a constant 12 m/s (the final velocity) during these 3 s, the distance would be 36 m. The actual distance traveled during the 3 s is 18 m—half of 36 m. This makes sense, as the velocity was 0 m/s at the start of the problem and increased steadily to 12 m/s.

# QuickCheck 2.13

A car moves along a straight stretch of road. The following graph shows the car's position as a function of time:

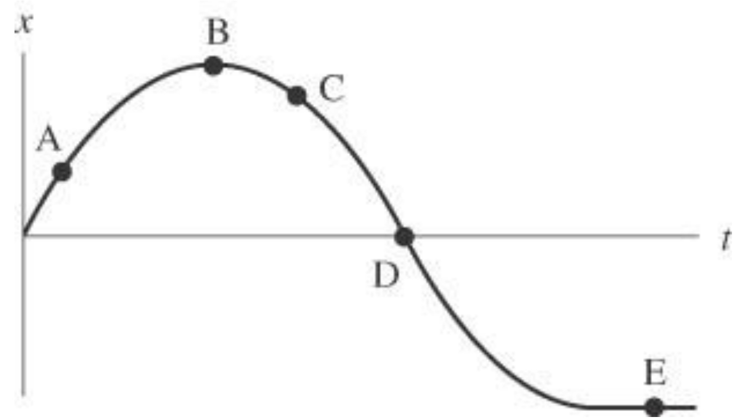



At what point (or points) do the following conditions apply?

- The displacement is zero.
- The speed is zero.
- The speed is increasing.
- The speed is decreasing.

# QuickCheck 2.13

A car moves along a straight stretch of road. The following graph shows the car's position as a function of time:




At what point (or points) do the following conditions apply?

- The displacement is zero. D
- The speed is zero. B, E
- The speed is increasing. C
- The speed is decreasing. A

# Section 2.4 Acceleration

# Acceleration

- We define a new motion concept to describe an object whose velocity is changing.
  - The ratio of $\Delta v_x/\Delta t$ is the *rate of change of velocity.*
  - The ratio of $\Delta v_x/\Delta t$ is the *slope of a velocity-versus-time graph.*

$$a_x = \frac{\Delta v_x}{\Delta t}$$

Definition of acceleration as the rate of change of velocity

# Units of Acceleration

- In our SI unit of velocity, 60 mph = 27 m/s.
- The Corvette speeds up to 27 m/s in $\Delta t$ = 3.6 s.

$$a_{\text{Corvette}\,x} = \frac{\Delta v_x}{\Delta t} = \frac{27\ \text{m/s}}{3.6\ \text{s}} = 7.5\ \frac{\text{m/s}}{\text{s}}$$

- Every second, the Corvette's velocity changes by 7.5 m/s.
- It is customary to abbreviate the acceleration units (m/s)/s as m/s$^2$, which we say as "meters per second squared."

**TABLE 2.2** Performance data for vehicles

| Vehicle | Time to go from 0 to 60 mph |
|---|---|
| 2011 Chevy Corvette | 3.6 s |
| 2012 Chevy Sonic | 9.0 s |

# Example 2.6 Animal acceleration

Lions, like most predators, are capable of very rapid starts. From rest, a lion can sustain an acceleration of 9.5 m/s$^2$ for up to one second. How much time does it take a lion to go from rest to a typical recreational runner's top speed of 10 mph?

**PREPARE** We can start by converting to SI units. The speed the lion must reach is

$$v_{\mathrm{f}} = 10 \text{ mph} \times \frac{0.45 \text{ m/s}}{1.0 \text{ mph}} = 4.5 \text{ m/s}$$

The lion can accelerate at 9.5 m/s$^2$, changing its speed by 9.5 m/s per second, for only 1.0 s—long enough to reach 9.5 m/s. It will take the lion less than 1.0 s to reach 4.5 m/s, so we can use $a_x$ = 9.5 m/s$^2$ in our solution.

# Example 2.6 Animal acceleration (cont.)

**SOLVE** We know the acceleration and the desired change in velocity, so we can rearrange Equation 2.8 to find the time:

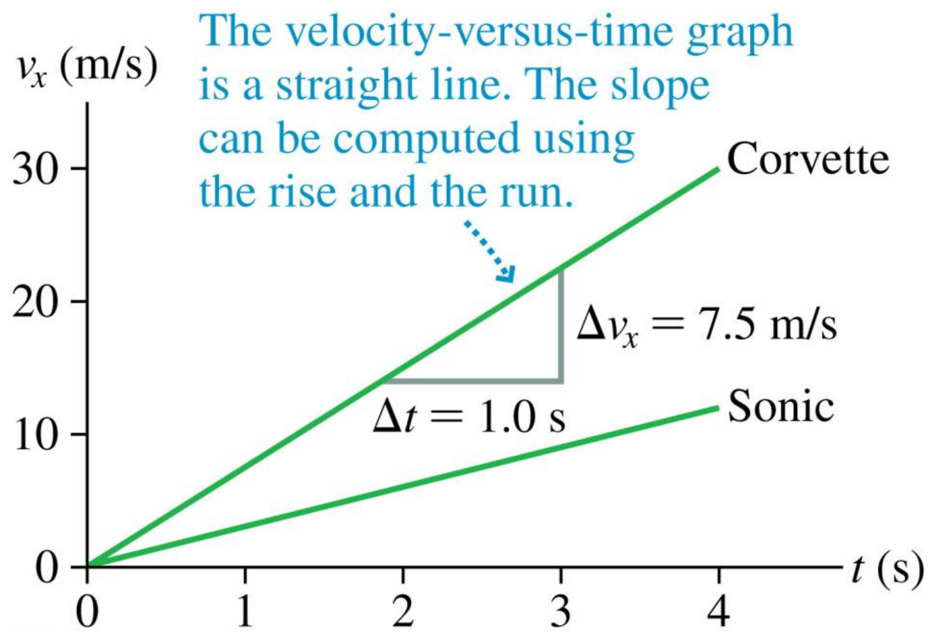
$$\Delta t = \frac{\Delta v_x}{a_x} = \frac{4.5\ \text{m/s}}{9.5\ \text{m/s}^2} = 0.47\ \text{s}$$

**ASSESS** The lion changes its speed by 9.5 meters per second in one second. So it's reasonable (if a bit intimidating) that it will reach 4.5 m/s in just under half a second.

# Representing Acceleration

**TABLE 2.3** Velocity data for the Sonic and the Corvette

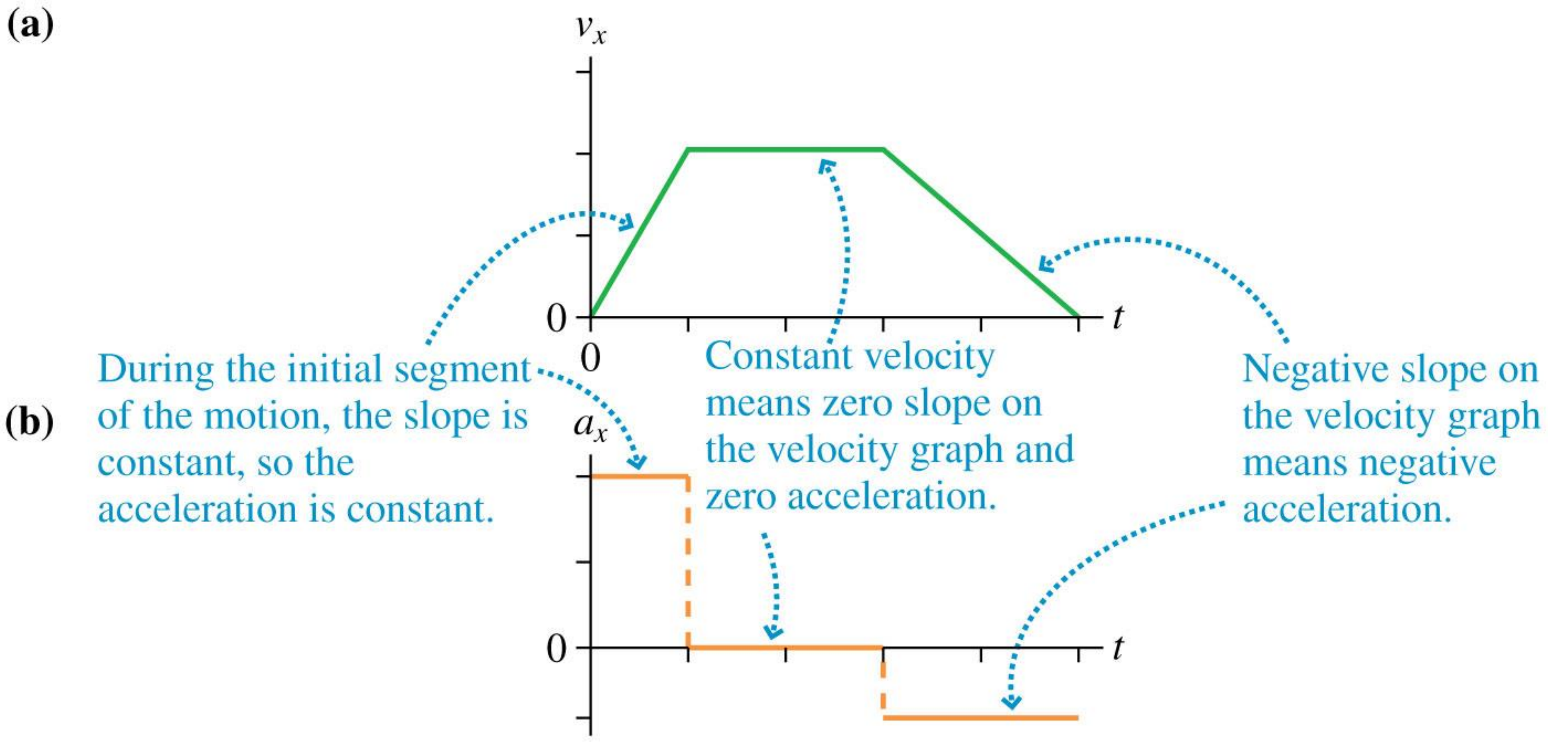
| Time (s) | Velocity of Sonic (m/s) | Velocity of Corvette (m/s) |
|---|---|---|
| 0 | 0 | 0 |
| 1 | 3.0 | 7.5 |
| 2 | 6.0 | 15.0 |
| 3 | 9.0 | 22.5 |
| 4 | 12.0 | 30.0 |




- **An object's acceleration is the slope of its velocity-versus-time graph.**

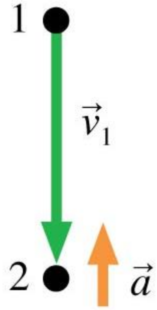
# Representing Acceleration

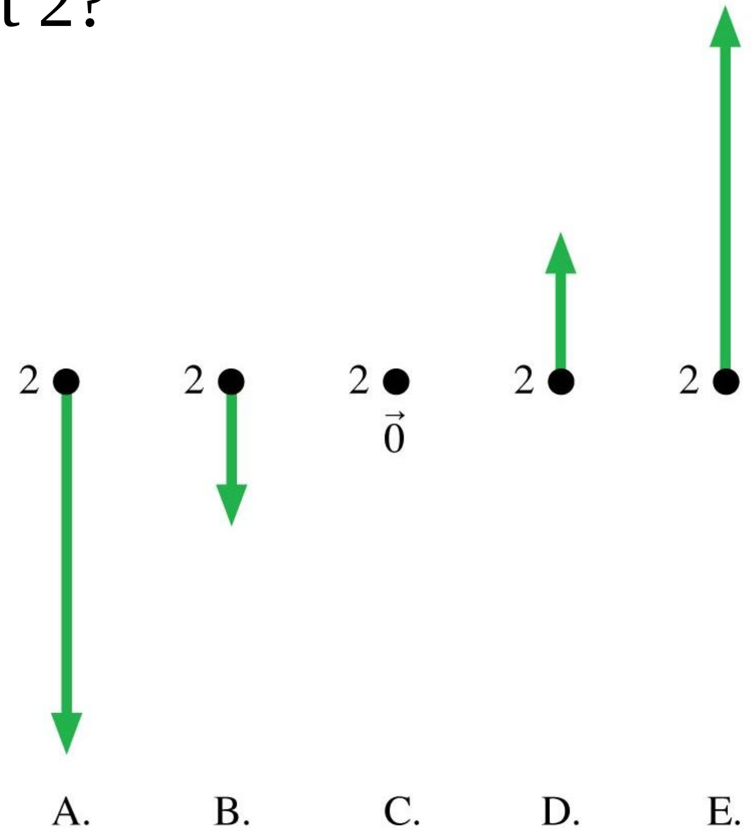
- We can find an acceleration graph from a velocity graph.



# QuickCheck 2.12

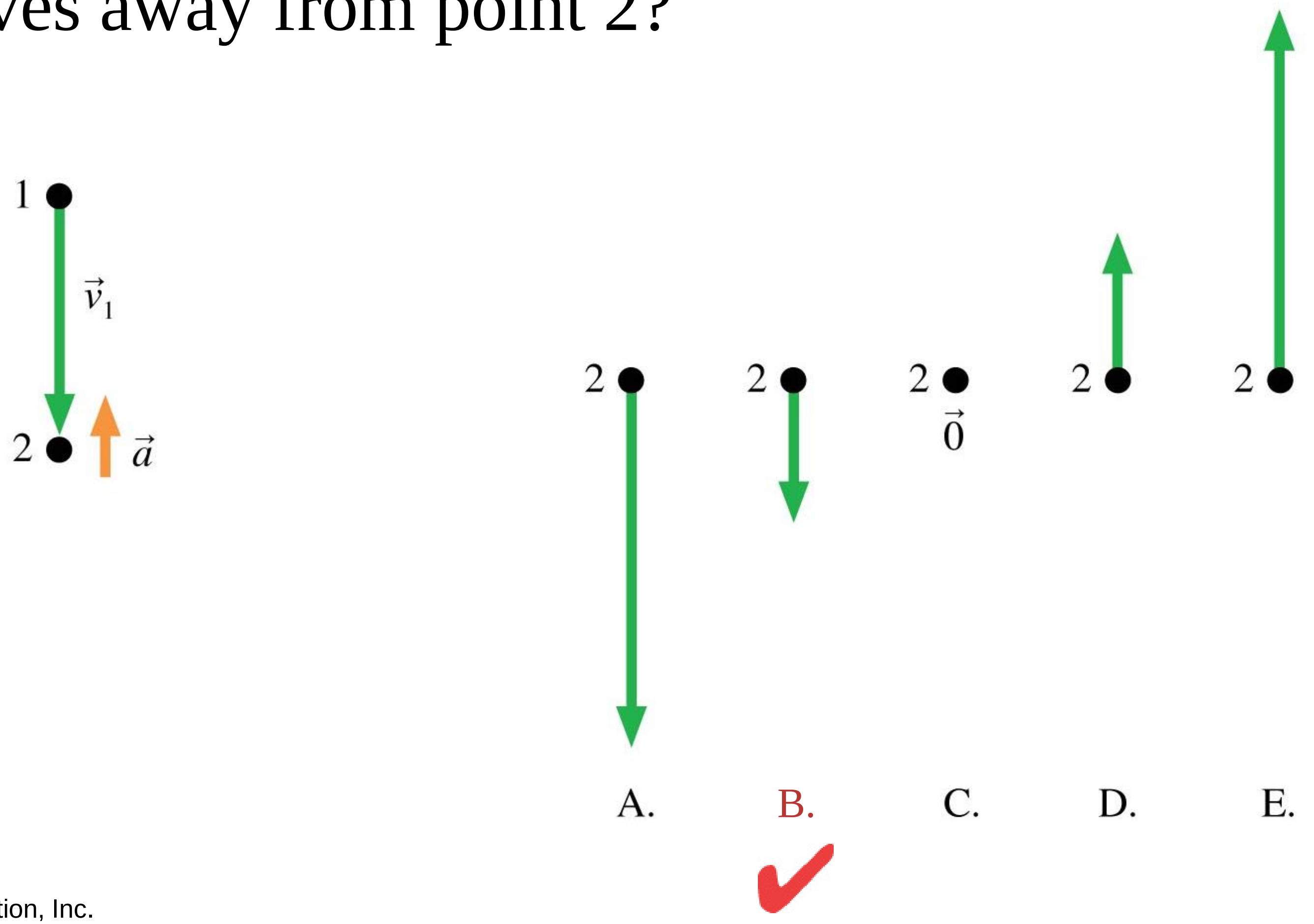
A particle has velocity $\vec{v}_1$ as it moves from point 1 to point 2. The acceleration is shown. What is its velocity vector $\vec{v}_2$ as it moves away from point 2?



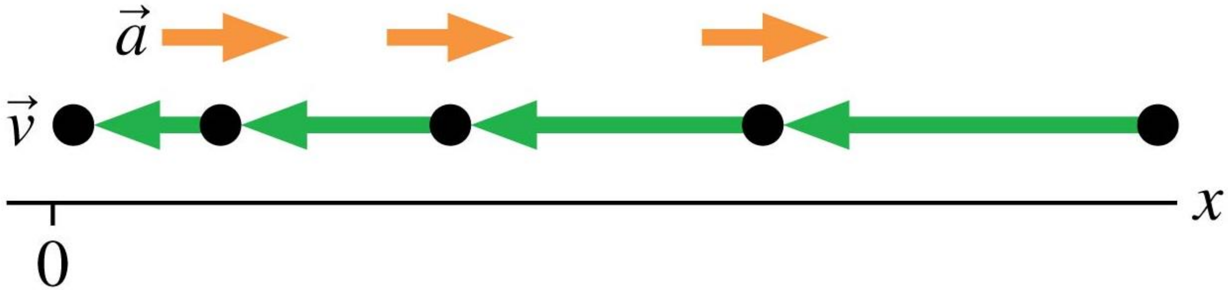



# QuickCheck 2.12

A particle has velocity $\vec{v}_1$ as it moves from point 1 to point 2. The acceleration is shown. What is its velocity vector $\vec{v}_2$ as it moves away from point 2?



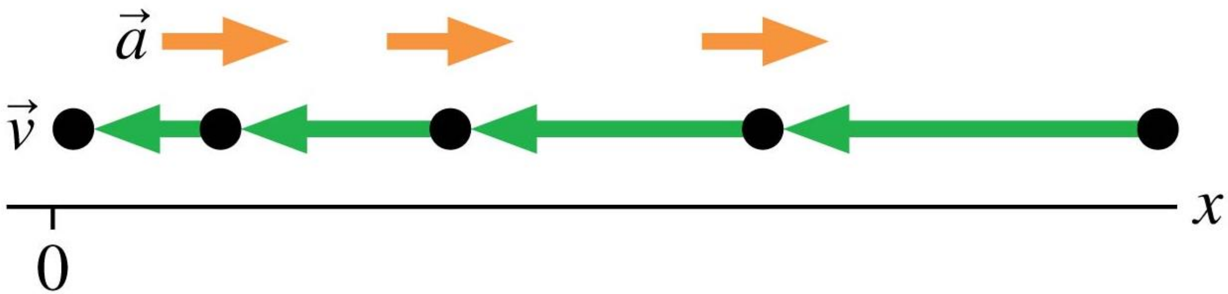
# QuickCheck 2.14




The motion diagram shows a particle that is slowing down. The sign of the position $x$ and the sign of the velocity $v_x$ are:

A. Position is positive, velocity is positive.
B. Position is positive, velocity is negative.
C. Position is negative, velocity is positive.
D. Position is negative, velocity is negative.

# QuickCheck 2.14




The motion diagram shows a particle that is slowing down. The sign of the position $x$ and the sign of the velocity $v_x$ are:

A. Position is positive, velocity is positive.

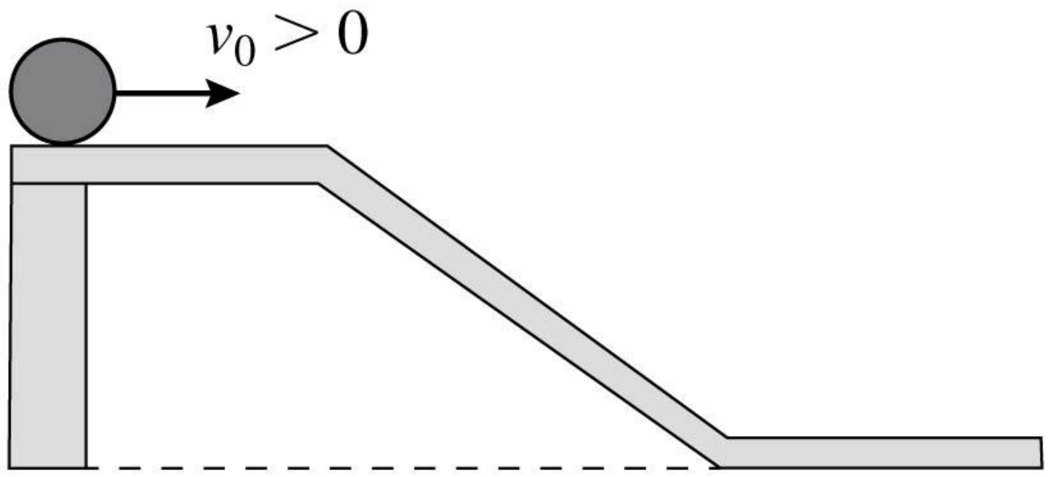
✔ B. Position is positive, velocity is negative.

C. Position is negative, velocity is positive.

D. Position is negative, velocity is negative.

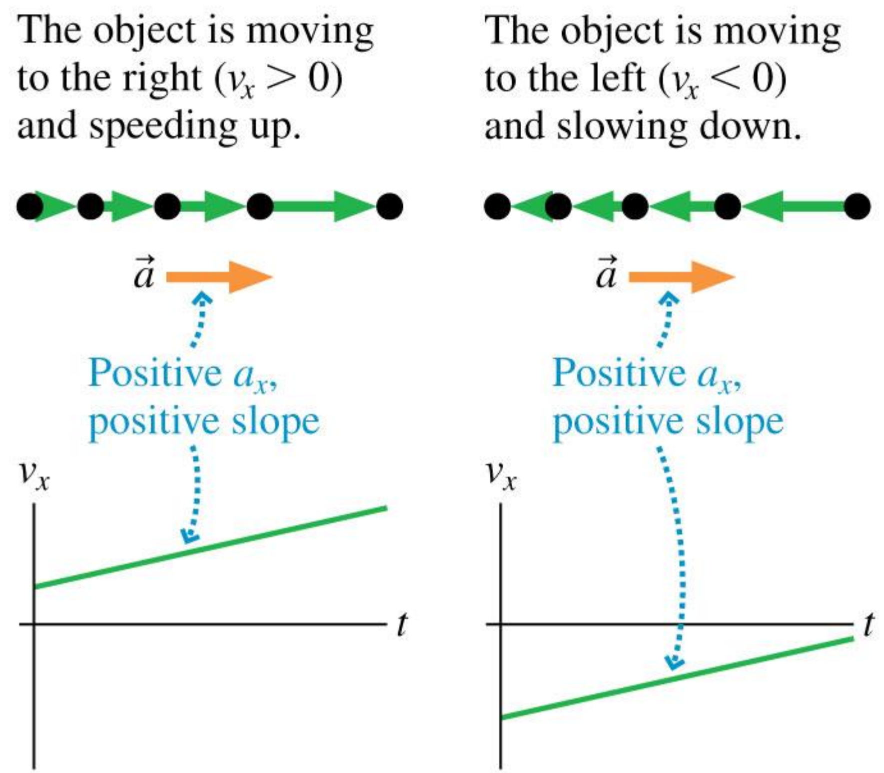
# Example Problem

A ball moving to the right traverses the ramp shown below. Sketch a graph of the velocity versus time, and, directly below it, using the same scale for the time axis, sketch a graph of the acceleration versus time.



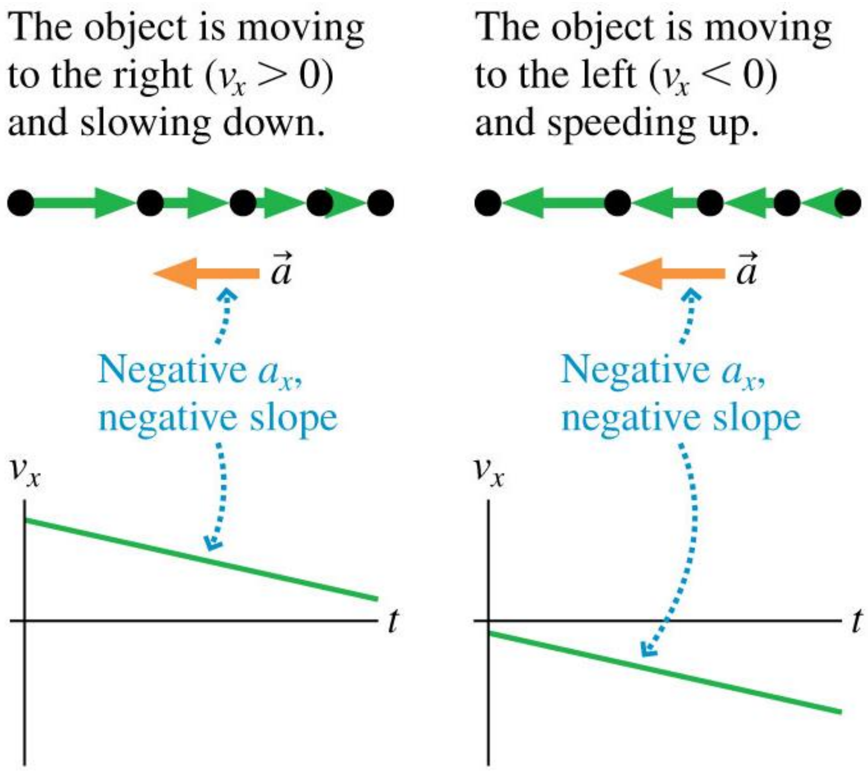
# The Sign of the Acceleration

An object can move right or left (or up or down) while either speeding up or slowing down. Whether or not an object that is slowing down has a negative acceleration depends on the direction of motion.

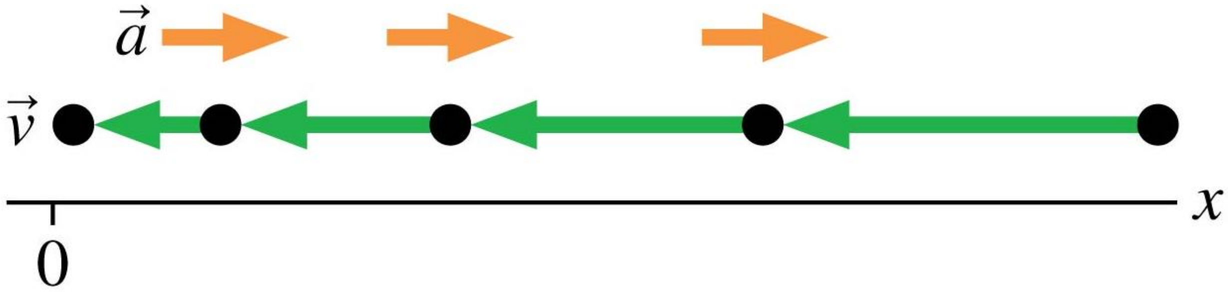


# The Sign of the Acceleration (cont.)

An object can move right or left (or up or down) while either speeding up or slowing down. Whether or not an object that is slowing down has a negative acceleration depends on the direction of motion.



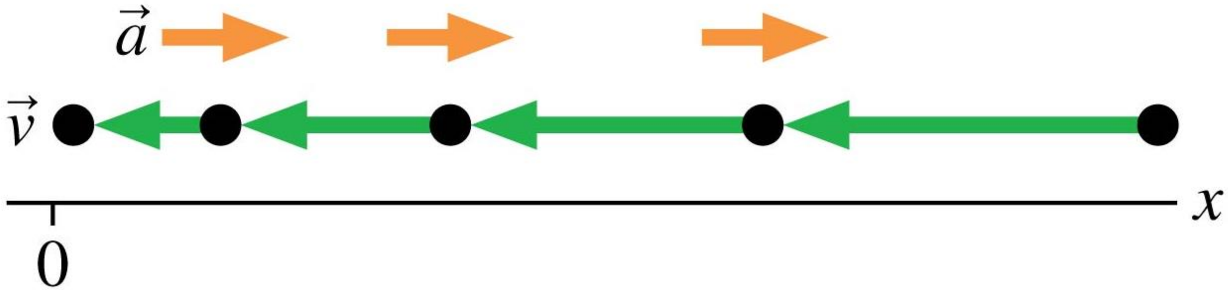
# QuickCheck 2.15




The motion diagram shows a particle that is slowing down. The sign of the acceleration $a_x$ is:

A. Acceleration is positive.
B. Acceleration is negative.

# QuickCheck 2.15



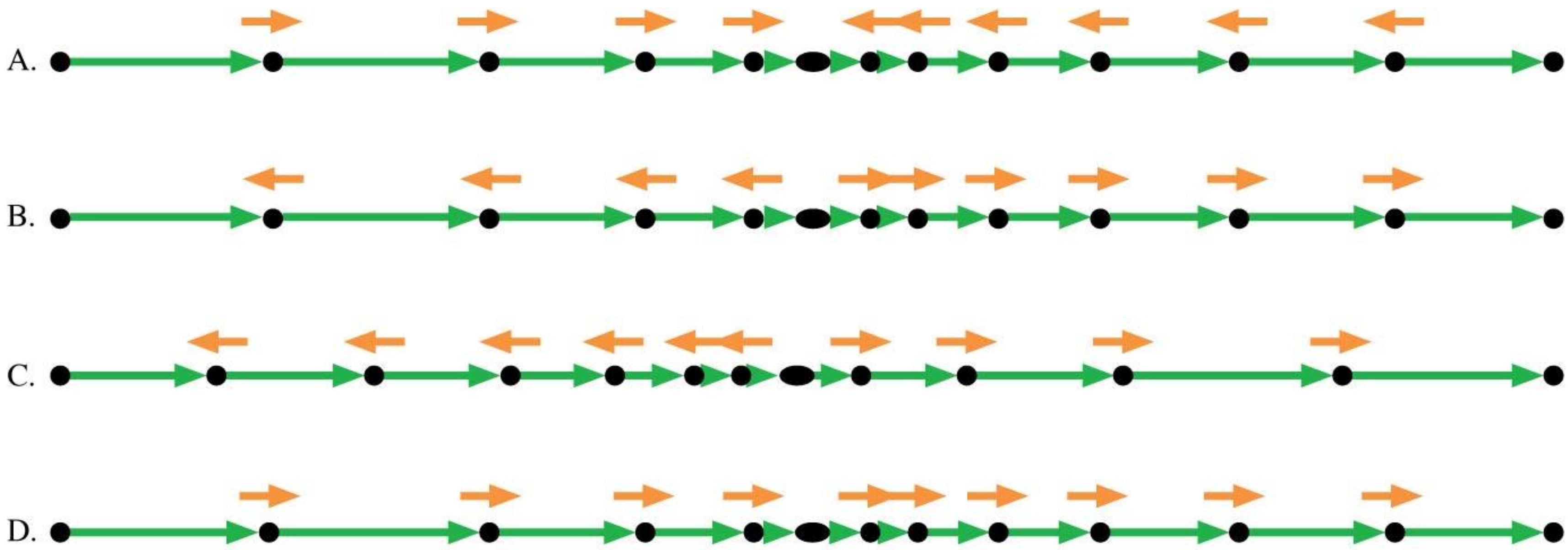

The motion diagram shows a particle that is slowing down. The sign of the acceleration $a_x$ is:

✔ A. Acceleration is positive.

B. Acceleration is negative.

# QuickCheck 2.16

A cyclist riding at 20 mph sees a stop sign and actually comes to a complete stop in 4 s. He then, in 6 s, returns to a speed of 15 mph. Which is his motion diagram?



# QuickCheck 2.16

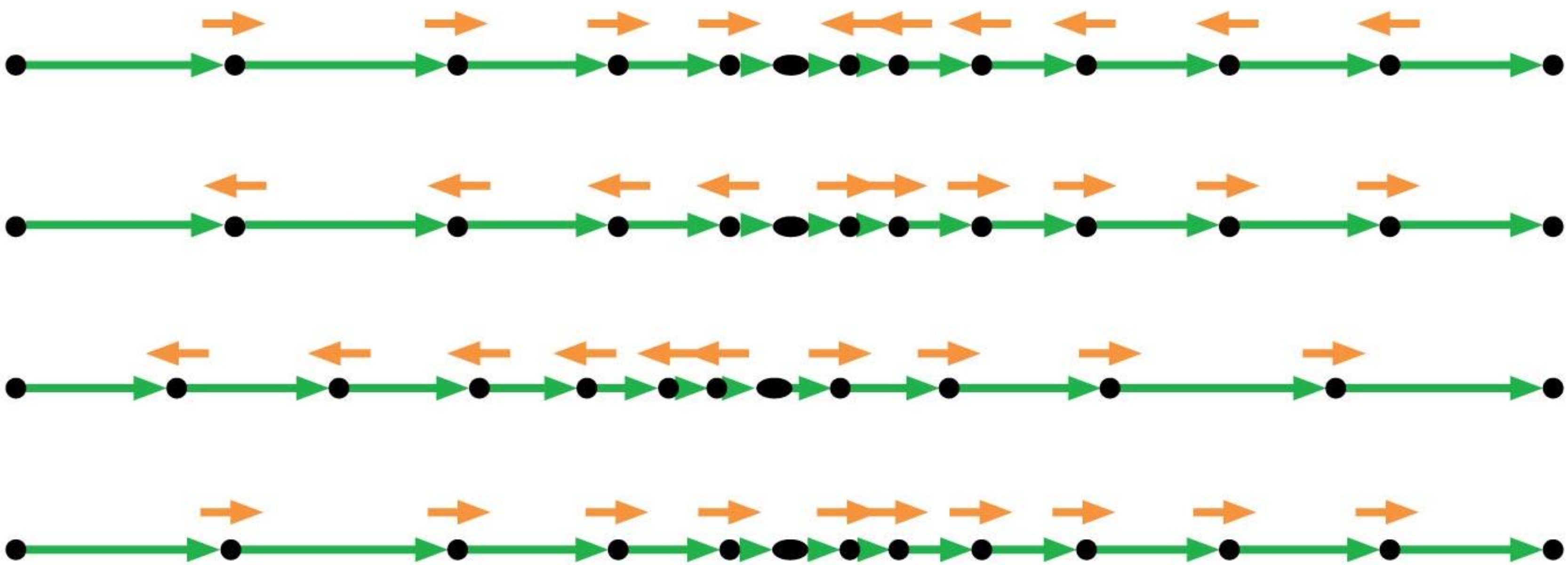
A cyclist riding at 20 mph sees a stop sign and actually comes to a complete stop in 4 s. He then, in 6 s, returns to a speed of 15 mph. Which is his motion diagram?

A.

B.

C.

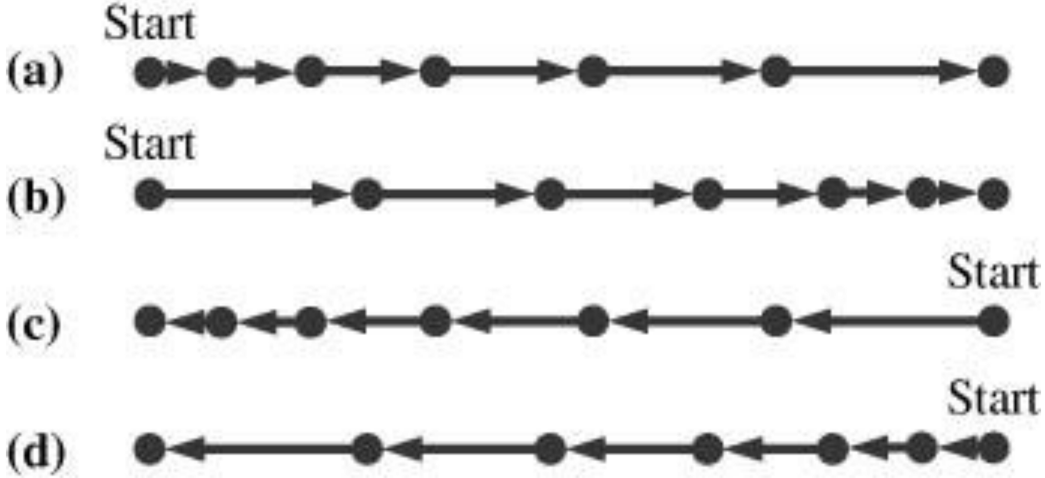
D.



# QuickCheck 2.17

These four motion diagrams show the motion of a particle along the *x*-axis.

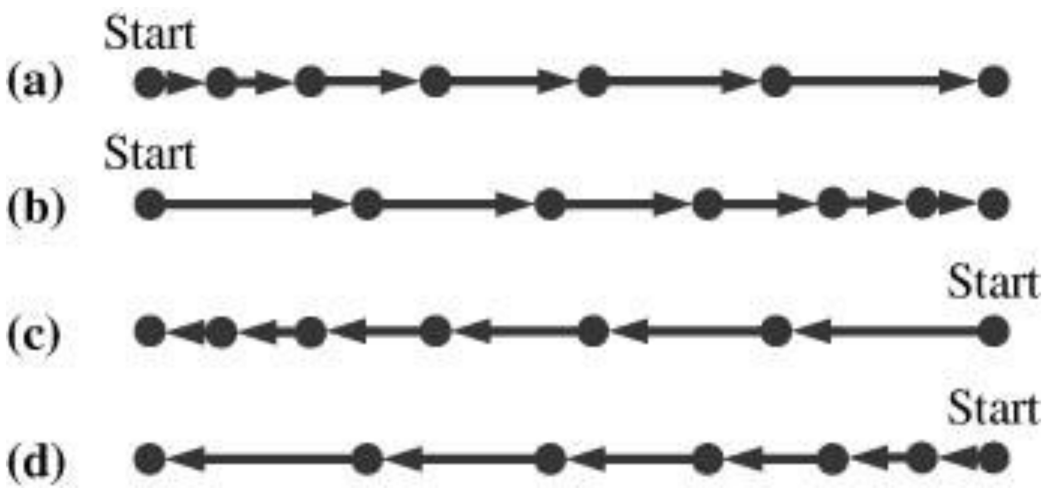
1. Which motion diagrams correspond to a positive acceleration?
2. Which motion diagrams correspond to a negative acceleration?



# QuickCheck 2.17

These four motion diagrams show the motion of a particle along the *x*-axis.

1. Which motion diagrams correspond to a positive acceleration?
2. Which motion diagrams correspond to a negative acceleration?




Positive

Negative

Positive

Negative

# QuickCheck 2.18

Mike jumps out of a tree and lands on a trampoline. The trampoline sags 2 feet before launching Mike back into the air.

At the very bottom, where the sag is the greatest, Mike's acceleration is

A. Upward.
B. Downward.
C. Zero.

# QuickCheck 2.18

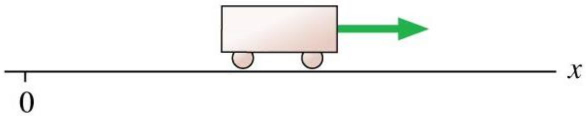
Mike jumps out of a tree and lands on a trampoline. The trampoline sags 2 feet before launching Mike back into the air.

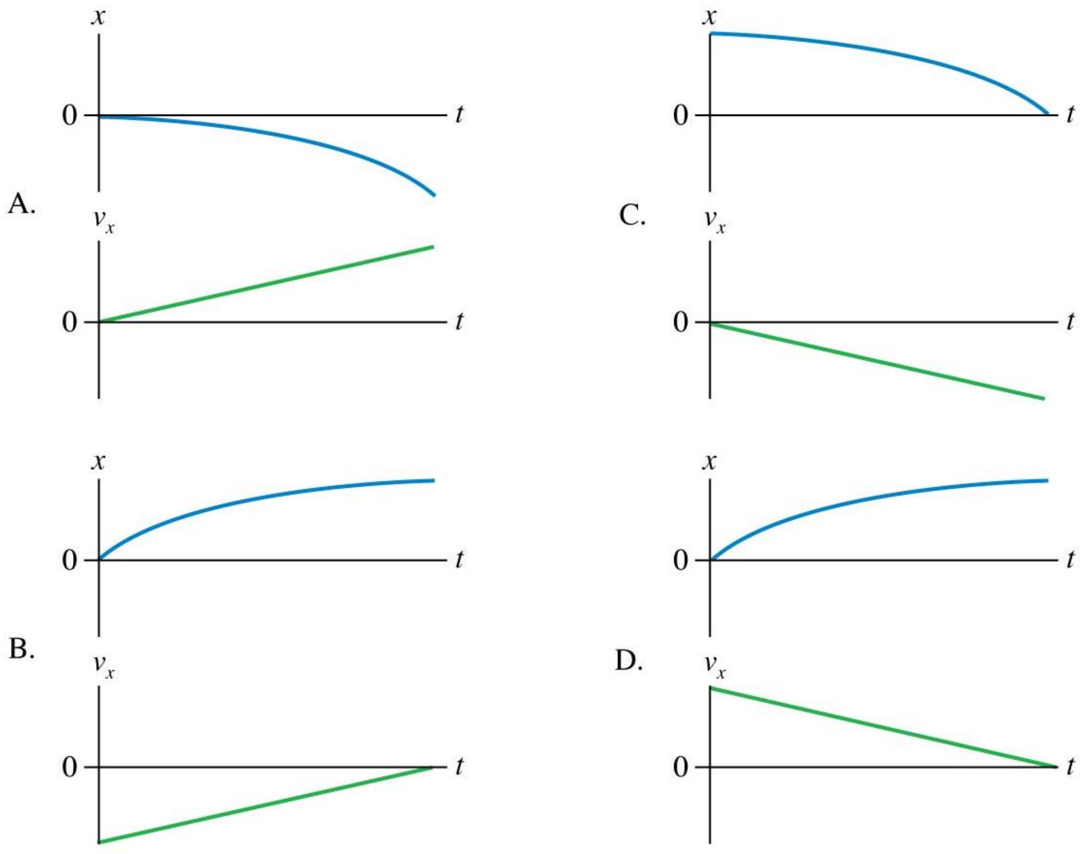
At the very bottom, where the sag is the greatest, Mike's acceleration is

✔ A. Upward.
B. Downward.
C. Zero.

# QuickCheck 2.19

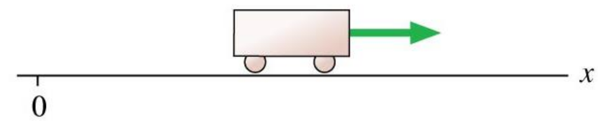
A cart slows down while moving away from the origin. What do the position and velocity graphs look like?

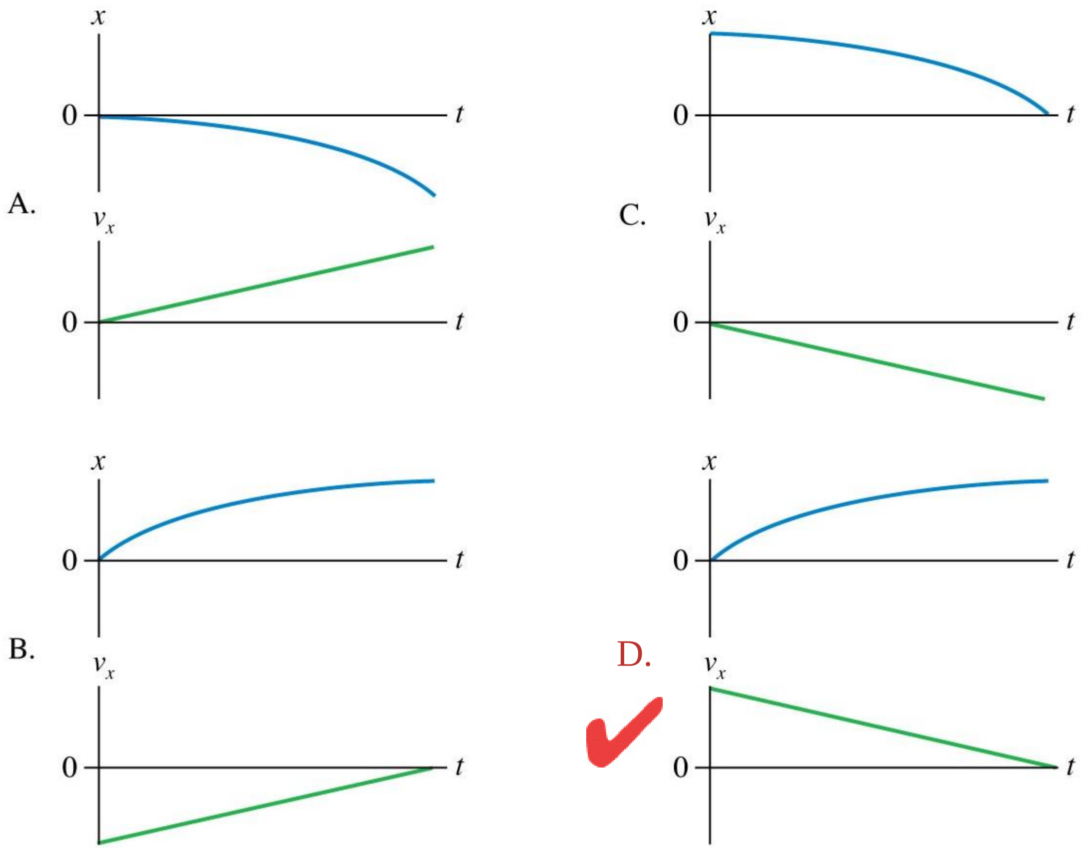


# QuickCheck 2.19

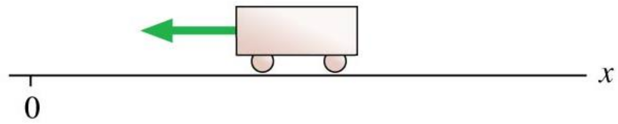
A cart slows down while moving away from the origin. What do the position and velocity graphs look like?

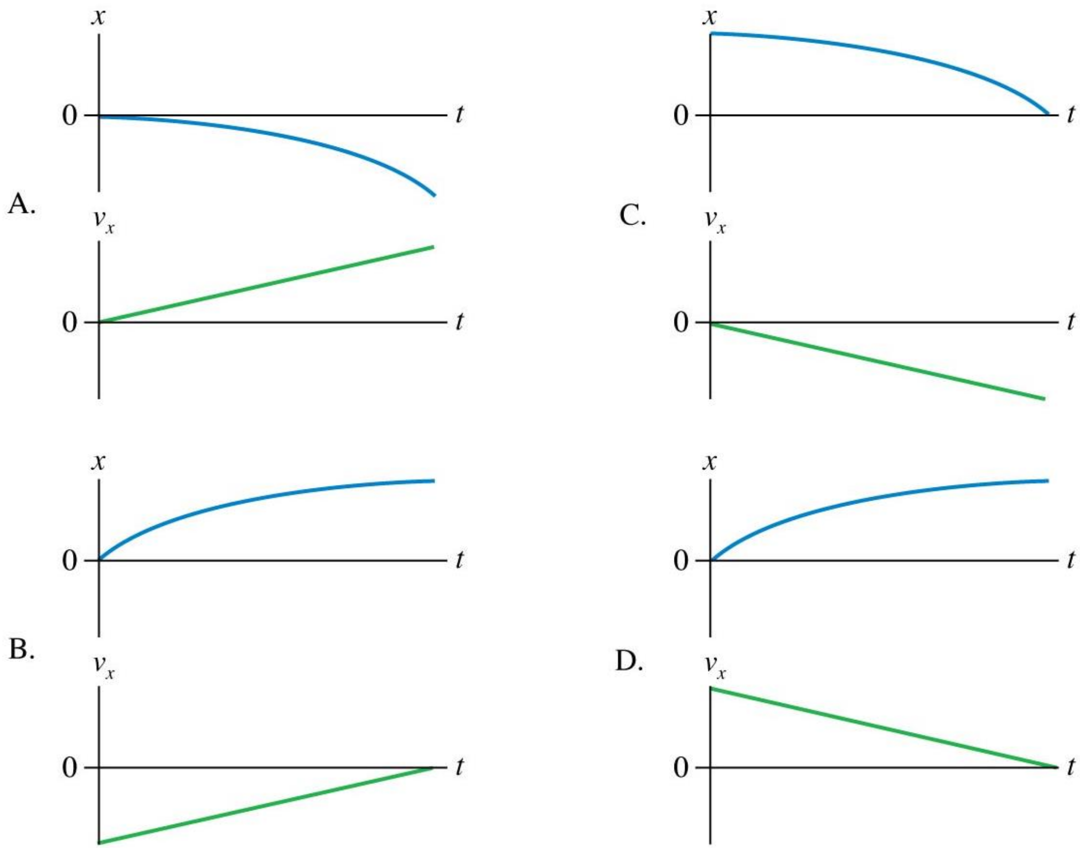


# QuickCheck 2.20

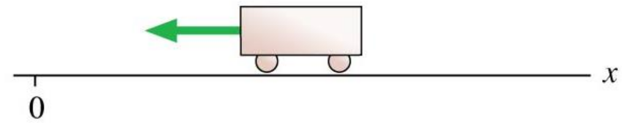
A cart speeds up toward the origin. What do the position and velocity graphs look like?

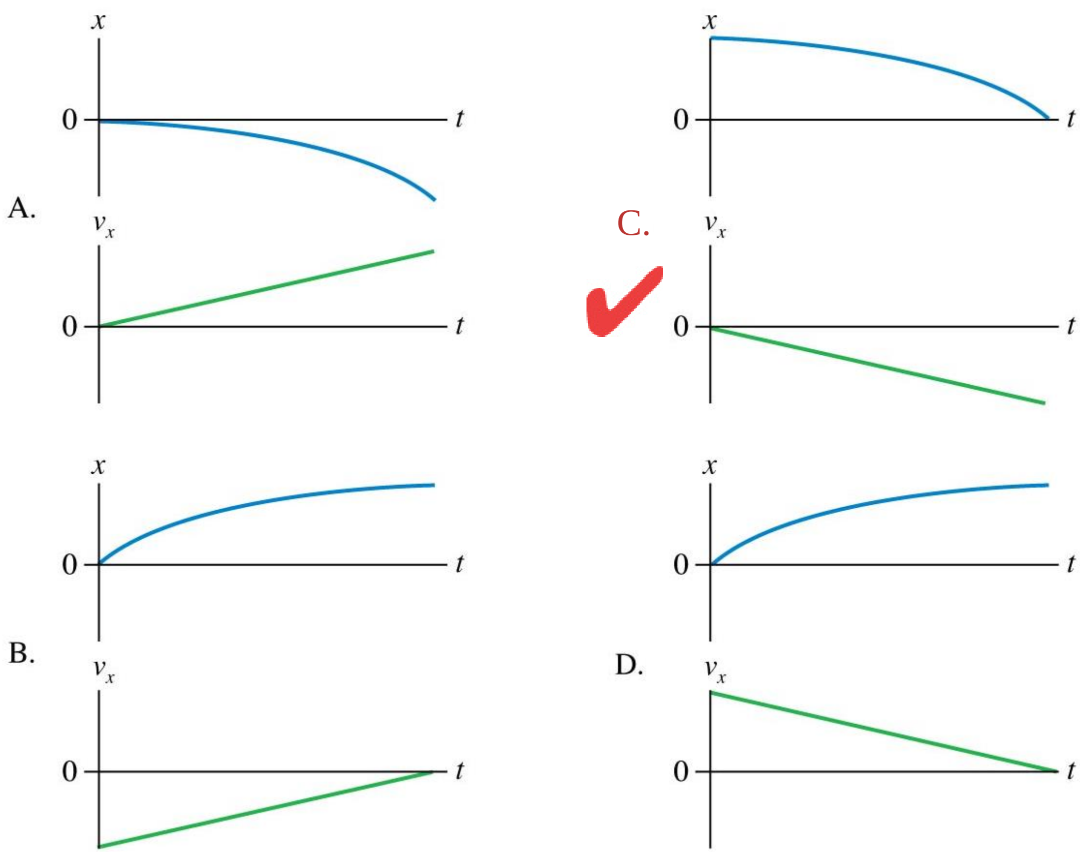


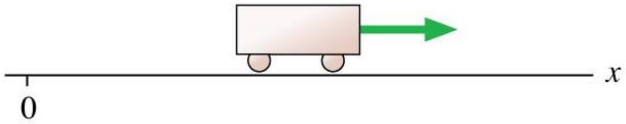
# QuickCheck 2.20

A cart speeds up toward the origin. What do the position and velocity graphs look like?

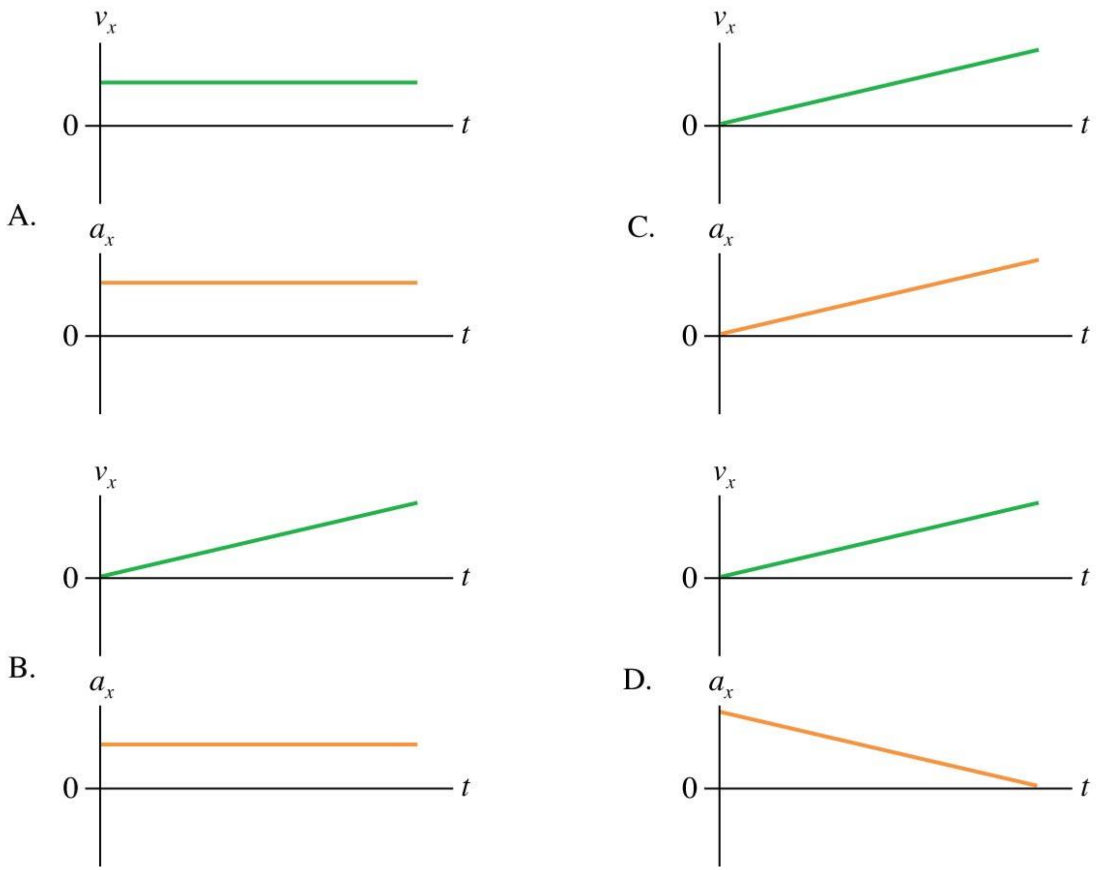


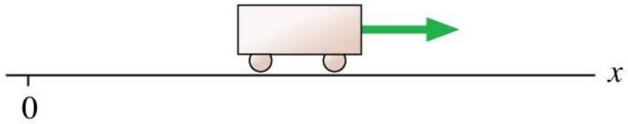
# QuickCheck 2.21



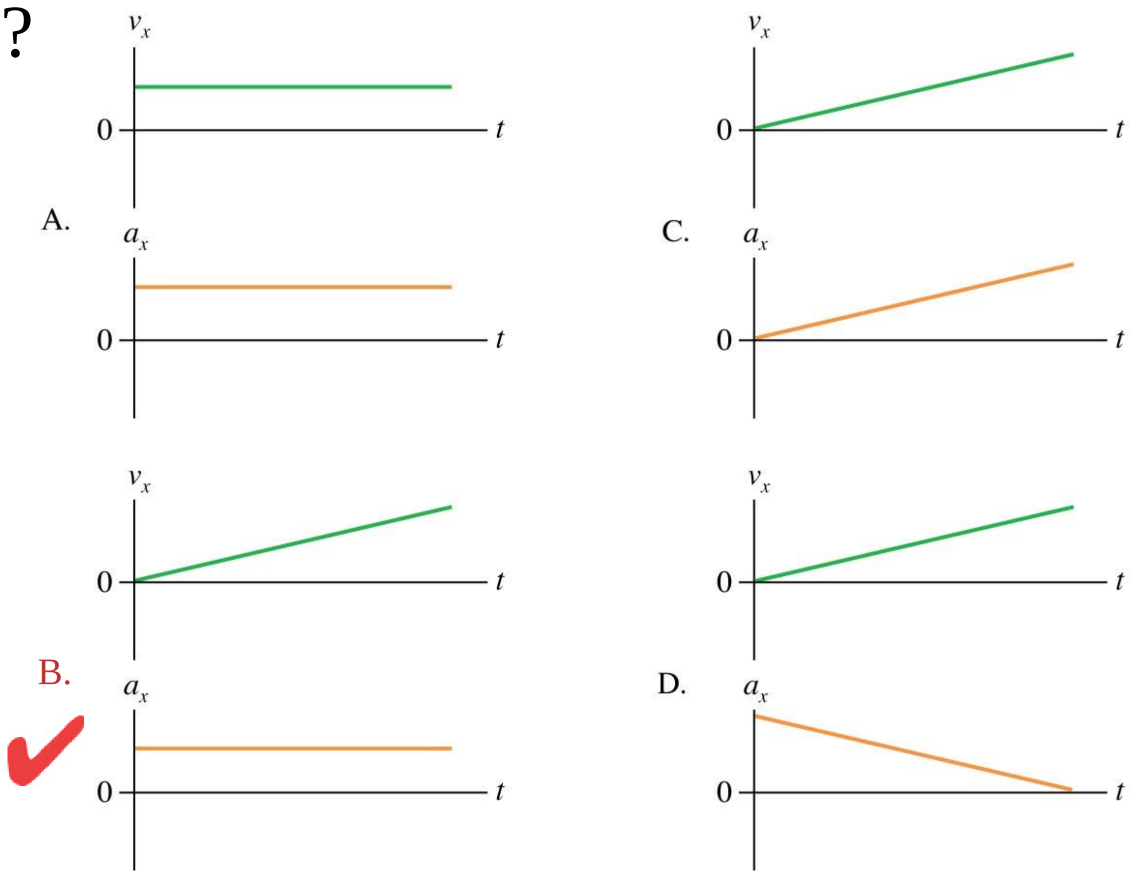

A cart *speeds up* while moving away from the origin. What do the velocity and acceleration graphs look like?



# QuickCheck 2.21

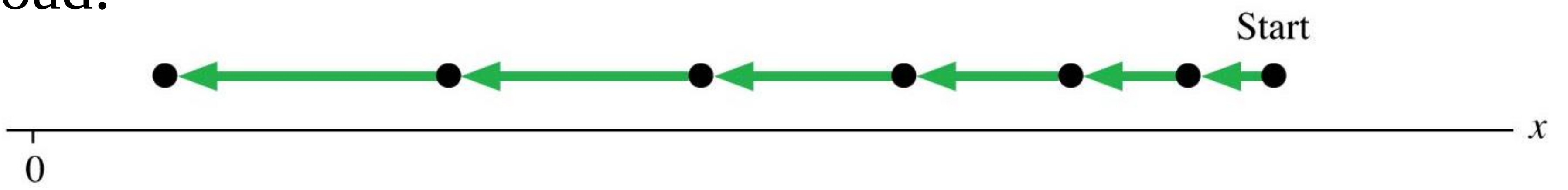



A cart *speeds up* while moving away from the origin. What do the velocity and acceleration graphs look like?



# QuickCheck 2.22

Here is a motion diagram of a car speeding up on a straight road:

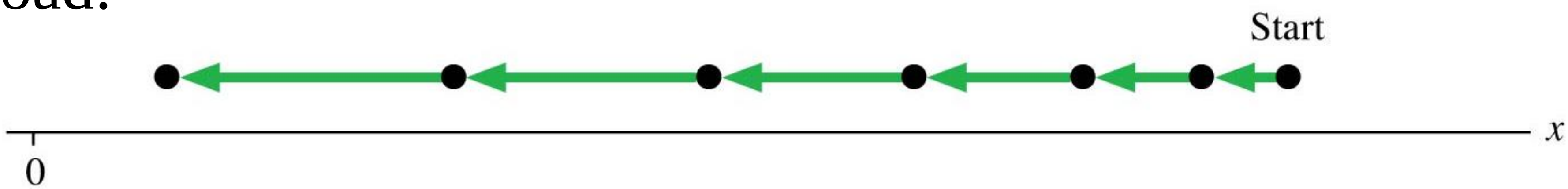



The sign of the acceleration $a_x$ is

A. Positive.
B. Negative.
C. Zero.

# QuickCheck 2.22

Here is a motion diagram of a car speeding up on a straight road:

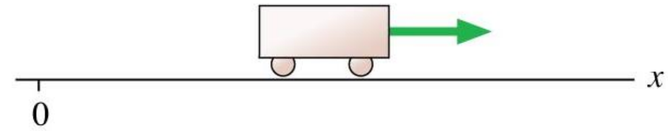


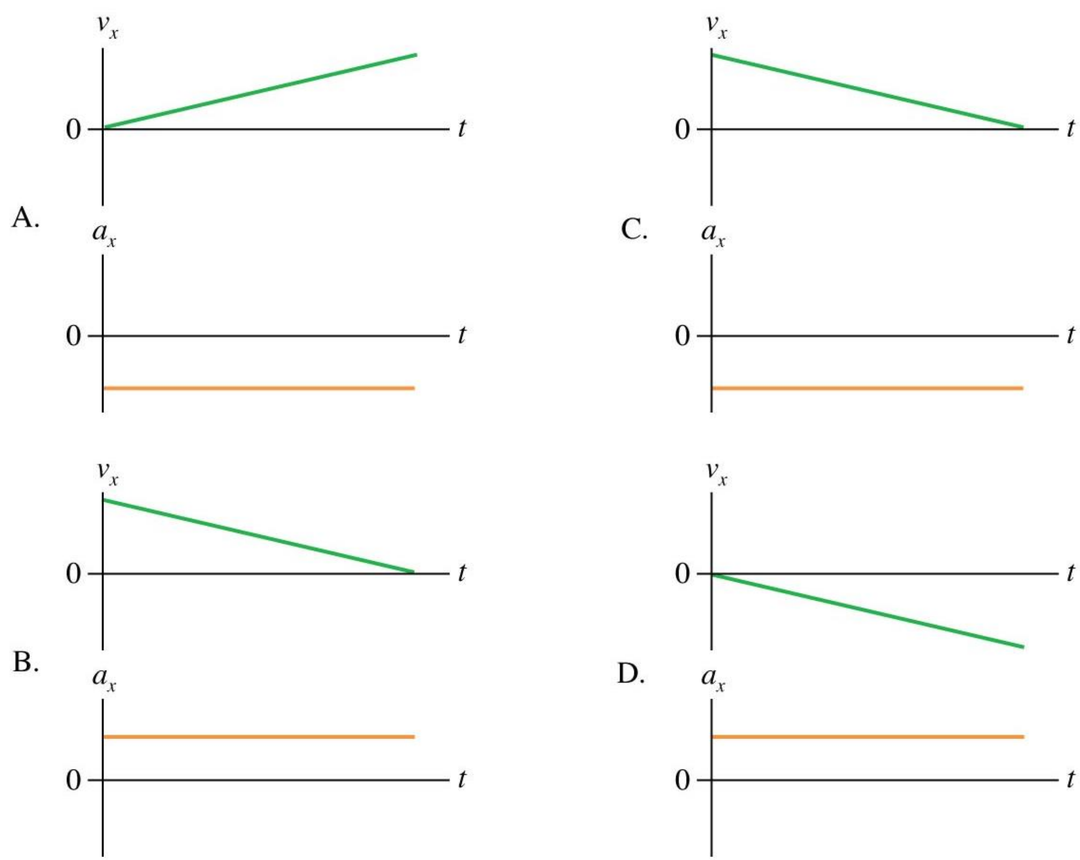

The sign of the acceleration $a_x$ is

A. Positive.
B. Negative. ✔ Speeding up means $v_x$ and $a_x$ have the same sign.
C. Zero.

# QuickCheck 2.23

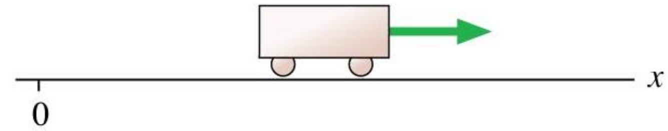
A cart *slows down* while moving away from the origin. What do the velocity and acceleration graphs look like?

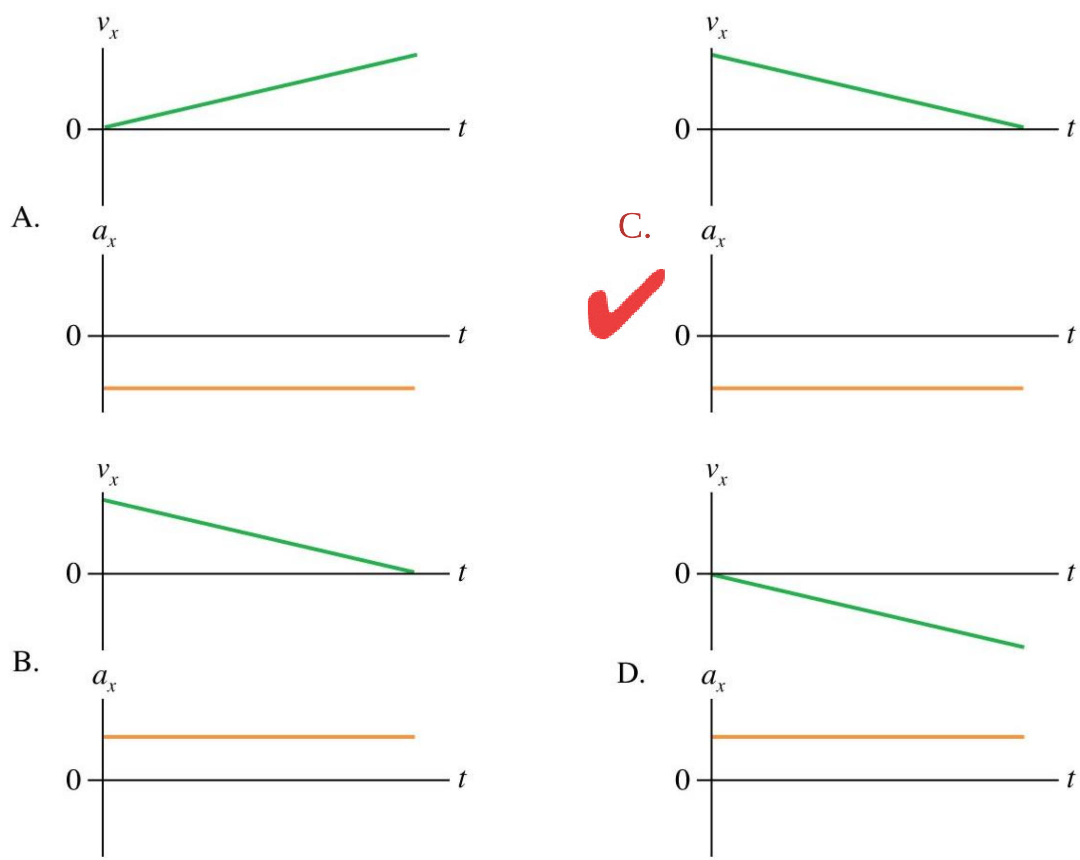


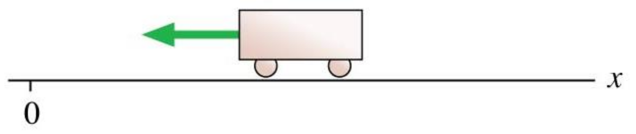
# QuickCheck 2.23

A cart *slows down* while moving away from the origin. What do the velocity and acceleration graphs look like?

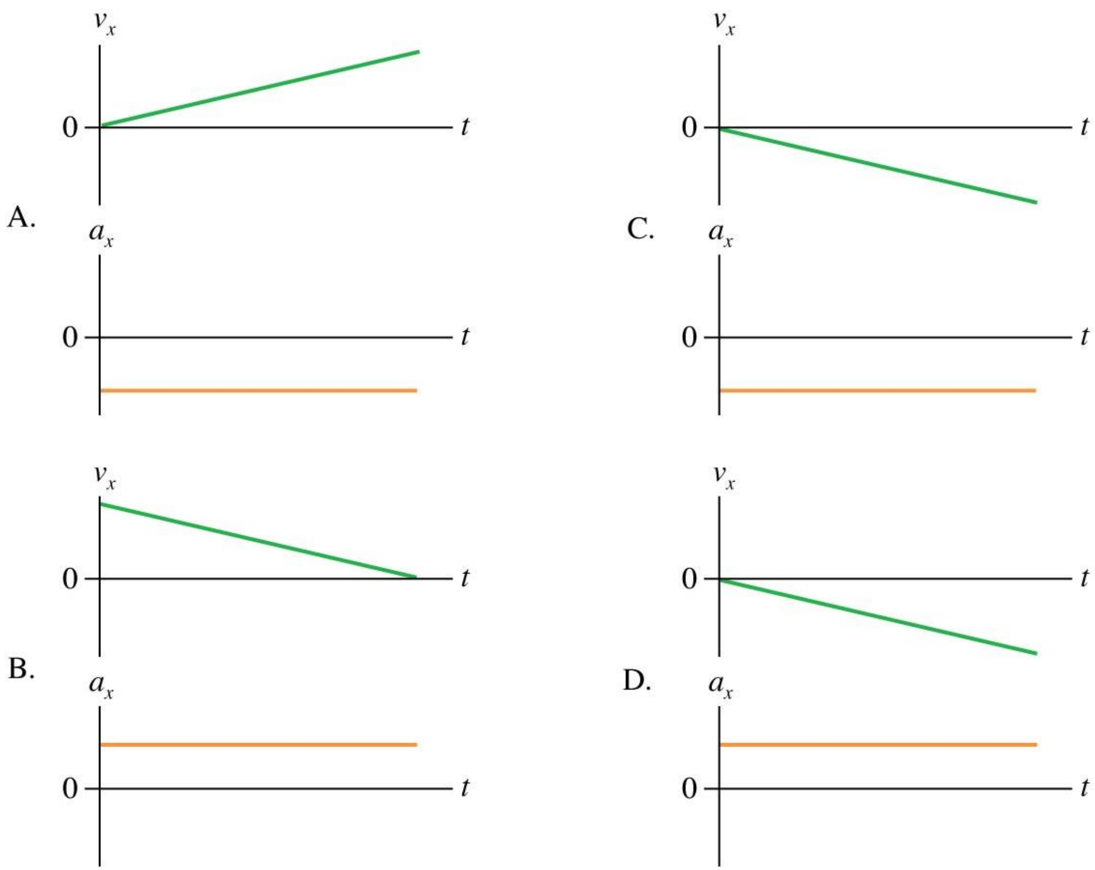


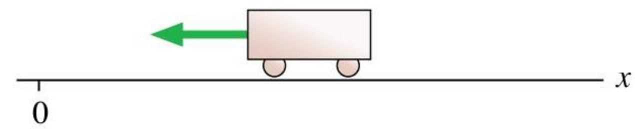
# QuickCheck 2.24




A cart *speeds up* while moving toward the origin. What do the velocity and acceleration graphs look like?

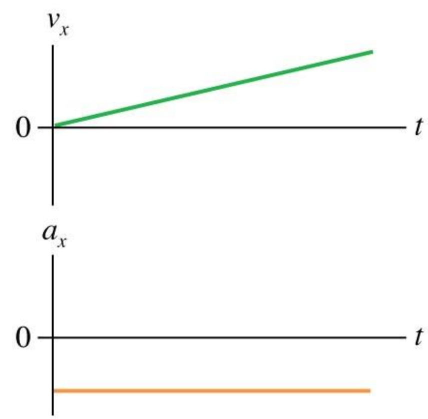


# QuickCheck 2.24

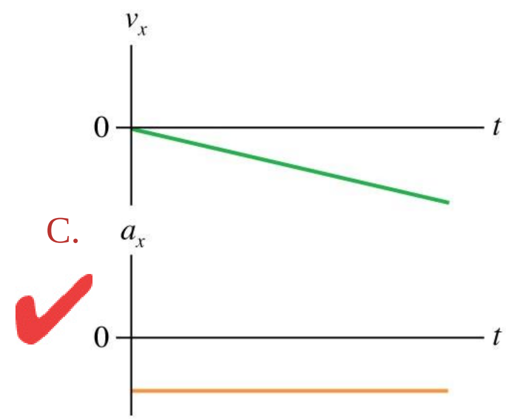



A cart *speeds up* while moving toward the origin. What do the velocity and acceleration graphs look like?

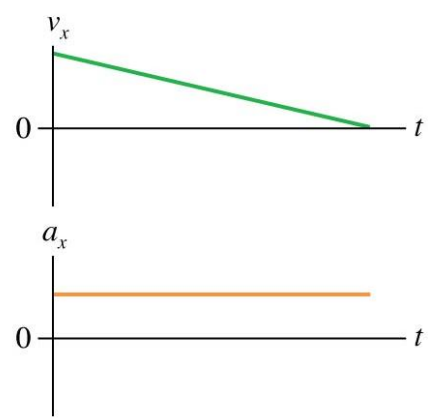
A.



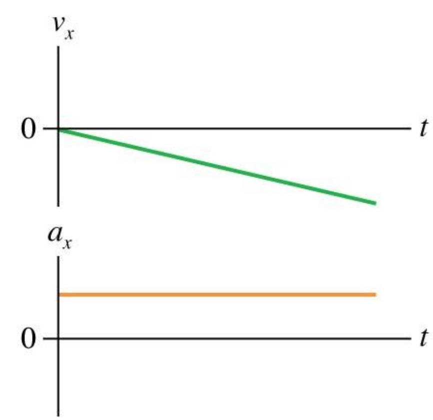

C. ✔




B.

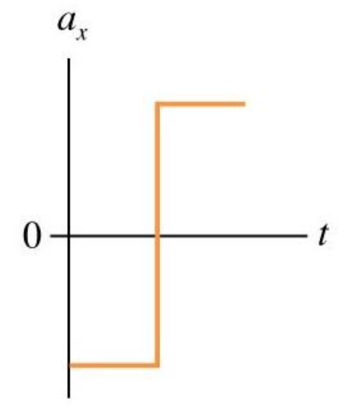


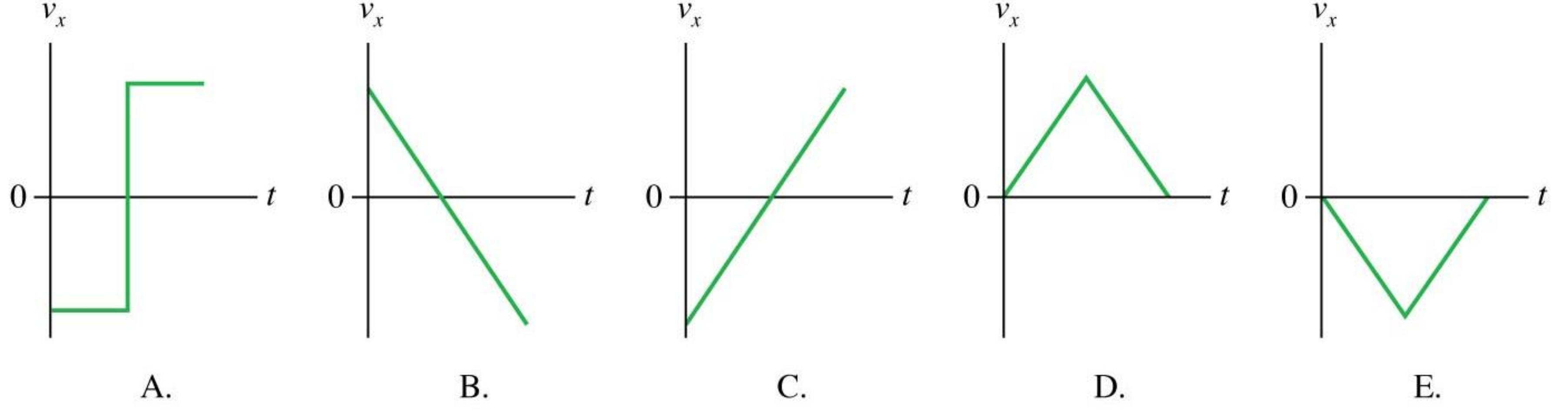

D.



# QuickCheck 2.25

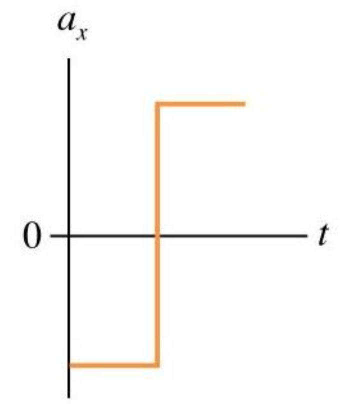
Which velocity-versus-time graph goes with this acceleration graph?

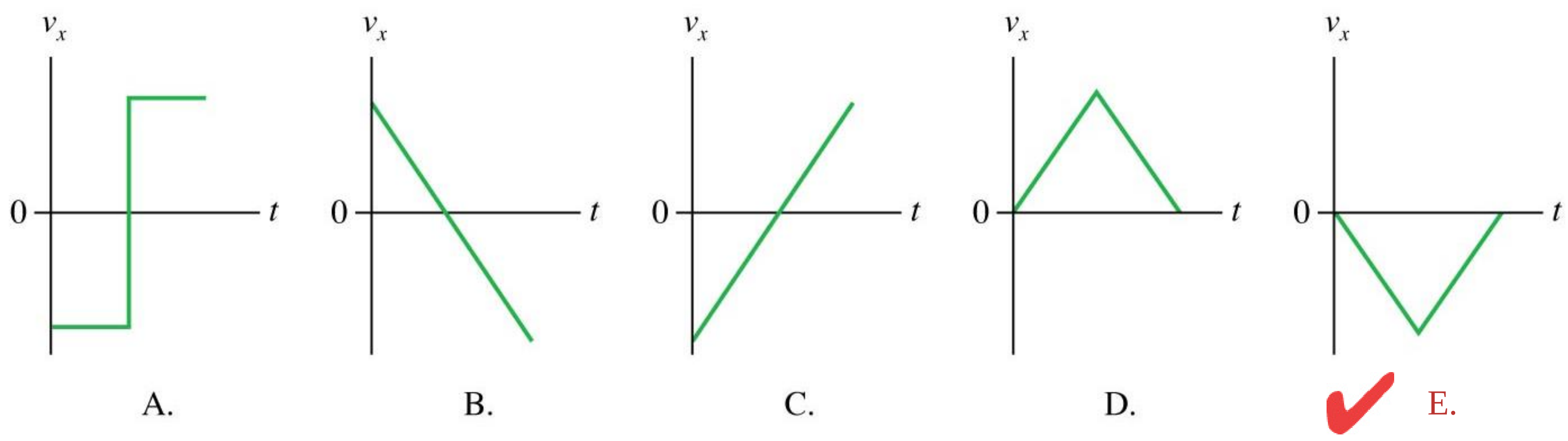


# QuickCheck 2.25

Which velocity-versus-time graph goes with this acceleration graph?



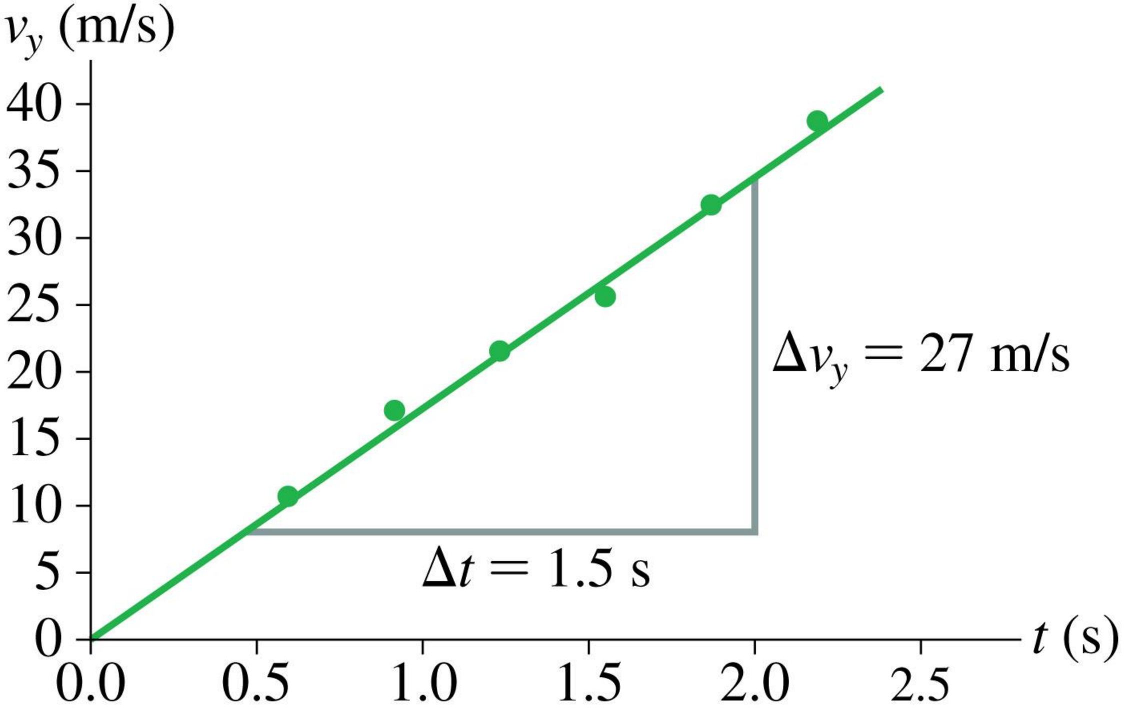



# Section 2.5 Motion with Constant Acceleration

# Motion with Constant Acceleration

- We can use the slope of the graph in the velocity graph to determine the acceleration of the rocket.

$$a_y = \frac{\Delta v_y}{\Delta t} = \frac{27 \text{ m/s}}{1.5 \text{ s}} = 18 \text{ m/s}^2$$



# Constant Acceleration Equations

- We can use the acceleration to find $(v_x)_f$ at a later time $t_f$.

$$a_x = \frac{\Delta v_x}{\Delta t} = \frac{(v_x)_f - (v_x)_i}{\Delta t}$$

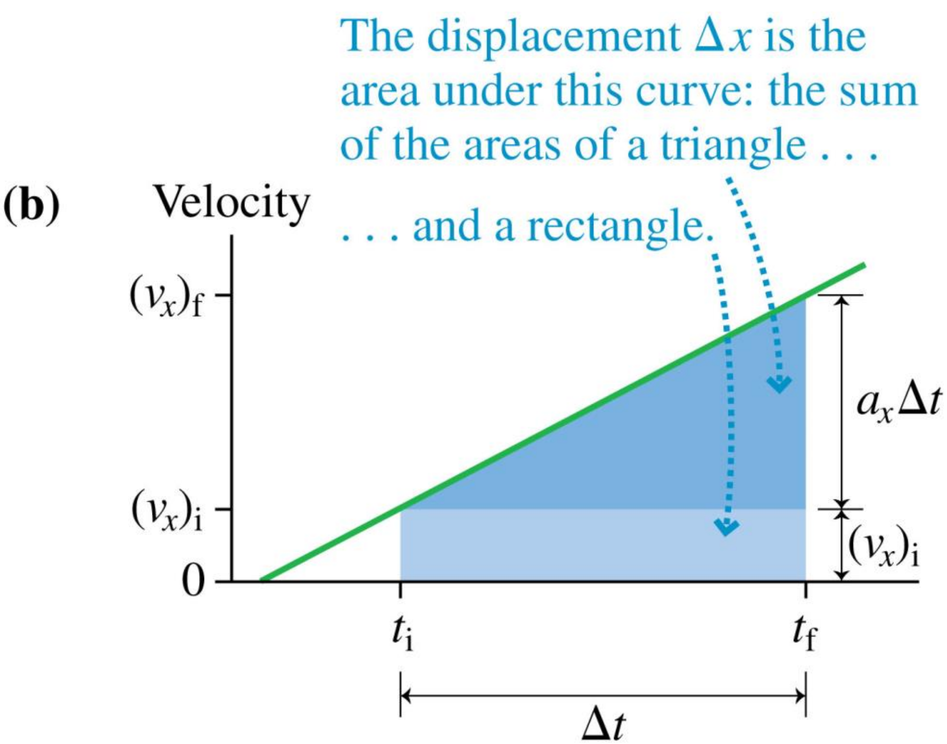
$$(v_x)_f = (v_x)_i + a_x \Delta t$$

Velocity equation for an object with constant acceleration

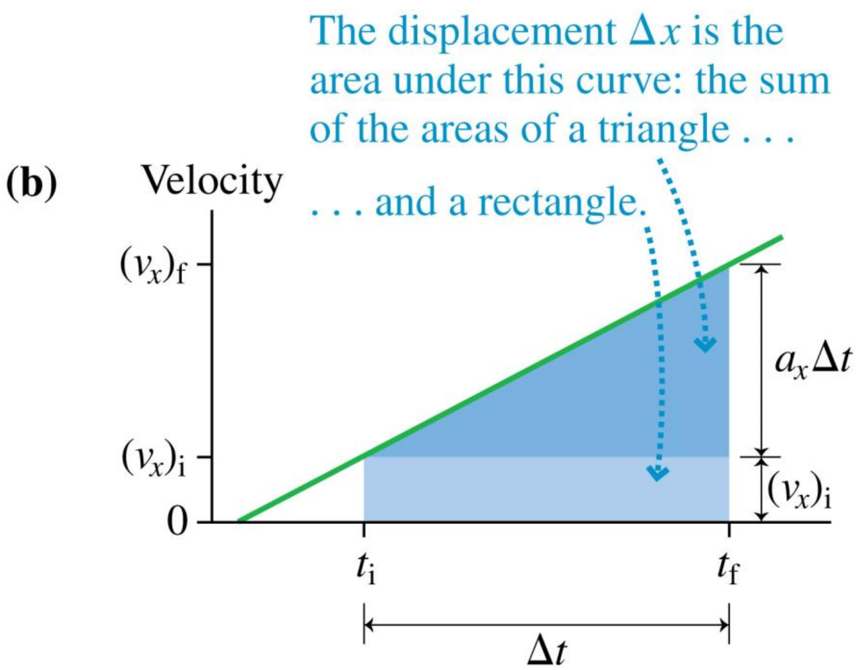
- We have expressed this equation for motion along the *x*-axis, but it is a general result that will apply to any axis.

# Constant Acceleration Equations

- The velocity-versus-time graph for constant-acceleration motion is a straight line with value $(v_x)_i$ at time $t_i$ and slope $a_x$.
- The displacement $\Delta x$ during a time interval $\Delta t$ is the area under the velocity-versus-time graph shown in the shaded area of the figure.



# Constant Acceleration Equations




- The shaded area can be subdivided into a rectangle and a triangle. Adding these areas gives

$$x_f = x_i + (v_x)_i\,\Delta t + \frac{1}{2}a_x(\Delta t)^2$$

Position equation for an object with constant acceleration

# Constant Acceleration Equations

- Combining Equation 2.11 with Equation 2.12 gives us a relationship between displacement and velocity:

$$(v_x)_f^2 = (v_x)_i^2 + 2a_x \Delta x$$

Relating velocity and displacement for constant-acceleration motion

- $\Delta x$ in Equation 2.13 is the displacement (not the distance!).

# Constant Acceleration Equations

For **motion with constant acceleration**:

- Velocity changes steadily:

Final and initial velocity (m/s)

$$(v_x)_f = (v_x)_i + a_x \Delta t \qquad \textbf{(1)}$$

Acceleration (m/s$^2$) Time interval (s)

- The position changes as the square of the time interval:

Final and initial position (m) Time interval (s)

$$x_f = x_i + (v_x)_i\, \Delta t + \tfrac{1}{2} a_x (\Delta t)^2 \qquad \textbf{(2)}$$

Initial velocity (m/s) Acceleration (m/s$^2$)

- We can also express the change in velocity in terms of **distance, not time**:

Final and initial velocity (m/s)

$$(v_x)_f^2 = (v_x)_i^2 + 2 a_x \Delta x \qquad \textbf{(3)}$$

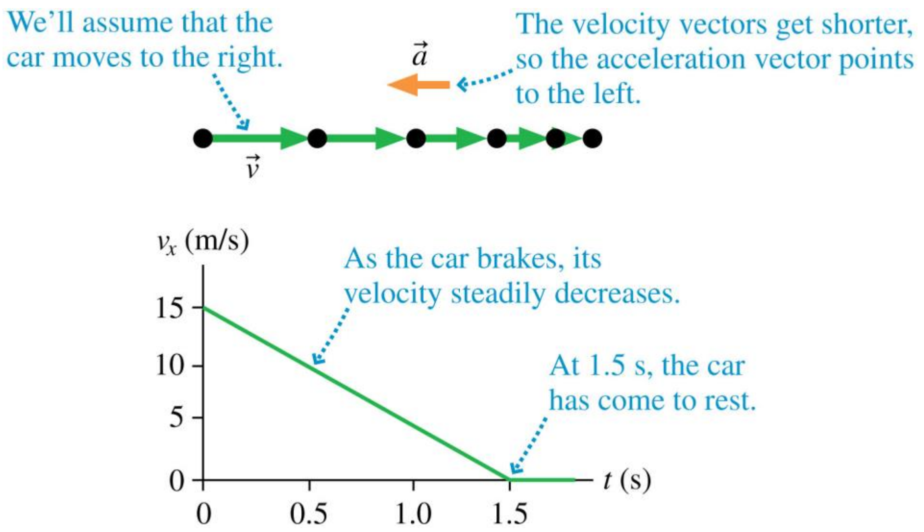
Acceleration (m/s$^2$) Change in position (m)

Text: p. 43

# Example 2.8 Coming to a stop

As you drive in your car at 15 m/s (just a bit under 35 mph), you see a child's ball roll into the street ahead of you. You hit the brakes and stop as quickly as you can. In this case, you come to rest in 1.5 s. How far does your car travel as you brake to a stop?

**PREPARE** The problem statement gives us a description of motion in words. To help us visualize the situation, FIGURE 2.30 illustrates the key features of the motion with a motion diagram and a velocity graph. The graph is based on the car slowing from 15 m/s to 0 m/s in 1.5 s.



# Example 2.8 Coming to a stop (cont.)

**SOLVE** We've assumed that your car is moving to the right, so its initial velocity is $(v_x)_i = +15$ m/s. After you come to rest, your final velocity is $(v_x)_f = 0$ m/s. We use the definition of acceleration from Synthesis 2.1:

$$a_x = \frac{\Delta v_x}{\Delta t} = \frac{(v_x)_f - (v_x)_i}{\Delta t} = \frac{0 \text{ m/s} - 15 \text{ m/s}}{1.5 \text{ s}} = -10 \text{ m/s}^2$$

An acceleration of $-10$ m/s$^2$ (really $-10$ m/s per second) means the car slows by 10 m/s every second.

Now that we know the acceleration, we can compute the distance that the car moves as it comes to rest using the second constant acceleration equation in Synthesis 2.1:

$$\begin{aligned} x_f - x_i &= (v_x)_i \Delta t + \tfrac{1}{2} a_x (\Delta t)^2 \\ &= (15 \text{ m/s})(1.5 \text{ s}) + \tfrac{1}{2}(-10 \text{ m/s}^2)(1.5 \text{ s})^2 = 11 \text{ m} \end{aligned}$$

# Example 2.8 Coming to a stop (cont.)

**ASSESS** 11 m is a little over 35 feet. That's a reasonable distance for a quick stop while traveling at about 35 mph. The purpose of the Assess step is not to prove that your solution is correct but to use common sense to recognize answers that are clearly wrong. Had you made a calculation error and ended up with an answer of 1.1 m—less than 4 feet—a moment's reflection should indicate that this couldn't possibly be correct.

# Example Problem: Reaching New Heights

Spud Webb, height 5'7", was one of the shortest basketball players to play in the NBA. But he had in impressive vertical leap; he was reputedly able to jump 110 cm off the ground.

To jump this high, with what speed would he leave the ground?

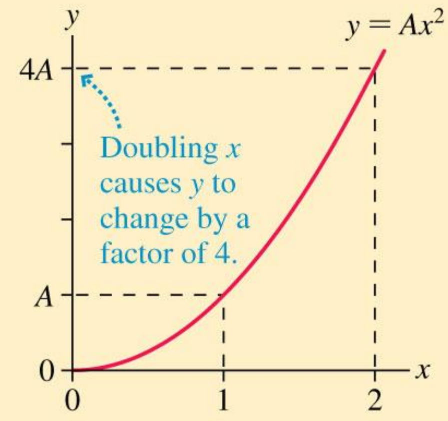
# Quadratic Relationships

## Quadratic relationships

Two quantities are said to have a **quadratic relationship** if $y$ is proportional to the square of $x$. We write the mathematical relationship as

$$y = Ax^2$$

$y$ is proportional to $x^2$




The graph of a quadratic relationship is a parabola.

**SCALING** If $x$ has the initial value $x_1$, then $y$ has the initial value $y_1 = A(x_1)^2$. Changing $x$ from $x_1$ to $x_2$ changes $y$ from $y_1$ to $y_2$. The ratio of $y_2$ to $y_1$ is

$$\frac{y_2}{y_1} = \frac{A(x_2)^2}{A(x_1)^2} = \left(\frac{x_2}{x_1}\right)^2$$

The ratio of $y_2$ to $y_1$ is the square of the ratio of $x_2$ to $x_1$. If $y$ is a quadratic function of $x$, a change in $x$ by some factor changes $y$ by the square of that factor:

- If you increase $x$ by a factor of 2, you increase $y$ by a factor of $2^2 = 4$.
- If you decrease $x$ by a factor of 3, you decrease $y$ by a factor of $3^2 = 9$.

Generally, we can say that:

**Changing $x$ by a factor of $c$ changes $y$ by a factor of $c^2$.**

Exercise 19

Text: p. 44

# Example 2.9 Displacement of a drag racer

A drag racer, starting from rest, travels 6.0 m in 1.0 s. Suppose the car continues this acceleration for an additional 4.0 s. How far from the starting line will the car be?

**PREPARE** We assume that the acceleration is constant, and the initial speed is zero, so the displacement will scale as the square of the time.

**SOLVE** After 1.0 s, the car has traveled 6.0 m; after another 4.0 s, a total of 5.0 s will have elapsed. The initial elapsed time was 1.0 s, so the elapsed time increases by a factor of 5. The displacement thus increases by a factor of $5^2$, or 25. The total displacement is

$$\Delta x = 25(6.0 \text{ m}) = 150 \text{ m}$$

**ASSESS** This is a big distance in a short time, but drag racing is a fast sport, so our answer makes sense.

# Section 2.6 Solving One-Dimensional Motion Problems

# Problem-Solving Strategy

The first step in solving a seemingly complicated problem is to break it down into a series of smaller steps. In worked examples in the text, we use a problem-solving strategy that consists of three steps: *prepare, solve,* and *assess*. Each of these steps has important elements that you should follow when you solve problems on your own.

Text: p. 45

# Problem-Solving Strategy (cont.)

**PREPARE** The Prepare step of a solution is where you identify important elements of the problem and collect information. It's tempting to jump right to the Solve step, but a skilled problem solver will spend the most time on preparation, which includes:

- **Drawing a picture.** This is often the most important part of a problem. The picture lets you model the problem and identify the important elements. As you add information to your picture, the outline of the solution will take shape. For the problems in this chapter, a picture could be a motion diagram or a graph—or perhaps both.
- **Collecting necessary information.** The problem's statement may give you some values of variables. Other information may be implied, or looked up in a table, or estimated or measured.
- **Doing preliminary calculations.** Some calculations, such as unit conversions, are best done in advance.

Text: p. 45

# Problem-Solving Strategy (cont.)

**SOLVE** The Solve step of a solution is where you actually do the mathematics or reasoning necessary to arrive at the answer needed. This is the part of the problem-solving strategy that you likely think of as "solving problems." But don't make the mistake of starting here! The Prepare step will help you be certain you understand the problem before you start putting numbers in equations.

Text: p. 45

# Problem-Solving Strategy (cont.)

**ASSESS** The Assess step of your solution is very important. Once you have an answer, you should check to see whether it makes sense. Ask yourself:

- **Does my solution answer the question that was asked?** Make sure you have addressed all parts of the question and clearly written down your solutions.
- **Does my answer have the correct units and number of significant figures?**
- **Does the value I computed make physical sense?** In this book all calculations use physically reasonable numbers. If your answer seems unreasonable, go back and check your work.
- **Can I estimate what the answer should be to check my solution?**
- **Does my final solution make sense in the context of the material I am learning?**

Text: p. 45

# The Pictorial Representation

**TACTICS BOX 2.2** **Drawing a pictorial representation**

1. **Sketch the situation.** Not just any sketch: Show the object at the *beginning* of the motion, at the *end,* and at any point where the character of the motion changes. Very simple drawings are adequate.
2. **Establish a coordinate system.** Select your axes and origin to match the motion.
3. **Define symbols.** Use the sketch to define symbols representing quantities such as position, velocity, acceleration, and time. *Every* variable used later in the mathematical solution should be defined on the sketch.

We will generally combine the pictorial representation with a **list of values,** which will include:

- *Known information*. Make a table of the quantities whose values you can determine from the problem statement or that you can find quickly with simple geometry or unit conversions.
- *Desired unknowns*. What quantity or quantities will allow you to answer the question?

Exercise 21

Text: p. 46

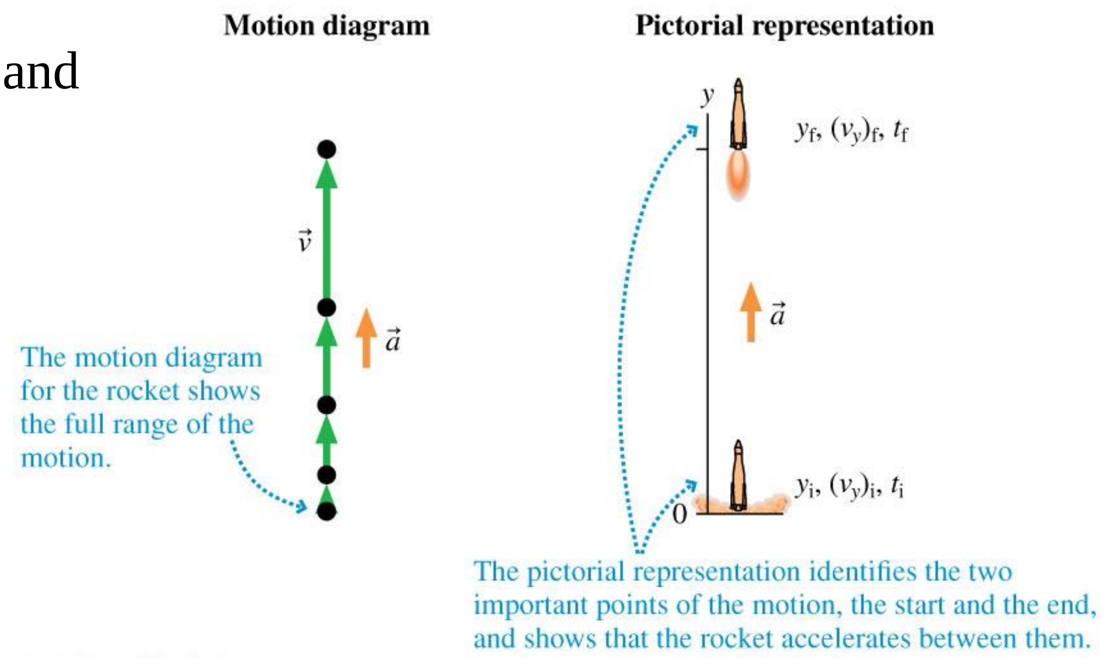
# The Visual Overview

- The **visual overview** will consist of some or all of the following elements:
  - A *motion diagram*. A good strategy for solving a motion problem is to start by drawing a motion diagram.
  - A *pictorial representation*, as defined above.
  - A *graphical representation*. For motion problems, it is often quite useful to include a graph of position and/or velocity.
  - A *list of values*. This list should sum up all of the important values in the problem.

# Example 2.11 Kinematics of a rocket launch

A Saturn V rocket is launched straight up with a constant acceleration of 18 m/s$^2$. After 150 s, how fast is the rocket moving and how far has it traveled?

**PREPARE** FIGURE 2.32 shows a visual overview of the rocket launch that includes a motion diagram, a pictorial representation, and a list of values. The visual overview shows the whole problem in a nutshell. The motion diagram illustrates the motion of the rocket. The pictorial representation (produced according to Tactics Box 2.2) shows axes, identifies the important points of the motion, and defines variables. Finally, we have included a list of values that gives the known and unknown quantities. In the visual overview we have taken the statement of the problem in words and made it much more precise. The overview contains everything you need to know about the problem.



# Example 2.11 Kinematics of a rocket launch (cont.)

**SOLVE** Our first task is to find the final velocity. Our list of values includes the initial velocity, the acceleration, and the time interval, so we can use the first kinematic equation of Synthesis 2.1 to find the final velocity:

$$(v_y)_f = (v_y)_i + a_y\,\Delta t = 0\text{ m/s} + (18\text{ m/s}^2)(150\text{ s})$$

$$= 2700\text{ m/s}$$

**List of values**

Known

$y_i = 0$ m
$(v_y)_i = 0$ m/s
$t_i = 0$ s
$a_y = 18$ m/s$^2$
$t_f = 150$ s

Find

$(v_y)_f$ and $y_f$

The list of values makes everything concrete. We define the start of the problem to be at time 0 s, when the rocket has a position of 0 m and a velocity of 0 m/s. The end of the problem is at time 150 s. We are to find the position and velocity at this time.

# Example 2.11 Kinematics of a rocket launch (cont.)

**SOLVE**

The distance traveled is found using the second equation in Synthesis 2.1:

$$y_f = y_i + (v_y)_i\,\Delta t + \tfrac{1}{2}a_y(\Delta t)^2$$

$$= 0\text{ m} + (0\text{ m/s})(150\text{ s}) + \tfrac{1}{2}(18\text{ m/s}^2)(150\text{ s})^2$$

$$= 2.0 \times 10^5\text{ m} = 200\text{ km}$$

**List of values**

Known

$y_i = 0$ m
$(v_y)_i = 0$ m/s
$t_i = 0$ s
$a_y = 18$ m/s$^2$
$t_f = 150$ s

Find

$(v_y)_f$ and $y_f$

The list of values makes everything concrete. We define the start of the problem to be at time 0 s, when the rocket has a position of 0 m and a velocity of 0 m/s. The end of the problem is at time 150 s. We are to find the position and velocity at this time.

# Problem-Solving Strategy for Motion with Constant Acceleration

**PROBLEM-SOLVING STRATEGY 2.1** **Motion with constant acceleration**

MP®

Problems involving constant acceleration—speeding up, slowing down, vertical motion, horizontal motion—can all be treated with the same problem-solving strategy.

**PREPARE** Draw a visual overview of the problem. This should include a motion diagram, a pictorial representation, and a list of values; a graphical representation may be useful for certain problems.

**SOLVE** The mathematical solution is based on the three equations in Synthesis 2.1.

- Though the equations are phrased in terms of the variable $x$, it's customary to use $y$ for motion in the vertical direction.
- Use the equation that best matches what you know and what you need to find. For example, if you know acceleration and time and are looking for a change in velocity, the first equation is the best one to use.
- Uniform motion with constant velocity has $a = 0$.

**ASSESS** Is your result believable? Does it have proper units? Does it make sense?

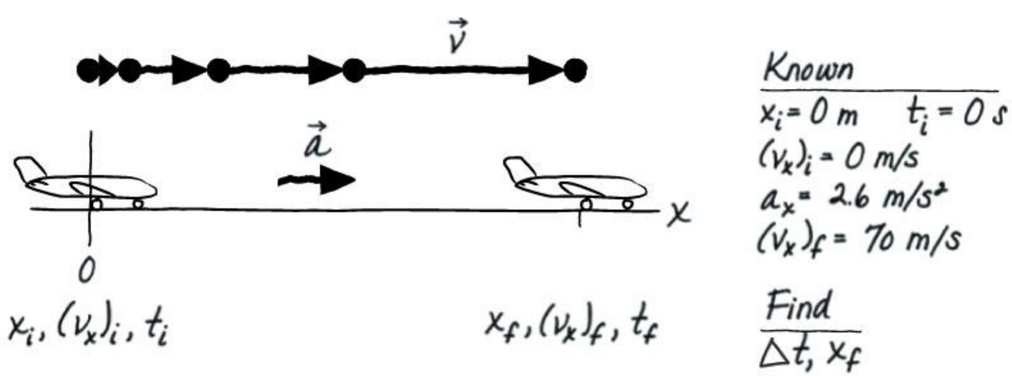
Exercise 25

Text: p. 48

# Example 2.12 Calculating the minimum length of a runway

A fully loaded Boeing 747 with all engines at full thrust accelerates at 2.6 $m/s^2$. Its minimum takeoff speed is 70 m/s. How much time will the plane take to reach its takeoff speed? What minimum length of runway does the plane require for takeoff?

**PREPARE** The visual overview of FIGURE 2.33 summarizes the important details of the problem. We set $x_i$ and $t_i$ equal to zero at the starting point of the motion, when the plane is at rest and the acceleration begins. The final point of the motion is when the plane achieves the necessary takeoff speed of 70 m/s. The plane is accelerating to the right, so we will compute the time for the plane to reach a velocity of 70 m/s and the position of the plane at this time, giving us the minimum length of the runway.



# Example 2.12 Calculating the minimum length of a runway (cont.)

**SOLVE** First we solve for the time required for the plane to reach takeoff speed. We can use the first equation in Synthesis 2.1 to compute this time:

$$(v_x)_\mathrm{f} = (v_x)_\mathrm{i} + a_x \, \Delta t$$

$$70 \text{ m/s} = 0 \text{ m/s} + (2.6 \text{ m/s}^2) \, \Delta t$$

$$\Delta t = \frac{70 \text{ m/s}}{2.6 \text{ m/s}^2} = 26.9 \text{ s}$$

We keep an extra significant figure here because we will use this result in the next step of the calculation.

# Example 2.12 Calculating the minimum length of a runway (cont.)

**SOLVE**

Given the time that the plane takes to reach takeoff speed, we can compute the position of the plane when it reaches this speed using the second equation in Synthesis 2.1:

$$x_f = x_i + (v_x)_i\,\Delta t + \tfrac{1}{2}a_x(\Delta t)^2$$

$$= 0\text{ m} + (0\text{ m/s})(26.9\text{ s}) + \tfrac{1}{2}(2.6\text{ m/s}^2)(26.9\text{ s})^2$$

$$= 940\text{ m}$$

Our final answers are thus that the plane will take 27 s to reach takeoff speed, with a minimum runway length of 940 m.

# Example 2.12 Calculating the minimum length of a runway (cont.)

**ASSESS** Think about the last time you flew; 27 s seems like a reasonable time for a plane to accelerate on takeoff. Actual runway lengths at major airports are 3000 m or more, a few times greater than the minimum length, because they have to allow for emergency stops during an aborted takeoff. (If we had calculated a distance far greater than 3000 m, we would know we had done something wrong!)

# Example Problem: Champion Jumper

The African antelope known as a springbok will occasionally jump straight up into the air, a movement known as a **pronk**. The speed when leaving the ground can be as high as 7.0 m/s.

If a springbok leaves the ground at 7.0 m/s:

A. How much time will it take to reach its highest point?
B. How long will it stay in the air?
C. When it returns to earth, how fast will it be moving?

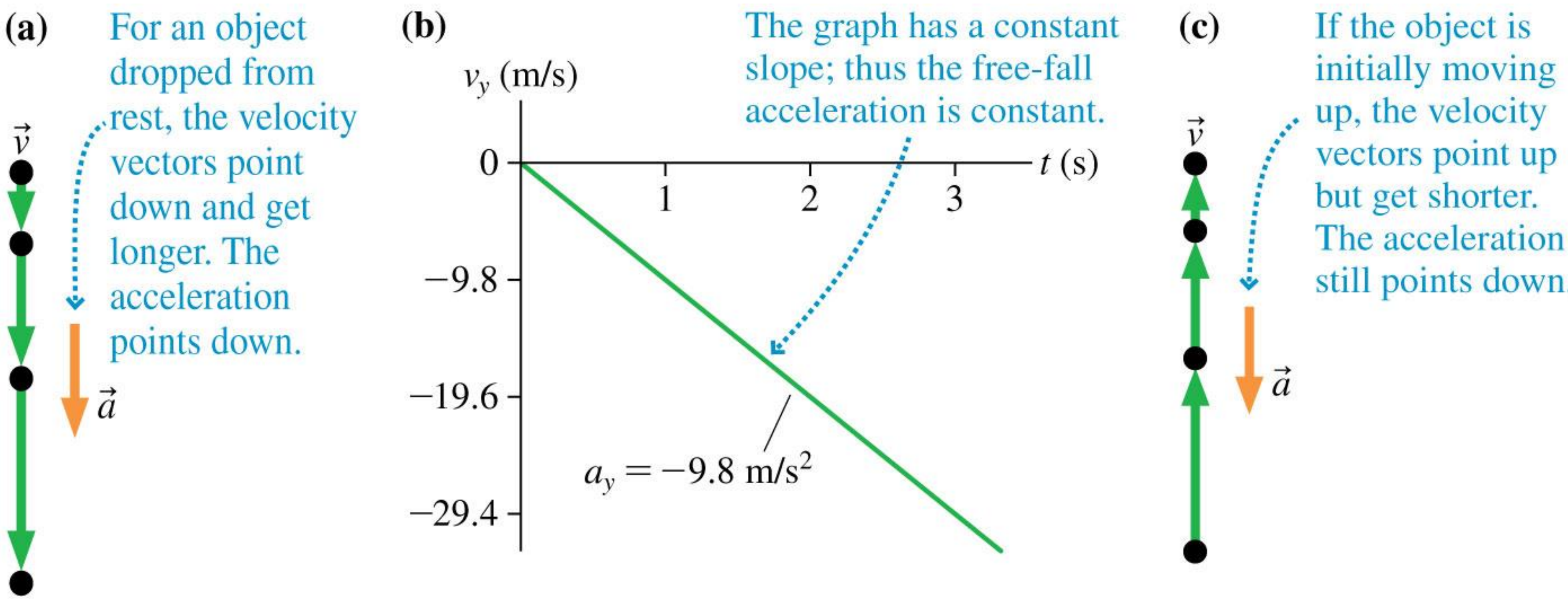
# Section 2.7 Free Fall

# Free Fall

- If an object moves under the influence of gravity only, and no other forces, we call the resulting motion **free fall**.
- **Any two objects in free fall, regardless of their mass, have the same acceleration.**
- On the earth, air resistance is a factor. For now we will restrict our attention to situations in which air resistance can be ignored.

Apollo 15 lunar astronaut David Scott performed a classic experiment on the moon, simultaneously dropping a hammer and a feather from the same height. Both hit the ground at the exact same time—something that would not happen in the atmosphere of the earth!

# Free Fall




- The figure shows the motion diagram for an object that was released from rest and falls freely. The diagram and the graph would be the same for all falling objects.

# Free Fall

- **The free-fall acceleration always points down**, no matter what direction an object is moving.
- Any object moving under the influence of gravity only, and no other force, is in free fall.

$$\vec{a}_{\text{free fall}} = (9.80\ \text{m/s}^2,\ \text{vertically downward})$$

Standard value for the acceleration of an object in free fall

# Free Fall

- $g$, by definition, is always positive. **There will never be a problem that uses a negative value for $g$.**
- Even though a falling object speeds up, it has negative acceleration ($-g$).
- Because free fall is motion with constant acceleration, we can use the kinematic equations for constant acceleration with $a_y = -g$.
- $g$ is not called "gravity." $g$ is the *free-fall acceleration*.
- $g = 9.80\ \text{m/s}^2$ only on earth. Other planets have different values of $g$.
- We will sometimes compute acceleration in units of $g$.

# QuickCheck 2.26

A ball is tossed straight up in the air. At its very highest point, the ball's instantaneous acceleration $a_y$ is

A. Positive.
B. Negative.
C. Zero.

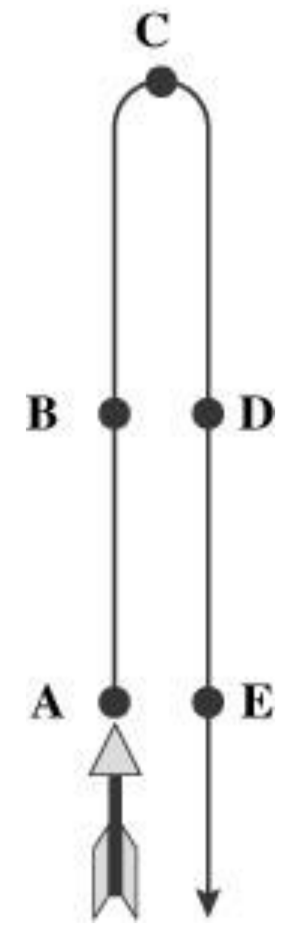
# QuickCheck 2.26

A ball is tossed straight up in the air. At its very highest point, the ball's instantaneous acceleration $a_y$ is

A. Positive.
✔ B. Negative.
C. Zero.

# QuickCheck 2.27

An arrow is launched vertically upward. It moves straight up to a maximum height, then falls to the ground. The trajectory of the arrow is noted. At which point of the trajectory is the arrow's acceleration the greatest? The least? Ignore air resistance; the only force acting is gravity.



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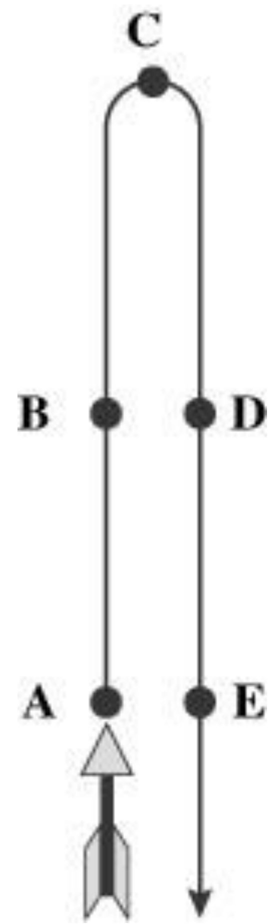
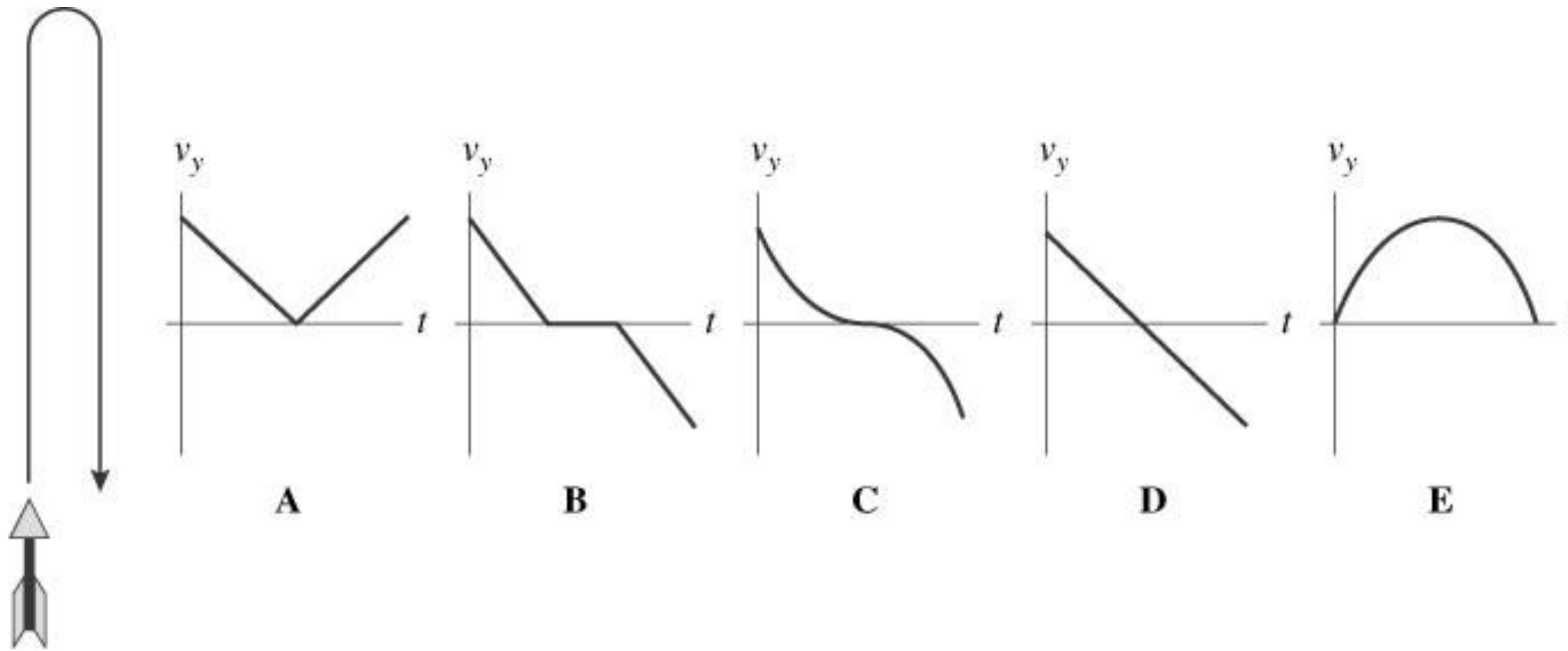



Same at all points.

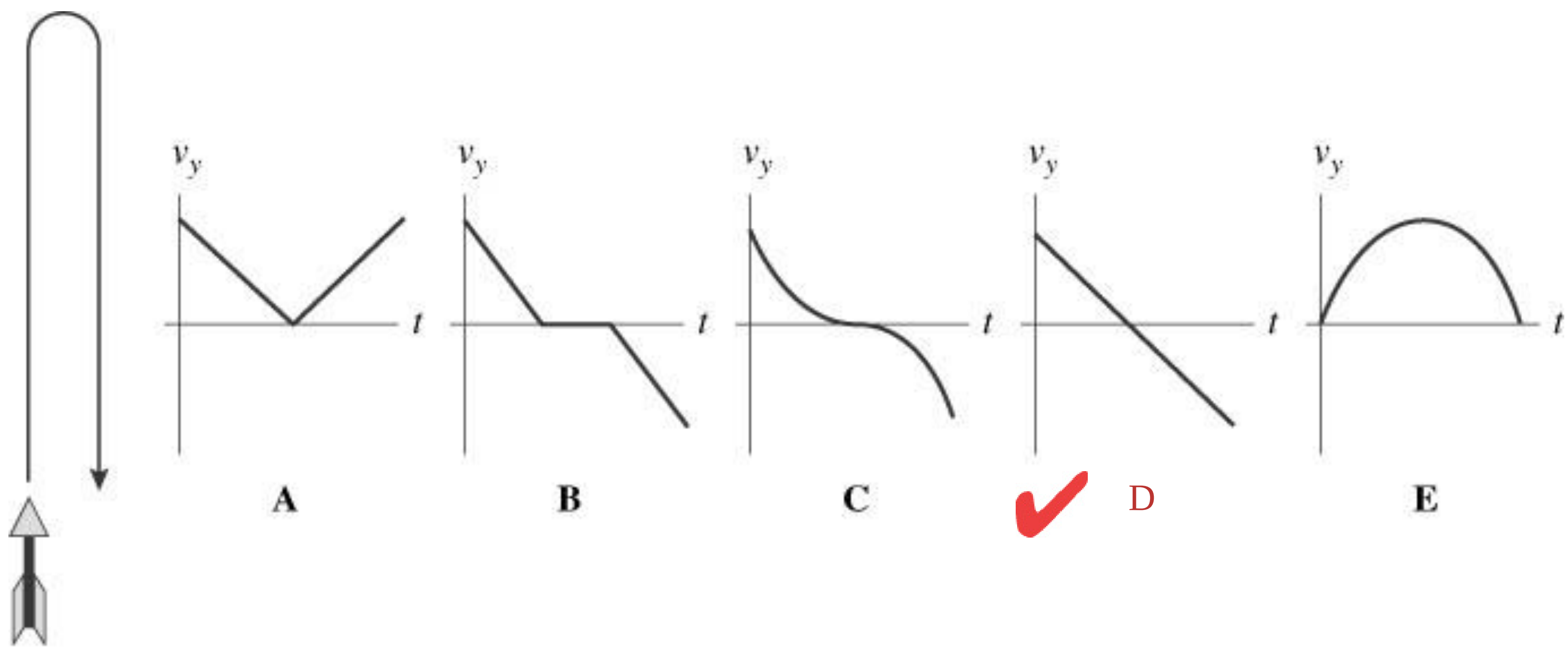
# QuickCheck 2.28

An arrow is launched vertically upward. It moves straight up to a maximum height, then falls to the ground. The trajectory of the arrow is noted. Which graph best represents the vertical velocity of the arrow as a function of time? Ignore air resistance; the only force acting is gravity.



# QuickCheck 2.28

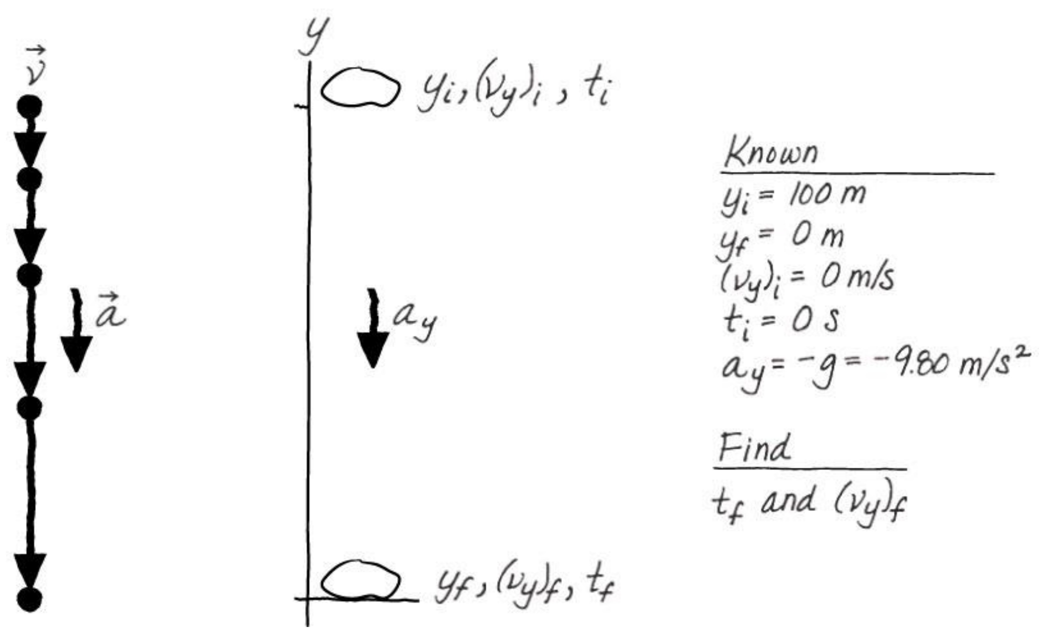
An arrow is launched vertically upward. It moves straight up to a maximum height, then falls to the ground. The trajectory of the arrow is noted. Which graph best represents the vertical velocity of the arrow as a function of time? Ignore air resistance; the only force acting is gravity.



# Example 2.14 Analyzing a rock's fall

A heavy rock is dropped from rest at the top of a cliff and falls 100 m before hitting the ground. How long does the rock take to fall to the ground, and what is its velocity when it hits?

**PREPARE** FIGURE 2.36 shows a visual overview with all necessary data. We have placed the origin at the ground, which makes $y_i = 100$ m.



# Example 2.14 Analyzing a rock's fall (cont.)

**SOLVE** Free fall is motion with the specific constant acceleration $a_y = -g$. The first question involves a relation between time and distance, a relation expressed by the second equation in Synthesis 2.1. Using $(v_y)_i = 0$ m/s and $t_i = 0$ s, we find

$$y_f = y_i + (v_y)_i\,\Delta t + \tfrac{1}{2}a_y\,(\Delta t)^2 = y_i - \tfrac{1}{2}g\,(\Delta t)^2 = y_i - \tfrac{1}{2}g t_f^{\,2}$$

We can now solve for $t_f$:

$$t_f = \sqrt{\frac{2(y_i - y_f)}{g}} = \sqrt{\frac{2(100\text{ m} - 0\text{ m})}{9.80\text{ m/s}^2}} = 4.52\text{ s}$$

Now that we know the fall time, we can use the first kinematic equation to find $(v_y)_f$:

$$\begin{aligned}(v_y)_f &= (v_y)_i - g\,\Delta t = -g t_f = -(9.80\text{ m/s}^2)(4.52\text{ s})\\ &= -44.3\text{ m/s}\end{aligned}$$

# Example 2.14 Analyzing a rock's fall (cont.)

**ASSESS** Are the answers reasonable? Well, 100 m is about 300 feet, which is about the height of a 30-floor building. How long does it take something to fall 30 floors? Four or five seconds seems pretty reasonable. How fast would it be going at the bottom? Using an approximate version of our conversion factor 1 m/s ≈ 2 mph, we find that 44.3 m/s ≈ 90 mph. That also seems like a pretty reasonable speed for something that has fallen 30 floors. Suppose we had made a mistake. If we misplaced a decimal point we could have calculated a speed of 443 m/s, or about 900 mph! This is clearly *not* reasonable. If we had misplaced the decimal point in the other direction, we would have calculated a speed of 4.3 m/s ≈ 9 mph. This is another unreasonable result, because this is slower than a typical bicycling speed.

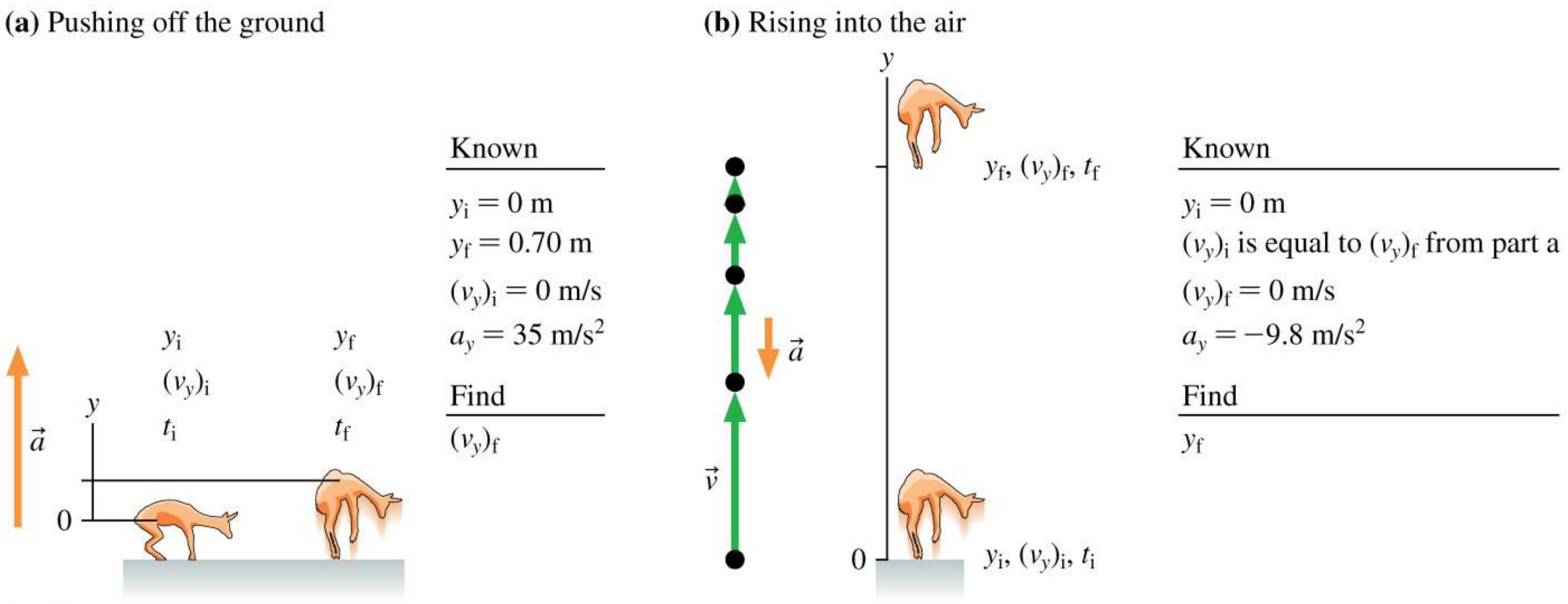
# Example 2.16 Finding the height of a leap

A springbok is an antelope found in southern Africa that gets its name from its remarkable jumping ability. When a springbok is startled, it will leap straight up into the air—a maneuver called a "pronk." A springbok goes into a crouch to perform a pronk. It then extends its legs forcefully, accelerating at 35 $m/s^2$ for 0.70 m as its legs straighten. Legs fully extended, it leaves the ground and rises into the air.

a. At what speed does the springbok leave the ground?

b. How high does it go?

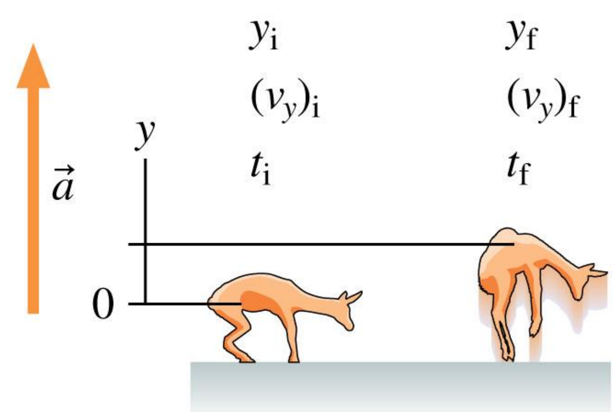
# Example 2.16 Finding the height of a leap (cont.)



# Example 2.16 Finding the height of a leap (cont.)

**PREPARE** We begin with the visual overview shown in FIGURE 2.38, where we've identified two different phases of the motion: the springbok pushing off the ground and the springbok rising into the air. We'll treat these as two separate problems that we solve in turn. We will "re-use" the variables $y_i$, $y_f$, $(v_y)_i$, and $(v_y)_f$ for the two phases of the motion.

For the first part of our solution, in Figure 2.38a we choose the origin of the $y$-axis at the position of the springbok deep in the crouch. The final position is the top extent of the push, at the instant the springbok leaves the ground. We want to find the velocity at this position because that's how fast the springbok is moving as it leaves the ground.




Known

$y_i = 0$ m
$y_f = 0.70$ m
$(v_y)_i = 0$ m/s
$a_y = 35$ m/s$^2$

Find

$(v_y)_f$

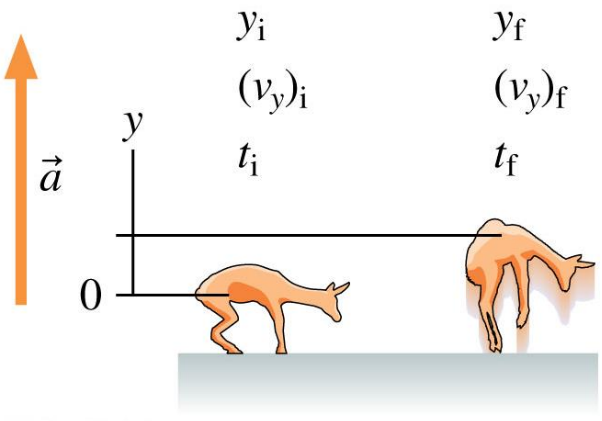
# Example 2.16 Finding the height of a leap (cont.)

**SOLVE** a. For the first phase, pushing off the ground, we have information about displacement, initial velocity, and acceleration, but we don't know anything about the time interval. The third equation in Synthesis 2.1 is perfect for this type of situation. We can rearrange it to solve for the velocity with which the springbok lifts off the ground:

$$(v_y)_f^2 = (v_y)_i^2 + 2a_y\Delta y = (0\ \text{m/s})^2 + 2(35\ \text{m/s}^2)(0.70\ \text{m}) = 49\ \text{m}^2/\text{s}^2$$

$$(v_y)_f = \sqrt{49\ \text{m}^2/\text{s}^2} = 7.0\ \text{m/s}$$

The springbok leaves the ground with a speed of 7.0 m/s.




Known

$y_i = 0$ m
$y_f = 0.70$ m
$(v_y)_i = 0$ m/s
$a_y = 35$ m/s$^2$

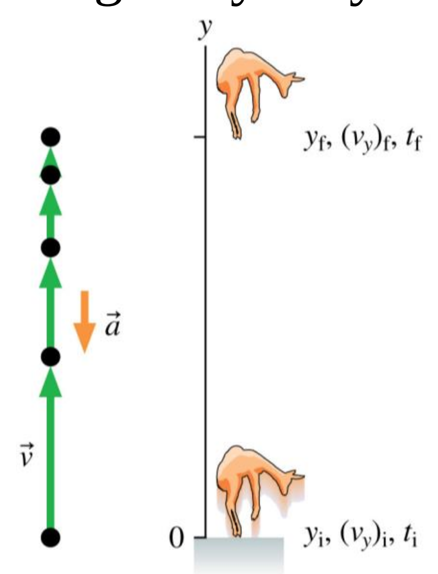
Find

$(v_y)_f$

# Example 2.16 Finding the height of a leap (cont.)

Figure 2.38b essentially starts over—we have defined a new vertical axis with its origin at the ground, so the highest point of the springbok's motion is a distance above the ground. The table of values shows the key piece of information for this second part of the problem: The initial velocity for part b is the final velocity from part a.

After the springbok leaves the ground, this is a free-fall problem because the springbok is moving under the influence of gravity only. We want to know the height of the leap, so we are looking for the height at the top point of the motion. This is a turning point of the motion, with the instantaneous velocity equal to zero. Thus $y_f$, the height of the leap, is the springbok's position at the instant $(v_y)_f = 0$.




**Known**

$y_i = 0$ m

$(v_y)_i$ is equal to $(v_y)_f$ from part a

$(v_y)_f = 0$ m/s

$a_y = -9.8$ m/s$^2$

**Find**

$y_f$

# Example 2.16 Finding the height of a leap (cont.)

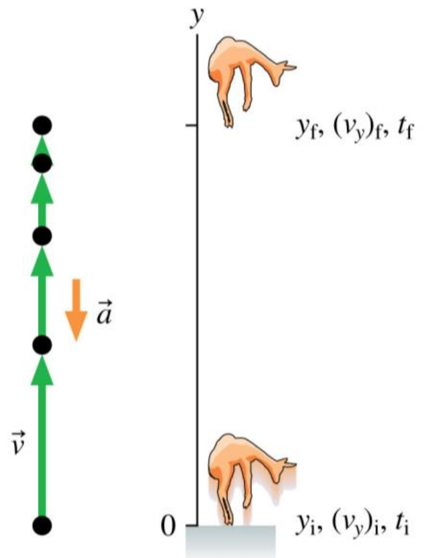
**SOLVE** b. Now we are ready for the second phase of the motion, the vertical motion after leaving the ground. The third equation in Synthesis 2.1 is again appropriate because again we don't know the time. Because $y_i = 0$, the springbok's displacement is $\Delta y = y_f - y_i = y_f$, the height of the vertical leap. From part a, the initial velocity is $(v_y)_i = 7.0$ m/s, and the final velocity is $(v_y)_f = 0$. This is free-fall motion, with $a_y = -g$; thus

$$(v_y)_f^2 = 0 = (v_y)_i^2 - 2g\Delta y = (v_y)_i^2 - 2gy_f$$

which gives $(v_y)_i^2 = 2gy_f$

Solving for $y_f$, we get a jump height of

$$y_f = \frac{(7.0\text{ m/s})^2}{2(9.8\text{ m/s}^2)} = 2.5\text{ m}$$




**Known**

$y_i = 0$ m
$(v_y)_i$ is equal to $(v_y)_f$ from part a
$(v_y)_f = 0$ m/s
$a_y = -9.8$ m/s$^2$

**Find**

$y_f$

# Example 2.16 Finding the height of a leap (cont.)

**ASSESS** 2.5 m is a remarkable leap—a bit over 8 ft—but these animals are known for their jumping ability, so this seems reasonable.

# Example Problem

Passengers on the Giant Drop, a free-fall ride at Six Flags Great America, sit in cars that are raised to the top of a tower. The cars are then released for 2.6 s of free fall. How fast are the passengers moving at the end of this speeding up phase of the ride? If the cars in which they ride then come to rest in a time of 1.0 s, what is the acceleration (magnitude and direction) of this slowing down phase of the ride? Given these numbers, what is the minimum possible height of the tower?

# Summary: General Strategies

## Problem-Solving Strategy

Our general problem-solving strategy has three parts:

**PREPARE** Set up the problem:

- Draw a picture.
- Collect necessary information.
- Do preliminary calculations.

**SOLVE** Do the necessary mathematics or reasoning.

**ASSESS** Check your answer to see if it is complete in all details and makes physical sense.

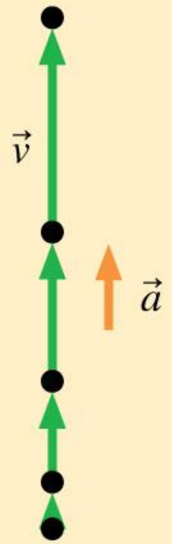
Text: p. 55

# Summary: General Strategies

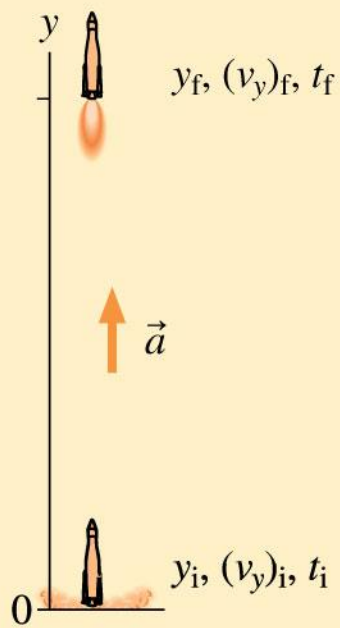
## Visual Overview

A visual overview consists of several pieces that completely specify a problem. This may include any or all of the elements below:

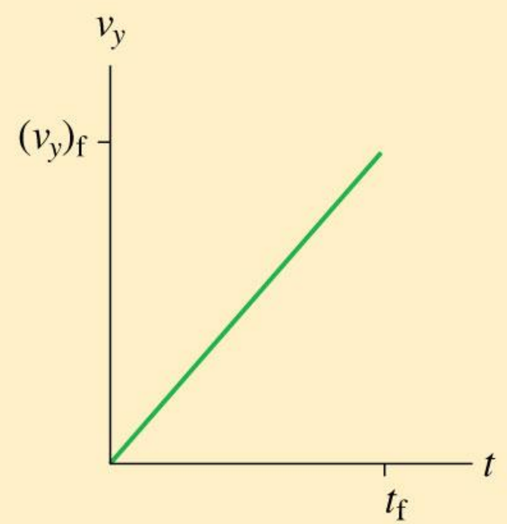
**Motion diagram**




**Pictorial representation**




**Graphical representation**




**List of values**

Known

$y_i = 0$ m

$(v_y)_i = 0$ m/s

$t_i = 0$ s

$a_y = 18$ m/s$^2$

$t_f = 150$ s

Find

$(v_y)_f$ and $y_f$

Text: p. 55

# Summary: Important Concepts

**Velocity** is the rate of change of position:

$$v_x = \frac{\Delta x}{\Delta t}$$

**Acceleration** is the rate of change of velocity:

$$a_x = \frac{\Delta v_x}{\Delta t}$$

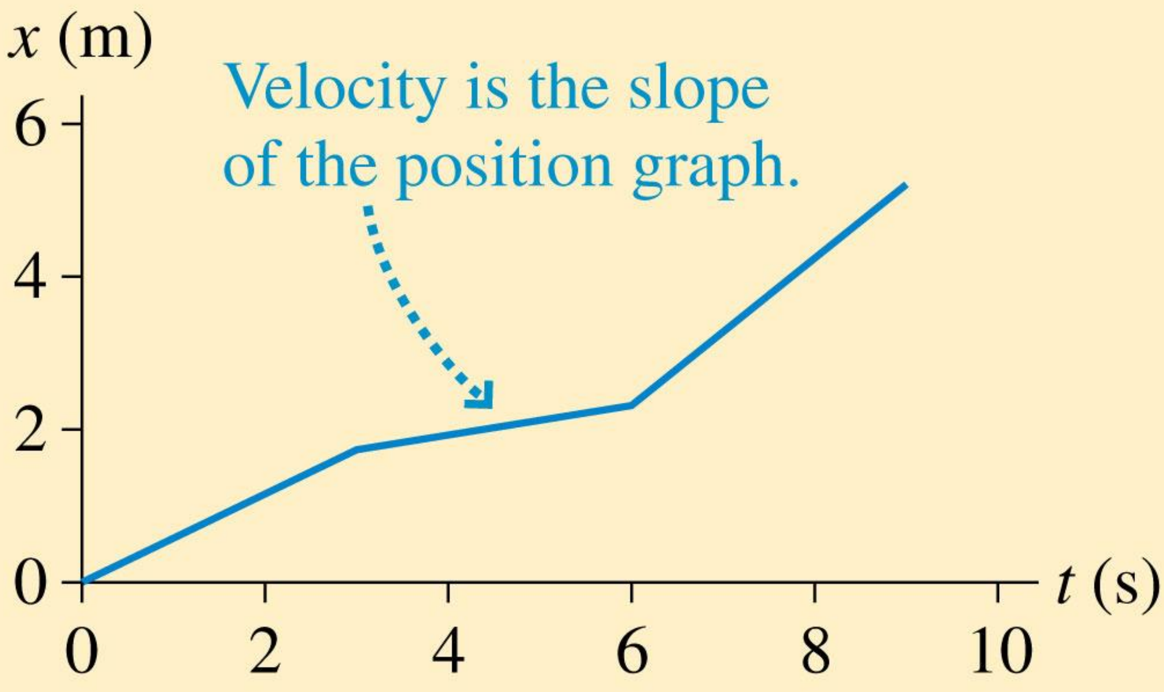
The units of acceleration are $m/s^2$.

An object is speeding up if $v_x$ and $a_x$ have the same sign, slowing down if they have opposite signs.

Text: p. 55

# Summary: Important Concepts

A **position-versus-time graph** plots position on the vertical axis against time on the horizontal axis.



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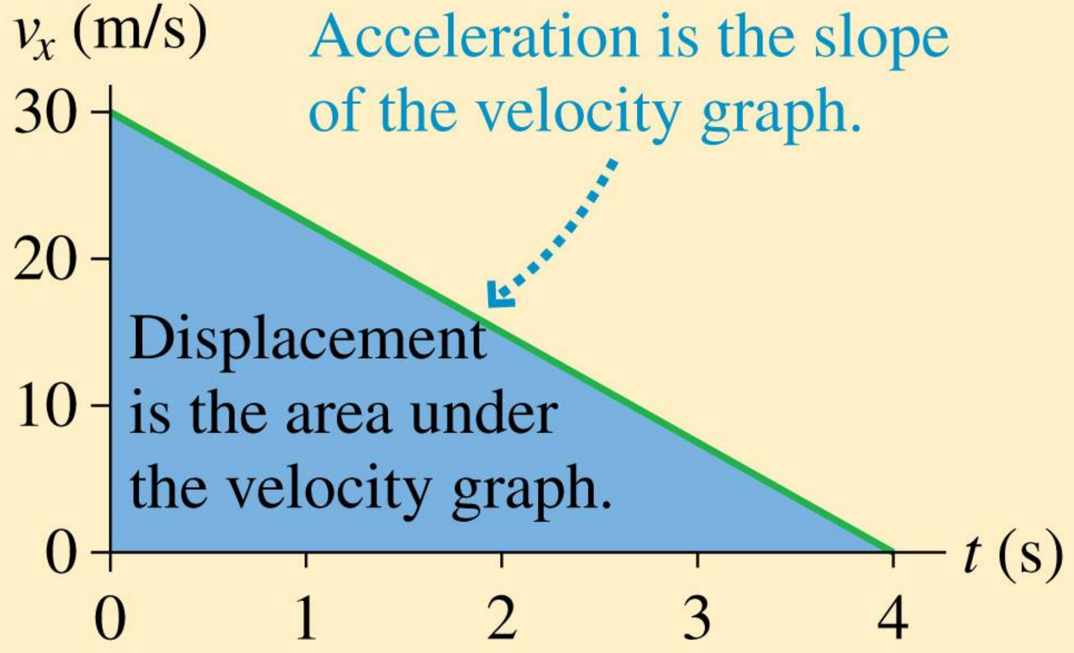
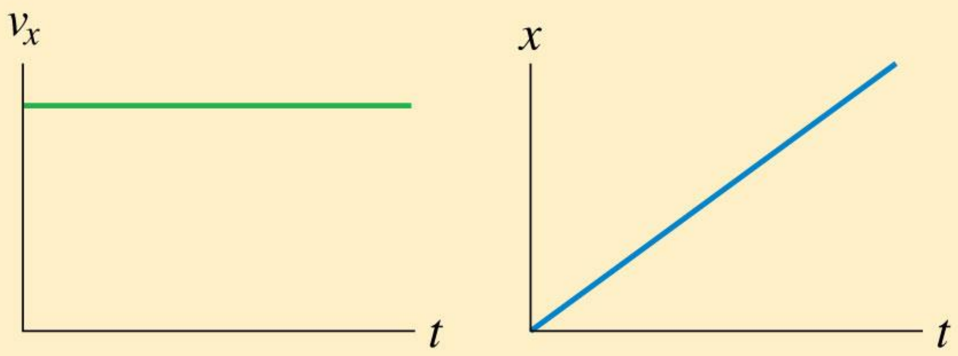



Text: p. 55

# Summary: Applications

## Uniform motion

An object in uniform motion has a constant velocity. Its velocity graph is a horizontal line; its position graph is linear.




Kinematic equation for uniform motion:

$$x_f = x_i + v_x \, \Delta t$$

Uniform motion is a special case of constant-acceleration motion, with $a_x = 0$.

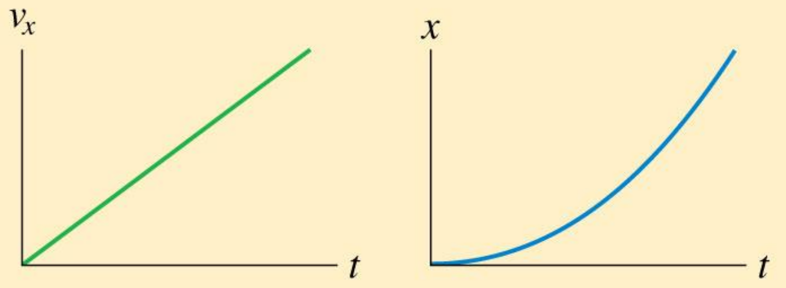


Text: p. 55

# Summary: Applications

## Motion with constant acceleration

An object with constant acceleration has a constantly changing velocity. Its velocity graph is linear; its position graph is a parabola.




Kinematic equations for motion with constant acceleration:

$$(v_x)_\text{f} = (v_x)_\text{i} + a_x\,\Delta t$$

$$x_\text{f} = x_\text{i} + (v_x)_\text{i}\,\Delta t + \tfrac{1}{2}a_x(\Delta t)^2$$

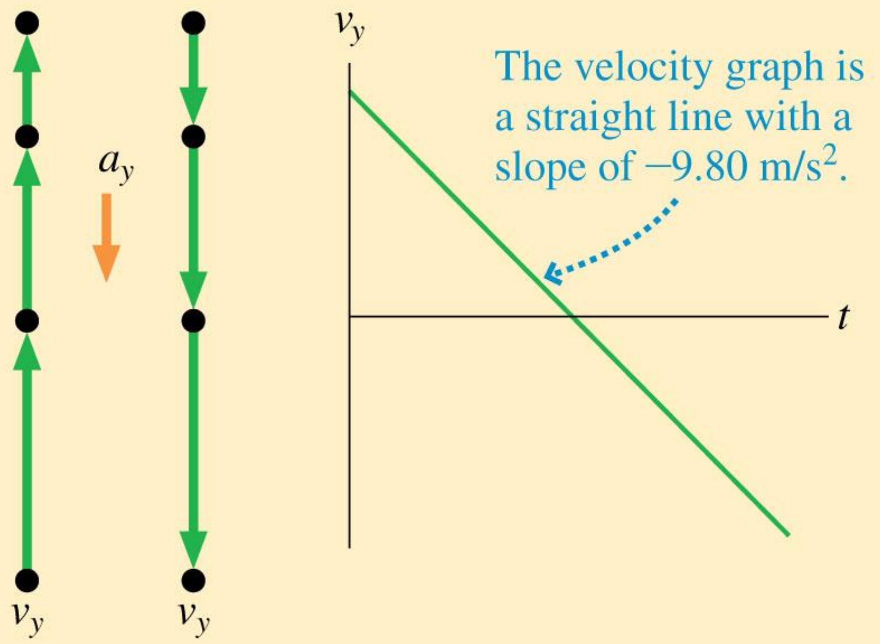
$$(v_x)_\text{f}^2 = (v_x)_\text{i}^2 + 2a_x\,\Delta x$$

Text: p. 55

# Summary: Applications

## Free fall

Free fall is a special case of constant-acceleration motion. The acceleration has magnitude $g = 9.80\ \text{m/s}^2$ and is always directed vertically downward whether an object is moving up or down.




Text: p. 55

# Summary

## GENERAL STRATEGIES

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**ASSESS** Check your answer to see if it is complete in all details and makes physical sense.

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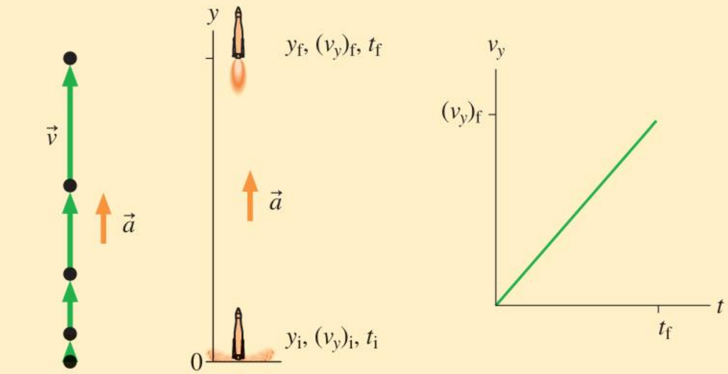


Known

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$(v_y)_i = 0$ m/s

$t_i = 0$ s

$a_y = 18$ m/s$^2$

$t_f = 150$ s

Find

$(v_y)_f$ and $y_f$

Text: p. 55

# Summary

## IMPORTANT CONCEPTS

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$$v_x = \frac{\Delta x}{\Delta t}$$

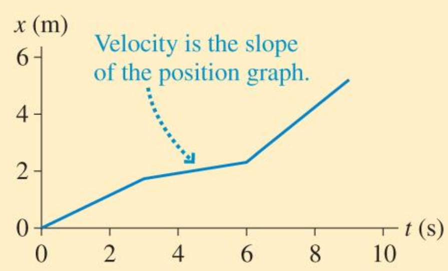
**Acceleration** is the rate of change of velocity:

$$a_x = \frac{\Delta v_x}{\Delta t}$$

The units of acceleration are $m/s^2$.

An object is speeding up if $v_x$ and $a_x$ have the same sign, slowing down if they have opposite signs.

A **position-versus-time graph** plots position on the vertical axis against time on the horizontal axis.

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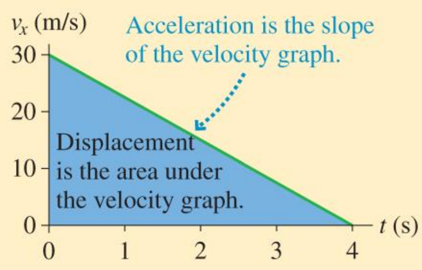


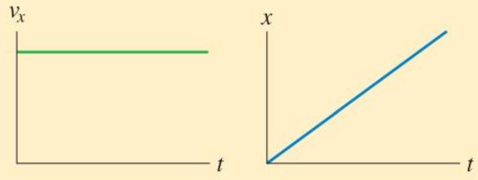

Text: p. 55

# Summary

## APPLICATIONS

### Uniform motion

An object in uniform motion has a constant velocity. Its velocity graph is a horizontal line; its position graph is linear.



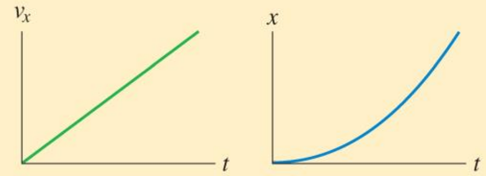

Kinematic equation for uniform motion:

$$x_f = x_i + v_x \Delta t$$

Uniform motion is a special case of constant-acceleration motion, with $a_x = 0$.

### Motion with constant acceleration

An object with constant acceleration has a constantly changing velocity. Its velocity graph is linear; its position graph is a parabola.




Kinematic equations for motion with constant acceleration:

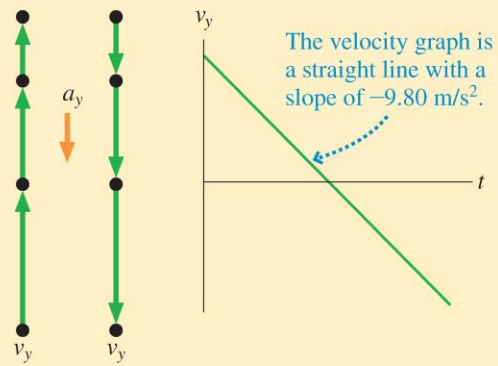
$$(v_x)_f = (v_x)_i + a_x \Delta t$$

$$x_f = x_i + (v_x)_i \Delta t + \frac{1}{2}a_x(\Delta t)^2$$

$$(v_x)_f^2 = (v_x)_i^2 + 2a_x \Delta x$$

### Free fall

Free fall is a special case of constant-acceleration motion. The acceleration has magnitude $g = 9.80\ \text{m/s}^2$ and is always directed vertically downward whether an object is moving up or down.

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